Homework5

Control Theory

April 11, 2020

Deadline is April 24, 2020, 23:59 (MSK)

All results should be beautiful packed in one pdf and loaded to Github in PR. Plots should be signed. Do not forget to describe and explain your results.

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1. Select your variant.

   Open link: Just Link

   Change name and email and press button "run".

   You will see your variant on the right side. Use it for all tasks.

   Put name and email that you use for generation and your variant in the report.

2. Consider classical benchmark system in control theory - inverted pendulum on a cart (Figure 1). It is nonlinear under-actuated system that has the following dynamics

$$(M+m)\ddot{x} - ml\cos(\theta)\ddot{\theta} + ml\sin(\theta)\dot{\theta}^2 = F \tag{1}$$

$$-\cos(\theta)\ddot{x} + l\ddot{\theta} - g\sin(\theta) = 0 \tag{2}$$

   where $g = 9.81$ is gravitational acceleration.

   The system dynamics can be written in state space form:

$$\dot{z} = f(z) + g(z)u$$
$$y = h(z) = \begin{bmatrix} x & \theta \end{bmatrix}^T$$

   where $z = \begin{bmatrix} x & \theta & \dot{x} & \dot{\theta} \end{bmatrix}^T$ is the state vector of the system, y is the output vector. The dynamics of the system around unstable equilibrium of the pendulum ($\bar{z} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$) can be described by a linear system that is obtained from linearization of the nonlinear dynamics around $\bar{z}$.

$$\delta\dot{z} = A\delta z + B\delta u$$
$$\delta y = C\delta z$$

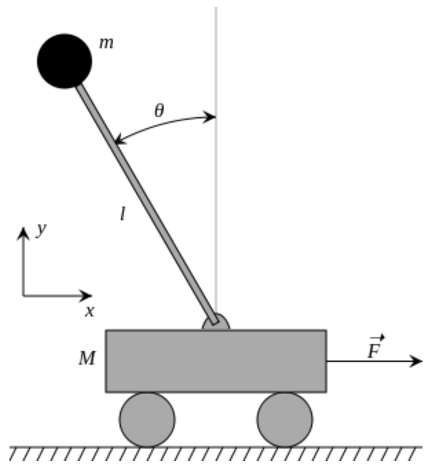

Figure 1: A schematic drawing of the inverted pendulum on a cart. The rod is considered massless. The mass of the cart and the point mass at the end of the rod are denoted by M and m. The rod has a length l.

Variants for the task:

(a) $M = 7.5, m = 4.4, l = 1.2$
(b) $M = 3.4, m = 8.2, l = 0.89$
(c) $M = 3.6, m = 3.6, l = 1.01$
(d) $M = 5.3, m = 3.2, l = 1.15$
(e) $M = 4.2, m = 5.5, l = 2.1$
(f) $M = 15.1, m = 1.2, l = 0.35$
(g) $M = 11.6, m = 2.7, l = 0.57$

(A) prove that it is possible to design state observer of the linearized system
(B) for open loop state observer, is the error dynamics stable?
(C) design Luenberger observer for linearized system using both pole placement and LQR methods
(D) design state feedback controller for linearized system
(E) simulate nonlinear system with Luenberger observer and state feedback controller that uses estimated states ($u = K\hat{x}$). Make sure that the system is stabilized for various initial conditions around $\bar{z}$.
(F) add white gaussian noise to the output ($\delta y = C\delta z + v$). What happens to the state estimation?
(G) add white gaussian noise to the dynamics ($\delta \dot{z} = A\delta z + B\delta u + w$). What happens to the state estimation and control system?

(H) implement Kalman Filter (you can use libraries with KF if this task is not for you to get some points in next ones)

(I) generate some data and show that your implementation of KF is correct

(J) using KF function implement LQG controller

Suggestions are the same. Good luck!