4.3. MSE for BLP:

Let $g(x) = \alpha + \beta \cdot x$ be a BLP for Y

~~We have already shown, the~~

Let $h(x) = \hat{\alpha} + \hat{\beta} \cdot x$ be a BLP for $\tilde{Y}$

We have already shown, that $\beta = \hat{\beta}$, see 4.2

$\alpha = \hat{\alpha}$ as well, because $E[Y] = E[\tilde{Y}]$ and

$Cov[X, \tilde{Y}] = Cov[X, Y] + Cov[X, U] = Cov[X, Y]$

Therefore $g(x) = h(x)$

Using alternative formula for MSE (FOAS 2.1.23):

$MSE(Y|X) = V[Y] + (E(Y) - g(x))^2$

$MSE(\tilde{Y}|X) = V[Y+U] + (E(Y) - g(x))^2$, given $E(Y+U) = E(Y)$

$MSE(\tilde{Y}|X) - MSE(Y|X) = V[Y+U] - V[Y] = V[U]$,

given independence of Y and U.

$$\boxed{\begin{array}{l} MSE(\tilde{Y}|X) - MSE(Y|X) = V[U] > 0 \\ \text{because } V[U] > 0 \end{array}}$$