Homework4

Control Theory

March 26, 2020

Deadline is April 10, 2020, 23:59 (MSK)

All results should be beautiful packed in one pdf and loaded to Github in PR. Plots should be signed. Do not forget to describe and explain your results.

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1. Select your variant.

   Open link: Another Link

   Change name and email and press button "run".

   You will see your variant on the right side. Use it for all tasks.

   Put name and email that you use for generation and your variant in the report.

2. Consider classical benchmark system in control theory - inverted pendulum on a cart (Figure 1). It is nonlinear under-actuated system that has the following dynamics

$$(M+m)\ddot{x} - ml\cos(\theta)\ddot{\theta} + ml\sin(\theta)\dot{\theta}^2 = F \tag{1}$$

$$-\cos(\theta)\ddot{x} + l\ddot{\theta} - g\sin(\theta) = 0 \tag{2}$$

   where $g = 9.81$ is gravitational acceleration.

   Variants for the task:

   (a) $M = 5.3, m = 3.2, l = 1.15$
   (b) $M = 4.2, m = 5.5, l = 2.1$
   (c) $M = 15.1, m = 1.2, l = 0.35$
   (d) $M = 11.6, m = 2.7, l = 0.57$
   (e) $M = 7.5, m = 4.4, l = 1.2$
   (f) $M = 3.4, m = 8.2, l = 0.89$
   (g) $M = 3.6, m = 3.6, l = 1.01$

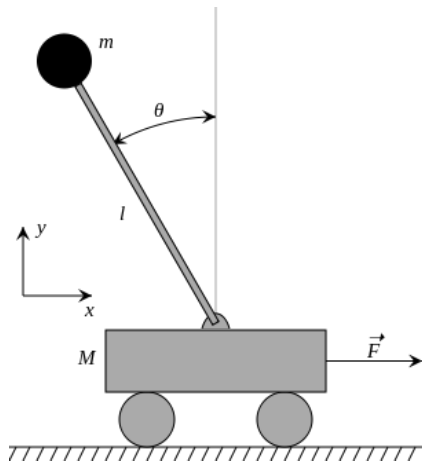

Figure 1: A schematic drawing of the inverted pendulum on a cart. The rod is considered massless. The mass of the cart and the point mass at the end of the rod are denoted by M and m. The rod has a length l.

(A) write equations of motion of the system in manipulator form

$$M(q)\ddot{q} + n(q,\dot{q}) = Bu$$

where $u = F$, $q = \begin{bmatrix} x & \theta \end{bmatrix}^T$ is vector of generalized coordinates;

(B) write dynamics of the system in control affine nonlinear form

$$\dot{z} = f(z) + g(z)u$$

where $z = \begin{bmatrix} x & \theta & \dot{x} & \dot{\theta} \end{bmatrix}^T$ is vector of states of the system;

(C) linearize nonlinear dynamics of the systems around equilibrium point $\bar{z} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$

$$\delta\dot{z} = A\delta z + B\delta u$$

(D) check stability of the linearized system using any method you like;

(E) check if linearized system is controllable; if not - try another variant or change values of your variant and find controllable.

(F) (for the controllable system) design state feedback controller for linearized system using pole placement method. Assess the performance of the controller for variety of initial conditions. Justify the choice of initial conditions. Solve the task by two ways: using root-locus and with python. Compare them;

(G) (for the controllable system) design linear quadratic regulator for linearized system. Assess the performance of the controller for variety of initial conditions. Justify the choice of initial conditions;

Suggestions are the same. Good luck!