4.1 $\beta_{Y\tilde{X}} = \dfrac{Cov[\tilde{X}, Y]}{V[\tilde{X}]} = \dfrac{Cov[X+U_X, Y]}{V[X+U_X]} =$

$$= \frac{Cov[X,Y] + Cov[U_X, Y]}{V[X] + V[U_X]} =$$ because U_X is independent of X

$$= \frac{Cov[X,Y]}{V[X] + V[U_X]}$$ because U_X is independent of Y

$$\boxed{\beta_{Y\tilde{X}} = \frac{Cov[X,Y]}{V[X] + V[U_X]} < \frac{Cov[X,Y]}{V[X]} = \beta_{XY}}$$

given $V[U_X] > 0$, $Cov[X,Y] > 0$, $V[X] > 0$

4.2 $\beta_{\tilde{Y}X} = \dfrac{Cov[X, Y+U_Y]}{V[X]} = \dfrac{Cov[X,Y] + Cov[X, U_Y]}{V[X]}$

$$= \frac{Cov[X,Y]}{V[X]} = \beta_{YX},$$ because X and U_Y are independent, $\Rightarrow Cov[X, U_Y] = 0$