problem6

February 13, 2020

1 Problem 6

In this notebook you can find my solutions of the problem 6 of the Assignment 1.

[1]:
```
from pylab import *
from scipy.integrate import *
```

1.1 Stability

Let us check if the given system is stable or not.

[2]:
```
a = np.array([[0, 1], [1/2, 1]]) # matrix A
b = np.array([[0, 0], [3/4, -5/4]])

np.linalg.eig(a)
```

[2]:
```
(array([-0.3660254,  1.3660254]), array([[-0.9390708 , -0.59069049],
        [ 0.34372377, -0.80689822]]))
```

As you can see the system has one eigenvalue with positive real part. That means the system is not stable.

1.2 ODE solution

This function solves the ODE and draws the plot of it.

[3]:
```
def solve_ode(f, init,  t = linspace(0, 5, 1000)):
    # solving the ode
    result = odeint(f, init, t)

    x0 = result[:, 0]

    # draw a plot
    plot(t,x0,lw=2)
    xlabel('t')
    ylabel('x')
    grid()

    return t, x0
```

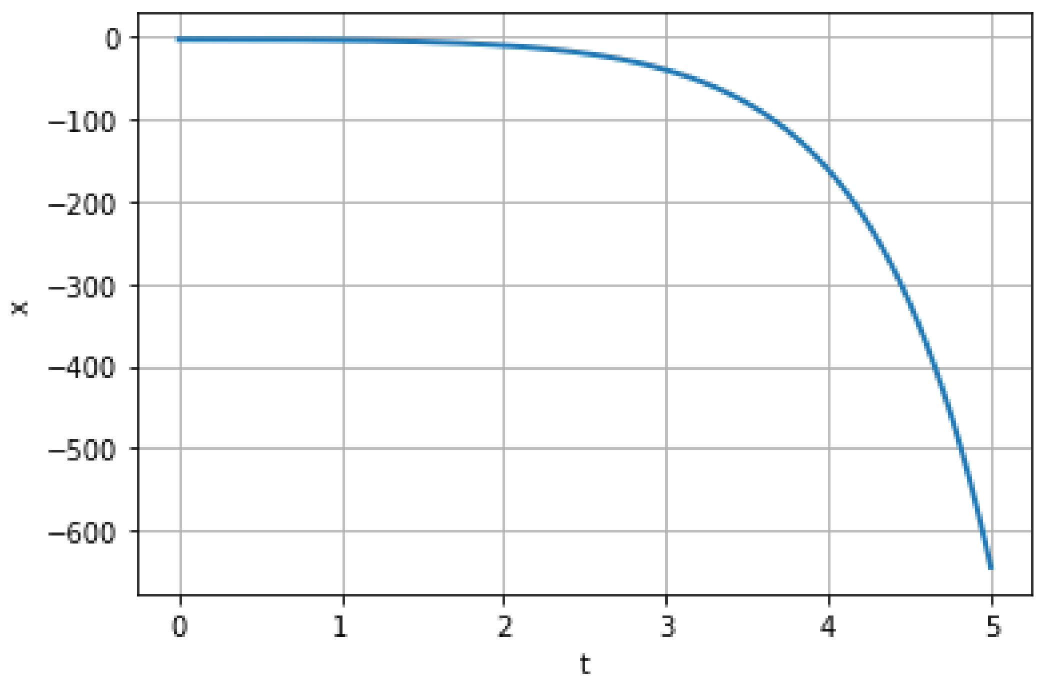
Solve the given ODE

$$4x'' - 4x' + 5t - 2x = 3, \quad x(0) = -3, x'(0) = 0$$

Let us define the function that represents our ODE. Since pylab does not allow solving equations rather than in standard form. But x and y can be vectors, so we can easily solve it in vector form.

[4]:
```
# define the ODE function
def f(x, t):
    x0, x1 = x
    return [x1, x1 - 5/4 * t + 1/2 * x0 + 3/4]
```

[5]:
```
init = [-3, 0]
ode = solve_ode(f, init)
```



1.3 State Space solver

This function gets two matrices as input, solve the given SS and draws its plot.

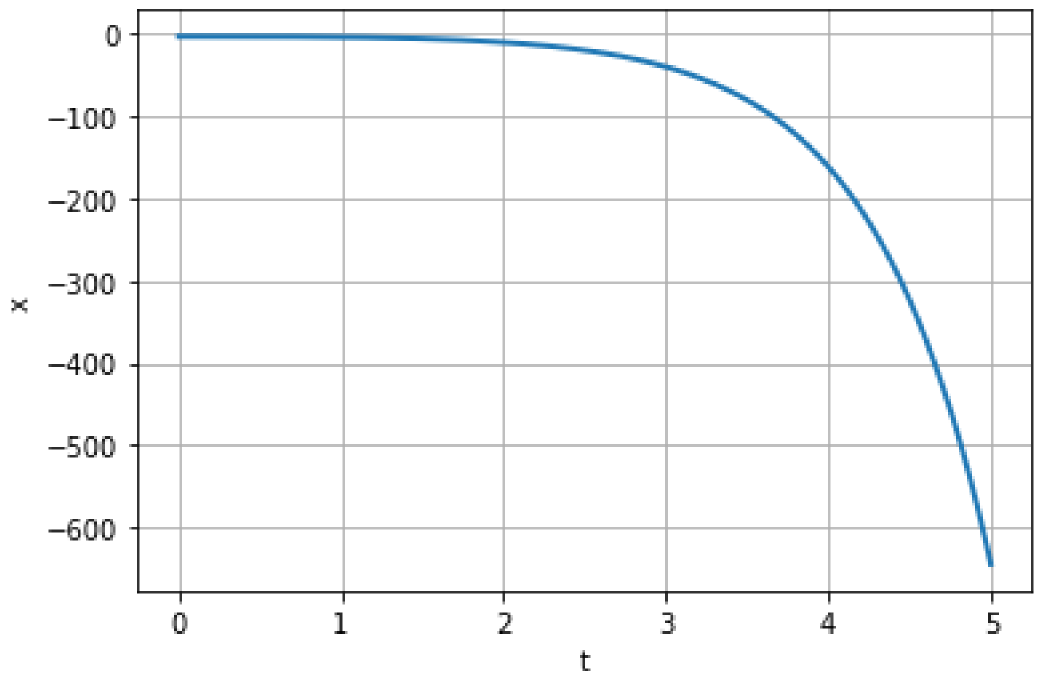
[6]:
```
def ss_solve(A, B, f, init):
    t = linspace(0, 5, 1000)

    # solving the ode
    result = odeint(f, init, t)
    x0 = result[:, 0]
```

```
    # draw a plot
    plot(t,x0,lw=2)
    xlabel('t')
    ylabel('x')
    grid()
    return (t, x0)
```

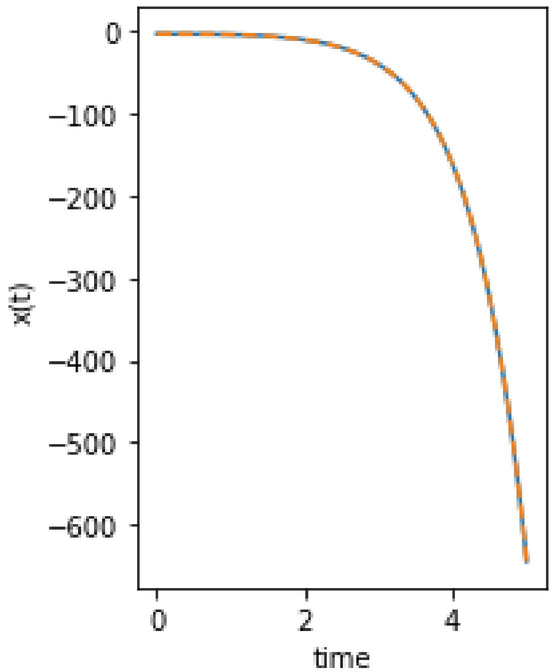
[7]:
```
def f_ss(x, t):
    x = np.reshape(x, (2, -1))
    return ((a.dot(x)) + b.dot([[1], [t]])).T.tolist()[0]
```

[8]:
```
ss = ss_solve(a, b, f_ss, [-3, 0]) # this is the same graph
```



[9]:
```
import matplotlib.pyplot as plt
# ode based model
plt.subplot(121)
plt.plot(*ode)
plt.plot(*ss, '--')
plt.xlabel('time')
plt.ylabel('x(t)')
```

[9]: Text(0, 0.5, 'x(t)')



As you can see the solutions are the same.