Control Theory: Assignment 2

Due on March 9, 2020 at 11:59pm

Mike Ivanov

Artem Bakhanov (B18-03)

Contents

1 Introduction

I created a GitHub repository here. All the problems are solved by me, Artem Bakhanov, a student of Innopolis University. My variant is **g**.

2 Problem 2 (TF)

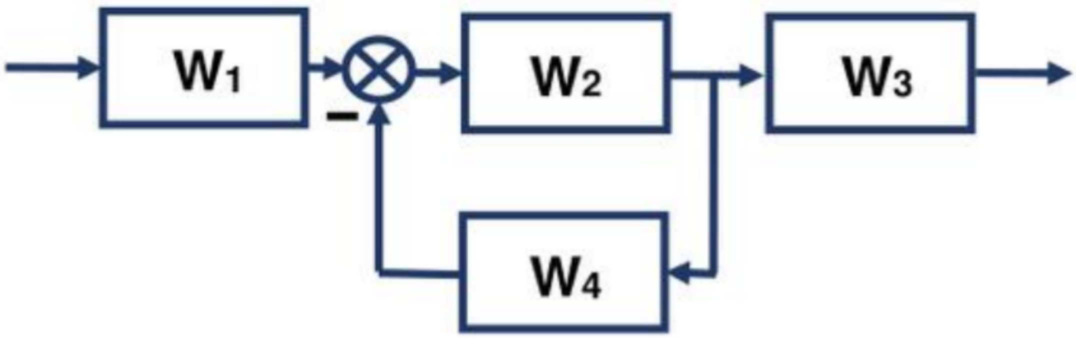


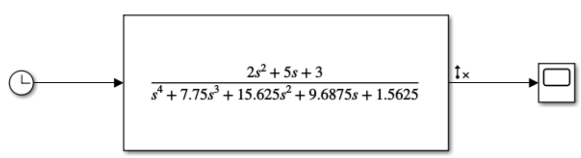
Figure 1: Initial schema

2.1 Calculations

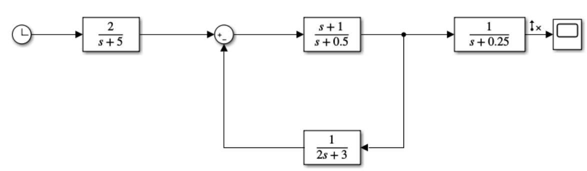
Two blocks in the middle can be substituted with one with value $T = \frac{W_2}{1+W_2W_4}$. There is a plus in the denominator because the feedback is negative. Then the whole system is just three consecutive blocks, which can be transformed to one block with value equal to $W = \frac{W_1W_2W_3}{1+W_2W_4}$. Then we just substitute these transfer functions in this expression. So we get:

$$W = \frac{\frac{2}{s+5}\frac{s+1}{s+0.5}\frac{1}{s+0.25}}{1+\frac{s+1}{s+0.5}\frac{1}{2s+3}} = \frac{2s^2+5s+3}{s^4+7.75s^3+15.625s^2+9.6875s+1.5625} \tag{1}$$

2.2 Simulink



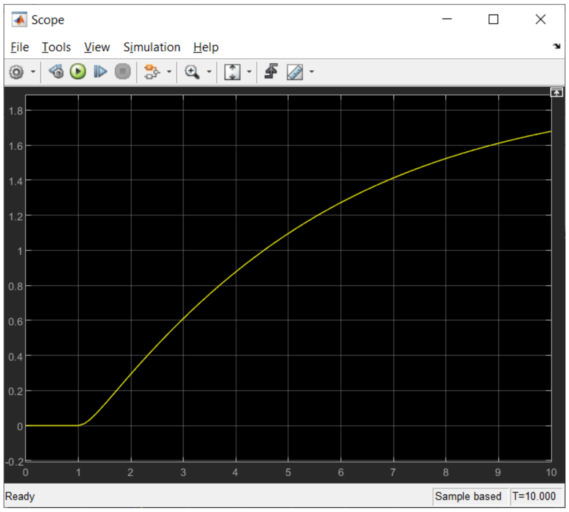
(a) Original system



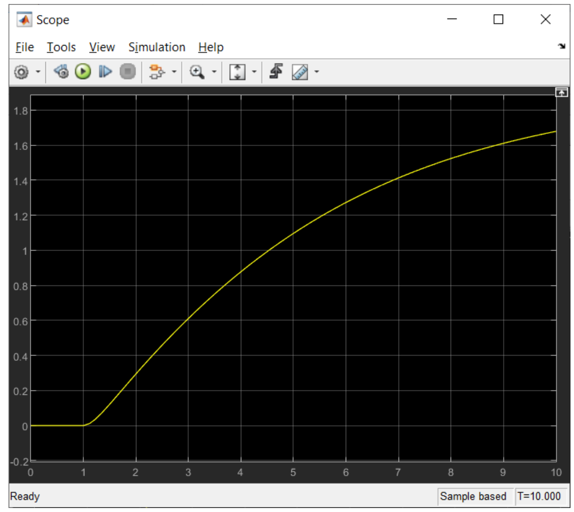
(b) Simplified system

Figure 2: Simulink Schemas

Note that I change clock input to step block with step time 1 for step response; Pulse Generator with amplitude = 1000000000 (almost infinity), period = 100 sec, and pulse width 0.01 sec (almost small) for impulse response; Sin Wave block for frequency response. As you can see the systems are identical. It means that all calculations was correct.

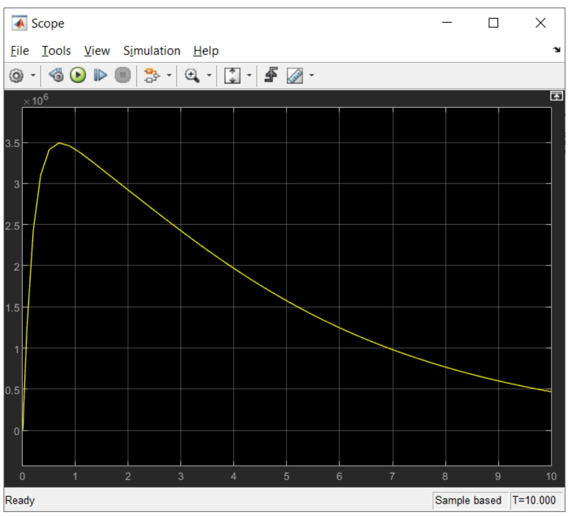


(a) Original system

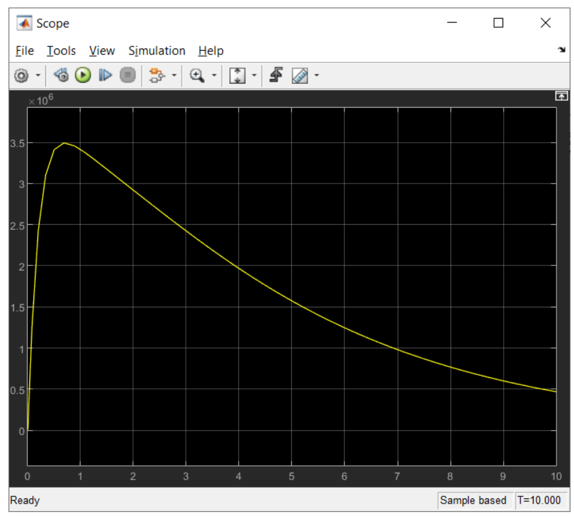


(b) Simplified system

Figure 3: Step responses

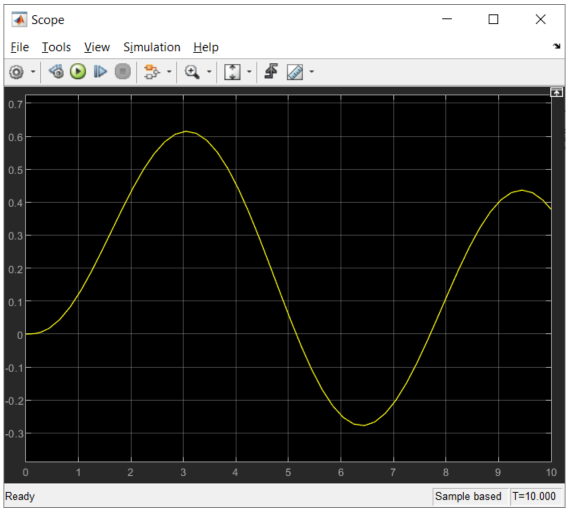


(a) Original system

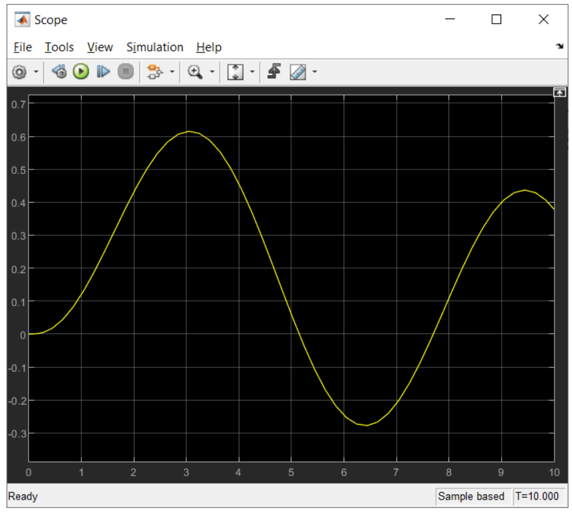


(b) Simplified system

Figure 4: Impulse responses



(a) Original system



(b) Simplified system

Figure 5: Frequency responses

2.3 Bode and Pole-Zero plots

I chose step input.

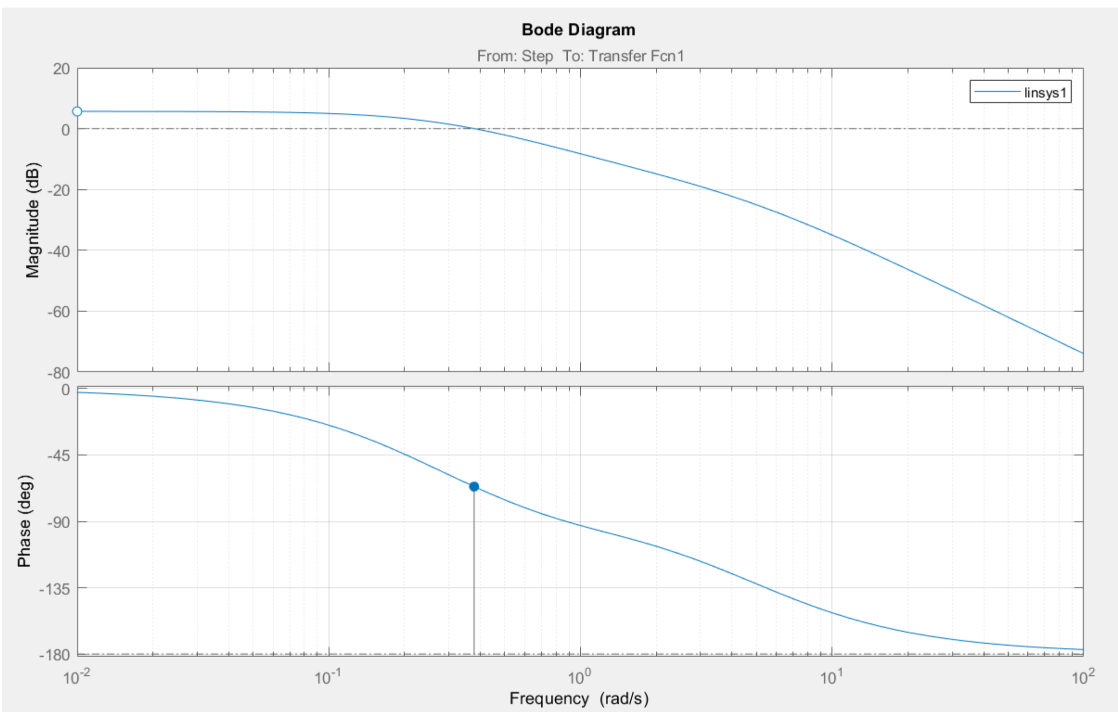


Figure 6: Bode plot

The system is stable since both phase margin and gain margin are positive.

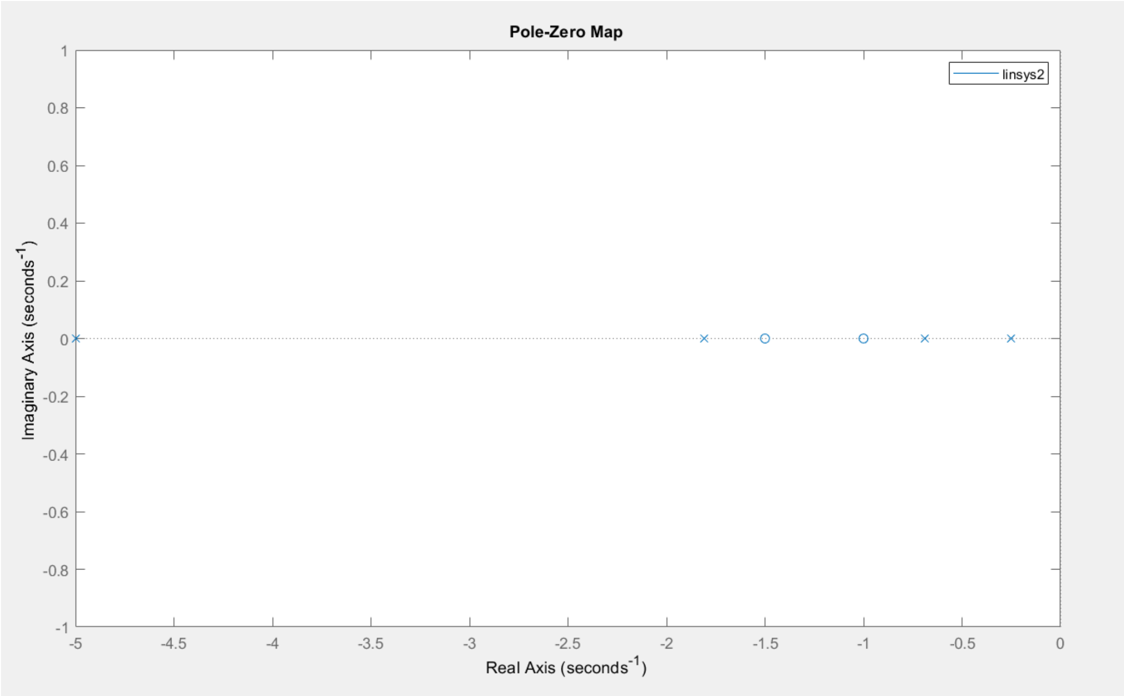


Figure 7: Pole-Zero plot

Pole-Zero plot just confirms that the system is stable. All the poles are situated in the left half plane.

3 TTF

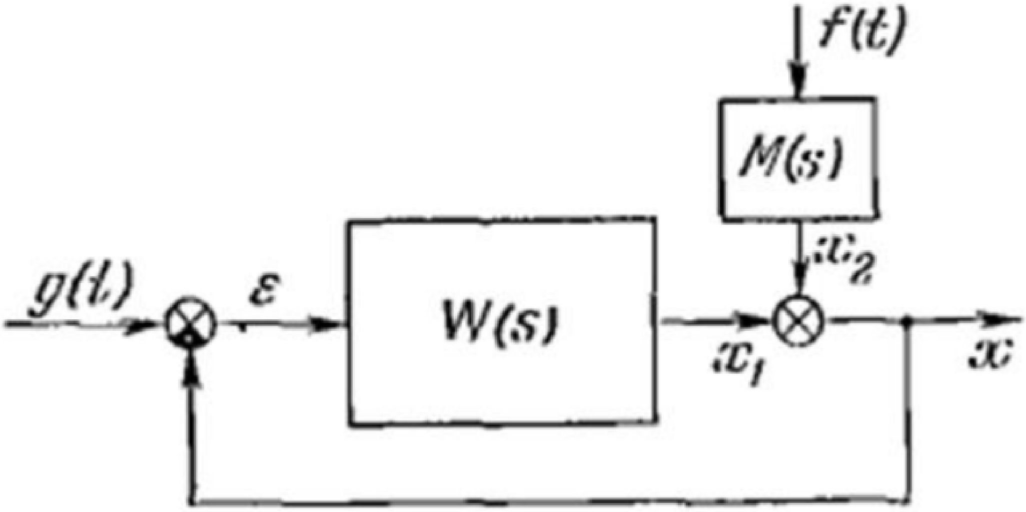


Figure 8: The given system

In this case, the total transfer function is

$$T = \frac{W}{1+W}G + \frac{M}{1+W}F \tag{2}$$

Let us just substitute the functions here:

$$T = \frac{\frac{2}{s^2+2}}{1+\frac{2}{s^2+2}}G + \frac{\frac{s+2}{2s+3}}{1+\frac{2}{s^2+2}}F = \frac{2}{s^2+4}G + \frac{s^3+2s^2+2s+4}{2s^3+3s^2+8s+12}F \tag{3}$$

4 SS2TF(1)

In this task I used a formula that was given during one of the labs. The transfer function can be calculated as followed:

$$H = C(sI-A)^{-1}B + D \tag{4}$$

I wrote a program that evaluates this expression. The program looks like:

Listing 1: Finding TF from the given SS in Matlab

```
% first solution using built-in function ss2tf
A = [3, 1; -2 2];
B = [2; 0];
C = [1, 3];
D = [1];
[b, a] = ss2tf(A, B, C, D);
disp(b);
disp(a);

% second solution using common sense
syms s;
K = C * inv(s * eye(2) - A) * B + D;
disp(simplify(K));
```

I got the following solution:

$$H = \frac{s^2-3s-8}{s^2-5s+8} \tag{5}$$

5 SS2TF(2)

Using the same approach as I have used in the previous task I got:

$$T_1 = \frac{s^2-s-18}{s^2-3s-10} \tag{6}$$

$$T_2 = \frac{6s^2-13s-68}{s^2-3s-10} \tag{7}$$

The first one is transfer function for first input (u_1), the second one is for the second input (u_2). Note that I used matrix approach that is general for any number of inputs and outputs. So, as a result we get a vector with all the transfer function we need.

6 TF System

In this task for the first part I assume that $f = 0$ and ignore this input. For the second part I ignore x and assume that it is equal to 0. As you can see, transfer function for x:

$$T_x = ((W_5W_6 + W_3) \cdot W_7W_4 + W_1 + W_8W_5) \cdot W_2 \tag{8}$$

for f:

$$T_f = (W_6W_7W_4 + W_8) \cdot W_2 \tag{9}$$

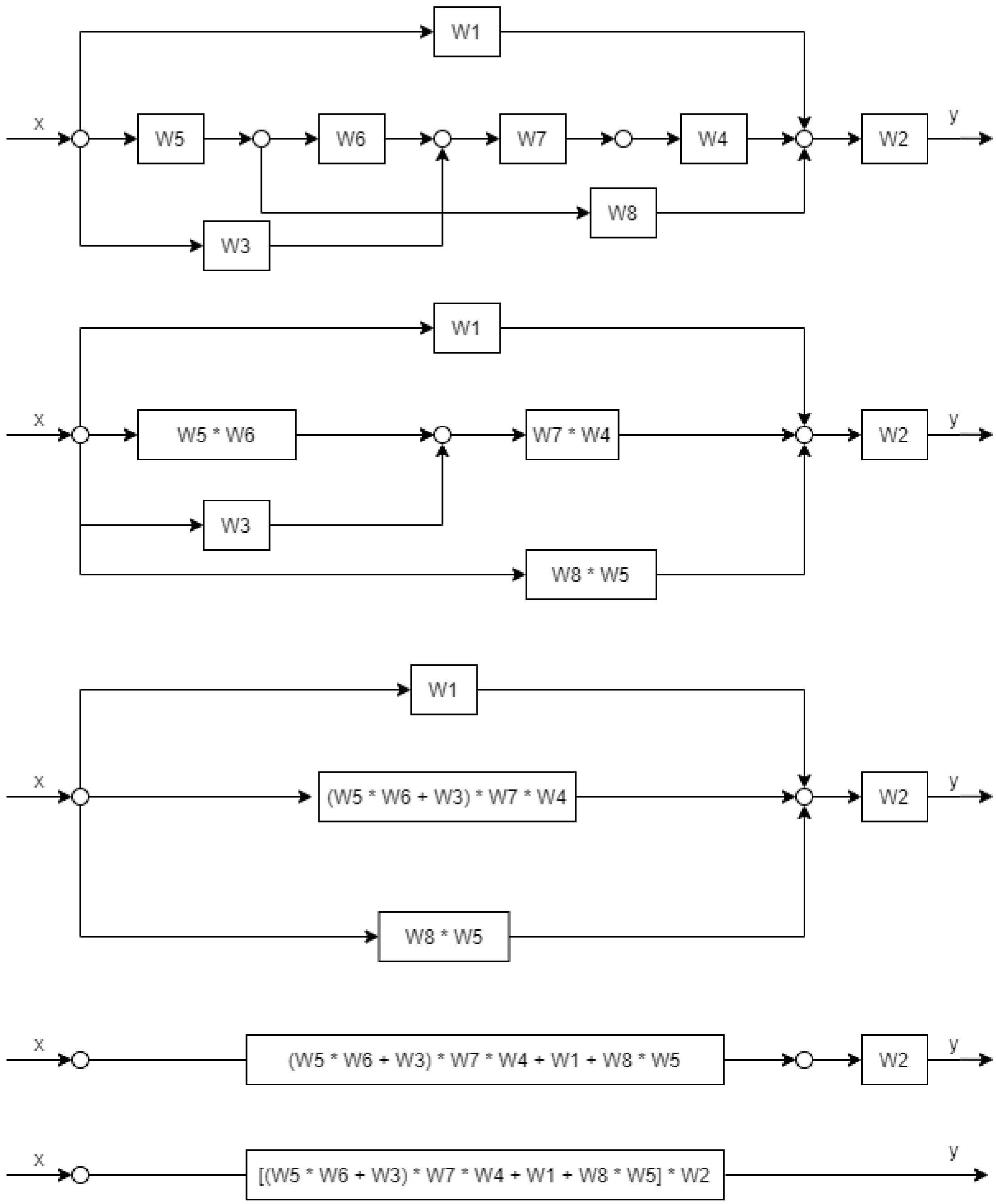


Figure 9: Simplifying the system for x

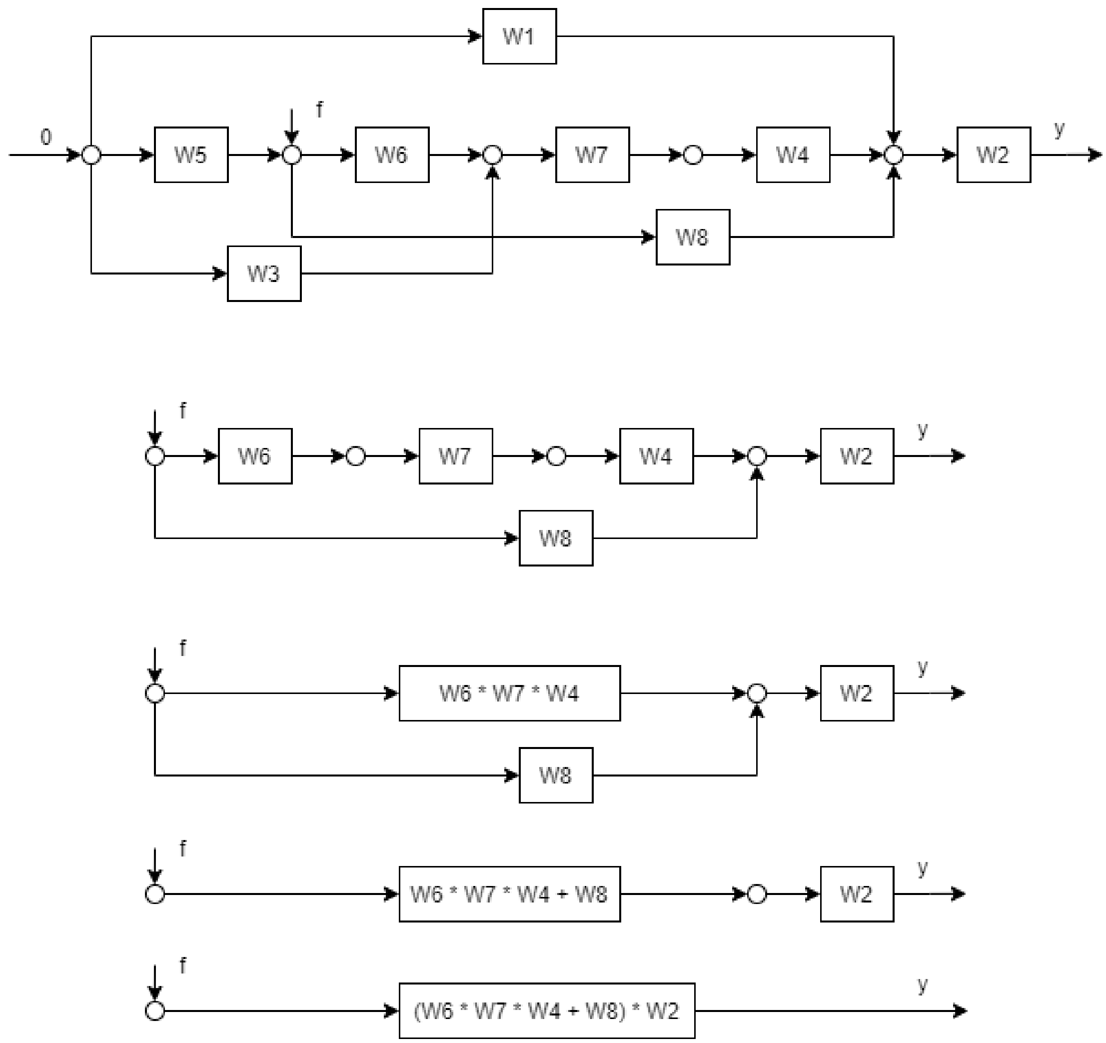


Figure 10: Simplifying the system for f

Total transfer function is:

$$T = T_x x + T_f f = ((W_5 W_6 + W_3) \cdot W_7 W_4 + W_1 + W_8 W_5) \cdot W_2 \cdot x + (W_6 W_7 W_4 + W_8) \cdot W_2 \cdot f \tag{10}$$