Project

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k_1}{J_1}x_1 - \frac{c_1}{J_1}x_2 + \frac{k_1}{J_1}x_3 + \frac{1}{J_1}(u+\varphi) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{k_1}{J_2}x_1 - \frac{k_1+k_2}{J_2}x_3 - \frac{c_2}{J_2}x_4 \end{cases} \quad \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \alpha_{21}x_1 + \alpha_{22}x_2 + \alpha_{23}x_3 + b(u+\varphi) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \alpha_{41}x_1 + \alpha_{43}x_3 + \alpha_{44}x_4 \end{cases}$$

$y_1 = x_3$

$\dot{y}_1 = y_2 = \dot{x}_3 = x_4$

$\dot{y}_2 = y_3 = \dot{x}_4 = \alpha_{41}x_1 + \alpha_{43}\dot{x}_3 + \alpha_{44}x_4$

$\dot{y}_3 = y_4 = \ddot{x}_{44} = \alpha_{41}x_2 + \alpha_{43}x_4 + \alpha_{44}(\alpha_{41}x_1 + \alpha_{43}x_3 + \alpha_{44}x_4)$

$\dot{y}_4 = f(x) + \alpha_{41}b(u+\varphi)$

$u = \frac{-1}{\alpha_{41}b} f(x) + v \quad \Big| \Rightarrow \dot{y}_4 = \alpha_{41}b(v+\varphi)$

Cont. controller 4-sm:

$v_4 = -1.1\lfloor y_1 \rceil^{\frac{4}{5}}_{\frac{1}{5}} - 1.9\lfloor y_2 \rceil^{\frac{3}{4}}_{\frac{1}{4}} - 2.6\lfloor y_3 \rceil^{\frac{2}{3}}_{\frac{1}{3}} - 2.8\lfloor y_4 \rceil^{\frac{1}{2}}_{\frac{1}{2}} + v$

$v = -0{,}002\lfloor G \rceil^{0} = -0.002\lfloor y_1 \rceil^{0}$

Discont. controller 4-sm:

$$v_{4d} = -1.1\lfloor y_4 + 3\left(y_3^6 + y_2^4 + |y_1|^3\right)^{\frac{1}{12}} \rceil \lfloor y_3 + \left(y_2^4 + |y_1|^3\right)^{\frac{1}{6}} \rceil \lfloor y_2 + 0.5|y_1|^{\frac{3}{4}} \rceil \lfloor y_1 \rceil^{0} \rceil^{0} \rceil^{0}$$

for continuous controllers:

$L_c > |\dot{\varphi}|$

$\varphi = -m\sin\left(\frac{t}{d}\right) + 0.1d\cos(30mt), \quad d = 1, \ m = 9$

$\dot{\varphi} = -\frac{m}{d}\cos\left(\frac{t}{d}\right) - 3md\sin(30mt)$

$L_c = \frac{m}{d} + 3md = \frac{9}{1} + 3 \cdot 9 \cdot 1 = 36$

for discontinuous controllers:

$L_d > |\varphi|$

$L_d = m + 0.1d \approx 10$

Project

$\sigma = y_2 + c y_1 = \sigma_1 \qquad c = d = 1$

$\dot{\sigma}_1 = \sigma_2 = y_3 + c y_2$

$\dot{\sigma}_2 = \sigma_3 = y_4 + c y_3$

$\dot{\sigma}_3 = \dot{y}_4 + c y_4 = \alpha_{41} b (v + \varphi) + y_4$

$v = -\frac{1}{\alpha_{41} b} y_4 + \theta \;\Big|\; \Rightarrow \dot{\sigma}_3 = \alpha_{41} b (\theta + \varphi)$

3-SM Cont. controller:

$\theta_c = v_{3c} = -1.3 L^{\frac{3}{4}} \lceil \sigma_1 \rfloor^{\frac{1}{4}} - 2.2 L^{\frac{2}{3}} \lceil \sigma_2 \rfloor^{\frac{1}{3}} - 3 L^{\frac{1}{2}} \lceil \sigma_3 \rfloor^{\frac{1}{2}} + \vartheta$

$\dot{\vartheta} = -0.09 L \lceil \sigma_1 \rfloor^{0}$

3-SM Discont. controller:

$v_{3d} = -1.1 L \lceil \sigma_3 + 2 (|\sigma_2|^3 + |\sigma_1|^2)^{\frac{1}{6}} \lceil \sigma_2 + |\sigma_1|^{\frac{2}{3}} \lceil \sigma_1 \rfloor^{0} \rfloor^{0} \rfloor^{0} = \theta_d$

$\gamma = y_3 + c_1 y_2 + c_2 y_1 = \gamma_1$

$\langle \lambda_1 = -9, \lambda_2 = -1 \Rightarrow c_1 = 10, c_2 = 9 \rangle$

$\dot{\gamma}_1 = \gamma_2 = y_4 + c_1 y_3 + c_2 y_2$

$\dot{\gamma}_2 = \dot{y}_4 + c_1 y_4 + c_2 y_3 = \alpha_{41} b (v + \varphi) + c_1 y_4 + c_2 y_3$

$v = \theta - \frac{1}{\alpha_{41} b} (c_1 y_4 + c_2 y_3) \;\Big|\; \Rightarrow \dot{\gamma}_2 = \alpha_{41} b (\theta + \varphi)$

2-SM Cont. controller:

$\theta_c = v_{2c} = -2.7 L^{\frac{2}{3}} \lceil \gamma_1 \rfloor^{\frac{1}{3}} - 5.345 L^{\frac{1}{2}} \lceil \gamma_2 \rfloor^{\frac{1}{2}} + \vartheta \qquad \dot{\vartheta} = -1.1 L \lceil \gamma_1 \rfloor^{0}$

2-SM Discont. controller: $v_{2d} = -1.1 L \lceil \gamma_1 \rfloor^{0} - 1.5 \sqrt{L} \lceil \gamma_2 \rfloor^{0} = \theta_d$

$\psi = y_4 + c_1 y_3 + c_2 y_2 + c_3 y_1 = \psi_1$

$\langle \lambda_1 = -1, \lambda_2 = -9, \lambda_3 = -1 \Rightarrow c_1 = 11, c_2 = 19, c_3 = 9 \rangle$

$\dot{\psi}_1 = \dot{y}_4 + c_1 y_4 + c_2 y_3 + c_3 y_2 = \alpha_{41} b (\theta + \varphi)$

$v = \theta - c_1 y_4 - c_2 y_3 - c_3 y_2$

1-SM Cont. controller:

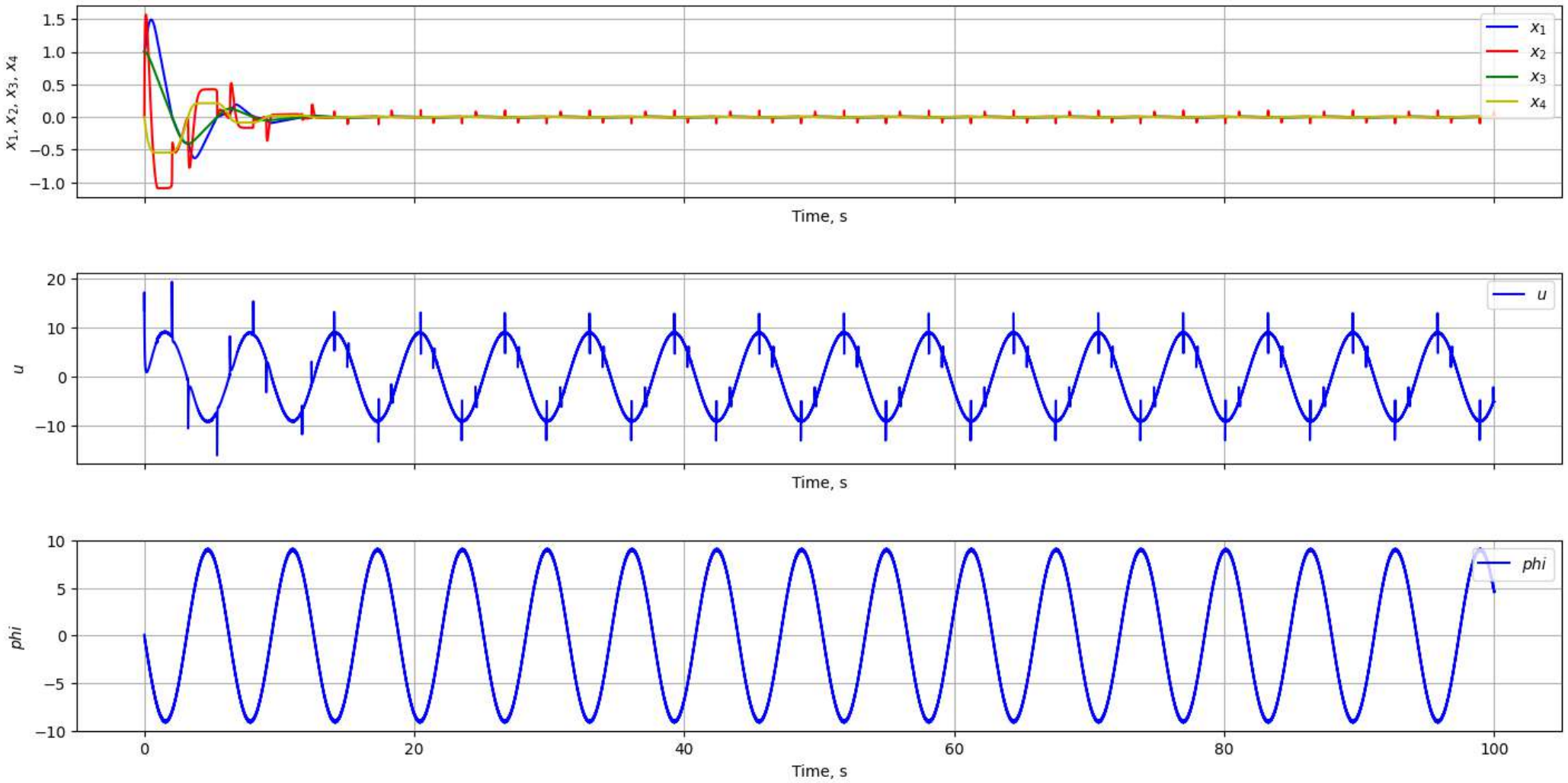
$\theta_c = v_{1c} = -1.5 \sqrt{L} \lceil \psi_1 \rfloor^{\frac{1}{2}} + \vartheta, \qquad \dot{\vartheta} = -1.1 L \lceil \psi_1 \rfloor^{0}$

1-SM. Discont. controller: $v_{1d} = -1.1 L \lceil \psi_1 \rfloor^{0} = \theta_d$

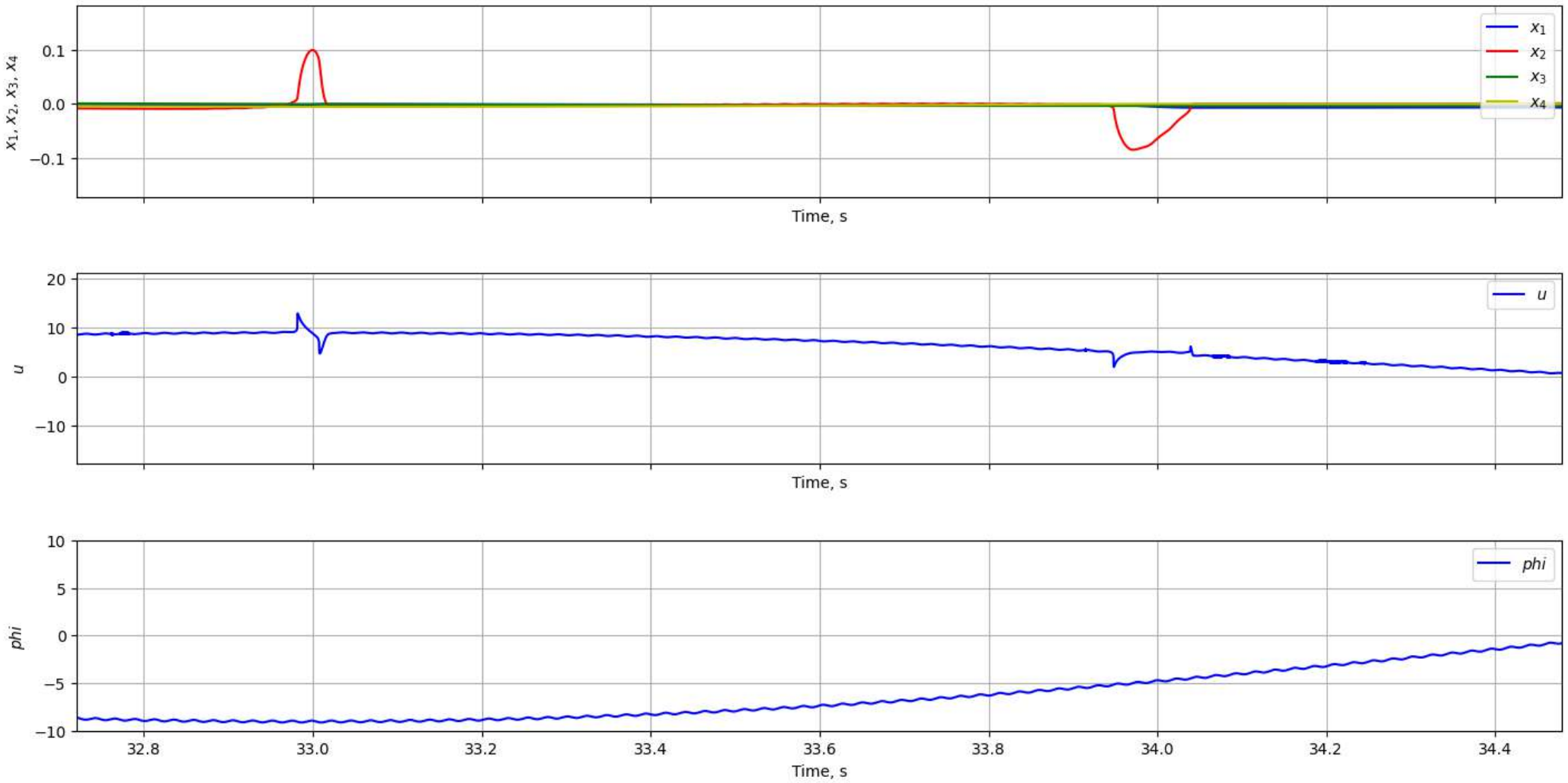
System parameters

```
# sim params
t1 = 100.0
dt = 0.0001

# system params
k1 = 0.5
k2 = 0.5
c1 = 0.012195
c2 = 0.00272
J1 = 0.29462
J2 = 0.292045

# birth month $ day
m = 9
d = 1

a21 = -k1 / J1
a22 = -c1 / J1
a23 = k1 / J1
a41 = k1 / J2
a43 = -(k1 + k2) / J2
a44 = -c2 / J2

b = 1 / J1

# Lipschitz constants for cont.
# and disc. controllers
Lc = 50
Ld = 10
```

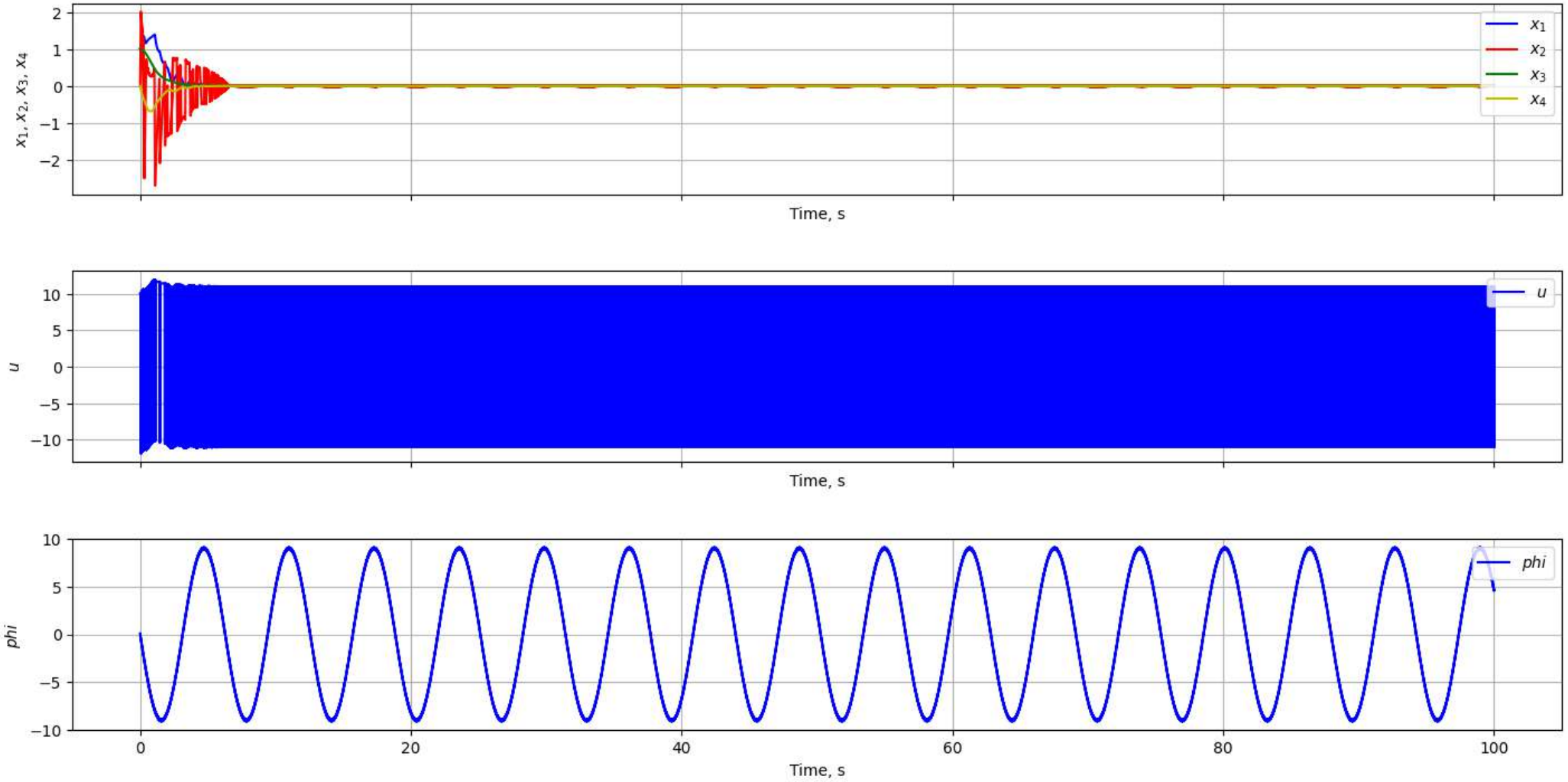
4-sm cont. controller



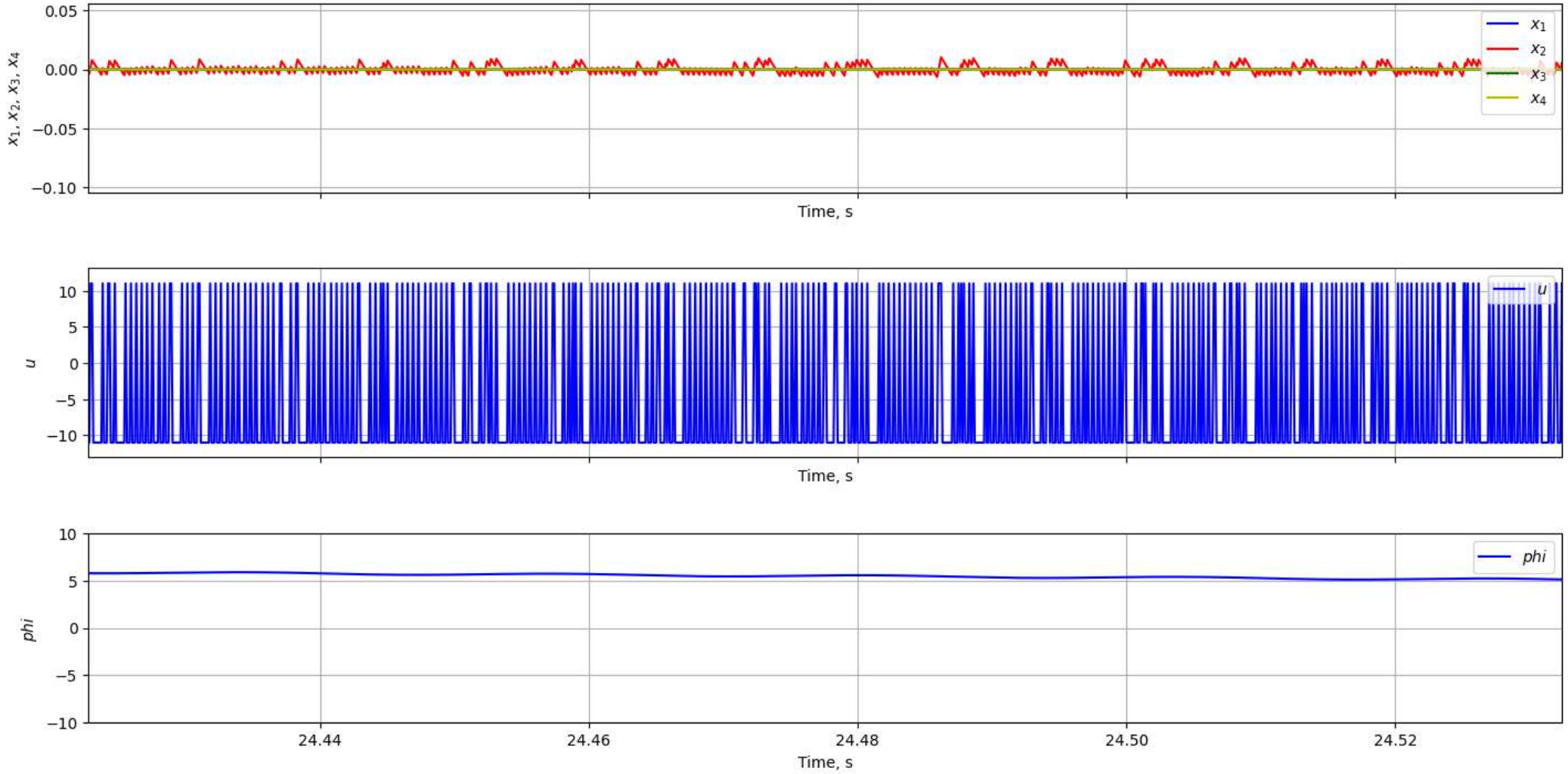
4-sm cont. controller (zoom)



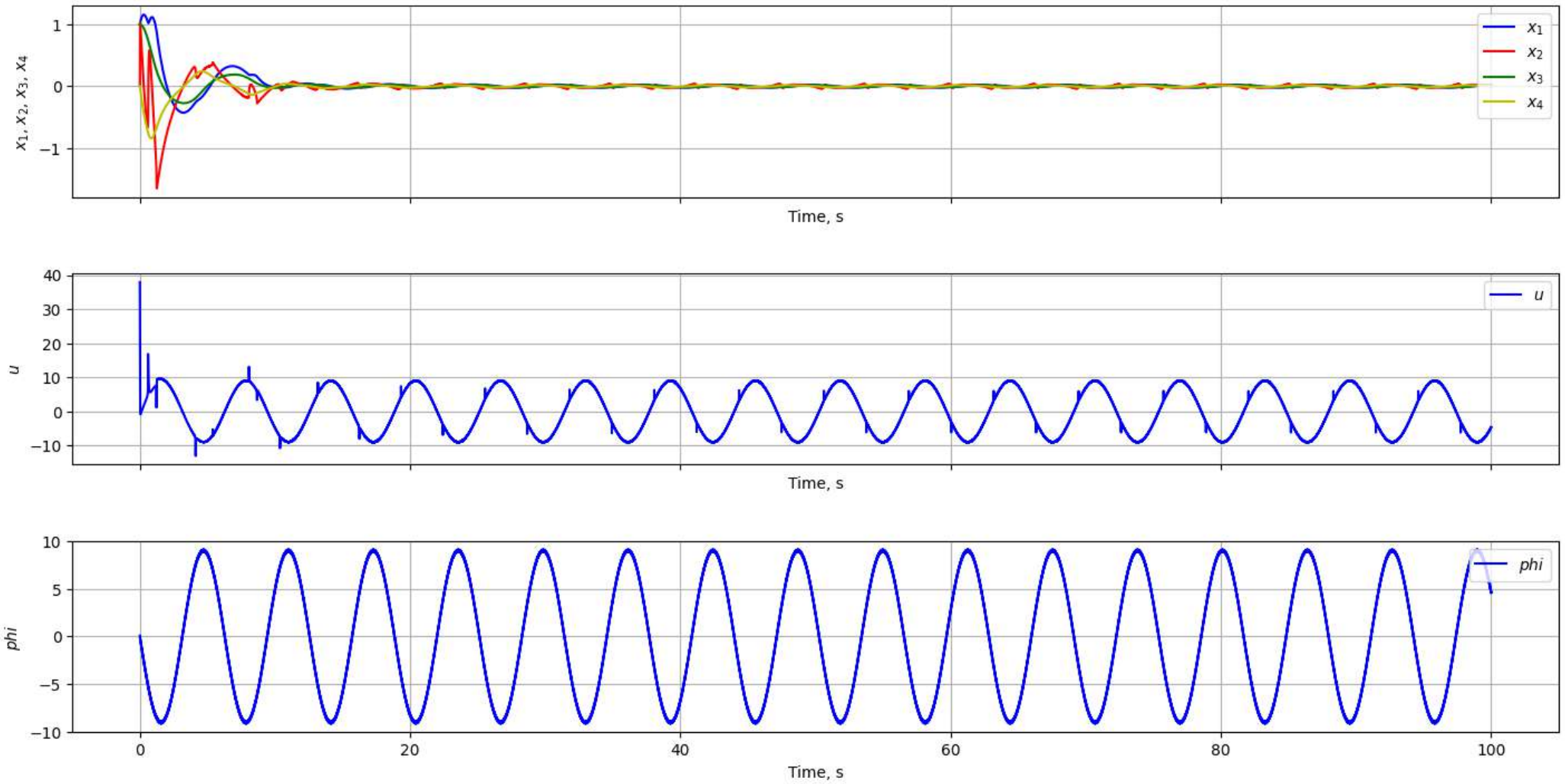
4-sm discont. controller



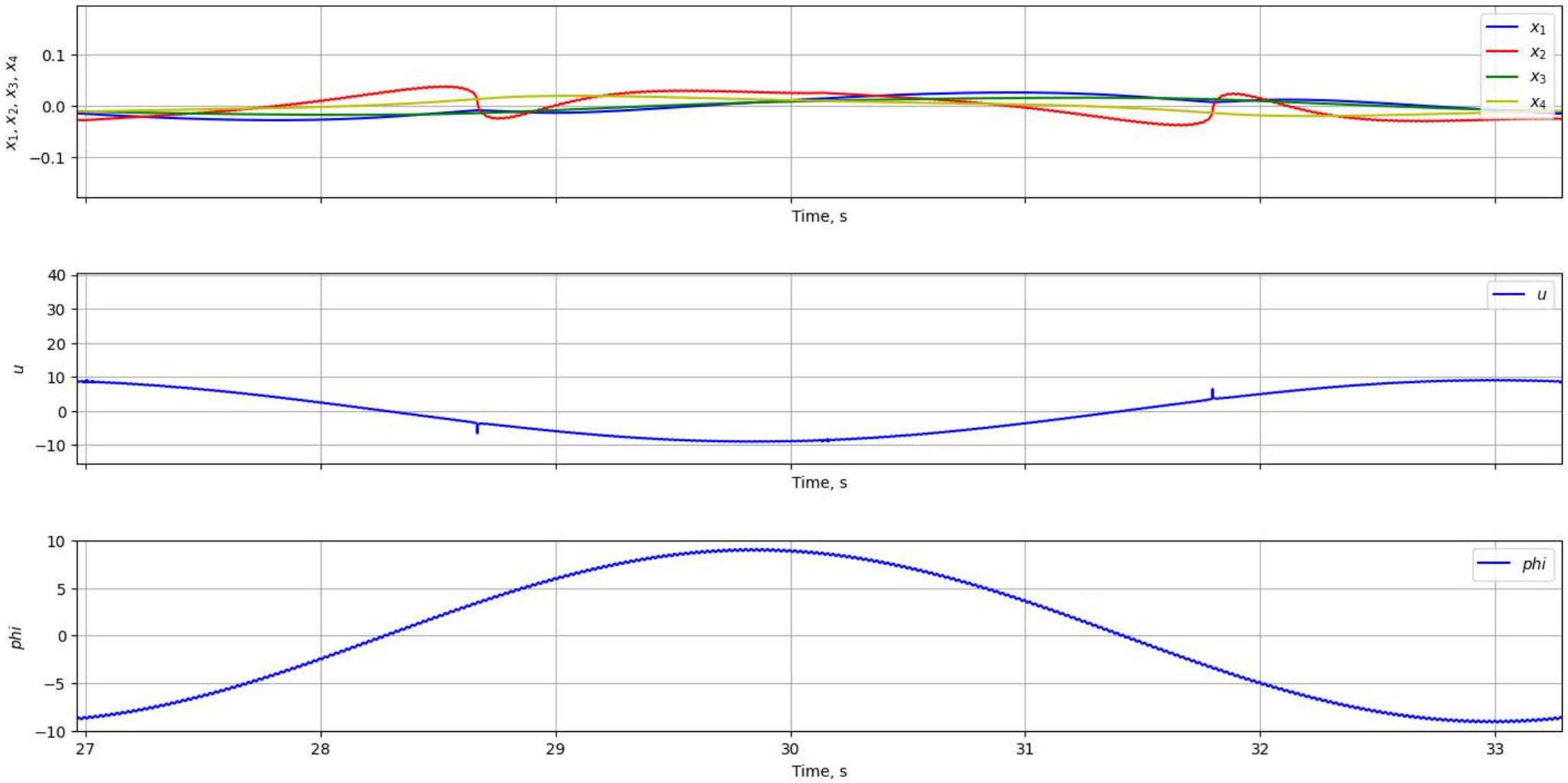
4-sm discont. controller (zoom)



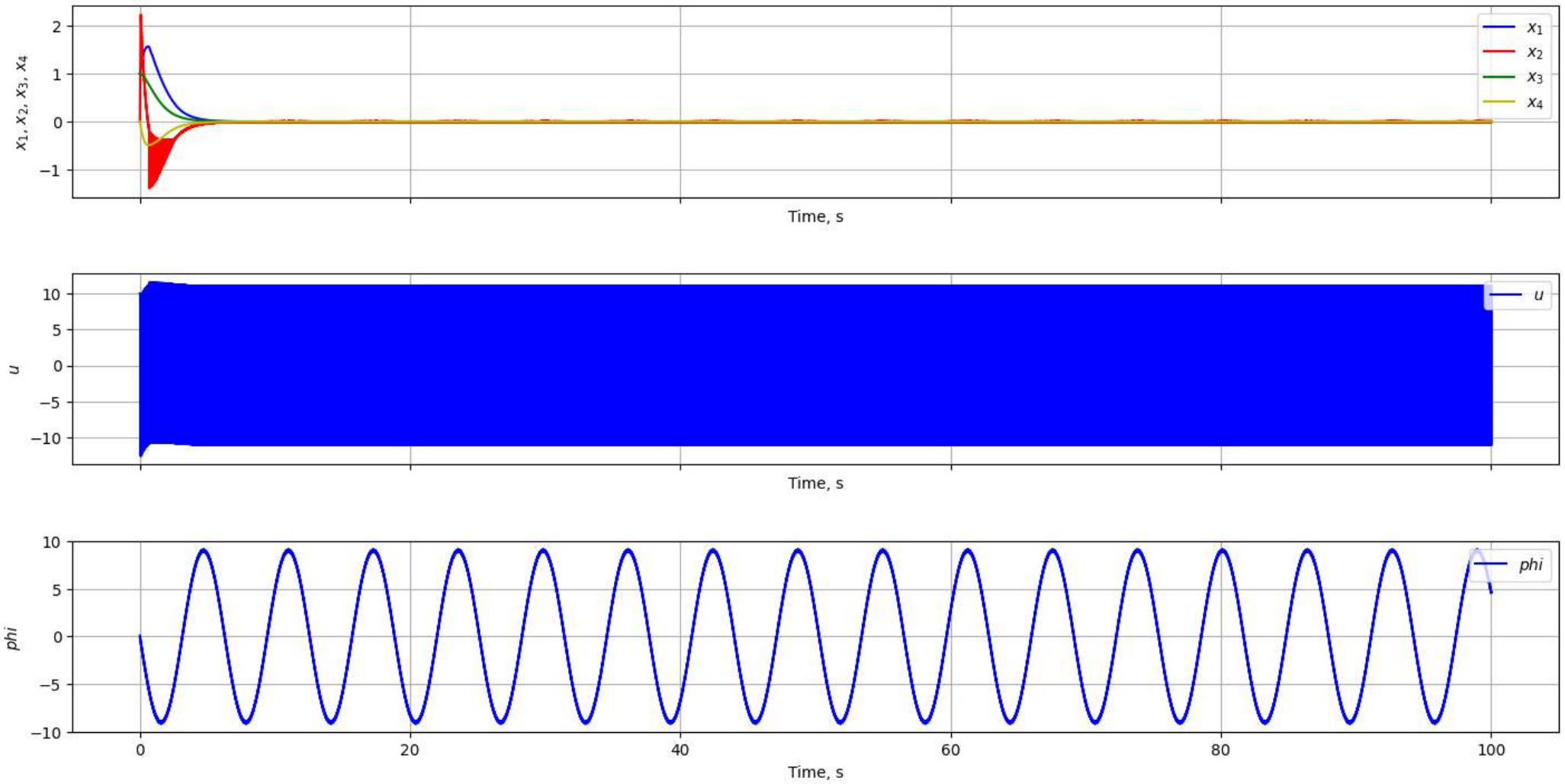
3-sm cont. controller



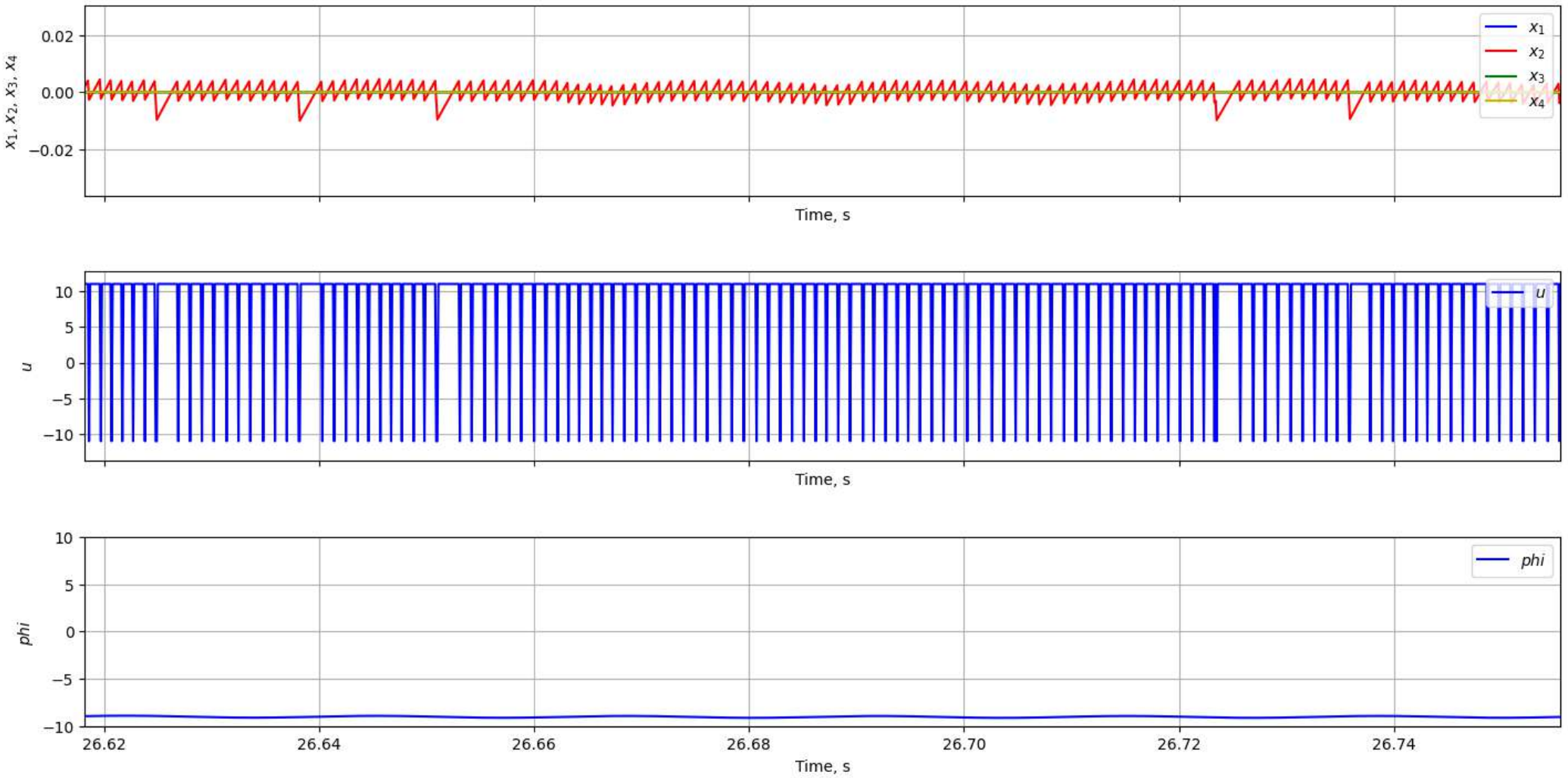
3-sm cont. controller (zoom)



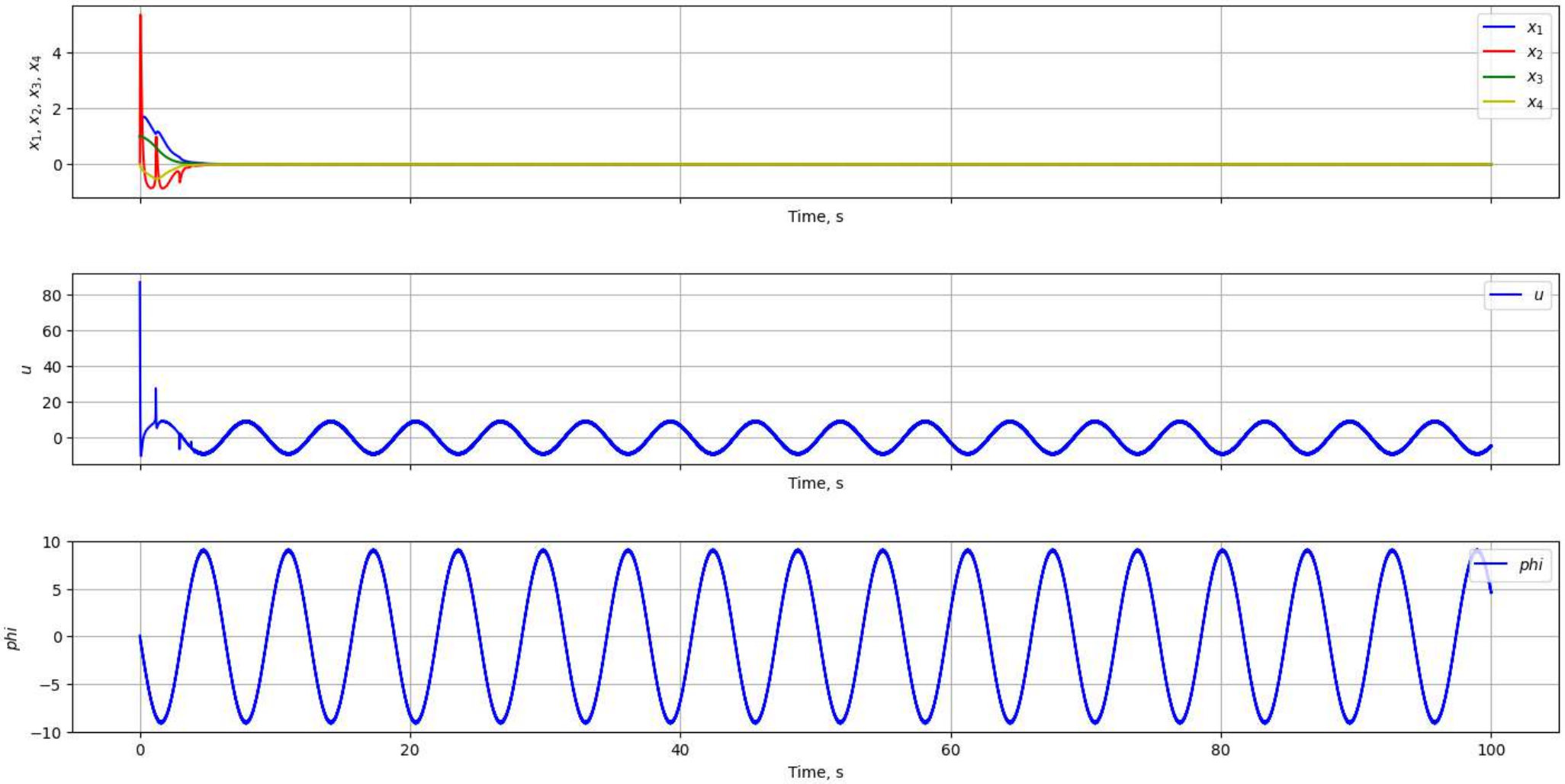
3-sm discont. controller



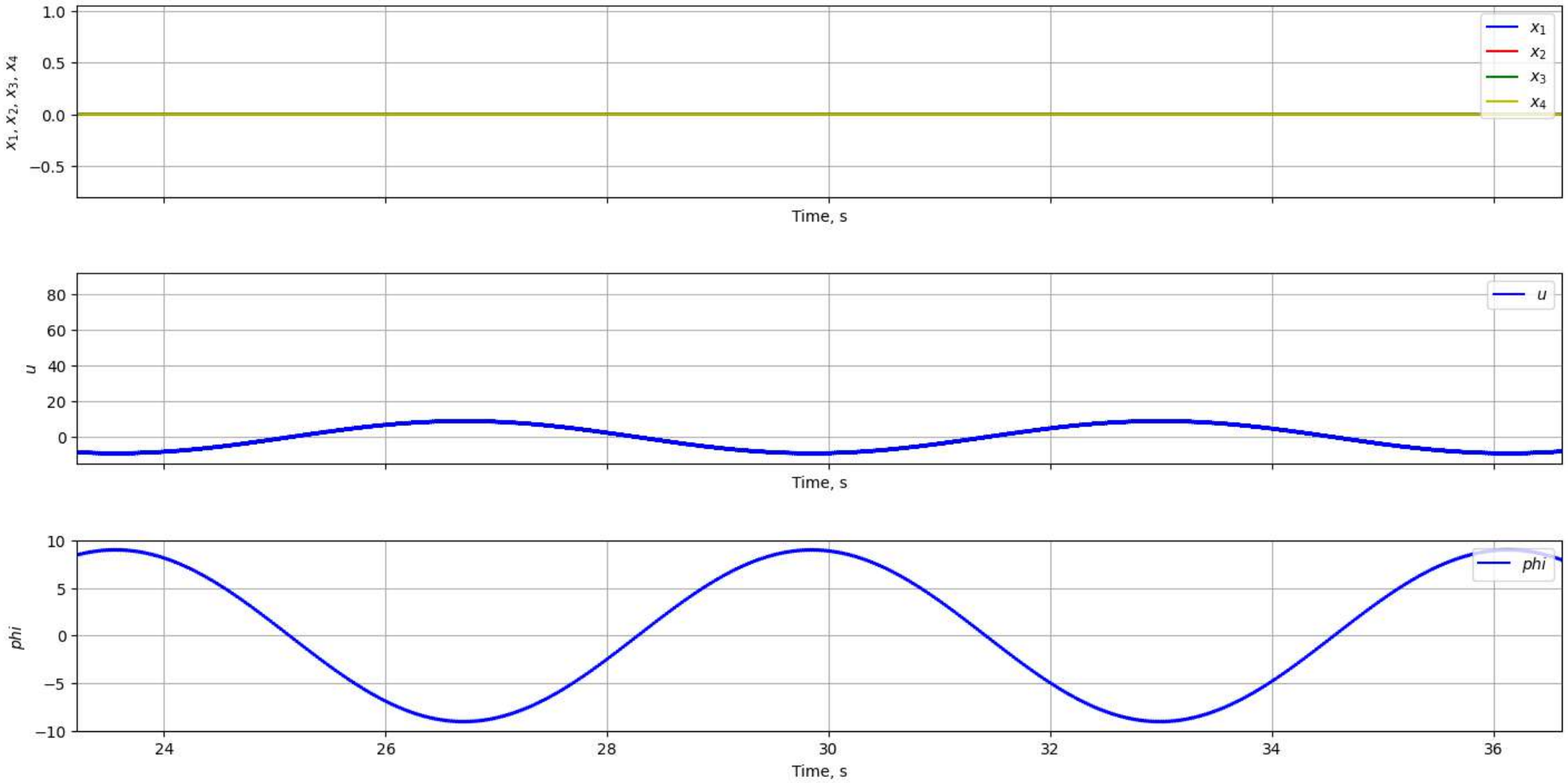
3-sm discont. controller (zoom)



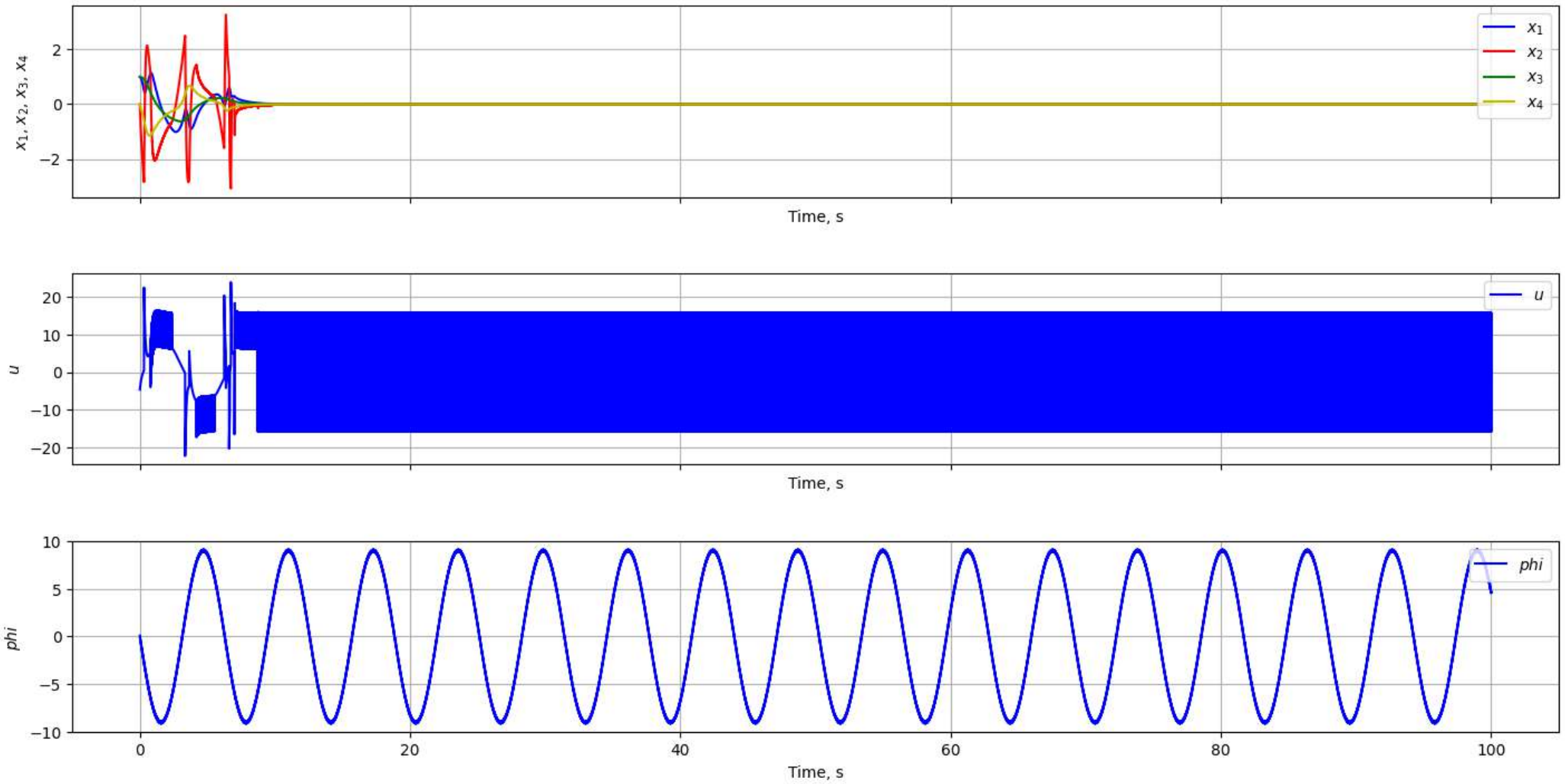
2-sm cont. controller



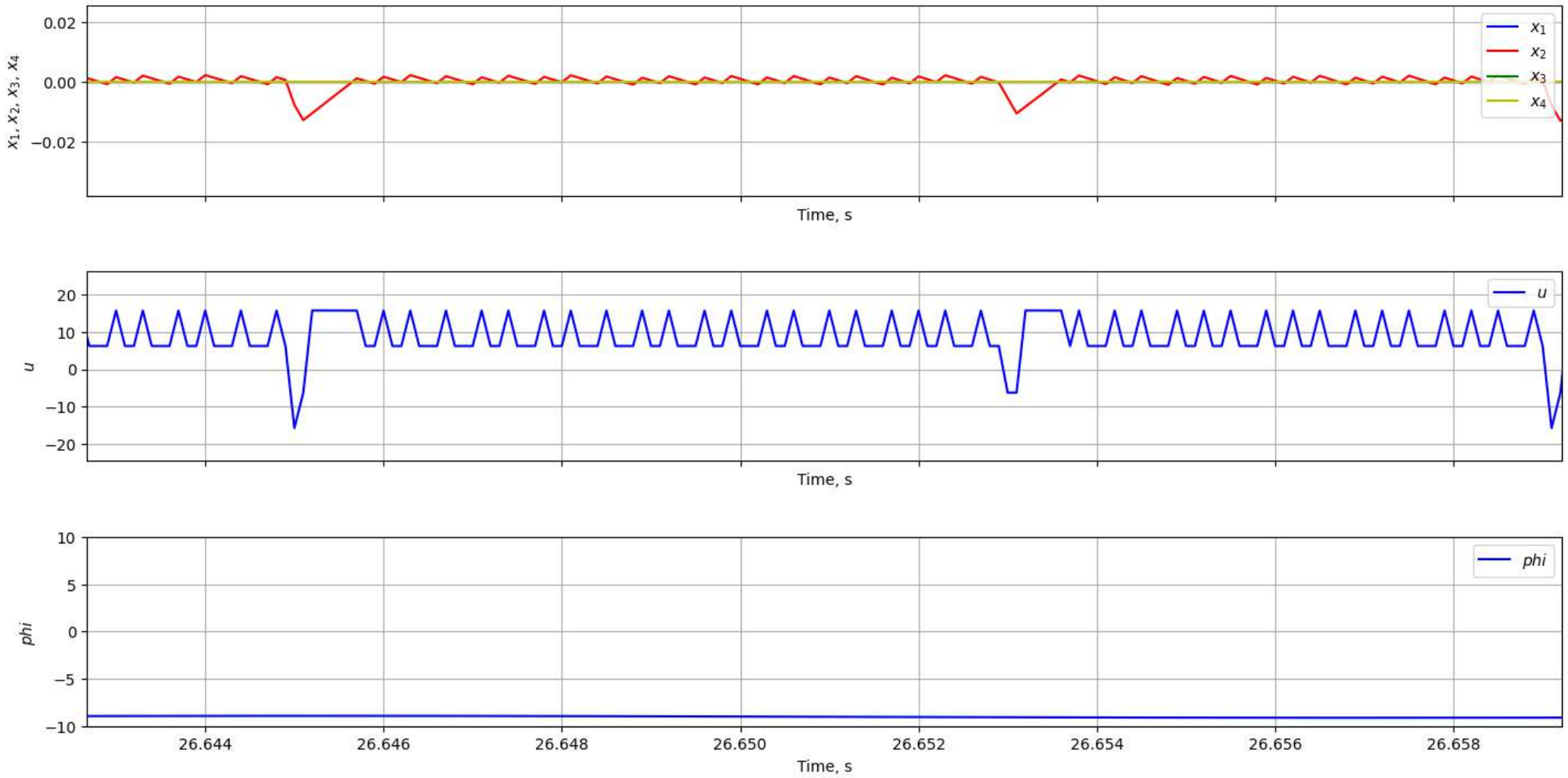
2-sm cont. controller (zoom)



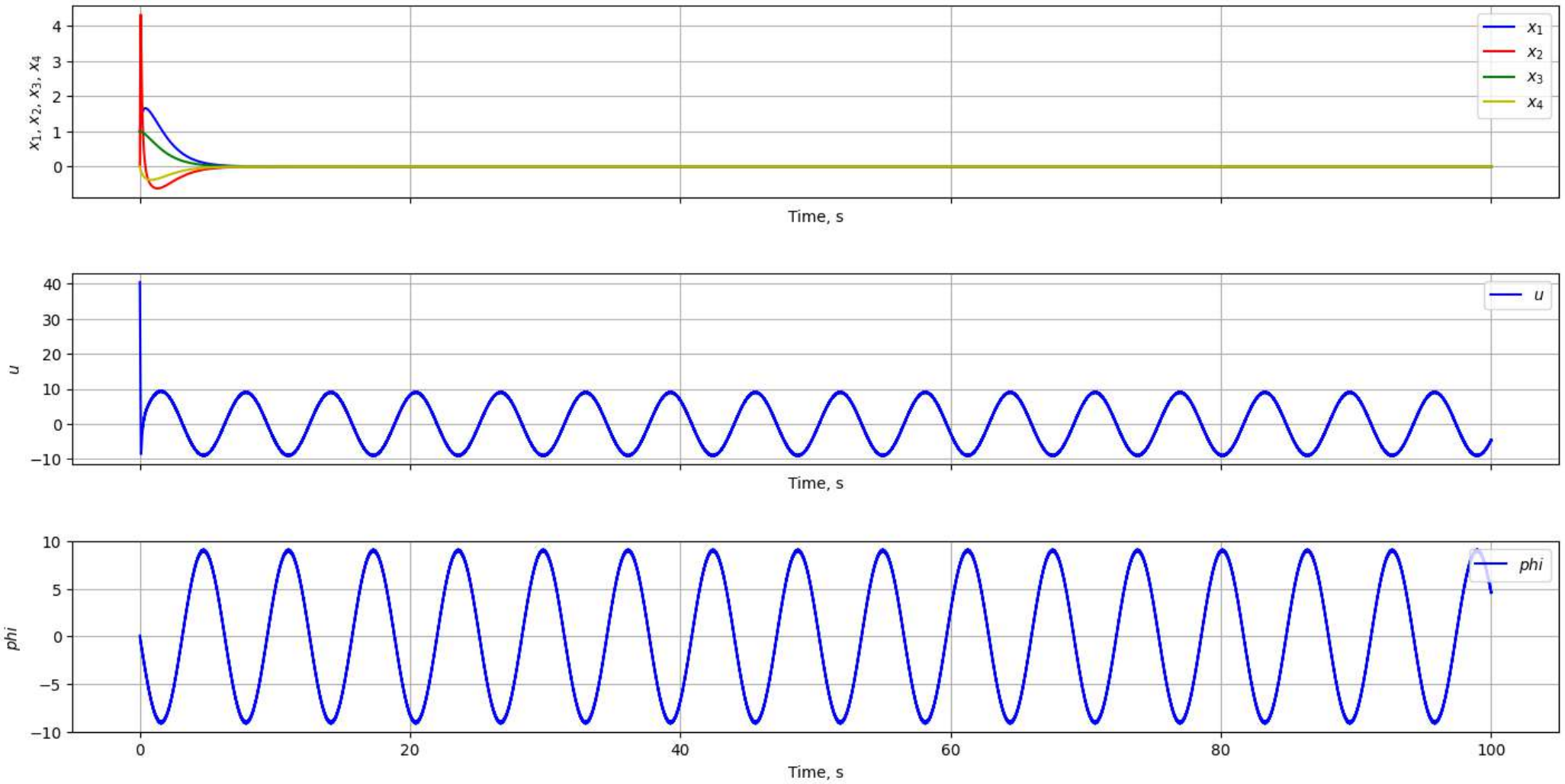
2-sm discont. controller



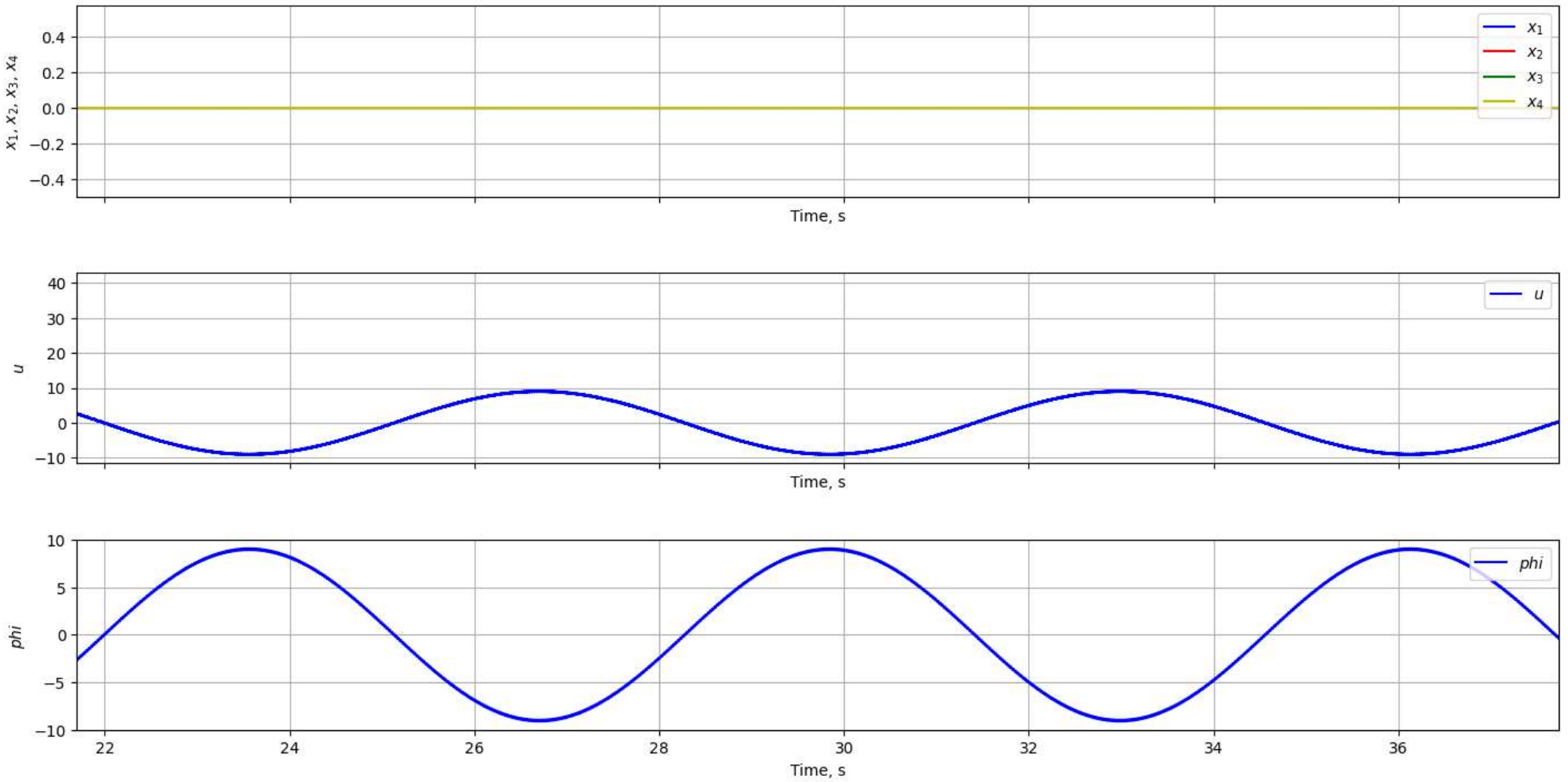
2-sm discont. controller (zoom)



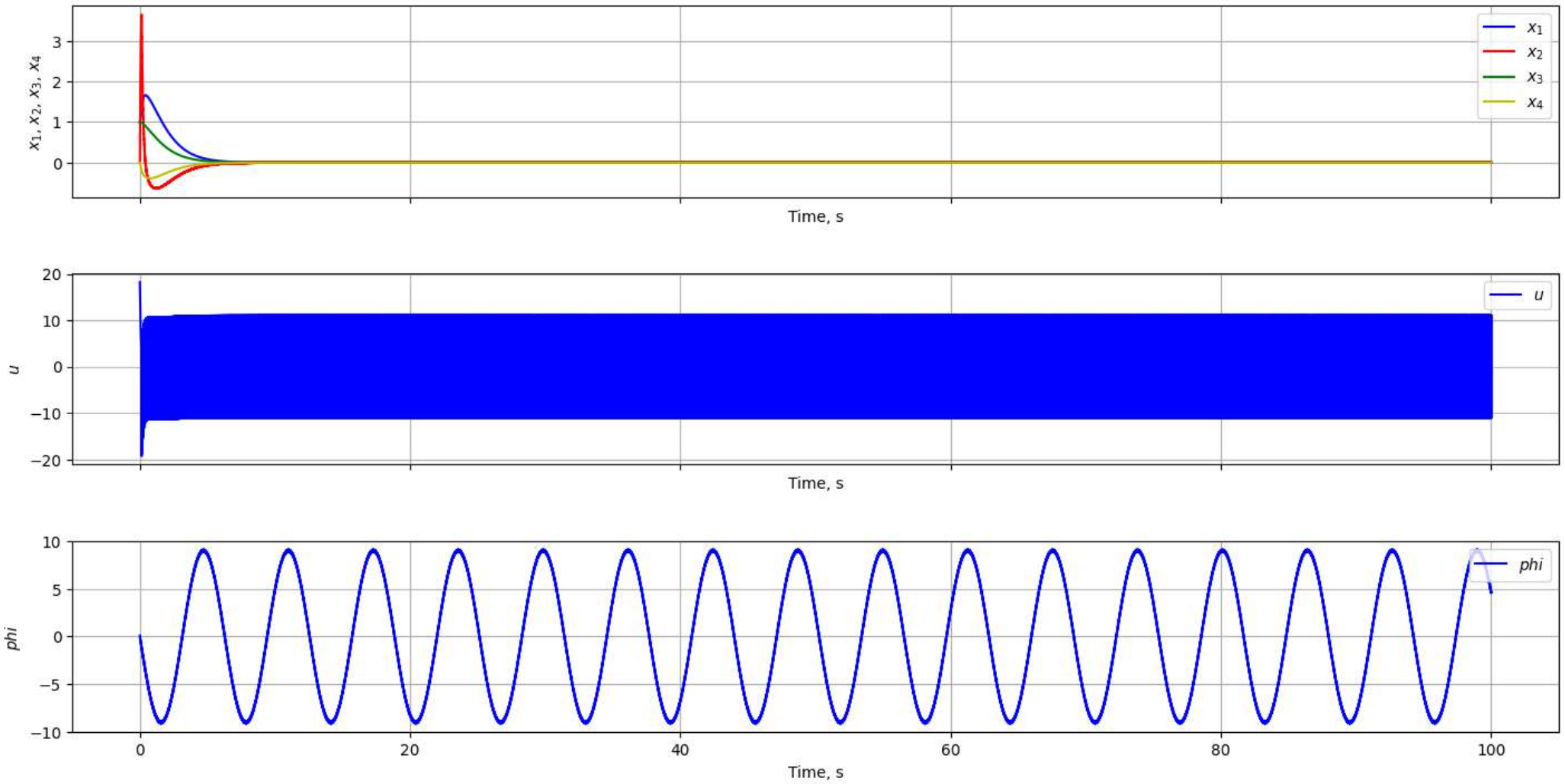
1-sm cont. controller



1-sm cont. controller (zoom)



1-sm discont. controller



1-sm discont. controller (zoom)