Market Timing with Bi-Objective Cost-Sensitive Learning

Robert James
U Sydney & NGS Super

Jessica Wai Yin Leung
Monash U

Artem Prokhorov
U Sydney & CEBDA & CIREQ

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Cost-sensitive learning framework

- Suppose you have a standard classification problem:
  - direction of return y_t: $\{\uparrow, \downarrow\}$

$$p_{\uparrow} = \frac{1}{1 + e^{-f(\mathbf{x}_t;\boldsymbol{\theta})}}; \quad p_{\downarrow} = 1 - p_{\uparrow}$$

  - $\hat{p}_{\uparrow} > 0.5 \Rightarrow$ predict $\hat{y}_t \uparrow$; $\quad \hat{p}_{\downarrow} > 0.5 \Rightarrow$ predict $\hat{y}_t \downarrow$.
  - cost matrix:

		$\uparrow$	$\downarrow$	$\hat{y}_t$
y_t	$\uparrow$	0	c	
	$\downarrow$	c	0	

  - assumes:
    - ★ c same for false positives and false negatives
    - ★ c time-constant

Cost-sensitive learning framework

- **Cost-sensitive** classification problem:
  - direction of return y_t: $\{\uparrow, \downarrow\}$

$$p_\uparrow = \frac{1}{1 + e^{-f(\mathbf{x}_t;\boldsymbol{\theta})}}; \quad p_\downarrow = 1 - p_\uparrow$$

  - $\hat{p}_\uparrow > 0.5 \Rightarrow$ predict $\hat{y}_t \uparrow$; $\quad \hat{p}_\downarrow > 0.5 \Rightarrow$ predict $\hat{y}_t \downarrow$.
  - cost matrix:

		$\uparrow$	$\downarrow$	$\hat{y}_t$
y_t	$\uparrow$	0	$c^{\mathbf{fn}}$	
	$\downarrow$	$c^{\mathbf{fp}}$	0	

  - assumes:
    - ★ $c^{\mathbf{fp}} \neq c^{\mathbf{fn}}$
    - ★ time-constant

Cost-sensitive learning framework

- **Cost-sensitive example-dependent** classification problem:
  - direction of return y_t: $\{\uparrow, \downarrow\}$

$$p_{\uparrow} = \frac{1}{1 + e^{-f(\mathbf{x}_t;\boldsymbol{\theta})}}; \quad p_{\downarrow} = 1 - p_{\uparrow}$$

  - $\hat{p}_{\uparrow} > 0.5 \Rightarrow$ predict $\hat{y}_t \uparrow$; $\quad \hat{p}_{\downarrow} > 0.5 \Rightarrow$ predict $\hat{y}_t \downarrow$.
  - cost matrix:

		$\uparrow$	$\downarrow$	$\hat{y}_t$
y_t	$\uparrow$	0	$c_{\mathbf{t}}^{\mathbf{fn}}$	
	$\downarrow$	$c_{\mathbf{t}}^{\mathbf{fp}}$	0	

  - assumes:
    - ★ $c_{\mathbf{t}}^{\mathbf{fp}} \neq c_{\mathbf{t}}^{\mathbf{fn}}$
    - ★ each time-dependent

Options prices as costs

- As a basis, look at options that are trading ATM at time t:
  - **false positive** $\Leftrightarrow \{\hat{y}_t \uparrow, y_t \downarrow\}$

    which option hedges against that?
  - **false negative** $\Leftrightarrow \{\hat{y}_t \downarrow, y_t \uparrow\}$

    which option hedges against that?
- Many prices and strikes, OTM, ITM, at different implied exposures

Options prices as costs

- As a basis, look at options that are trading ATM at time t:
  - **false positive** $\Leftrightarrow \{\hat{y}_t \uparrow, y_t \downarrow\}$

    which option hedges against that? – **European put**
  - **false negative** $\Leftrightarrow \{\hat{y}_t \downarrow, y_t \uparrow\}$

    which option hedges against that? – **European call**
- Many prices and strikes, OTM, ITM, at different implied exposures

Options prices as costs

- Let $p_t^{(p)}$ and $p_t^{(c)}$ be generic (p)ut and (c)all prices then

$$c_{\mathbf{t}}^{\mathbf{fp}} = p_t^{(p)} : \text{ price of the put that did not get exercised}$$

$$c_{\mathbf{t}}^{\mathbf{fn}} = p_t^{(c)} : \text{ price of the call that did not get exercised}$$

- There is a distribution of $p_t^{(p)}$ and $p_t^{(c)}$ with different strike prices K
- Need an exposure-controlled measure of cost

How to control exposure?

- Consider put price $p_t^{(p)}$, strike price K, expiry h days ahead;
- Let $F(P_{t+h})$ denote the distribution of asset price P_{t+h}; then,

$$p_t^{(p)}(K) = e^{-rh} \int_0^K (K - P_{t+h}) dF(P_{t+h})$$

- After some algebra:

$$\frac{dp_t^{(p)}(K)}{dK} = e^{-rh} F(K)$$

- So asset price cdf is linked to option price derivative wrt strike price
- This suggests a way to control exposure.

How to control exposure?

- Estimate the derivative nonparametrically from a cross-section of $(p_t^{(p)}, K)$
- For left tail (small K's), look at three contiguous put prices $p_t^{(p)}(K_{i-1}) < p_t^{(p)}(K_i) < p_t^{(p)}(K_{i+1})$ with strikes $K_{i-1} < K_i < K_{i+1}$
- Implicit risk level $\hat{\alpha} = \hat{F}(K_i)$ is

$$e^{rh}\left[\frac{1}{2}\left(\nu_1 \frac{p_t^{(p)}(K_{i+1}) - p_t^{(p)}(K_i)}{K_{i+1} - K_i} + \nu_2 \frac{p_t^{(p)}(K_i) - p_t^{(p)}(K_{i-1})}{K_i - K_{i-1}}\right)\right],$$

where $\nu_1, \nu_2 =$ weights to account for strike prices not equidistant.

How to control exposure?

- Similar for call options:

$$\frac{dp_t^{(c)}(K)}{dK} = e^{-rh}(1 - F(K))$$

- Estimate the derivative from a cross-section of $(p_t^{(c)}, K)$
- For right tail (large K's), look at three contiguous put prices $p_t^{(c)}(K_{i-1}) < p_t^{(c)}(K_i) < p_t^{(c)}(K_{i+1})$ with strikes $K_{i-1} > K_i > K_{i+1}$
- Implicit risk level $1 - \hat{\alpha} = 1 - \hat{F}(K_i)$ is

$$e^{rh}\left[\frac{1}{2}\left(\nu_1 \frac{p_t^{(c)}(K_{i+1}) - p_t^{(c)}(K_i)}{K_i - K_{i+1}} + \nu_2 \frac{p_t^{(c)}(K_i) - p_t^{(c)}(K_{i-1})}{K_{i-1} - K_i}\right)\right]$$

How to control exposure?

- Result: $\hat{\alpha} \Leftrightarrow \{p_t^{(p)}, K\}$ and $1 - \hat{\alpha} \Leftrightarrow \{p_t^{(c)}, K\}$
- Find $\{p_t^{(p)}, K\}$ and $\{p_t^{(c)}, K\}$ for the desired risk level α you want, e.g., 0.05.
- Denote the prices by

$$\left\{p_t^{(p)}(\alpha), K(\alpha)\right\} \qquad \left\{p_t^{(c)}(1-\alpha), K(1-\alpha)\right\}$$

- $p_t^{(p)}(\alpha)$ and $p_t^{(c)}(1-\alpha)$ are risk-conrolled versions of $c_t^{\mathbf{fp}}$ and $c_t^{\mathbf{fn}}$; need exposure-controlled;
- Option-implied ES:

$$c_t^{\mathbf{fp}} = CVaR_{t,t+h}^{(\alpha)} = P_t - K(\alpha) + e^{r_t(h)} \frac{p_t^{(p)}(\alpha)}{\alpha}$$

$$c_t^{\mathbf{fn}} = CVaR_{t,t+h}^{(1-\alpha)} = K(1-\alpha) - P_t + e^{r_t(h)} \frac{p_t^{(c)}(1-\alpha)}{\alpha}$$

Option-implied CVaR misclassification costs

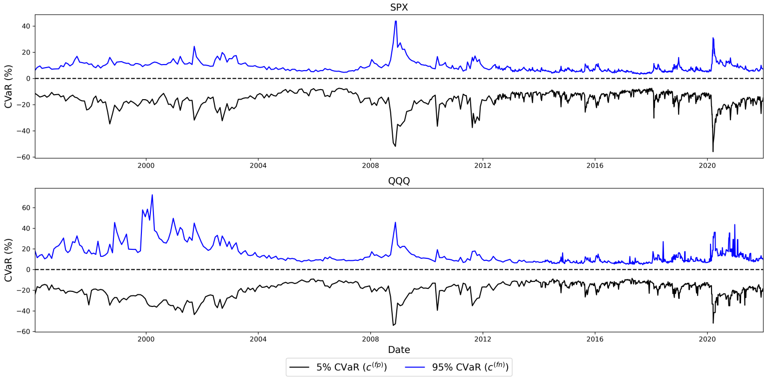

Figure: Option Implied CVaR Misclassification Costs for Two Market Trackers

Note: asymmetries, crises; prices in % of P_t; signs opposite for illustration

Return to cost-sensitive machine learning

- We look at rolling 10-day and 30-day ahead predictions using:
  - Logistic Regression [**LR**]
  - Gradient Boosting [**GB**]
- Both models use the score $f(\mathbf{x}_t; \boldsymbol{\theta})$ and probabilities

$$p_{\uparrow} = p(y_t \uparrow | \mathbf{x}_t; \boldsymbol{\theta}) = \frac{1}{1 + e^{-f(\mathbf{x}_t; \boldsymbol{\theta})}}$$

$$p_{\downarrow} = 1 - p_{\uparrow}$$

Cost-sensitive learning framework

- Traditional log-loss (cross-entropy) objective

$$L_1(\mathbf{y}, \mathbf{x}; \boldsymbol{\theta}) = \sum_{t=1}^{T} -[y_t \log(p(y_t = 1|\mathbf{x}_t; \boldsymbol{\theta})) + (1 - y_t) \log(1 - p(y_t = 1|\mathbf{x}_t; \boldsymbol{\theta}))]$$

- Cost-sensitive average expected cost [AEC] objective

$$L_2(\mathbf{y}, \mathbf{x}; \boldsymbol{\theta}) = \frac{1}{T} \sum_{t=1}^{T} y_t(1 - p_t(y_t = 1|\mathbf{x}_t; \boldsymbol{\theta})) \underbrace{\boxed{c_t^{\mathbf{fn}}}}_{CVaR_t^{(0.05)}} + (1 - y_t) p_t(y_t = 1|\mathbf{x}_t; \boldsymbol{\theta}) \underbrace{\boxed{c_t^{\mathbf{fp}}}}_{CVaR_t^{(0.95)}}$$

- The inclusion of $c_t^{\mathbf{fn}}$ and $c_t^{\mathbf{fp}}$ makes the LR and GB models sensitive to time-varying misclassification costs.
  - E.g., in a financial crisis investors may want costs be higher than average, increase with volatility, and have $c_t^{\mathbf{fp}} > c_t^{\mathbf{fn}}$.

Bi-objective optimization

- Combine log-loss and AEC using weights u_1 and u_2 expressing an investor's importance of each objective. Use $u_1 = u_2$ as a base.
- Since log-loss and AEC are on different scales, use normalizations η_1 and η_2 so actual weights are $w_1 = \eta_1 u_1$ and $w_2 = \eta_2 u_2$.
- Find η_1 and η_2 by using the nadir and utopia points (see, e.g., Mausser 2006).
    - Utopia points are
    $$s_i^U = \min_{\theta} L_i(\mathbf{y}, \mathbf{x}; \boldsymbol{\theta}), \quad i \in \{1, 2\}$$
    - Nadir points are
    $$s_i^N = \max_{j \in \{1,2\}} L_i(\mathbf{y}, \mathbf{x}; \boldsymbol{\theta}_j)$$
    - Then, the normalization factors are given by
    $$\eta_i = \frac{1}{s_i^N - s_i^U} \tag{1}$$
- Train bi-objective model using LR and GB

$$L(\mathbf{y}, \mathbf{x}; \boldsymbol{\theta}) = w_1 L_1(\mathbf{y}, \mathbf{x}; \boldsymbol{\theta}) + w_2 L_2(\mathbf{y}, \mathbf{x}; \boldsymbol{\theta})$$

Data

- S&P500, NASDAQ100 indices.
  - Training data = a rolling window of 2000 trading days (results are robust to wider window).
  - Asset prices from Compustat; options prices from OptionMetrics; risk free rates from Kenneth French library.
  - Option cross-section is cleaned using standard filters (see, e.g., Bali et al. 2023, Bollerslev et al. 2015)
- Include standard predictors that capture information about volatility, economic and financial conditions, technical indicators of momentum and trend, and behaviour of past returns (Welch & Goyal 2008).
  - VIX (VXN for NASDAQ100) and VRP are also included.
  - 30 lags of daily returns are included.

LR and GB models

- **LR**: elastic net logistic regression models:
  1. AEC LR: minimizes the average expected cost.
  2. Bi-LR: minimizes the bi-objective function.
  3. LR (benchmark): minimizes log-loss; includes tail asymmetry $[CVaR_t^{(0.05)} - CVaR_t^{(0.95)}]$ as a predictor.
- **GB**: gradient boosting models:
  1. AEC GB: Minimizes the Average Expected Cost Objective function.
  2. Bi-GB: Minimizes our weighted sum bi-objective function.
  3. GB (benchmark): minimizes log-loss; includes tail asymmetry $[CVaR_t^{(0.05)} - CVaR_t^{(0.95)}]$ as a predictor.
- Models are estimated and variables are engineered to ensure there is no look-ahead bias at time t.

Hyperparameter optimization

$$\text{AEC LR:} \quad \min_{\boldsymbol{\beta}} \; L_2(\mathbf{y}, \mathbf{x}; \boldsymbol{\beta}) + \lambda \left[(1-\alpha)\|\boldsymbol{\beta}\|_2/2 + \alpha\|\boldsymbol{\beta}\|_1\right]$$

$$\text{Bi-LR:} \quad \min_{\boldsymbol{\beta}} \; w_1 L_1(\mathbf{y}, \mathbf{x}; \boldsymbol{\beta}) + w_2 L_2(\mathbf{y}, \mathbf{x}; \boldsymbol{\beta}) + \lambda \left[(1-\alpha)\|\boldsymbol{\beta}\|_2/2 + \alpha\|\boldsymbol{\beta}\|_1\right]$$

$$\text{LR:} \quad \min_{\boldsymbol{\beta}} \; L_1(\mathbf{y}, \mathbf{x}; \boldsymbol{\beta}) + \lambda \left[(1-\alpha)\|\boldsymbol{\beta}\|_2/2 + \alpha\|\boldsymbol{\beta}\|_1\right]$$

$$\text{GB:} \quad f(\mathbf{x}_t; \boldsymbol{\theta}) = \sum_{k=1}^{K} h_k(\mathbf{x}_t; \boldsymbol{\theta}_k), \quad g_{tk} = \left.\frac{\partial \ell(y_t, \xi)}{\partial \xi}\right|_{\xi = f^{(k-1)}(\mathbf{x}_t; \boldsymbol{\theta})}$$

$$f^{(k)}(\mathbf{x}_t; \boldsymbol{\theta}) := f^{(k-1)}(\mathbf{x}_t; \boldsymbol{\theta}) + \nu h_k(\mathbf{x}_t; \boldsymbol{\theta}_k)$$

Hyperparameter optimization

- We use the Optuna framework (Akiba et al. 2019) together with 5-fold purged and embargoed cross validation (De Prado 2018).
  - Optuna implements an efficient hyper-parameter sampling strategy to search regions concentrated around the best values.
  - We let Optuna use 125 iterations.
- Our hyper-parameter tuning process makes it more likely that differences in model performance are likely attributable to our bi-objective loss function and not arbitrary or sub-optimal hyper-parameter choices.
- Hyperparameters are re-tuned at the start of each week.

Hyperparameter optimization

Table: Hyperparameter Search Space

(i) Logistic Regression			
Hyper-parameter	Low	High	Choices
λ (log scale)	0.01	10	
α	0.01	1	
(ii) Gradient Boosting			
Hyper-parameter	Low	High	Choices
n_estimators			[50, 100, 200]
num_leaves	2	16	
learning_rate	0.01	0.1	
subsample			[0.5, 0.7, 0.9]
min_child_samples	20	200	
max_bin	20	255	
drop_rate (log scale)	0.025	0.3	
reg_alpha (log scale)	0.01	1	
reg_lambda	0.01	1	

Market timing strategy

- Cost-sensitive learning emphasizes the investment results more than predictive accuracy.
- Backtest: equal-weighted equity index market timing strategy using our forecasts.
  - Long the S&P500(SPDR S&P 500 ETF)/NASDAQ (INVESCO QQQ) when $\hat{y} = 1$.
  - Long Core iShares U.S. Aggregate Bond ETF when $\hat{y} = 0$.
- Transaction costs assumed at 0.3% of value traded (Petraki 2020), include 0.1% of the mid price for slippage.
- To mitigate path dependence we average the return paths for 30 different starting days at the beginning of the sample.
  - All results are reported for the average investment strategy return path
- We only trade on a forecast at time $t + 1$ to emulate real-world execution times.
- The re-balancing period matches the forecast horizon (30-days)

Market timing strategy (LR)

Table: Logistic Regression Market Timing Strategy
***, **, * represent statistical significance form the studentized time series bootstrap test of Ledoit & Wolf (2008) against the benchmark LR model at the 10%, 5% and 1% levels respectively.

	Buy & Hold	LR	AEC LR	Bi-LR
Annualised Return (%)	13.113	10.239	10.661	11.730
Annualised Std	20.357	13.262	12.291	12.094
Annualised Sharpe Ratio	0.686	0.776	0.858	0.946**
Annualised Sortino Ratio	1.068	1.205	1.335	1.478
Max Drawdown (%)	-53.931	-42.913	-38.557	-35.614
Max Drawdown Period (days)	353	276	248	248
MAR Ratio	0.243	0.239	0.277	0.329
Daily Var 5%	1.967	1.281	1.115	1.124
Daily Var 1%	3.871	2.679	2.435	2.382
Daily CVar 5%	3.156	2.107	1.929	1.901
Daily CVar 1%	5.297	3.618	3.456	3.394
Annualised Downside Deviation	13.091	8.726	8.014	8.673

Market timing strategy (GB)

Table: Gradient Boosting Market Timing Strategy

	Buy & Hold	GB	AEC GB	Bi-GB
Annualised Return (%)	13.113	14.119	10.432	10.812
Annualised Std	20.357	16.730	11.883	11.670
Annualised Sharpe Ratio	0.686	0.847	0.866	0.908
Annualised Sortino Ratio	1.068	1.306	1.323	1.385
Max Drawdown (%)	-53.931	-34.910	-25.786	-23.553
Max Drawdown Period (days)	353	258	106	221
MAR Ratio	0.243	0.404	0.405	0.459
Daily Var 5%	1.967	1.671	1.150	1.150
Daily Var 1%	3.871	3.037	2.306	2.307
Daily CVar 5%	3.156	2.591	1.906	1.876
Daily CVar 1%	5.297	4.351	3.283	3.264
Annualised Downside Deviation	13.091	10.836	7.837	8.089

International equity market strategy

- To further asses the robustness of our results we backtest a “transfer learning“ market timing strategy for the Canada, U.K, France, Germany, and Japan equity markets (see, e.g., Liu et al. 2020, Jiang et al. 2023)

- Long the equal weighted basket of iShares MSCI Canada, iShares MSCI Canada, iShares MSCI France, iShares MSCI Germany, and iShares MSCI Japan ETFs when SPX $\hat{y} = 1$. Long Core U.S. Aggregate Bond ETF when $\hat{y} = 0$.

- Since international stock market data was not used to train our cost-sensitive models these tests further evaluate out-of-sample performance.

International equity market strategy

Table: International Equity Market Timing Strategy
***, **, * represent statistical significance form the studentized time series bootstrap test of Ledoit & Wolf (2008) against the benchmark GBM model at the 10%, 5% and 1% levels respectively.

	(i) Logisitic Regression			
	Buy & Hold	LR	AEC LR	Bi-LR
Annualised Return (%)	4.618	7.068	6.183	7.066
Annualised Std	21.486	13.504	13.034	12.947
Annualised Sharpe Ratio	0.311	0.556	0.510	0.574
Annualised Sortino Ratio	0.477	0.857	0.782	0.883
Max Drawdown (%)	-60.449	-42.044	-33.089	-29.173
MAR Ratio	0.076	0.168	0.187	0.242
Daily CVar 5%	3.346	2.169	2.079	2.070
Daily CVar 1%	6.063	3.607	3.624	3.572
	(ii) Gradient Boosting			
	Buy & Hold	GB	AEC GB	Bi-GB
Annualised Return (%)	4.618	5.585	5.354	6.625
Annualised Std	21.486	16.464	11.237	11.516
Annualised Sharpe Ratio	0.311	0.402	0.505	0.596**
Annualised Sortino Ratio	0.477	0.608	0.766	0.898
Max Drawdown (%)	-60.449	-35.780	-26.232	-20.898
MAR Ratio	0.076	0.156	0.204	0.317
Daily CVar 5%	3.346	2.554	1.783	1.842
Daily CVar 1%	6.063	4.411	3.102	3.273

LR classification performance

Table: Out-of-sample performance of LR

	LR	AEC LR	Bi-LR
Precision	0.698	0.713	0.717
Recall	0.680	0.632	0.644
F1 (1)	0.689	0.670	0.678
NPV	0.354	0.368	0.376
Specificity	0.374	0.457	0.457
F1 (0)	0.363	0.408	0.412
MCC	0.053	0.084	0.097
Balanced Accuracy	0.527	0.544	0.551
ROCAUC	0.533	0.559	0.569
AUPRC	0.707	0.739	0.737
Log Loss	0.785	4.679	1.337
AEC	6.956	6.346	6.476

- Cost sensitive models are better at classifying negative future returns (higher F1 (0) score).
- Interestingly, the bi-objective LR tends to perform best across all classification performance metrics.

GB classification performance

Table: Out-of-sample performance of GB

	GB	AEC GB	Bi-GB
Precision	0.696	0.701	0.706
Recall	0.924	0.628	0.612
F1 (+1)	0.794	0.663	0.656
NPV	0.462	0.351	0.356
Specificity	0.140	0.428	0.457
F1 (0)	0.214	0.386	0.400
MCC	0.100	0.054	0.065
Balanced Accuracy	0.532	0.528	0.534
ROCAUC	0.565	0.539	0.552
AUPRC	0.714	0.701	0.707
Log Loss	0.632	1.192	0.774
AEC	5.931	6.420	6.745

- Cost sensitive models trade lower Recall for significantly higher Specificity.
- Interestingly, the bi-objective GB model has the highest out-of-sample AEC.

Summary

- Cost sensitive models are better at classifying negative future returns. They tend to trade recall for specificity, correctly classifying a larger fraction of all negative future returns.
- The option implied $CVaR_t^{(0.05)}$ is often higher than the option implied $CVaR_t^{(0.95)}$ so the cost of FP is higher than FN.
  - These cost dynamics likely explain why our cost-sensitive ML models are better at predicting negative returns.
- out-of-sample Log-loss is substantially higher for all AEC models, while the bi-objective models adequately balance the out-of-sample realized log-loss and AEC.
- Also, GB models do not comprehensively outperform the elastic-net LR out-of-sample, consistent with Iworiso & Vrontos (2020). This may have implications for practitioners.

Conference on Econometrics and Big Data Methods

- iCEBDA 2025 in Istanbul, 11-14 Sept

  Keynotes:

  - Guido Imbens (Stanford GSB)
  - Tommaso Proietti (Tor Vergata)

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