Concept Drift

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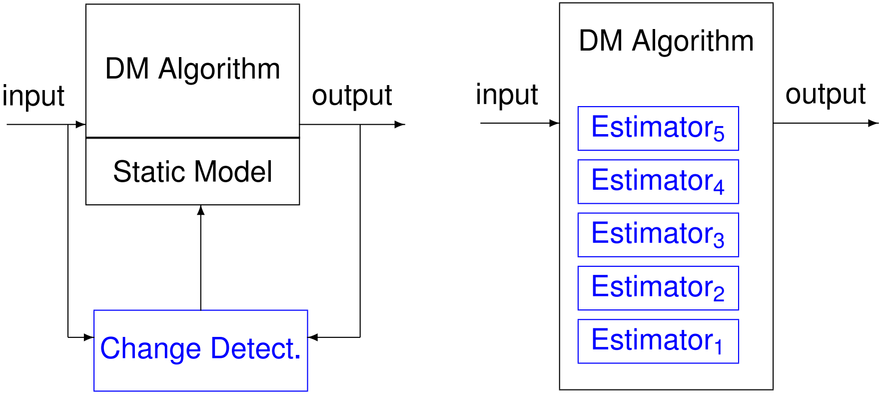
Data Streams

Big Data & Real Time

Data Mining Algorithms with Concept Drift.



Introduction.

Problem

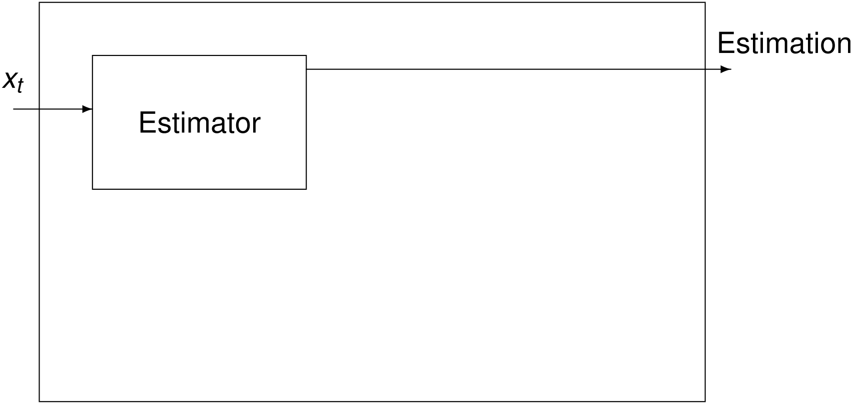
Given an input sequence $x_1, x_2, \cdots, x_t$ we want to output at instant t an alarm signal if there is a distribution change and also a prediction $\widehat{x}_{t+1}$ minimizing prediction error:

$$|\widehat{x}_{t+1} - x_{t+1}|$$

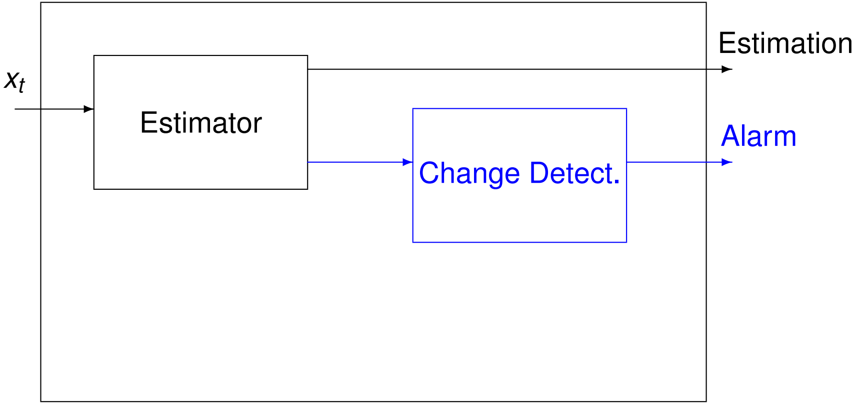
Outputs

- an estimation of some important parameters of the input distribution, and
- a signal alarm indicating that distribution change has recently occurred.

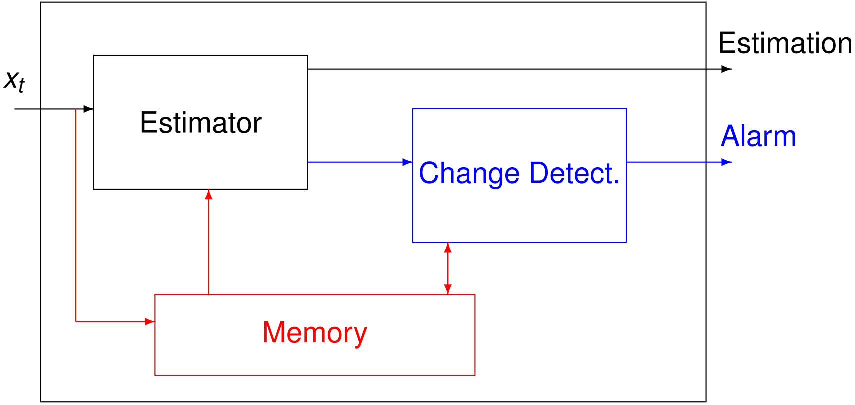
Change Detectors and Predictors



Change Detectors and Predictors



Change Detectors and Predictors



Concept Drift Evaluation

- Mean Time between False Alarms (MTFA)
- Mean Time to Detection (MTD)
- Missed Detection Rate (MDR)
- Average Run Length (ARL(θ))

> The design of a change detector is a compromise between detecting true changes and avoiding false alarms.

Data Stream Algorithmics

- High accuracy in the prediction
- Low mean time to detection (MTD), false positive rate (FAR) and missed detection rate (MDR)
- Low computational cost: minimum space and time needed
- Theoretical guarantees
- No parameters needed

Main properties of an optimal change detector and predictor system.

The CUSUM Test

- The cumulative sum (CUSUM algorithm), gives an alarm when the mean of the input data is significantly different from zero.
- The CUSUM test is memoryless, and its accuracy depends on the choice of parameters υ and h.

$$g_0 = 0, \qquad g_t = \max\left(0, g_{t-1} + \epsilon_t - \upsilon\right)$$

$$\text{if } g_t > h \text{ then alarm and } g_t = 0$$

Cumulative sum algorithm (CUSUM).

Page Hinckley Test

- The CUSUM test

$$g_0 = 0, \qquad g_t = \max\left(0, g_{t-1} + \epsilon_t - \upsilon\right)$$

$$\text{if } g_t > h \text{ then alarm and } g_t = 0$$

- The Page Hinckley Test

$$g_0 = 0, \qquad g_t = g_{t-1} + (\epsilon_t - \upsilon)$$

$$G_t = \min(g_t)$$

$$\text{if } g_t - G_t > h \text{ then alarm and } g_t = 0$$

Geometric Moving Average Test

- The CUSUM test

$$g_0 = 0, \qquad g_t = \max\left(0, g_{t-1} + \epsilon_t - \upsilon\right)$$

$$\text{if } g_t > h \text{ then alarm and } g_t = 0$$

- The Geometric Moving Average Test

$$g_0 = 0, \qquad g_t = \lambda g_{t-1} + (1 - \lambda)\epsilon_t$$

$$\text{if } g_t > h \text{ then alarm and } g_t = 0$$

The forgetting factor λ is used to give more or less weight to the last data arrived.

Statistical test

$$\hat{\mu}_0 - \hat{\mu}_1 \in N(0, \sigma_0^2 + \sigma_1^2), \text{ under } H_0$$

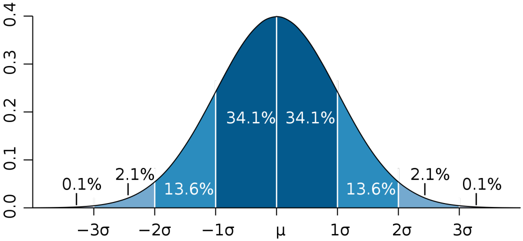
Example: Probability of false alarm of 5%

$$\Pr\left(\frac{|\hat{\mu}_0 - \hat{\mu}_1|}{\sqrt{\sigma_0^2 + \sigma_1^2}} > h\right) = 0.05$$

As $P(X < 1.96) = 0.975$ the test becomes

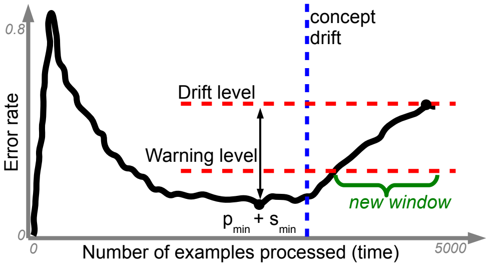
$$\frac{(\hat{\mu}_0 - \hat{\mu}_1)^2}{\sigma_0^2 + \sigma_1^2} > 1.96^2$$

Concept Drift



6 sigma

Concept Drift



Statistical Drift Detection Method
(Joao Gama et al. 2004)

ADWIN: Adaptive Data Stream Sliding Window

Let $W = \boxed{101010110111111}$

- Equal & fixed size subwindows: $\boxed{1010}\,1011011\,\boxed{1111}$
- Equal size adjacent subwindows: $1010101\,\boxed{1011}\boxed{1111}$
- Total window against subwindow: $\boxed{\boxed{10101011011}\,1111}$
- ADWIN: All adjacent subwindows:

$$\boxed{1}\boxed{01010110111111}$$

$$\boxed{1010}\boxed{10110111111}$$

$$\boxed{1010101}\boxed{10111111}$$

$$\boxed{1010101101}\boxed{11111}$$

$$\boxed{10101011011111}\boxed{1}$$

Data Stream Sliding Window

101100011110101 $\boxed{0111010}$

Sliding Window

We can maintain simple statistics over sliding windows, using $O(\frac{1}{\epsilon}\log^2 N)$ space, where

- N is the length of the sliding window
- ϵ is the accuracy parameter

M. Datar, A. Gionis, P. Indyk, and R. Motwani.
Maintaining stream statistics over sliding windows. 2002

Exponential Histograms

$M = 2$

	$\boxed{1010101}$	$\boxed{101}$	$\boxed{11}$	$\boxed{1}$	$\boxed{1}$	$\boxed{1}$
Content:	4	2	2	1	1	1
Capacity:	7	3	2	1	1	1

	$\boxed{1010101}$	$\boxed{101}$	$\boxed{11}$	$\boxed{11}$	$\boxed{1}$
Content:	4	2	2	2	1
Capacity:	7	3	2	2	1

	$\boxed{1010101}$	$\boxed{10111}$	$\boxed{11}$	$\boxed{1}$
Content:	4	4	2	1
Capacity:	7	5	2	1

Exponential Histograms

1010101	101	11	1	1

Content:	4	2	2	1	1
Capacity:	7	3	2	1	1

Error $<$ content of the last bucket W/M

$\epsilon = 1/(2M)$ and $M = 1/(2\epsilon)$

> $M \cdot \log(W/M)$ buckets to maintain the data stream sliding window

Exponential Histograms

1010101	101	11	1	1

Content:	4	2	2	1	1
Capacity:	7	3	2	1	1

To give answers in $O(1)$ time,
it maintain three counters LAST, TOTAL and VARIANCE.

$M \cdot \log(W/M)$ buckets to maintain the data stream sliding window

Algorithm ADaptive Sliding WINdow

ADWIN: ADAPTIVE WINDOWING ALGORITHM

Initialize W as an empty list of buckets
Initialize WIDTH, VARIANCE and TOTAL
for each $t > 0$
    do SETINPUT(x_t, W)
        output $\hat{\mu}_W$ as TOTAL/WIDTH and ChangeAlarm

SETINPUT(item e, List W)

INSERTELEMENT(e, W)
repeat DELETEELEMENT(W)
  until $|\hat{\mu}_{W_0} - \hat{\mu}_{W_1}| < \epsilon_{cut}$ holds
    for every split of W into $W = W_0 \cdot W_1$

Algorithm ADaptive Sliding WINdow

INSERTELEMENT(item e, List W)

create a new bucket b with content e and capacity 1
$W \leftarrow W \cup \{b\}$ (i.e., add e to the head of W)
update WIDTH, VARIANCE and TOTAL
COMPRESSBUCKETS(W)

DELETEELEMENT(List W)

remove a bucket from tail of List W
update WIDTH, VARIANCE and TOTAL
ChangeAlarm $\leftarrow$ **true**

Algorithm ADaptive Sliding WINdow

COMPRESSBUCKETS(List W)

```
Traverse the list of buckets in increasing order
    do If there are more than M buckets of the same capacity
        do merge buckets
           COMPRESSBUCKETS(sublist of W not traversed)
```

Algorithm ADaptive Sliding WINdow

Theorem

At every time step we have:

1. (False positive rate bound). *If μ_t remains constant within W, the probability that* ADWIN *shrinks the window at this step is at most δ.*
2. (False negative rate bound). *Suppose that for* some *partition of W in two parts $W_0 W_1$ (where W_1 contains the most recent items) we have $|\mu_{W_0} - \mu_{W_1}| > 2\epsilon_{cut}$. Then with probability* $1 - \delta$ ADWIN *shrinks W to W_1, or shorter.*

ADWIN tunes itself to the data stream at hand, with no need for the user to hardwire or precompute parameters.

Algorithm ADaptive Sliding WINdow

ADWIN using a Data Stream Sliding Window Model,

- can provide the exact counts of 1's in $O(1)$ time per point.
- tries $O(\log W)$ cutpoints
- uses $O(\frac{1}{\epsilon}\log W)$ memory words
- the processing time per example is $O(\log W)$ (amortized and worst-case).

Sliding Window Model

	1010101	101	11	1	1
Content:	4	2	2	1	1
Capacity:	7	3	2	1	1