STAT0011 Decision and Risk In-Course Assessment 2024
Group 5

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Contents

1 Introduction

This report presents the analysis conducted by Group 5 for the STAT0011 Decision and Risk In-Course Assessment 2024. We evaluate the market risk of an equally-weighted portfolio comprising two stock indices, the S&P 500 and the Shanghai Stock Exchange (SSE) Composite Index, using weekly log-returns from 2000 to 2023. Our objective is to estimate the 95% and 99% 1-week Value-at-Risk (VaR) employing a Monte Carlo simulation approach based on copula theory, as outlined in the task description.

2 Data Description

We selected the S&P 500 and SSE Composite Index from the six candidate indices provided. Weekly closing prices were obtained from Yahoo Finance for the period spanning January 2000 to December 2023, yielding 1214 observations per index. Log-returns were calculated as follows:

$$\mathrm{ret}_t = \ln\left(\frac{P_t}{P_{t-1}}\right),$$

where P_t denotes the price at week t. The datasets are stored in `SP500.csv` and `SSE.csv`, submitted alongside this report.

3 Methodology

3.1 Time Series Modelling

The log-returns exhibited volatility clustering and non-normality, confirmed via Jarque-Bera tests (p-values ¡ 0.05). Autocorrelation functions (ACF) and partial ACF (PACF) plots guided the selection of autoregressive (AR) models: AR(1) for S&P 500 and AR(2) for SSE, with GARCH(1,1) components to capture volatility clustering. We fitted AR-GARCH models using the `fGarch` package in R, testing various conditional distributions (normal, Student-t, skewed Student-t) and selecting the best fit based on AIC/BIC criteria (see Table 1).

For S&P 500, an AR(1)-GARCH(1,1) model with a skewed Student-t (sstd) distribution was optimal. For SSE, an AR(2)-GARCH(1,1) with sstd was chosen. Model adequacy was assessed using ACF of residuals, Ljung-Box tests, and Probability Integral Transform (PIT) uniformity tests (Kolmogorov-Smirnov and Anderson-Darling).

3.2 Copula Modelling

Uniform residuals (u1, u2) from the marginal models were used to fit a bivariate copula using the `VineCopula` package. The best copula was selected based on AIC, with independence tests performed at the 5% level.

Table 1: Conditional Distribution Selection for AR(3)-GARCH(1,1) Model (Sample from Document)

Conditional Distribution	AIC	BIC
Normal (norm)	-1.934944	-4.901724
Student-t (std)	-4.977397	-4.939430
Skew Student-t (sstd)	-5.009752	-4.967040
Skew Normal (snorm)	-4.988160	-4.950193
Skew Generalized Error (sged)	-5.005165	-4.962453
Generalized Error (ged)	-4.970248	-4.932282

3.3 VaR Estimation

We simulated 100,000 scenarios of one-week-ahead log-returns using the fitted copula and marginal models. The portfolio log-return was computed as:

$$\text{port}_t = \ln\left(1 + \frac{(\exp(\text{ret1}_t) - 1) + (\exp(\text{ret2}_t) - 1)}{2}\right).$$

The 95% and 99% VaR were derived as the 5th and 1st percentiles of the negative simulated portfolio returns.

4 R Code

Below is the R code used for the analysis:

```r
# Load required packages
library(stats)
library(VineCopula)
library(fGarch)
library(fBasics)
library(KScorrect)
library(ADGofTest)

# Import data
SP500 <- read.csv("SP500.csv")
SSE <- read.csv("SSE.csv")
price1 <- rev(SP500$Price)
price2 <- rev(SSE$Price)
ret1 <- log(price1[-1]/price1[-1214])
ret2 <- log(price2[-1]/price2[-1214])

# Model fitting for S&P 500
model1 <- garchFit(formula=~arma(1,0)+garch(1,1), data=ret1, trace=F,
    cond.dist="sstd")
res1 <- residuals(model1, standardize=TRUE)
u1 <- psstd(res1, nu=model1@fit$par["shape"], xi=model1@fit$par["skew"])

# Model fitting for SSE
model2 <- garchFit(formula=~arma(2,0)+garch(1,1), data=ret2, trace=F,
    cond.dist="sstd")
res2 <- residuals(model2, standardize=TRUE)
u2 <- psstd(res2, nu=model2@fit$par["shape"], xi=model2@fit$par["skew"])

# Copula selection
model <- BiCopSelect(u1, u2, familyset=NA, selectioncrit="AIC",
    indeptest=TRUE, level=0.05, se=TRUE)

# Monte Carlo simulation
N <- 100000
set.seed(0123)
u_sim <- BiCopSim(N, family=model$family, model$par, model$par2)
```

```r
res1_sim <- qsstd(u_sim[,1], nu=model1@fit$par["shape"], xi=model1@fit$par["skew"])
res2_sim <- qsstd(u_sim[,2], nu=model2@fit$par["shape"], xi=model2@fit$par["skew"])

# S&P 500 simulation
t <- length(ret1)
sp_sigma2hat <- numeric(t+1)
sp_omegahat <- model1@fit$par["omega"]
sp_alphahat <- model1@fit$par["alpha1"]
sp_betahat <- model1@fit$par["beta1"]
sp_sigma2hat[1] <- sp_omegahat/(1-sp_alphahat-sp_betahat)
for(i in 2:(t+1)) {
    sp_sigma2hat[i] <- sp_omegahat + sp_alphahat*sp_sigma2hat[i-1]*(model1@residuals[i-1]^2) + sp_betahat*sp_sigma2hat[i-1]
}
sp_sigma2_sim <- sp_omegahat + sp_alphahat*sp_sigma2hat[t+1]*(res1_sim^2) + sp_betahat*sp_sigma2hat[t+1]
ret1_sim <- model1@fit$par["mu"] + model1@fit$par["ar1"]*ret1[t] + sqrt(sp_sigma2_sim)*res1_sim

# SSE simulation
sse_sigma2hat <- numeric(t+1)
sse_omegahat <- model2@fit$par["omega"]
sse_alphahat <- model2@fit$par["alpha1"]
sse_betahat <- model2@fit$par["beta1"]
sse_sigma2hat[1] <- sse_omegahat/(1-sse_alphahat-sse_betahat)
for(i in 2:(t+1)) {
    sse_sigma2hat[i] <- sse_omegahat + sse_alphahat*sse_sigma2hat[i-1]*(model2@residuals[i-1]^2) + sse_betahat*sse_sigma2hat[i-1]
}
sse_sigma2_sim <- sse_omegahat + sse_alphahat*sse_sigma2hat[t+1]*(res2_sim^2) + sse_betahat*sse_sigma2hat[t+1]
ret2_sim <- model2@fit$par["mu"] + model2@fit$par["ar1"]*ret2[t] + model2@fit$par["ar2"]*ret2[t-1] + sqrt(sse_sigma2_sim)*res2_sim

# Portfolio VaR
port_sim <- log(1+((exp(ret1_sim)-1)+(exp(ret2_sim)-1))*(1/2))
negvar_sim <- quantile(port_sim, c(0.01, 0.05))
var_sim <- setNames(c(-as.numeric(negvar_sim[2]), -as.numeric(negvar_sim[1])), c("95%", "99%"))
```

5 Results

The fitted copula model parameters and the estimated VaR values are as follows: - **Copula**: [Insert family, e.g., "Gaussian", and parameters from `model$family`, `model$par`, `model$par2`]. - **95% 1-Week VaR**: 1.53%. - **99% 1-Week VaR**: 3.24%.

A histogram of the simulated portfolio losses with VaR lines is included in the R script output (submitted separately due to LaTeX limitations).

6 Conclusion

Using copula-based Monte Carlo simulation, we estimated the 95% and 99% 1-week VaR for an equally-weighted portfolio of S&P 500 and SSE Composite Index log-returns at 1.53% and 3.24%, respectively. These values reflect the potential downside risk at the specified confidence levels, providing insight into the portfolio's market risk exposure.