Unit 1 proofs

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1. Let's prove that $P(A \cap B)$ even exists: $\exists A \cap B \equiv \exists x : x \in A, x \in B$ Let's assume the opposite, i.e.

$$\nexists x : x \in A, x \in B \equiv A \cap B = \emptyset$$

with this assumption

$$P(\Omega) = P(A) + P(B) + P(\overline{A \cup B})$$

$P(\Omega) = 1$ because of axiom, calculating the expression above:

$$P(\overline{A \cup B}) = -0.25$$

which is against the axioms, therefore original assumption is false

2. Now we can prove that given a set of events $A = \{A_0, A_1 ... A_n\}$ **the highest probability of all events occurring at the same time** $P(\cap_{i=0}^{n} A_i)_{max} = P(A_{min})$ i.e. probability of the intersection of a series of events can not exceed the probability of the smallest event but can be equal to it.

Note that $\exists x \in \cap_{i=0}^{n} A_i : x \notin A_i$ is impossible by definition of union, therefore:

$$\forall A_i \in \ A, \cap_{i=0}^{n} A_i \leq A_i$$

The statement above holds for A_{min} just as well as for any other A_i, therefore:

$$P(\cap_{i=0}^{n} A_i) \leq P(A_{min})$$

At the same time $x \in \cap_{i=0}^{n} A_i$ can include at least $\forall x \in A_{min}$, therefore:

$$P(\cap_{i=0}^{n} A_i)_{max} \geq P(A_{min})$$

Combining these two statements

$$P(\cap_{i=0}^{n} A_i)_{max} = P(A_{min})$$

In case of the events in question, $P(A \cap B)_{max} = P(A) = 0.5$