Homework1

Control Theory

January 29, 2020

Deadline is February 14, 2020, 23:59 (MSK)

All results should be beautiful packed in a Pull Request.

1. Preparation.

Create PRIVATE repo on github for the course.

Add link of the repo to the spreadsheet with grades.

Add Mike (github account - L1va) as a collaborator for the repo.

Open link: Link to variants

Change name and surname and press button "run".

You will see your variant on the right side. Use it for all tasks in this homework.

2. Solve second order diff equation.

- (A) Draw a schema in Simulink (do not use transfer func block). As result paste a schema and a plot images in pdf.
- (B) Draw a schema in Simulink (use transfer func block). Put calculation of the transfer func, schema and a plot images, in report pdf.
- (C) Solve diff equation with matlab function (for example dsolve) and draw a plot in matlab. Put code and plot in the report.
- (D) Solve diff equation with Laplace transform in matlab. Put code in the report.

Variants for the task (x is output everywhere):

(a)
$$2x'' - 4x' - 2x + 6 = t$$
, $x'(0) = -5, x(0) = 3$

(b)
$$3x'' - 3x' - 9x = 6 - t$$
, $x'(0) = 0, x(0) = -3$

(c)
$$x'' + 2x' = -2x + 3\sin t$$
, $x'(0) = 5, x(0) = -1$

(d)
$$3x'' + x' = -2x + 3 + 5t$$
, $x'(0) = 7, x(0) = 9$

(e)
$$x'' - 4x' = 4x - 2\sin 2t$$
, $x'(0) = -2, x(0) = 2$

(f)
$$x'' + 2\sin 2t + 2x' = 4x + 1$$
, $x'(0) = 3, x(0) = 0$

(g)
$$x'' + x = x' + 3\sin 2t$$
, $x'(0) = 1, x(0) = -1$

(h)
$$x'' + 5 - 3t + 3x' = 5x - 9$$
, $x'(0) = 5, x(0) = -3$

(i)
$$x'' - 2x' + \sin 2t - 3 = x$$
, $x'(0) = -4$, $x(0) = 1$

(j)
$$4x'' - 4x' + 5t - 2x = 3$$
, $x'(0) = 0$, $x(0) = -3$

(k)
$$x'' = x' - 5x - 3\sin t$$
, $x'(0) = 0, x(0) = -5$

(1)
$$x'' = 3x' - 2x + 4\sin t$$
, $x'(0) = -1, x(0) = -2$

(m)
$$x'' - 4 - 2t = x' - 2x + 3$$
, $x'(0) = 5, x(0) = 1$

(n)
$$x'' - x' - 2x + 3 = \sin 2t$$
, $x'(0) = 2, x(0) = 5$

(o)
$$x'' + 2x' - 3x = \sin 4t$$
, $x'(0) = 3, x(0) = 2$

(p)
$$x'' - \sin t = x' - 2x + 3$$
, $x'(0) = -1$, $x(0) = 0$

(q)
$$x'' - 5x = x' + t + 2x + 3$$
, $x'(0) = 4$, $x(0) = 3$

(r)
$$x'' - 2x = 2x' - 3x + 3\sin 2t + 3$$
, $x'(0) = 5, x(0) = -1$

3. Find State Space Model of the system.

Variants for the task:

(a)
$$3x'' + 2x' - 3 = 2t - 2$$
, $y = 3x'$

(b)
$$x'' + 2x' - 3 = t + 5$$
, $y = x'$

(c)
$$x'' - x' + 5 = t + 1$$
, $y = 2x + x'$

(d)
$$2x'' + 2x' - 6 = 2t + 3$$
, $y = x'$

(e)
$$x'' + 2x' + 2x = t + 5$$
, $y = x' + t$

(f)
$$x'' + 2x' + x = t + 5$$
, $y = x' + 2t$

(g)
$$x'' + 5x' + x = t$$
, $y = x' + 2t$

(h)
$$x'' = t + 3$$
, $y = x + 2x'$

(i)
$$3x'' + 2x' - 3 = 2t - 2$$
, $y = 3x'$

(j)
$$x'' + 2x' - 3 = t + 5$$
, $y = x'$

(k)
$$x'' - x' + 5 = t + 1$$
, $y = 2x + x'$

(1)
$$2x'' + 2x' - 6 = 2t + 3$$
, $y = x'$

(m)
$$x'' + 2x' + 2x = t + 5$$
, $y = x' + t$

(n)
$$x'' + 3x' + 3x = t$$
, $y = x' + 2t$

(o)
$$x'' = t + 3$$
, $y = x + 2x'$

(p)
$$3x'' + 2x' - 3 = 2t - 2$$
, $y = 3x'$

(q)
$$3x'' + 3x' - 3 = 2t - 2$$
, $y = 3x'$

(r)
$$x'' + 2x' - 3 = t + 5$$
, $y = x'$

4. Find State Space Model of the system.

Variants for the task:

(a)
$$x'''' - 4x''' - x'' + 3x' + x = 3u_1 + u_2$$
, $y = x' + 2u_1$

(b)
$$x'''' + 6x''' - x'' = u_1 + 3u_2$$
, $y = 2x + 2x'$

(c)
$$2x'''' + x''' - 3x'' + 4x' - 3 = u_1 - 2u_2$$
, $y = 4x' - u_1$

(d)
$$3x'''' + 2x''' - x'' + 2x' - 3 = 3u_1 + 5u_2$$
, $y = x' + u_2$

(e)
$$x'''' - 2x''' + x'' - x' + 5 = u_1 + u_2$$
, $y = 2x + x' - 2u_1$

(f)
$$x'''' + 2x''' + 2x'' + 2x' - 6 = 2u_1 + 3u_2$$
, $y = x' + u_1 + 2u_2$

(g)
$$x'''' + 2x''' - x'' + 2x' + 2x = u_1 + 2u_2$$
, $y = x'$

(h)
$$3x'''' + 2x''' - x'' + 2x' - 3 = u_1 + 5u_2$$
, $y = x' + 2u_2$

(i)
$$x'''' - x''' + x'' - x' + 5 = u_1 + u_2$$
, $y = 2x + x' - u_1$

(j)
$$x'''' + 3x''' + 4x'' + 2x' - 6 = 2u_1 + 2u_2$$
, $y = x' + u_1 + 2u_2$

(k)
$$x'''' - x''' - x'' + 3x' + x = 3u_1 + u_2$$
, $y = x' + u_1$

(1)
$$x'''' + 2x''' - x'' = u_1 + 3u_2$$
, $y = x + 2x'$

(m)
$$2x'''' + x''' - 3x'' + 2x' - 3 = 2u_1 - 2u_2$$
, $y = 3x' - u_1$

(n)
$$3x'''' + 2x''' - x'' + 2x' - 3 = u_1 + 5u_2$$
, $y = x' + u_2$

(o)
$$x'''' - 2x''' + x'' - x' + 5 = u_1 + u_2$$
, $y = 2x + x' - u_1$

(p)
$$x'''' + 3x''' + 2x'' + 2x' - 6 = 2u_1 + 3u_2$$
, $y = x' + u_1 + 2u_2$

(q)
$$3x'''' + 2x''' - 3x'' + 2x' - 3 = u_1 + 5u_2$$
, $y = x' + u_2$

(r)
$$x'''' - x''' + 4x'' - x' + 5 = u_1 + u_2$$
, $y = x + x' - 2u_1$

5. Write a function in python that converts any ODE (power n):

$$a_k y^{(k)} + a_{k-1} y^{(k-1)} + \dots + a_2 \ddot{y} + a_1 \dot{y} + a_0 y = b_0$$

to the state space representation:

$$\dot{x} = Ax + b$$

6. Write functions in python that solves ODE and its state space representation. Test your functions on the ODE from task2. Draw plots. Use odeint from scipy.integrate library. Is the ODE stable? Does its solution converges or diverges?

Some suggestions:

All results should be in the PullRequest(PR) in a separate branch.

Do not forget to add Readme.md with description what the repo is, how to run the code, all dependencies installation, maybe other instructions. Put matlab/python versions, operating system that you use.

Report should contains calculations, pictures, maybe some descriptions, all - but only useful information.

For pdf generation, i suggest practising in usage LaTeX with overleaf.com for example.

Merge PR only after Mike approve it.

Do not leave the task for the last day. You can meet difficulties that you will not manage to solve in 30 minutes.