Control Theory: Assignment 1

Due on February 14, 2020 at 11:59pm

Mike Ivanov

Artem Bakhanov (B18-03)

1 Introduction

I created a GitHub repository here. All the problems are solved by me, Artem Bakhanov, a student of Innopolis University. My variant is **j**.

2 Problem 2 (SO ODE)

$$4x'' - 4x' + 5t - 2x = 3, \quad x'(0) = 0,\ x(0) = -3 \tag{1}$$

2.1 Simulink schema (without TF blocks)

Before transforming the equation to a Simuling schema I got the expression of x'' in terms of x', x, and t:

$$x'' = x' + \frac{1}{2}x - \frac{5}{4}t + \frac{3}{4} \tag{2}$$

Initial conditions are 0 and -3 (from left to right).

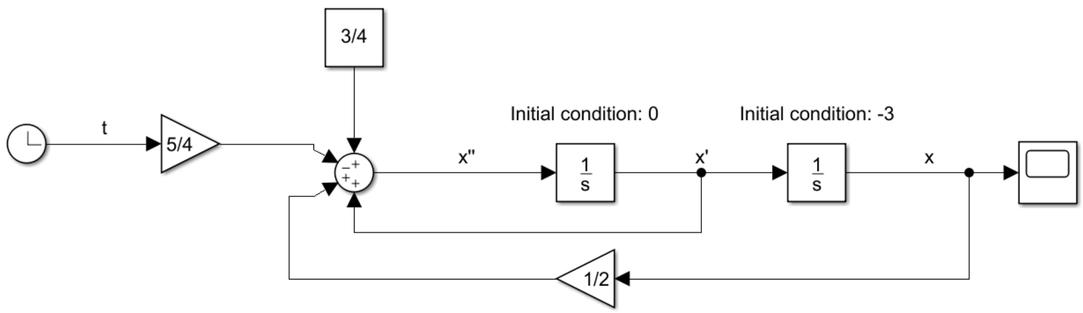

Figure 1: Simulink schema of the equation

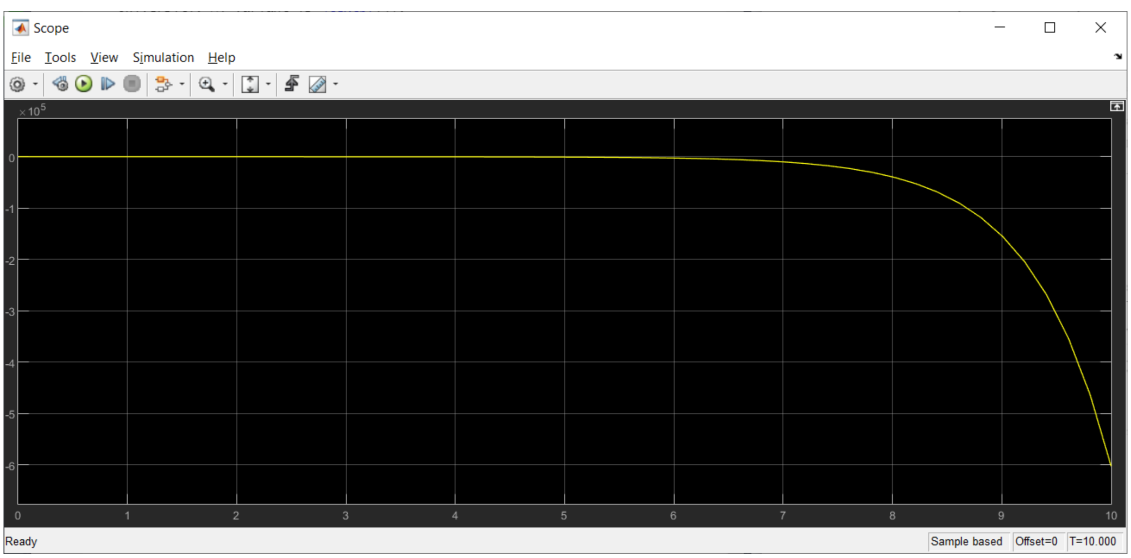

Figure 2: The plot of the solution

2.2 Simulink (with TF blocks)

2.3 Matlab solution of the ODE

Listing 1: Solution of the ODE in Matlab

```

% define the function
syms x(t)

% create symbolic functions for initial conditions
Dx = diff(x, t);

% define the ode
ode = 4 * diff(x, t, 2) - 4 * diff(x, t) + 5 * t - 2 * x == 3;

% define initial conditions
cond1 = x(0) == -3;
cond2 = Dx(0) == 0;
conds = [cond1 cond2];

xSol(t) = dsolve(ode, conds);
disp(xSol);

% draw a plot
figure
fplot(xSol, [0 10]);
```

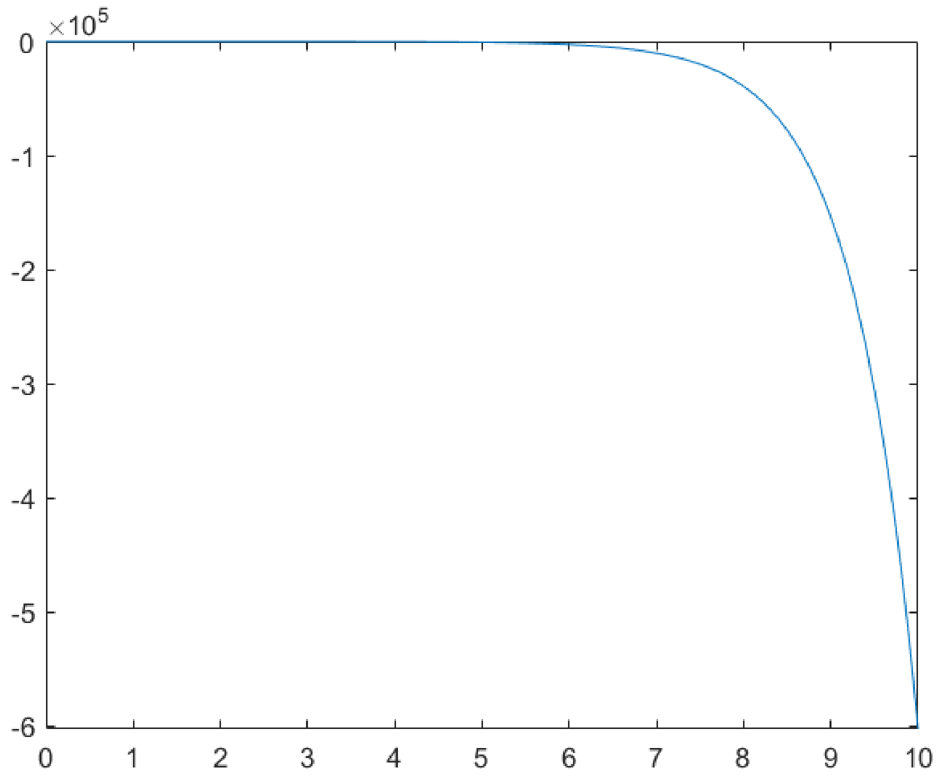

Figure 3: The plot of the Matlab solution

2.4 Laplace transform in Matlab

Listing 2: Solution of the ODE using Laplace transform in Matlab

```
% define symbolic variables, X is Laplace transform of the solution
syms s t X;

% define x' and x''
X1 = s * X - (-3);
X2 = s * X1 - 0;

% define right-hand function and find its Laplace transform
f = 3 - 5 * t;
F = laplace(f);

% solve for X
Sol = solve(4*X2 - 4 * X1 - 2 * X - F, X);

% find the inverse Laplace transform of X
sol = ilaplace(Sol, s, t);

disp(sol);
% plot the solution graph
figure
fplot(sol, [0 10]);
```

3 ODE2SS (1)

Find the SS model of a system described by the following ODE:

$$x'' + 2x' - 3 = t + 5, \quad y = x' \tag{3}$$

The SS model will look like

$$\dot{x} = Ax + Bu \tag{4}$$

$$y = Cx + Du \tag{5}$$

Let $x = \begin{bmatrix} x \\ x' \end{bmatrix}$, $u = \begin{bmatrix} 1 \\ t \end{bmatrix}$. Let us express x'':

$$x'' = 0x - 2x' + t + 8 \tag{6}$$

Then $A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 8 & 1 \end{bmatrix}$. It is easy to find matrices C and D. We are only interested in x'; thus, $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 0 & 0 \end{bmatrix}$. The final answer is:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 8 & 1 \end{bmatrix} u \tag{7}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \end{bmatrix} u \tag{8}$$

4 ODE2SS (2)

Find the SS model of a system described by the following ODE:

$$x'''' + 3x''' + 4x'' + 2x' - 6 = 2u_1 + 2u_2, \quad y = x' + u_1 + u_2 \tag{9}$$

The SS model will look like

$$\dot{x} = Ax + Bu \tag{10}$$
$$y = Cx + Du \tag{11}$$

Let $x = \begin{bmatrix} x \\ x' \\ x'' \\ x''' \end{bmatrix}$, $u = \begin{bmatrix} 1 \\ u_1 \\ u_2 \end{bmatrix}$. Then, obviously, $\dot{x} = \begin{bmatrix} x' \\ x'' \\ x''' \\ x'''' \end{bmatrix}$. Let us express x'''':

$$x'''' = 0x - 2x' - 4x'' - 3x''' + 6 + 2u_1 + 2u_2 \tag{12}$$

Then

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & -4 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 6 & 2 & 2 \end{bmatrix} \tag{13}$$

and

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \tag{14}$$

The final answer is:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & -4 & -3 \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 6 & 2 & 2 \end{bmatrix} u \tag{15}$$
$$y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} u \tag{16}$$

5 ODE2SS (Python)

The code is available in the file "ode2ss.py". You need to have NumPy installed on your computer to run the script.