1.1. $m\ddot{x} + k\dot{x}|\dot{x}| = u$

$x_1 = x \qquad \dot{x}_1 = x_2$

$x_2 = \dot{x} \qquad \dot{x}_2 = -\frac{k}{m} x_2 |x_2| + \frac{1}{m} u$

$\sigma = x_2 + c x_1$

$V = \frac{1}{2} \sigma^2$

$\dot{V} = \frac{1}{2} \cdot 2\sigma \cdot (\dot{x}_2 + c\dot{x}_1) = \sigma\left(c x_2 - \frac{k}{m} x_2 |x_2| + \frac{1}{m} u\right)$

$u = v - mcx_2$

$\dot{V} = \sigma\left(\frac{1}{m} v - \frac{k}{m} x_2 |x_2|\right) \le -\alpha\, \mathrm{sign}(\sigma)$

$\sigma \frac{1}{m}\left(v - kx_2|x_2|\right) \le -\alpha\, \mathrm{sign}\,\sigma$

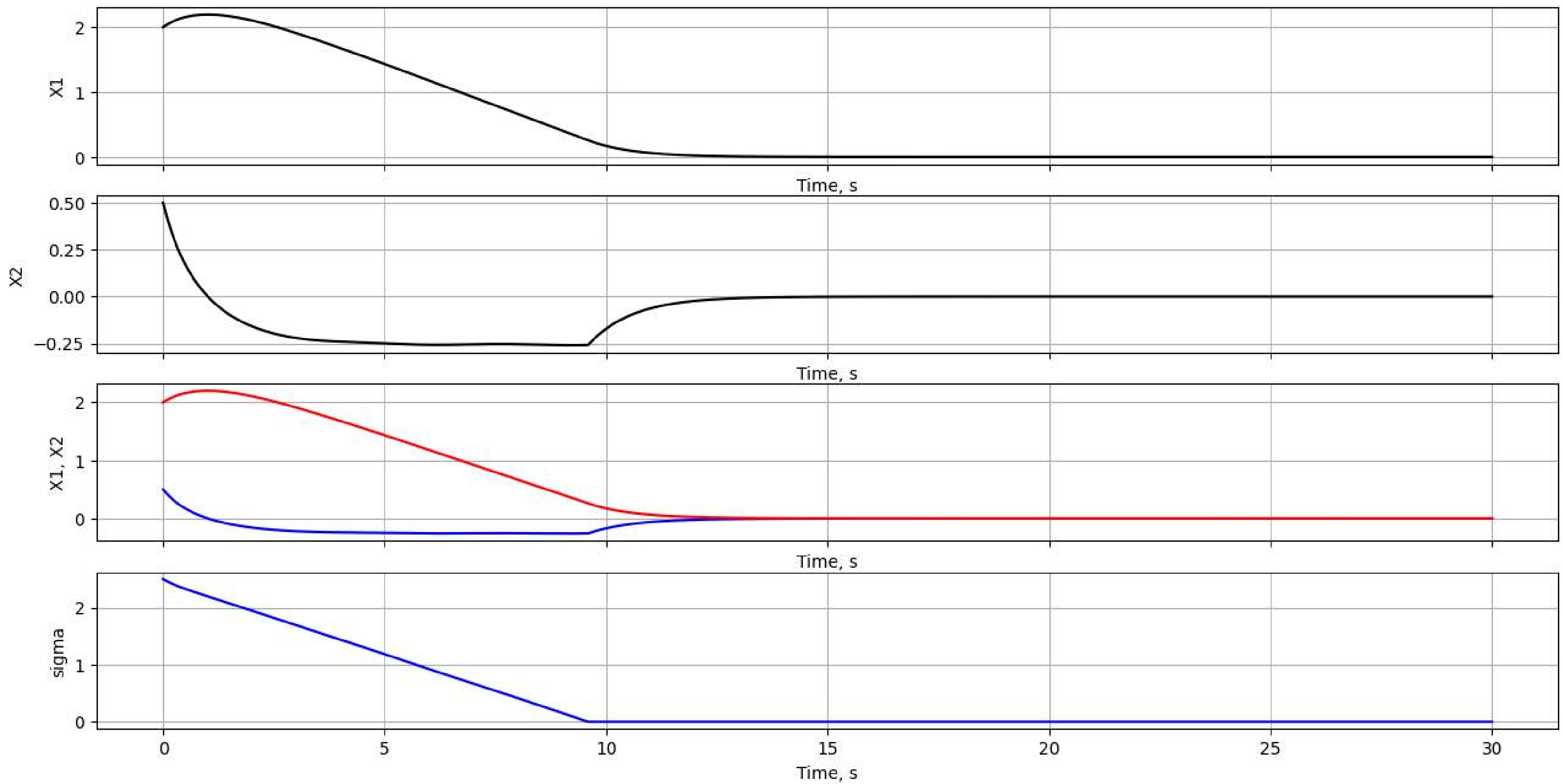
$v = -\mathrm{sign}(\sigma)$

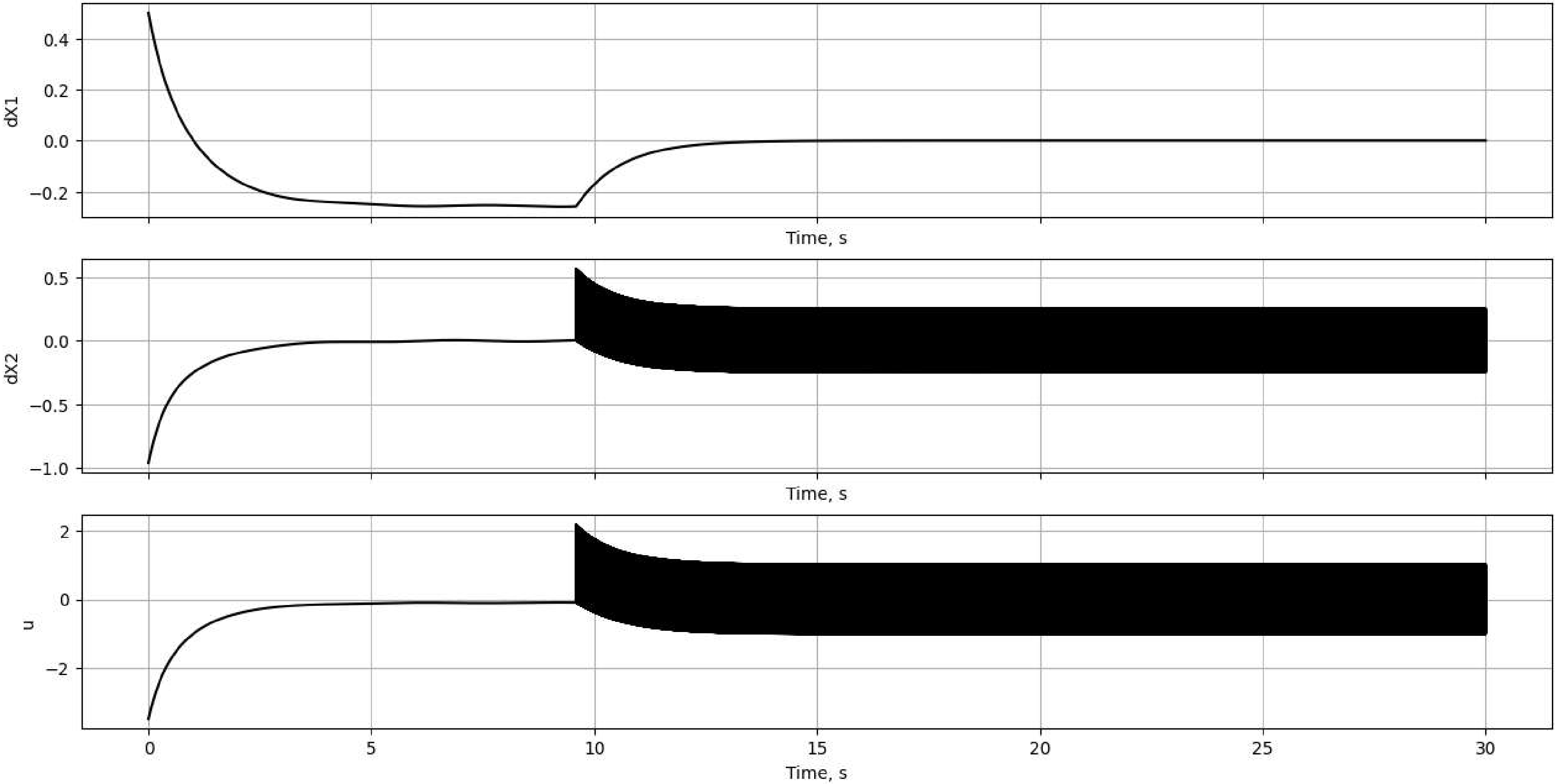
$\frac{1}{\cancel{m}} v > \frac{k}{\cancel{m}} x_2^2$

$\rho > k x_2^2 \quad \Rightarrow \quad v = -k x_2^2\, \mathrm{sign}(\sigma) - \mathrm{sign}\,\sigma$

$\dot{x}_1 = x_2$

$\dot{x}_2 = u + \text{stable\_part}$


X1
2
1
0
Time, s
X2
0.50
0.25
0.00
−0.25
Time, s
X1, X2
2
1
0
Time, s
sigma
2
1
0
0
5
10
15
20
25
30
Time, s



0.4
0.2
0.0
−0.2
dX1
Time, s
0.5
0.0
−0.5
−1.0
dX2
Time, s
2
0
−2
u
0
5
10
15
20
25
30
Time, s

1.2

①

$$\operatorname{sign} \sigma \approx \frac{\sigma}{|\sigma| + \varepsilon}$$

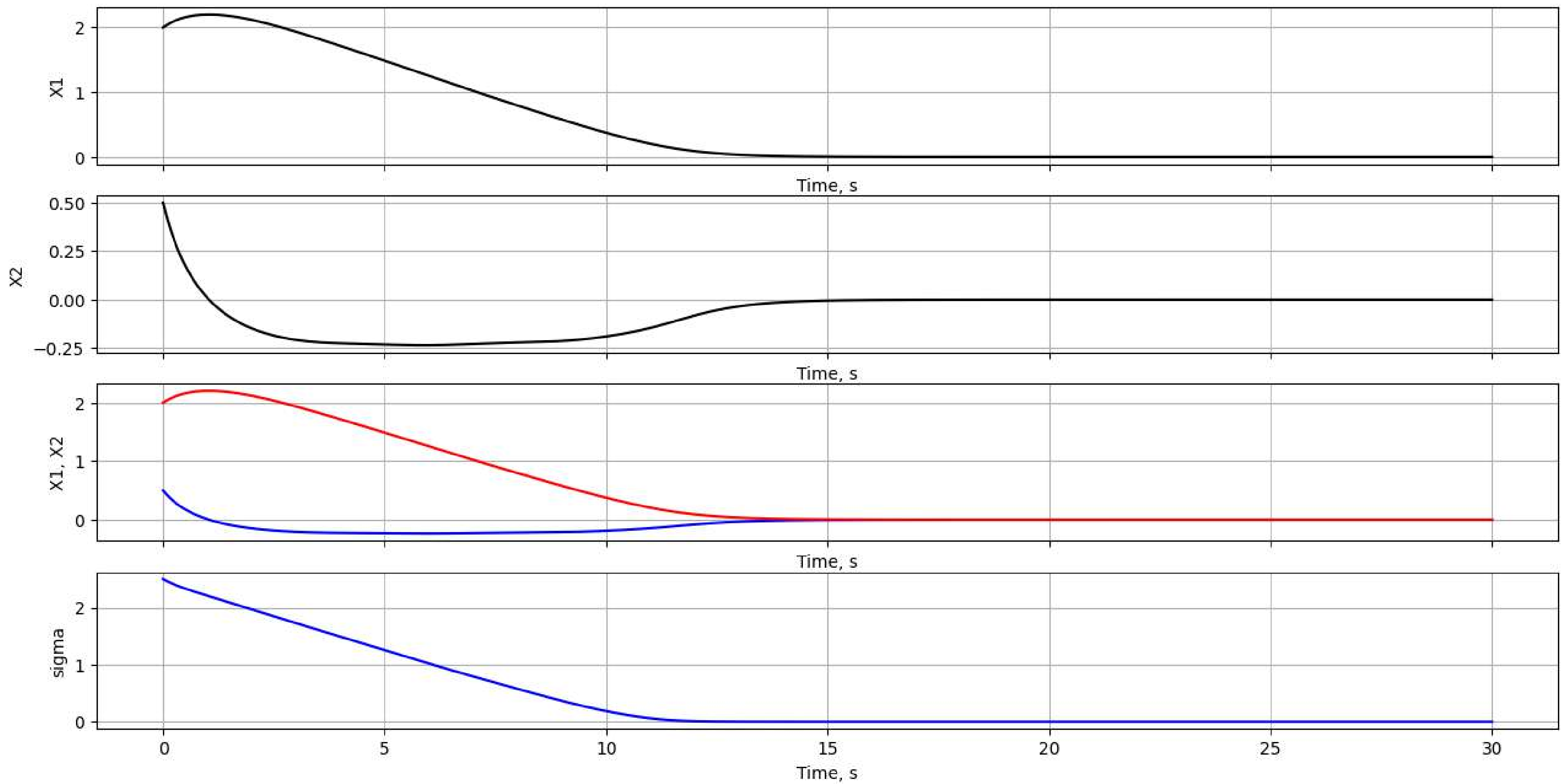
$$\Downarrow$$

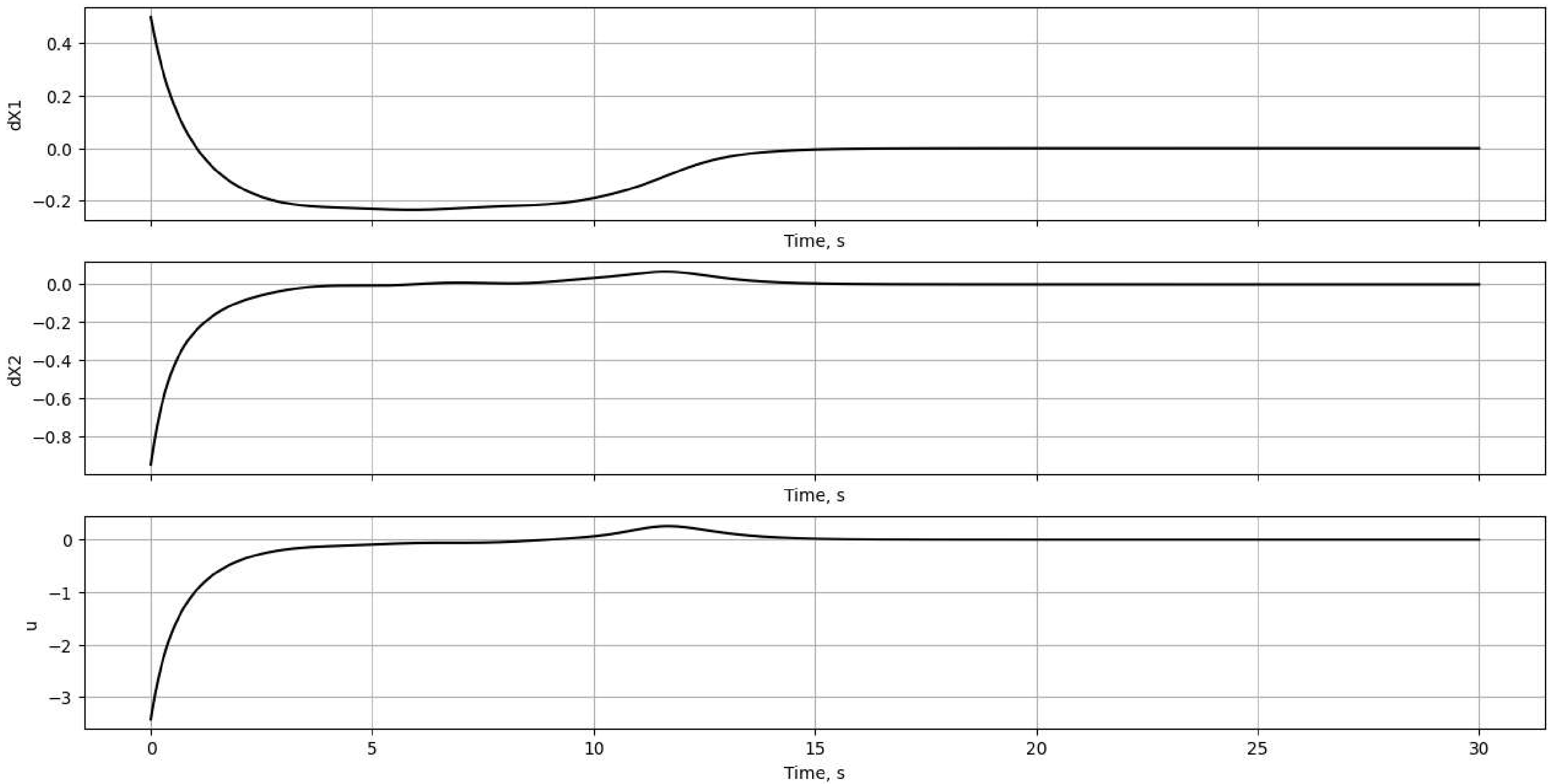
$$v = \frac{-\sigma}{|\sigma| + \varepsilon}$$

②

$$\operatorname{sig} \sigma \approx \begin{cases} 1, & \text{if } \sigma > \varepsilon \\ \dfrac{\sigma}{\varepsilon}, & \text{if } |\sigma| \le \varepsilon \\ -1, & \text{if } \sigma < -\varepsilon \end{cases}$$

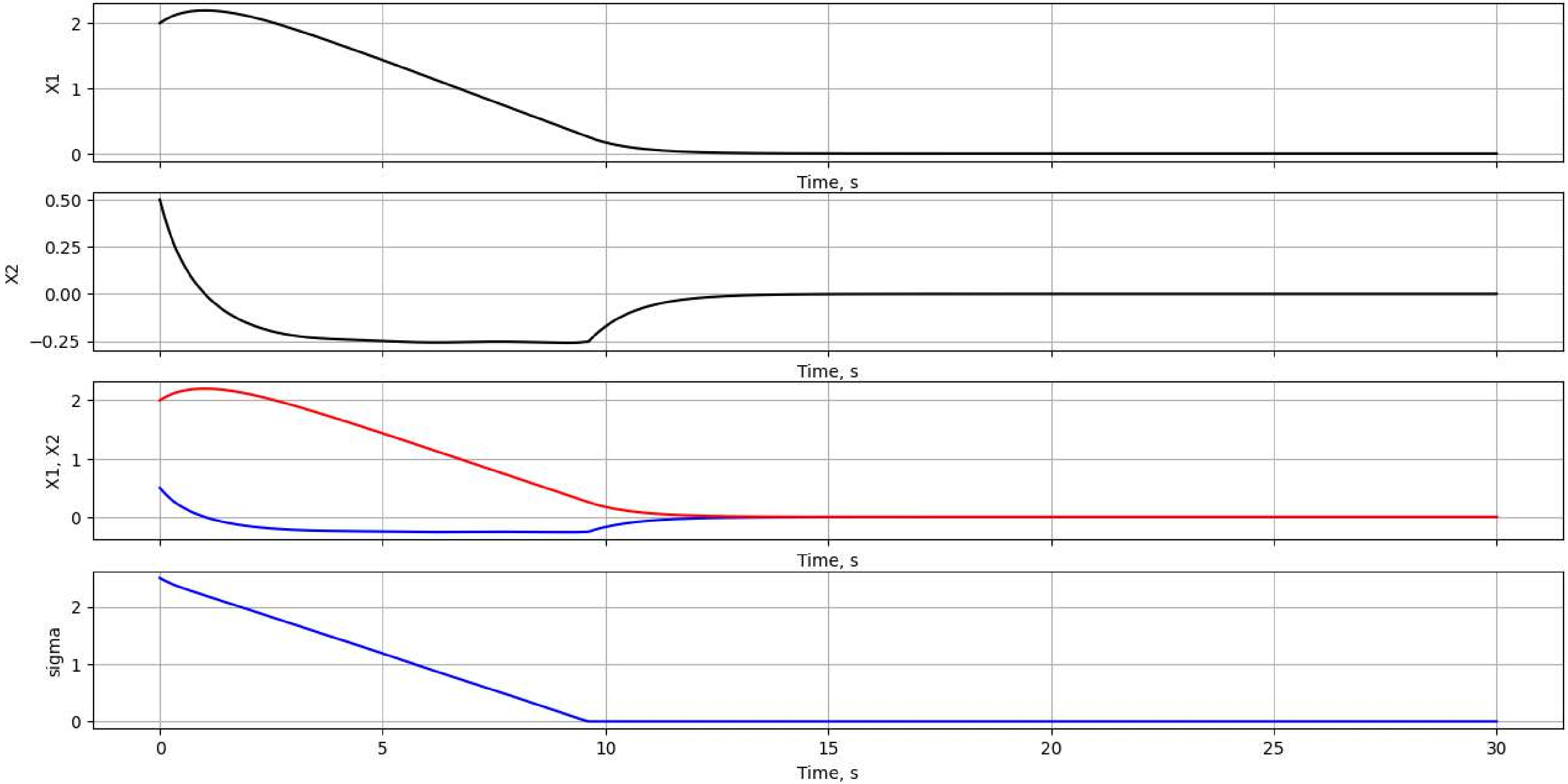
① $\operatorname{sign}\sigma = \frac{\sigma}{|\sigma|+\varepsilon}$, $\varepsilon = 0.1$

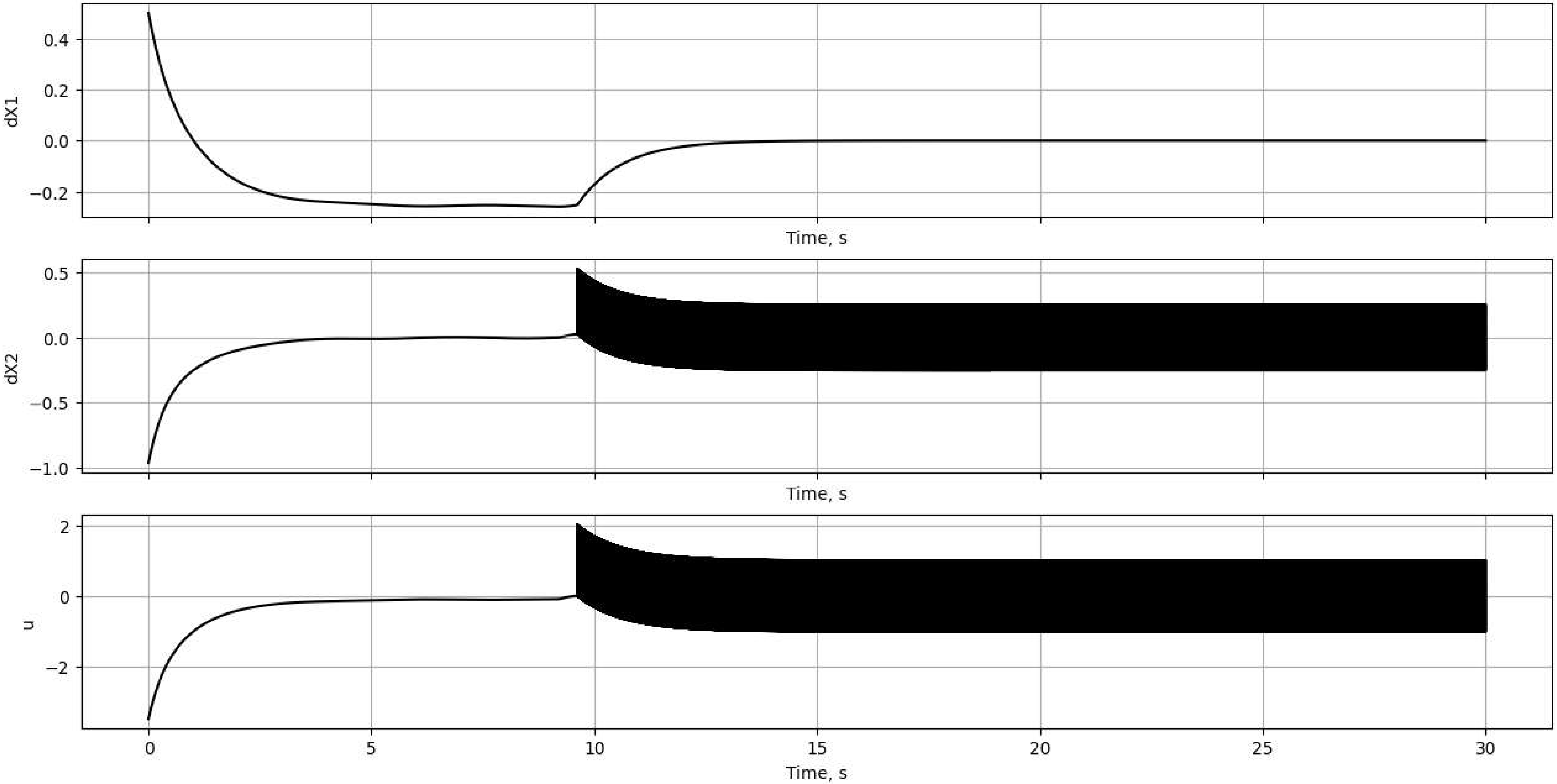




0.4
0.2
0.0
−0.2
dX1
Time, s
0.0
−0.2
−0.4
−0.6
−0.8
dX2
Time, s
0
−1
−2
−3
u
0
5
10
15
20
25
30
Time, s

$$② \quad \mathrm{sign}\,\sigma = \begin{cases} 1, \text{ if } \sigma > \varepsilon \\ \dfrac{\sigma}{\varepsilon}, \text{ if } |\sigma| \le \varepsilon \\ -1, \text{ if } \sigma < -\varepsilon \end{cases}$$





dX1
0.4
0.2
0.0
−0.2
Time, s
dX2
0.5
0.0
−0.5
−1.0
Time, s
u
2
0
−2
0
5
10
15
20
25
30
Time, s

L.3 $m\ddot{x} + k\dot{x}|\dot{x}| = u$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k}{m} x_2 |x_2| + \frac{1}{m} u \\ \dot{u} = v \end{cases}$$

$\sigma = x_2 + c x_1$

$s = \dot{\sigma} + c_1 \sigma$

Using observer

$\sigma_1 = \sigma$ $\qquad \dot{\hat{\sigma}}_1 = -k_1 [\hat{\sigma}_1 - \sigma] + \hat{\sigma}_2$

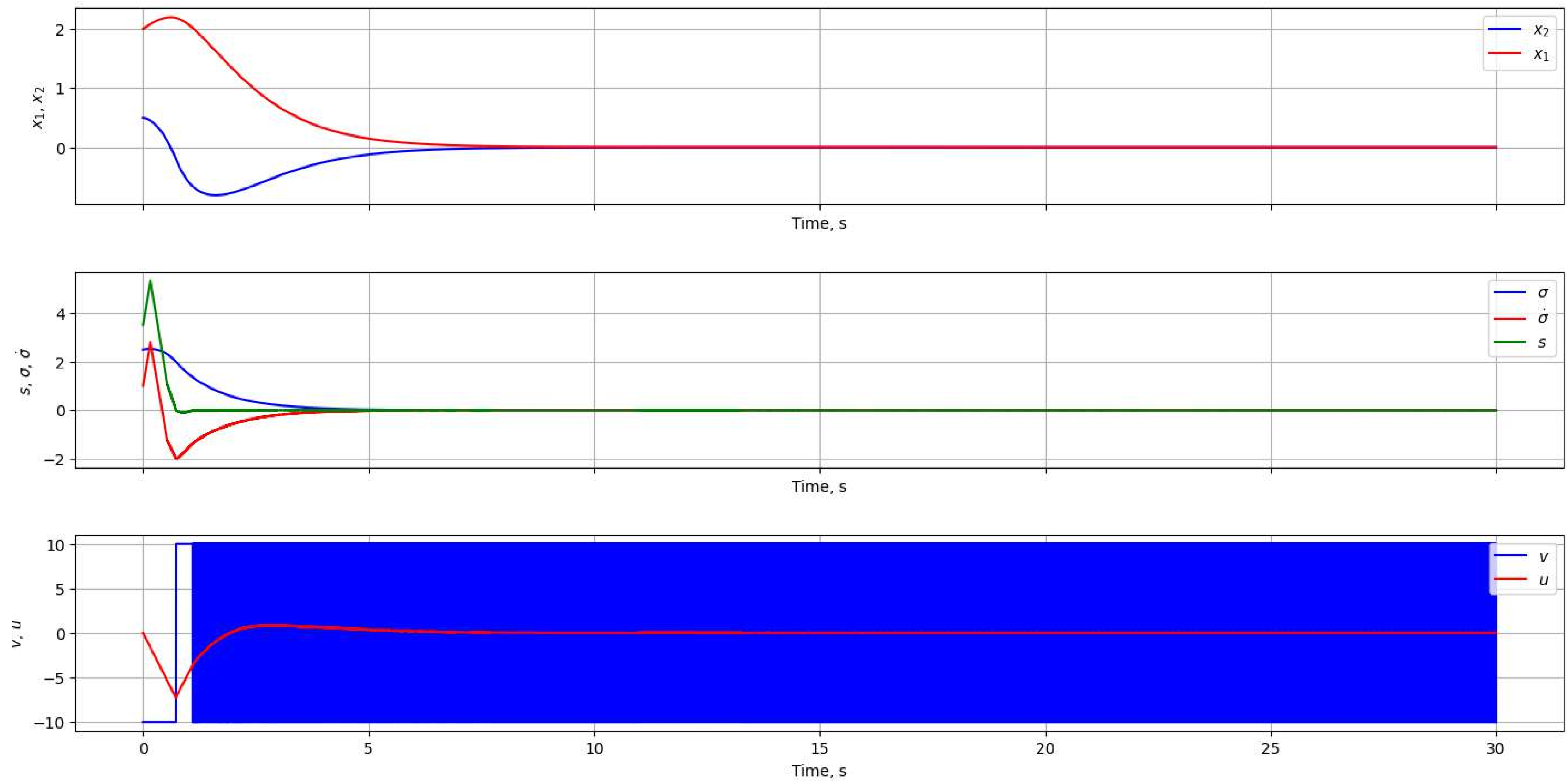
$\sigma_2 = \dot{\sigma}$ $\qquad \dot{\hat{\sigma}}_2 = -k_2 \operatorname{sign}(\hat{\sigma}_1 - \sigma)$

$k_1 = 1.5\sqrt{L}$

$k_2 = 1.1L$

$v = -1.1\hat{L}\operatorname{sign}(s)$ $\qquad \hat{L} \geq |\ddot{\sigma}|$

$u = \int_0^t v(x)\,dt$



x_2
x_1
x_1, x_2
Time, s
σ
$\dot{\sigma}$
s
$s, \sigma, \dot{\sigma}$
Time, s
v
u
v, u
Time, s

$m\ddot{x} + k\dot{x}|\dot{x}| = u$

$\dot{x}_1 = x_2$

$\dot{x}_2 = -\frac{k}{m} x_2 |x_2| + \frac{1}{m} u$

$\dot{\sigma} = f(t) + u \quad |\dot{f}| < L$ ①

$u = -k_1 \lceil \sigma \rfloor^{\frac{1}{2}} - k_2 \int_0^t \mathrm{sign}(\sigma(\tau))\, d\tau$

$\sigma = x_2 + c x_1$

$\dot{\sigma} = \dot{x}_2 + c\dot{x}_1 = -\frac{k}{m} x_2 |x_2| + \frac{1}{m} u + c x_2$

$u = -mcx_2 + v$

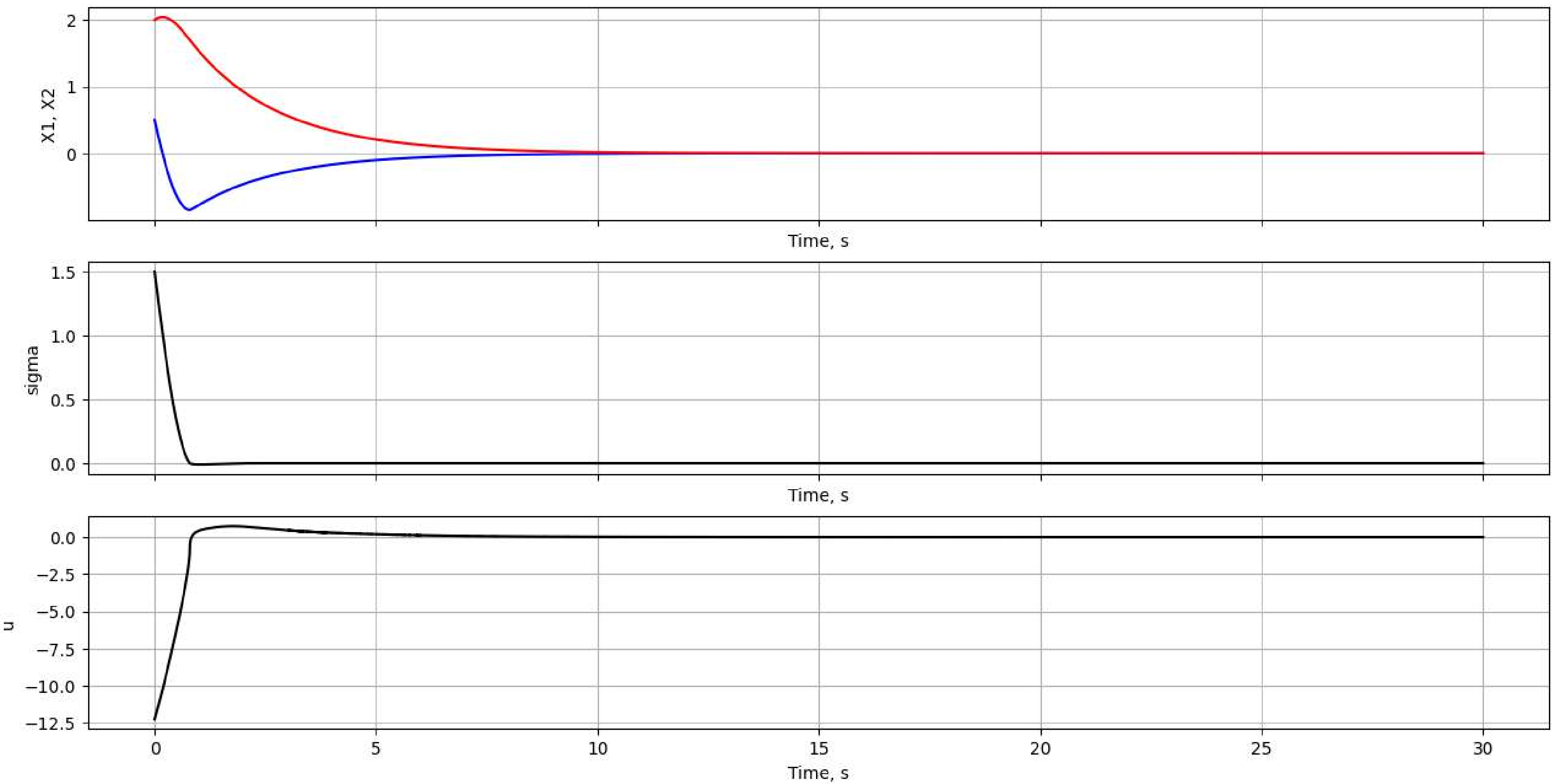
$\dot{\sigma} = -\frac{k}{m} x_2 |x_2| + \frac{1}{m} v$

$f(t) = -\frac{k}{m} x_2 |x_2| = -\frac{k}{m} x_2^2 \,\mathrm{sign}\, x_2$

$\dot{f}(t) = -\frac{2k}{m} x_2 \dot{x}_2 \,\mathrm{sign}(x_2), \quad x_2 \neq 0 \Rightarrow$

$\Rightarrow$ функция не огр. глобально $\Rightarrow$ функция не явл. глобально липшицевой.

$|x_2| \leq P, \ |\dot{x}_2| < R \Rightarrow |\dot{f}| < \frac{2kPR}{m} = L$ ①


2
1
0
X1, X2
Time, s
1.5
1.0
0.5
0.0
sigma
Time, s
0.0
−2.5
−5.0
−7.5
−10.0
−12.5
u
0
5
10
15
20
25
30
Time, s

1.5 $m\ddot{x} + k\dot{x}|\dot{x}| = u$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k}{m} x_2 |x_2| + \frac{1}{m} u \\ \dot{u} = v \end{cases}$$

Therminal sliding mode:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = u(x)$$

$$u(x) = -\alpha \, \mathrm{sign}(s(x))$$

$$s(x) = x_2 + \beta \lfloor x_1 \rceil^{\frac{1}{2}}$$

$\sigma = x_2 + c x_1$ $\qquad$ $\dot{\sigma}_1 = -\frac{k}{m} x_2 |x_2| + c x_2 + \frac{1}{m} u$

$\dot{\sigma} = \dot{x}_2 + c \dot{x}_1$ $\qquad$ $\dot{\sigma}_2 = \dots + v$ (cannot be measured)

Using observer

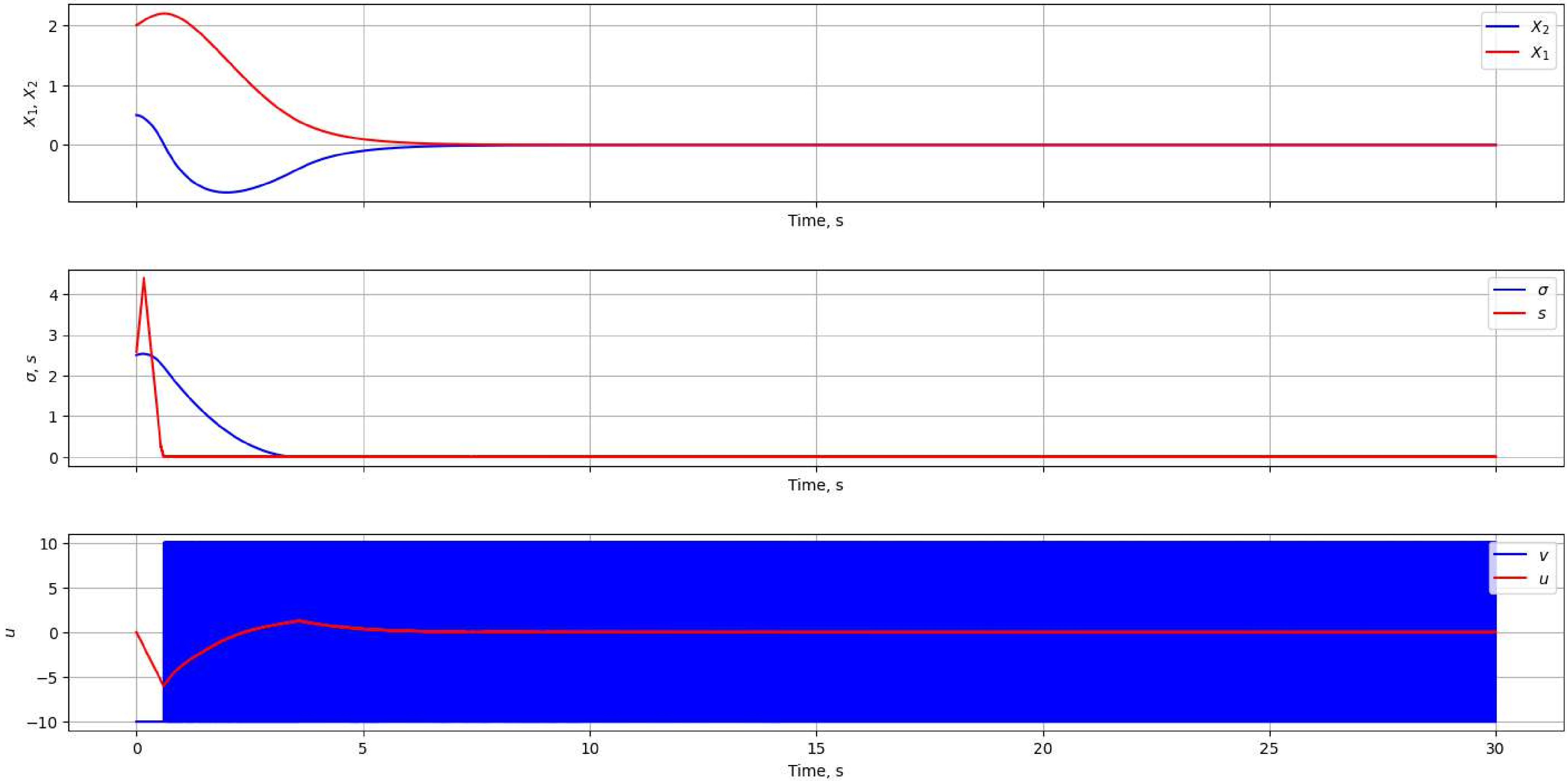
$$\dot{\hat{\sigma}}_1 = -k_1 \lfloor \hat{\sigma}_1 - \sigma \rceil^{\frac{1}{2}} + \hat{\sigma}_2$$

$$\dot{\hat{\sigma}}_2 = -k_2 \, \mathrm{sign}(\hat{\sigma}_1 - \sigma) \rightarrow \sigma_2 \text{ - estimation of } \dot{\sigma}$$

$$v(x) = -\alpha \, \mathrm{sign}(s)$$

$$s(x) = \hat{\sigma}_2 + \beta \lfloor \sigma_1 \rceil^{\frac{1}{2}}$$

$$u = \int_0^t v(x) \, dt - \frac{c}{m} x_2$$



X_2
X_1
X_1, X_2
Time, s
σ
s
σ, s
Time, s
v
u
u
Time, s

1.6

$$J\dot{\omega} = k_m i - T_L$$
$$L\dot{i} = -iR - k_b\omega + u$$
$$|T_L| < \varphi_1, \quad |\dot{T}_L| < \varphi_2$$

$x_1 = \omega \quad x_2 = i$

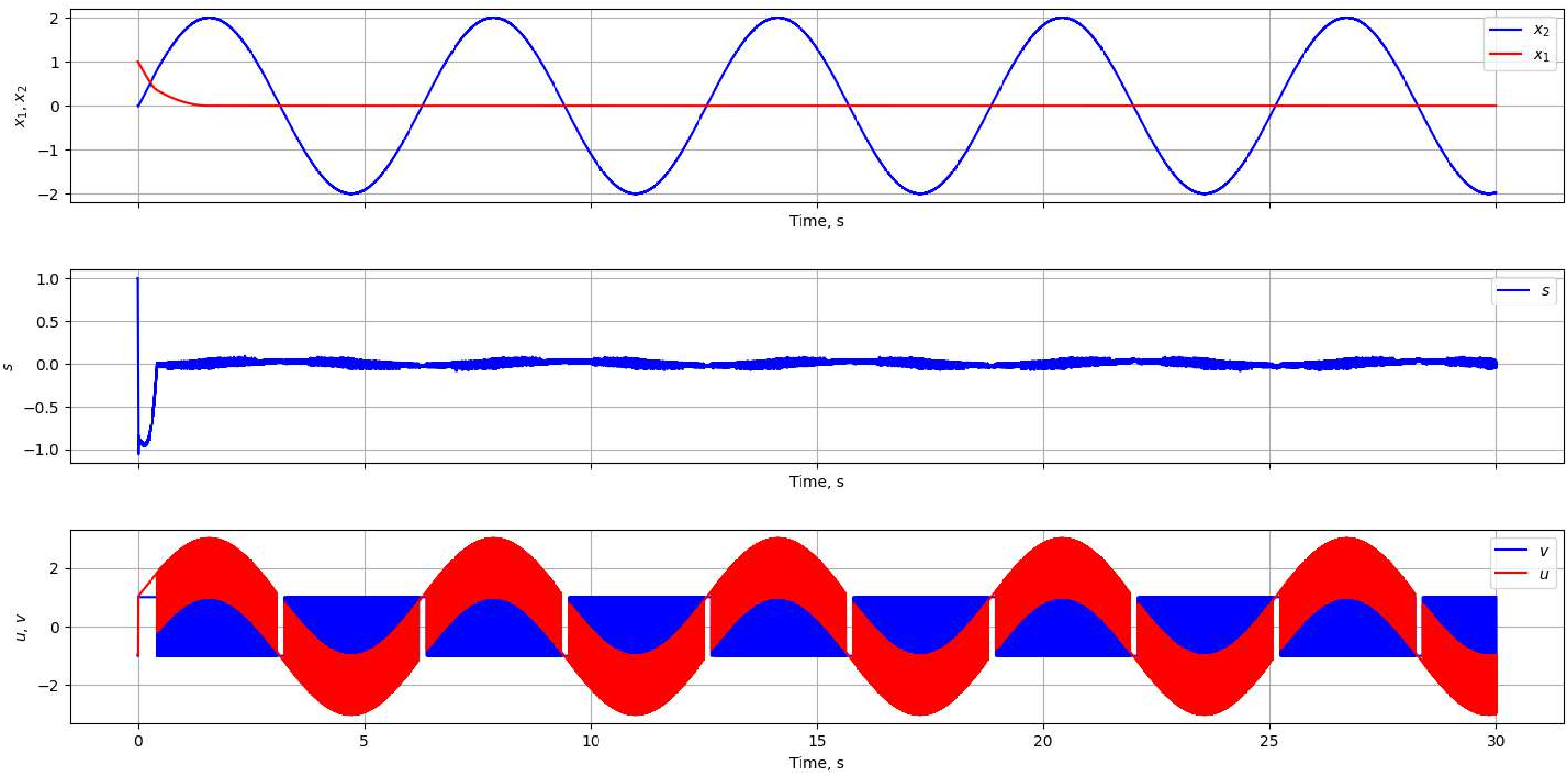
$$\begin{cases} \dot{x}_1 = \frac{1}{J}(k_m x_2 - T_L) \\ \dot{x}_2 = \frac{1}{L}(-Rx_2 - k_b x_1 + u) \end{cases} \Rightarrow \begin{array}{l} \dot{x}_1 = \frac{1}{J}(k_m x_2 - T_L) \\ \dot{x}_2 = \frac{1}{L}(V) \end{array}$$

$$u = V + Rx_2 + k_b x_1$$

$$z_1 = x_1$$
$$\dot{z}_1 = z_2 = \frac{1}{J}(k_m x_2 - T_L)$$
$$\dot{z}_2 = \frac{1}{J}(k_m \dot{x}_2 - \dot{T}_L) = \frac{k_m}{JL}v - \underbrace{\frac{1}{J}\dot{T}_L}_{\dot{f}}$$

$$\cancel{V_1 = \frac{k_m}{JL}v}$$

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = V_z - \frac{1}{J}\dot{T}_L = V_z + \dot{f}$$
$$V_z = -\lambda\,\mathrm{sign}(s) \quad \Big| \quad |\lambda| > \left|\frac{\varphi_2 L}{k_m}\right|$$
$$s = \hat{z}_2 + \beta\lfloor z_1 \rceil^{\frac{1}{2}} \quad \Big| \longrightarrow \text{Thermanal sliding mode}$$
$$\dot{\hat{z}}_1 = -k_1\lfloor \hat{z}_1 - z_1 \rceil^{\frac{1}{2}} + \hat{z}_2$$
$$\dot{\hat{z}}_2 = -k_2\,\mathrm{sign}(\hat{z}_1 - z_1)$$
$$k_1 : \hat{z}_1 > \dot{z}_1, \quad k_1 = 1.5\sqrt{L_c}$$
$$k_2 : \hat{z}_2 > \dot{z}_2, \quad k_2 = 1.1L_c \qquad z_1, z_2 \to 0 \Rightarrow x_1 \to 0$$



x_2
x_1
x_1, x_2
Time, s
s
s
Time, s
v
u
u, v
Time, s

1.7 $\begin{cases} J\dot{\omega} = k_m i - T_L \\ L\dot{i} = -iR - k_b\omega + u \end{cases}$ Only i is measured

$$\begin{cases} \dot{x}_1 = \frac{1}{J}\left(k_m x_2 - T_L\right) \\ \dot{x}_2 = \frac{1}{L}\left(-x_2 R - k_b x_1 + u\right) = \frac{1}{L} v \end{cases}$$

$$u = x_2 R + k_b x_1 + v$$

$$\dot{\hat{x}}_2 = -k_1 \lfloor \hat{x}_2 - x_2 \rceil^{\frac{1}{2}} + \ddot{\hat{x}}_2$$

$$\ddot{\hat{x}}_2 = -k_2 \operatorname{sign}\left(\hat{x}_2 - x_2\right)$$

$$x_1 = \frac{1}{k_B}\left(-L\dot{\hat{x}}_2 - R x_2 + u\right)$$

1.8 $J\dot{\omega} = k_m i - T_L$

$L\dot{i} = -iR - k_b\omega + u$

$$\begin{cases} \dot{x}_1 = \frac{1}{J}(k_m x_2 - T_L) \\ \dot{x}_2 = \frac{1}{L}(-k_b x_1 - R x_2 + u) = \frac{1}{L} v \end{cases}$$

$u = k_b x_1 + R x_2 + v$

• $z_1 = x_1$

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = \frac{1}{J}(k_m \dot{x}_2 - \dot{T}_L) = \frac{1}{J}\left(\frac{k_m}{L} v - \dot{T}_L\right) \end{cases}$$

$\sigma = \hat{z}_2 + c z_1$

$$\left.\begin{aligned} \dot{\hat{z}}_1 &= -k_1 \lfloor \hat{z}_1 - z_1 \rceil^{\frac{1}{2}} + \hat{z}_2 \\ \dot{\hat{z}}_2 &= -k_2 \,\mathrm{sign}(\hat{z}_1 - z_1) \end{aligned}\right| \to \hat{z}_2 \to \sigma$$

$\cancel{v = -\varphi_1 \lfloor \sigma \rceil^{\frac{1}{2}} \; k_2 \int_0^t \mathrm{sign}(\sigma(\tau))\,d\tau}$

$\cancel{s = \sigma_1 + c_1 \sigma_2}$

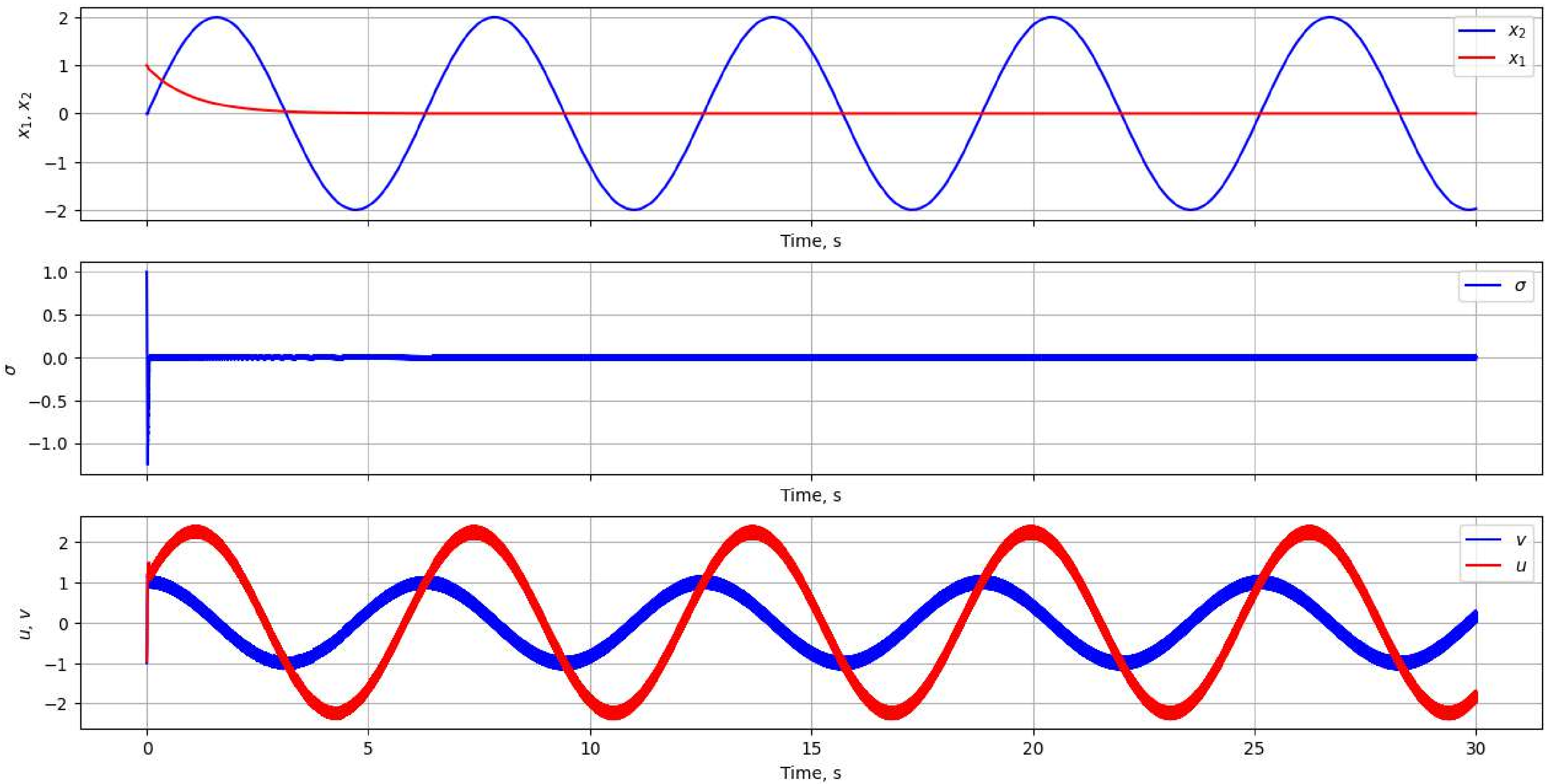
$\cancel{\dot{\hat{\sigma}}_1 = -k_1 \lfloor \hat{\sigma}_1 - \sigma_1 \rceil^{\frac{1}{2}} + \hat{\sigma}_2}$

$\cancel{\dot{\hat{\sigma}}_2 = -k_2 \,\mathrm{sign}(\hat{\sigma}_1 - \sigma_1)}$

only in sim

$\cancel{v = -}$

$v = -\varphi_1 \lfloor \sigma \rceil^{\frac{1}{2}} - \varphi_2 \int_0^t \mathrm{sign}(\sigma(\tau))\,d\tau$



x_2
x_1
x_1, x_2
Time, s
σ
Time, s
v
u
u, v
Time, s

1.9

$$J\dot{\omega} = k_m i - T_L$$

$$L\dot{i} = -iR - k_b\omega + u$$

$$\begin{cases} \dot{x}_1 = \frac{1}{J}\left(k_m x_2 - T_L\right) \\ \dot{x}_2 = \frac{1}{L}\left(-Rx_2 - k_b x_1 + u\right) = \frac{1}{L}v \end{cases}$$

$$u = Rx_2 + k_b x_1 + v$$

$$\boxed{\omega_c = 0.2\sin 2t}$$

$z_1 = x_1$

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = \ddot{x}_1 = \frac{1}{J}\left(k_m\dot{x}_2 - \dot{T}_L\right) = \frac{1}{J}\left(\frac{k_m}{L}v - \dot{T}_L\right) \end{cases}$$

$$\sigma = z_2 + c z_1$$

$$\underline{v = -\rho\,\mathrm{sign}(\sigma)}$$

$$\frac{k_m}{L}v - \dot{T}_L > 0 \Rightarrow v > \frac{\dot{T}_{Lm}L}{k_m}$$

$$\left|-\rho\,\mathrm{sign}(\sigma)\right| > \left|\frac{\dot{T}_{Lm}L}{k_m}\right|$$

$$\underline{\rho > \left|\frac{\dot{T}_{Lm}L}{k_m}\right|}$$

Строим наблюдатель для z_2, чтобы узнать $\sigma \to v$.

$$\dot{\hat{z}}_1 = -k_1\lceil\hat{z}_1 - z_1\rfloor^{\frac{1}{2}} + \hat{z}_2$$

$$\dot{\hat{z}}_2 = -k_2\,\mathrm{sign}(\hat{z}_1 - z_1)$$

next $\rightarrow$

$$z_1 = e = \omega - \omega^*$$

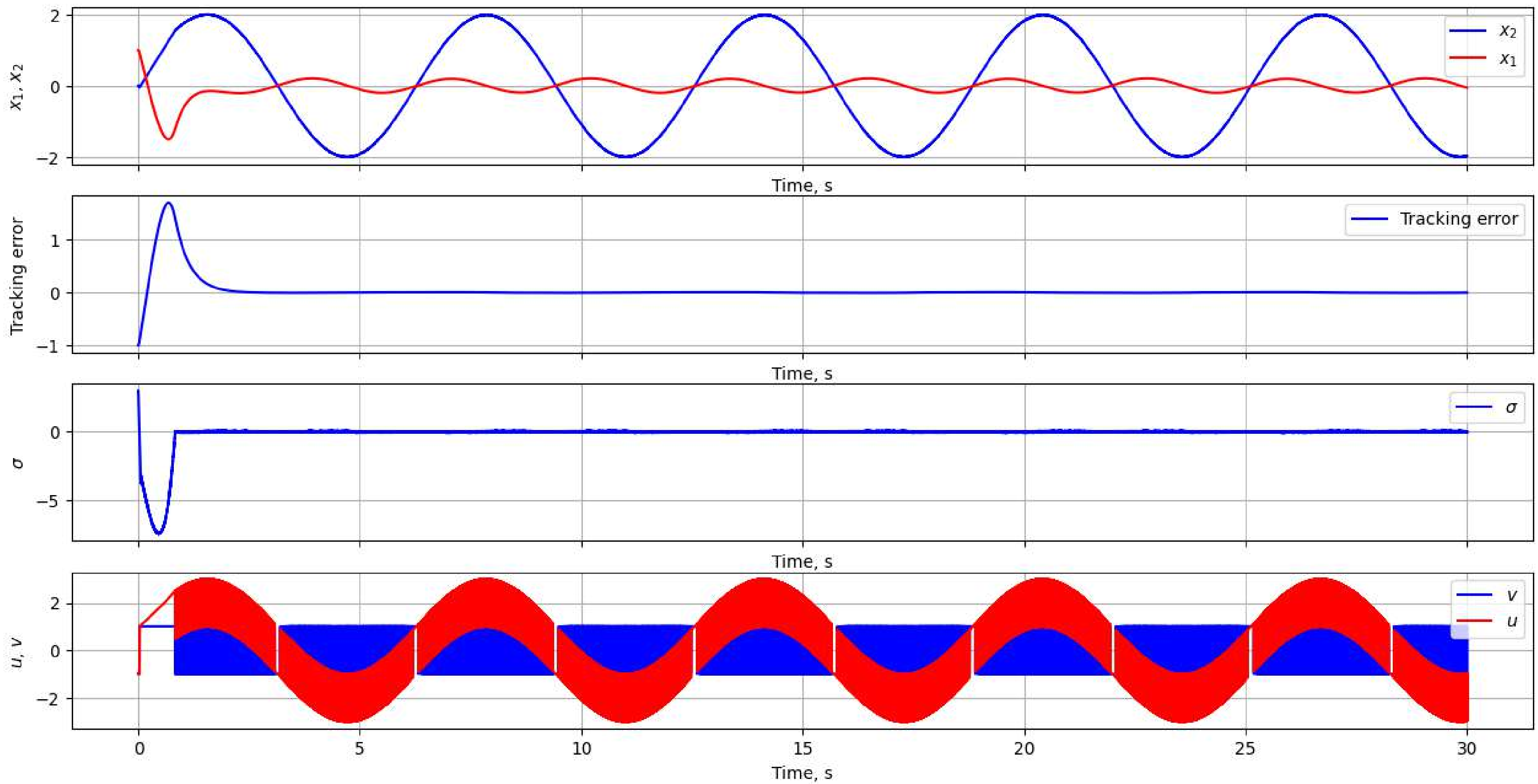
$$\dot{z}_1 = z_2 = \dot{e} = \dot{\omega} - \dot{\omega}^* = \frac{1}{J}\left(k_m x_2 - T_L \right) - 0.4\cos 2t$$

$$\dot{z}_2 = \frac{1}{J}\left(k_m \dot{x}_2 - \dot{T}_L \right) - 0.8\sin 2t = \frac{1}{J}\left(\frac{k_m}{L} v - \dot{T}_L \right) + 0.8\sin 2t$$

$$\sigma = \hat{z}_2 + c_1 z_1$$

$$v = -\rho\, \mathrm{sign}(\sigma)$$

$$\rho > \left| \frac{\varphi_2 L}{k_m} + 0.8 \right| = \frac{0.1 \cdot 0.5}{0.05} + 0.8 =$$



x_1, x_2
x_2
x_1
Time, s
Tracking error
Tracking error
Time, s
σ
σ
Time, s
u, v
v
u
Time, s

1.10 $\dot{z}_1 = \cancel{k_x}\, z_2$

$$\dot{z}_2 = \frac{1}{J}\left(\frac{k_m}{L} v - \dot{T}_L\right)$$

$$\dot{v} = \varphi$$

$\sigma = \hat{z}_2 + c z_1$, $\hat{z}_2$ is from observer

$\sigma_1 = \sigma$, $\sigma_2 = \dot{\sigma}$ $\qquad \dot{\sigma} = \underset{?}{\dot{z}_2} + c\dot{z}_1$

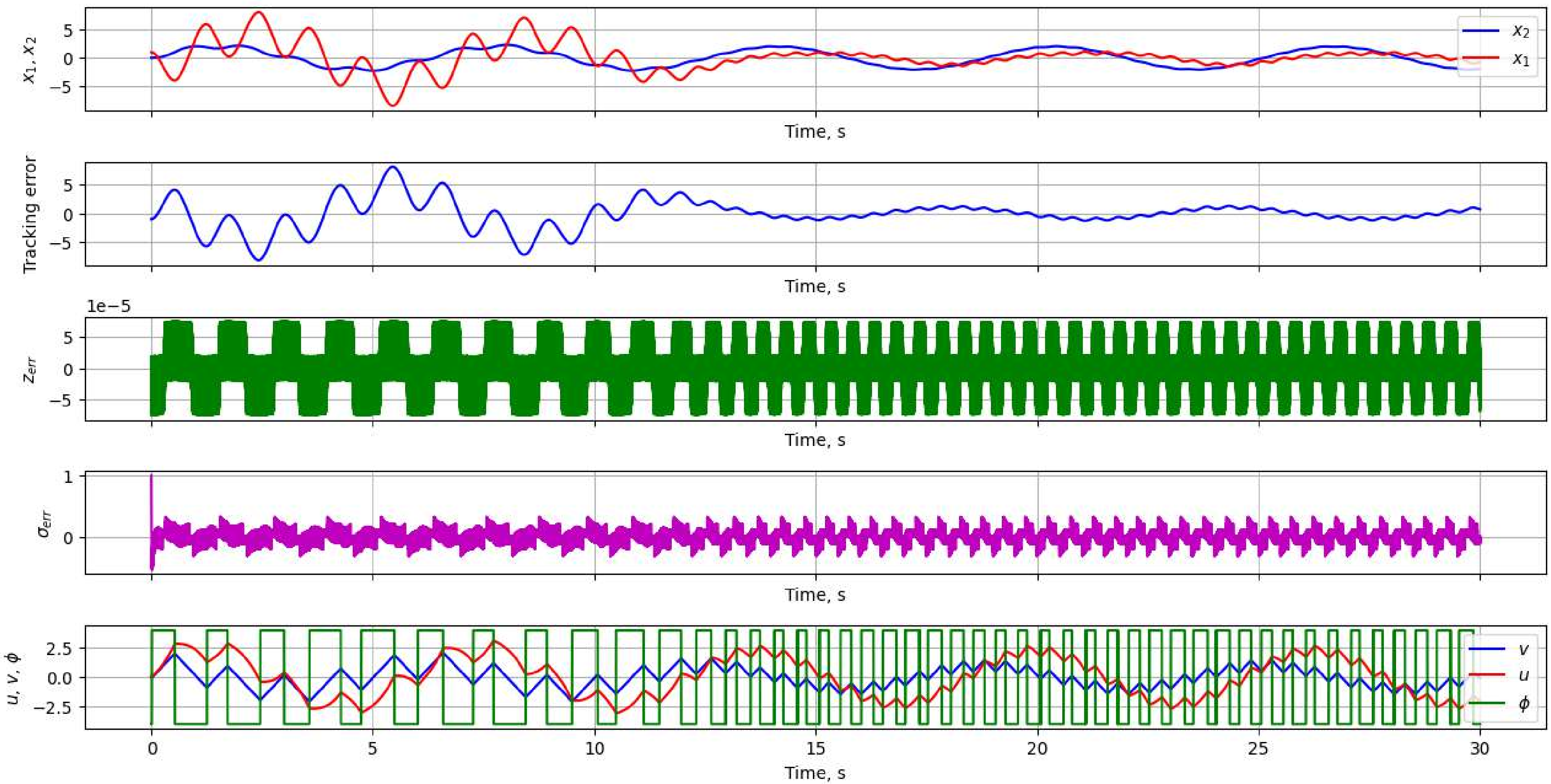
$$\dot{\hat{\sigma}}_1 = -k_1 [\hat{\sigma}_1 - \sigma]^{\frac{1}{2}} + \hat{\sigma}_2$$

$$\dot{\hat{\sigma}}_2 = -k_2 \operatorname{sign}(\hat{\sigma}_1 - \sigma)$$

$$s = \sigma_2 + c_1 \sigma_1$$

$$\varphi = -\rho \operatorname{sign}(s)$$

$$v = \int_0^t \varphi(x)\, dt \iff v_{n+1} = v_n + \varphi\, dt$$



x_1, x_2
x_2
x_1
Time, s
Tracking error
Time, s
1e−5
z_{err}
Time, s
σ_{err}
Time, s
u, v, ϕ
v
u
ϕ
Time, s

$$\dot{z}_1 = \frac{1}{J}\left(k_m x_2 - T_L\right) - 0.4\cos 2t$$

$$\dot{z}_2 = \frac{1}{J}\left(\frac{k_m}{L} v - \dot{T}_L\right) + 0.8\sin 2t$$

$$\sigma = \hat{z}_2 + c_1 z_1$$

$$\dot{\sigma} = \dot{\hat{z}}_2 + c_1 \dot{z}_1 = \dot{\hat{z}}_2 + c_1 z_2$$

$$\cancel{v = -k_1 \lfloor \sigma \rceil^{\frac{1}{2}} - k_2 \int_0^t \operatorname{sign}(\sigma(\tau))\, d\tau}$$

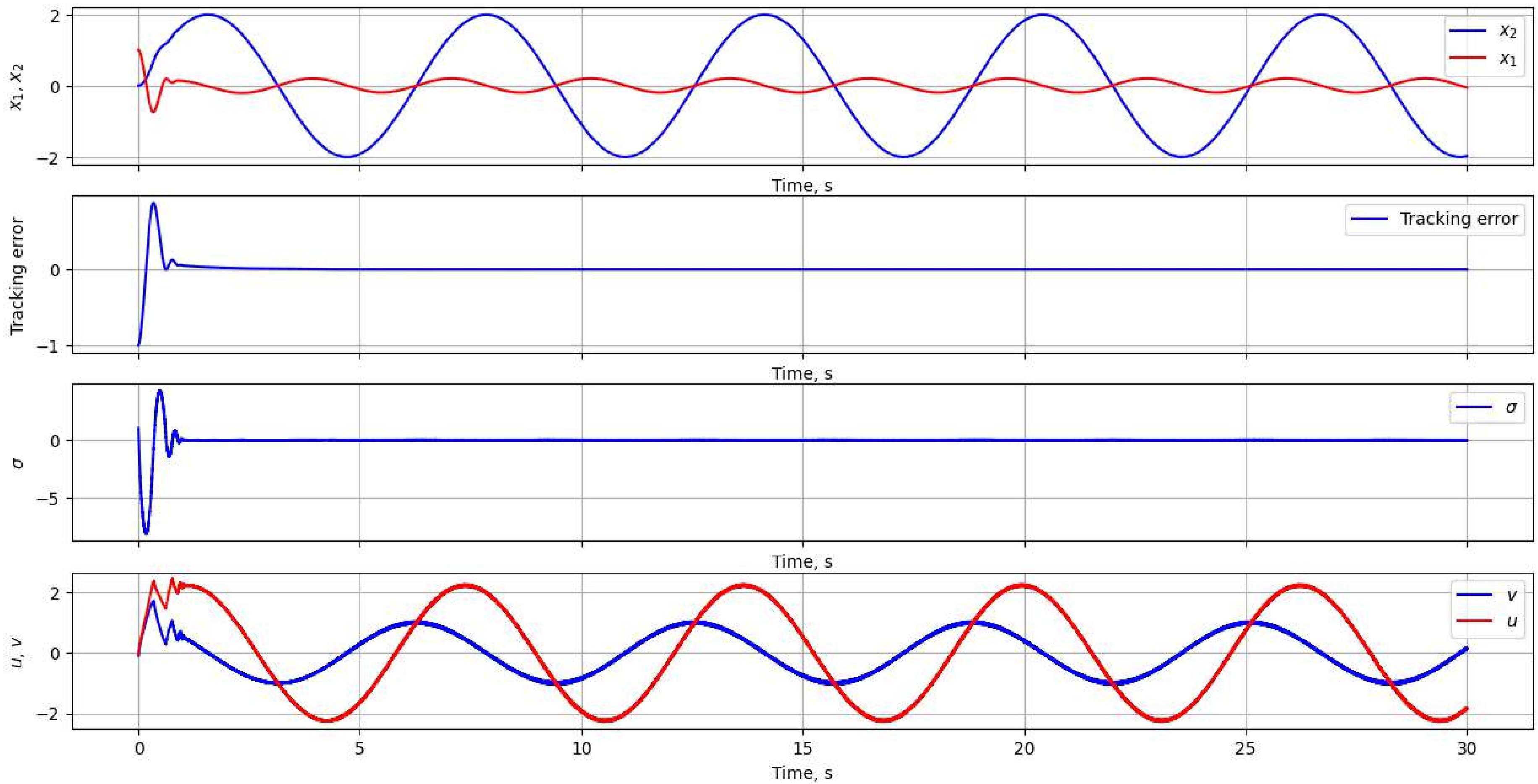
$$\left\{ \begin{aligned} \dot{\hat{\sigma}}_1 &= -k_1 \lfloor \hat{\sigma}_1 - \sigma_1 \rceil^{\frac{1}{2}} + \hat{\sigma}_2 \\ \dot{\hat{\sigma}}_2 &= -k_2 \operatorname{sign}(\hat{\sigma}_1 - \sigma_1) \end{aligned} \right\} \text{for sim only}$$

$$\cancel{s = \hat{\sigma}_2 + \beta \lfloor \sigma_1 \rceil^{\frac{1}{2}}}$$

$$\cancel{v = -\alpha \operatorname{sign}(s)}$$

$$\left\{ \begin{aligned} s &= \hat{\sigma}_2 + c_2 \sigma_1 \\ K &= -\varphi_1 \lfloor s \rceil^{\frac{1}{2}} - \varphi_2 \int_0^t \operatorname{sign}(s(\tau))\, d\tau \end{aligned} \right\} \text{for sim only}$$

$$v = -\varphi_1 \lfloor \sigma \rceil^{\frac{1}{2}} - \varphi_2 \int_0^t \operatorname{sign}(\sigma(\tau))\, d\tau$$



x_1, x_2
x_2
x_1
Tracking error
Tracking error
σ
σ
u, v
v
u
Time, s