

$$2.1 \text{ Bias} = E[z_n] - \lambda$$

$$E[z_n] = \frac{6}{n} E\left[\sum_{i=1}^n w_i\right] = \frac{6}{n} \cdot n \cdot E[w_i]$$

Given that w_i is uniformly distributed $[0, \lambda]$

$$E[w_i] = \lambda/2 \Rightarrow$$

$$E[z_n] = \frac{6}{n} \cdot n \cdot \frac{\lambda}{2} = 3\lambda$$

$$\boxed{\text{Bias} = 3\lambda - \lambda = 2\lambda}$$

$$2.2 \text{ } V[z_n] = V\left[\frac{6}{n} \sum_{i=1}^n w_i\right] = \frac{6^2}{n^2} V\left[\sum_{i=1}^n w_i\right]$$

Because of independence of w_i :

$$V\left[\sum_{i=1}^n w_i\right] = n V(w_i) = n \cdot (E(w_i^2) - E^2(w_i)) =$$

$$= n \cdot \left(\int_0^{\lambda} x^2 \cdot \frac{1}{\lambda} dx - \left(\frac{\lambda}{2}\right)^2 \right) = \frac{n}{12} \lambda^2$$

$$\boxed{V[z_n] = \frac{36}{n^2} \cdot \frac{n}{12} \lambda^2 = \frac{3}{n} \lambda^2}$$

$$2.3 \text{ } \text{MSE}(z_n) = E[(z_n - \lambda)^2] = \underbrace{V[z_n]}_{\frac{3}{n} \lambda^2} + \underbrace{(E[z_n] - \lambda)^2}_{3\lambda^2} \Rightarrow$$

$$\boxed{\text{MSE}(z_n) = \frac{3\lambda^2}{n} + 4\lambda^2 = \lambda^2 \left(4 + \frac{3}{n}\right)}$$