Control Theory: Assignment 2

Due on March 9, 2020 at 11:59pm

Mike Ivanov

Artem Bakhanov (B18-03)

Contents

1 Introduction **3**

2 Python **3**
2.1 A 3
2.2 B 3
2.3 C 4
2.4 D 4
2.5 E 5

3 PID Tuner **5**

Appendices **7**

A Python Code **7**

1 Introduction

I created a GitHub repository here. All the problems are solved by me, Artem Bakhanov, a student of Innopolis University. My variant is **g**.

2 Python

For the first task I used Python to calculate everything. You can find the code in Appendix 1 in the end of the document.

2.1 A

In this subtask I tested 2 trajectories: $\overset{*}{x} = 1$ and $\overset{*}{x} = 4$. Zero-conditions are $x(0) = .28789452$ and $x'(0) = .72404173$

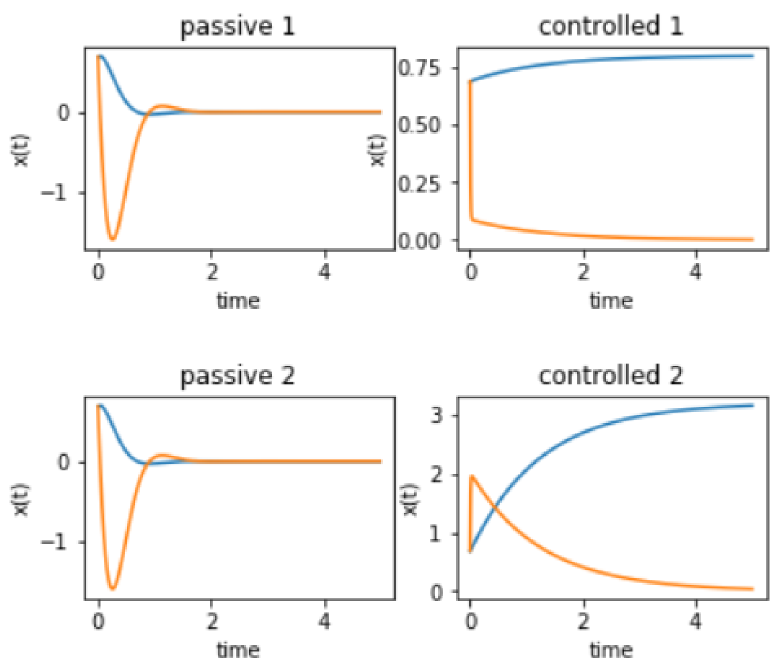


Figure 1: Two trajectories

2.2 B

For this part I used a process described on Wikipedia. I got parameters $k_p = 2000$ and $k_d = 300$. Trajectories are the same. I used step with amplitude 1 and starting time 0.5. On the plots below you can see that the PD controller works but not perfectly. I-component is missing.

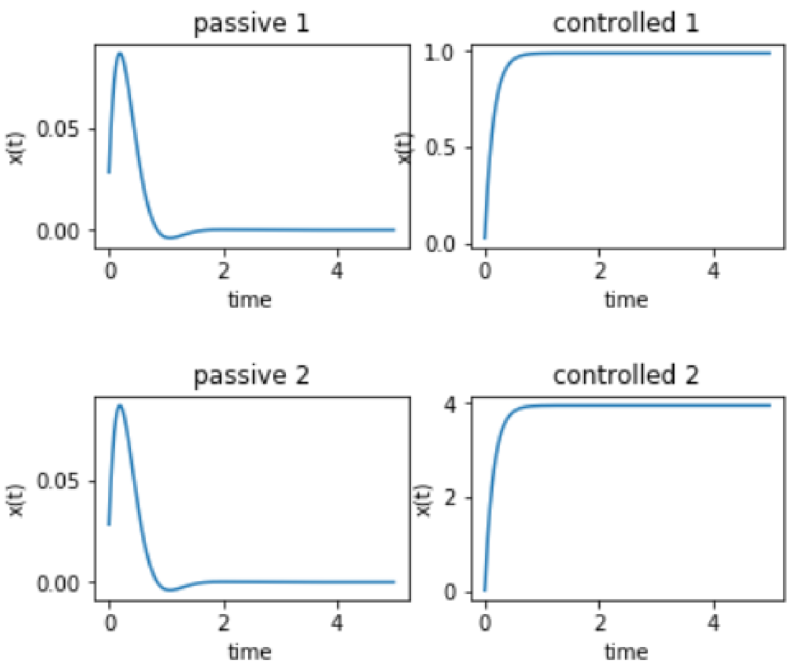


Figure 2: Tuned PD

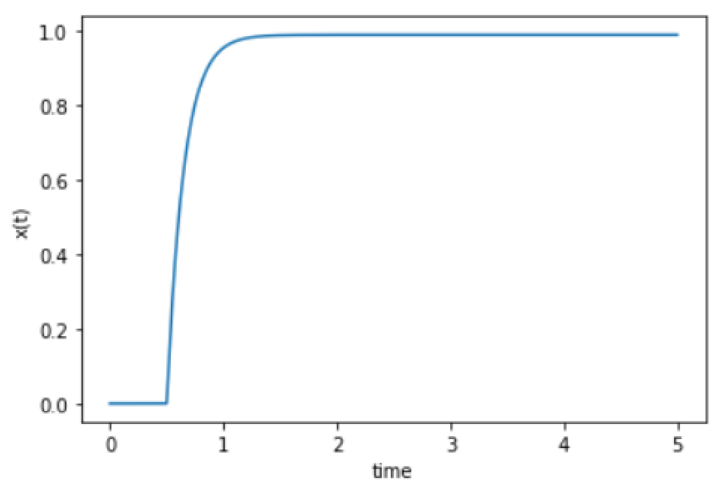


Figure 3: Step response. $t_0 = 0.5$

2.3 C

For this part I used built-in Matlab function `isstable`. The function proved that the controlled oscillator dynamics is stable for $k_p = 2000$ and $k_d = 300$. You can find the code below.

2.4 D

The solution is pretty the same but the parameters are different. You can look at my code in the Appendix 1. I did not get how we can do something with the system where we do not know the B matrix, so I assumed

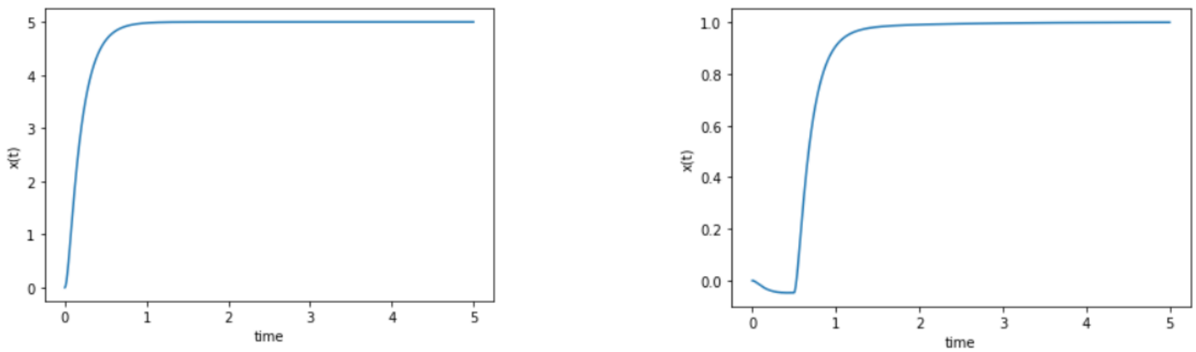
```
A = [0 1; -25 -7];
B = [0; 1];
C = [1 0];
D = 1;
system = ss(A, B, C, D);
controller = pid(2000, 0, 300);
cl = system * controller;
disp(isstable(cl));
```

Listing 1: Stability proof

that $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The problem is that I did not understand how to get the parameters, since it is now a numbers but matrices.

2.5 E

The PID controller is very similar to what was done in the task B. I found PID parameters by hand and got: 150, 130, and 25 for k_p, k_i, and k_d respectively.



(a) Desired x: 5

(b) Step response with amplitude 1 and time 0.5

Figure 4: Examples of tests.

3 PID Tuner

In this section I used Simulink for analyzing the system and PIDTuner to tune the PID controller in the system. The system looks like:

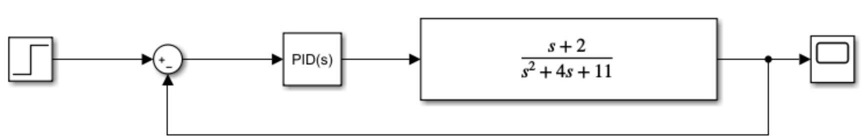


Figure 5: The system

The parameters that PIDTuner gave me: $k_p = 10.826321698108$, $k_i = 63.4181190369928$, and $k_d = -0.0076494311467772$. The step input (amplitude = 5) was used and the trajectory of the system is on the picture below.

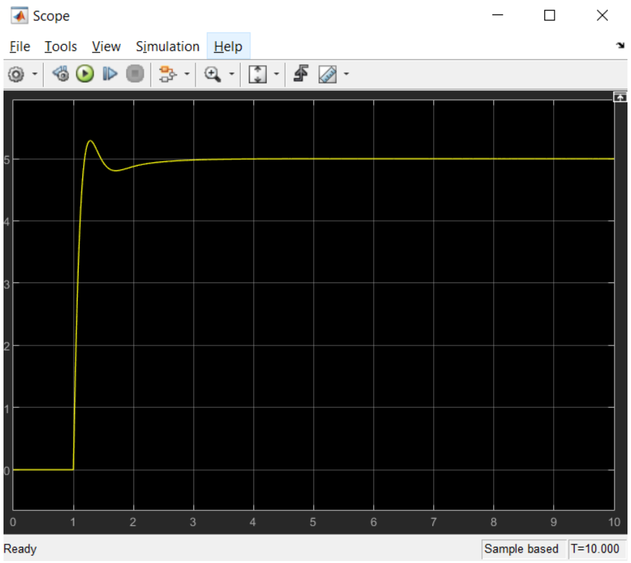


Figure 6: The system trajectory

Appendices

A Python Code

assignment3

April 1, 2020

[94]:
```
import numpy as np
from numpy.linalg import eig
from scipy.integrate import odeint
import matplotlib.pyplot as plt

time = np.linspace(0, 5, 1000)     # interval from 0 to 5
x0 = np.array([0.28789452, 0.72404173]) #np.random.rand(2)           # initial␣
 ↪state
```

1 Proportional-derivative (PD) control

Consider a second order linear ODE:

$$\ddot{x} + \mu\dot{x} + kx = u$$

We can use P control:

$$u = k_p(x^* - x)$$

or we can add a derivative term:

$$u = k_d(\dot{x}^* - \dot{x}) + k_p(x^* - x)$$

This is a **PD controller**.

Let us introduce $e = x^* - x$. Then we have:

$$u = k_d\dot{e} + k_p e$$

Variable e here is a **control error**.

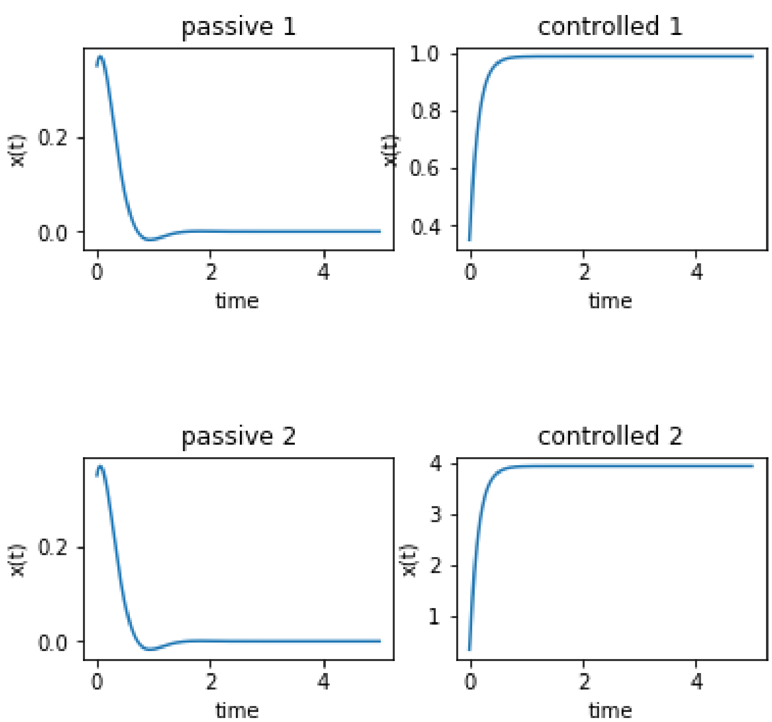
[95]:
```
mu = 7
k = 25

x_desired = 1
x_dot_desired = 0
```

1

```
kp = 2000
kd = 300
# kp = 100
# kd = 150

def Oscillator(x, t):
    return np.array([x[1], (- mu*x[1] - k*x[0])])

def Oscillator_Control(x, t):
    error     = x_desired     - x[0]
    error_dot = x_dot_desired - x[1]
    u = kp*error + kd*error_dot
    return np.array([x[1], (u - mu*x[1] - k*x[0])])

def StepFunction(t):
    return 0 if t < 0.5 else 1

def Oscillator_StepFunction(x, t):
    x_desired = StepFunction(t)
    error     = x_desired     - x[0]
    error_dot = x_dot_desired - x[1]
    u = kp*error + kd*error_dot
    return np.array([x[1], (u - mu*x[1] - k*x[0])])

x0 = np.random.rand(2)

solution = {"Oscillator1": odeint(Oscillator, x0, time), "Oscillator_Control1":
 ↪odeint(Oscillator_Control, x0, time)}
x_desired = 4
x_dot_desired = 0
solution["Oscillator2"] =  odeint(Oscillator, x0, time)
solution["Oscillator_Control2"] =  odeint(Oscillator_Control, x0, time)


plt.subplot(221)
plt.plot(time, solution["Oscillator1"].T[0].T)
plt.xlabel('time')
plt.ylabel('x(t)')
plt.title('passive 1')

plt.subplot(222)
plt.plot(time, solution["Oscillator_Control1"].T[0].T)
plt.xlabel('time')
plt.ylabel('x(t)')
plt.title('controlled 1')
```

2

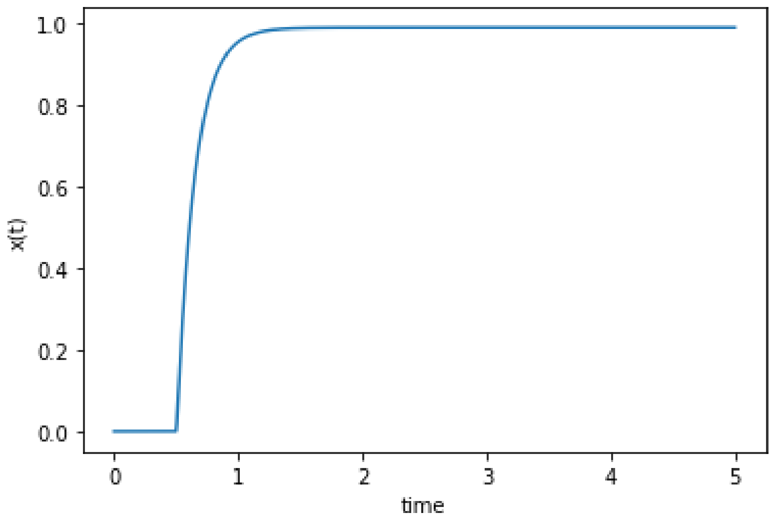
```
plt.show()

plt.subplot(223)
plt.plot(time, solution["Oscillator2"].T[0].T)
plt.xlabel('time')
plt.ylabel('x(t)')
plt.title('passive 2')

plt.subplot(224)
plt.plot(time, solution["Oscillator_Control2"].T[0].T)
plt.xlabel('time')
plt.ylabel('x(t)')
plt.title('controlled 2')
plt.show()
```



[96]:
```
x0 = np.zeros((2))
```

3

```
solution = odeint(Oscillator_StepFunction, x0, time)

plt.plot(time, solution.T[0].T)
plt.xlabel('time')
plt.ylabel('x(t)')
plt.show()
```

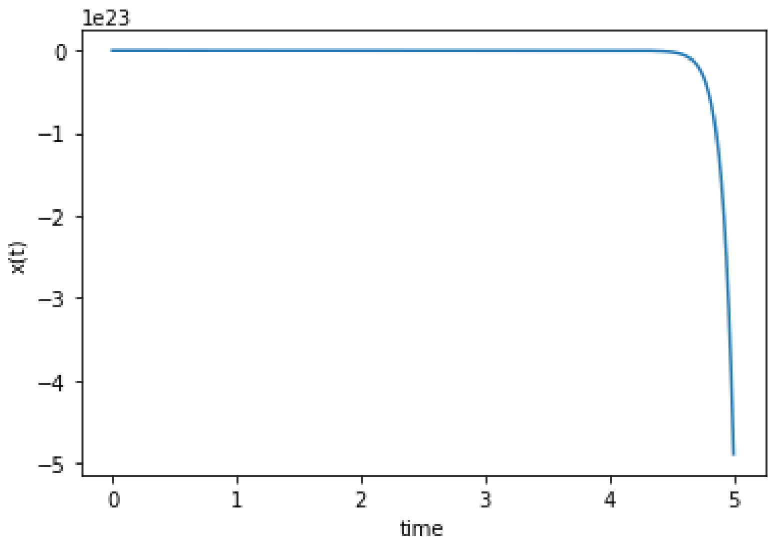


2 Task 2d

[97]:
```
A = np.array([[10, 3], [5, -5]]) # state matrix

x_desired = 10
x_dot_desired = 0
kp = 1;
kd = 2;
def LTV(x, t):
    error     = x_desired     - x[0]
    error_dot = x_dot_desired - x[1]
    u = kp*error + kd*error_dot
    x_ = A.dot(x)
    x_[1] -= u
    return x_
```

4

[98]:
```
solution = odeint(LTV, x0, time)

plt.plot(time, solution.T[0].T)
plt.xlabel('time')
plt.ylabel('x(t)')
plt.show()
```



3 Task 2e

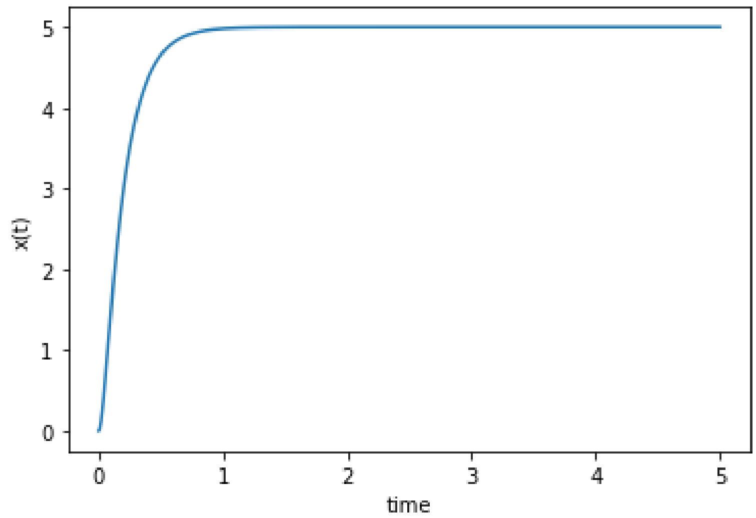
Here the PID controller is implemented

[239]:
```
x_desired = 5
x_dot_desired = 0
kp = 150
kd = 25
ki = 130.0
error_i = 0
errors = []
t_prev = 0
def Oscillator_Control(x, t):
    global error_i, ki, kd, kp, t_prev
    error     = x_desired     - x[0]
    error_dot = x_dot_desired - x[1]
```

5

```
    dt = t - t_prev
    t_prev = t
    error_i = error_i + dt * error
    errors.append(error_i)
    u = kp*error + kd*error_dot + ki*error_i
    return np.array([x[1], (u - mu*x[1] - k*x[0] - 9.8)])
```

```
[240]: error_i = 0
errors = []
t_prev = 0
x_desired = 5
x_dot_desired = 0
solution = odeint(Oscillator_Control, x0, time,)
plt.plot(time, solution.T[0].T)
plt.xlabel('time')
plt.ylabel('x(t)')
plt.show()
```

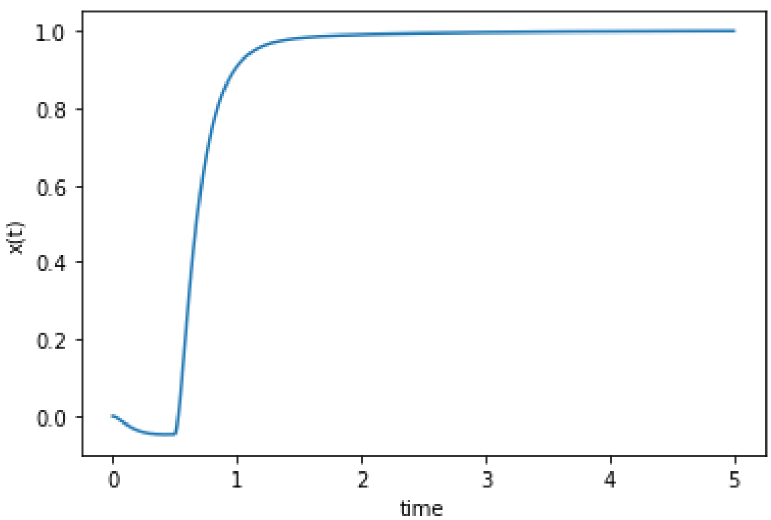



```
[241]: x_desired = 5
x_dot_desired = 0
kp = 150
kd = 25
ki = 130.0
error_i = 0
```

6

```
errors = []
t_prev = 0
def Oscillator_Control_Step(x, t):
    global error_i, ki, kd, kp, t_prev
    x_desired = StepFunction(t)
    error     = x_desired     - x[0]
    error_dot = x_dot_desired - x[1]
    dt = t - t_prev
    t_prev = t
    error_i = error_i + dt * error
    errors.append(error_i)
    u = kp*error + kd*error_dot + ki*error_i
    return np.array([x[1], (u - mu*x[1] - k*x[0] - 9.8)])
```

[242]:
```
error_i = 0
errors = []
t_prev = 0
solution = odeint(Oscillator_Control_Step, x0, time,)
plt.plot(time, solution.T[0].T)
plt.xlabel('time')
plt.ylabel('x(t)')
plt.show()
```



7