W203, Test 1, Question 4

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Q 4.2 Minimum P(X=Y)

Q 4.3 Minimum Covariance Cov[X,Y]
One of definitions of covariance:

$$Cov[X,Y] = E[X * Y] - E[X] * E[Y]$$

From marginal PDFs for both X and Y, we can calculate:

$$E[X] = E[Y] = \frac{1}{3} * 1 + \frac{1}{3} * 2 + \frac{1}{3} * 3 = 2$$

Therefore:

$$Cov[X,Y] = E[X * Y] - 4$$

Since expectation is a non-negative function, the smallest $E[XY]$ can be is 0, and in this case $Cov_min[X,Y] = -4$