Exponential form (lamba from two terms)

Equation of motion for the particles:

$$\frac{dx_{ip}}{dt} = (1-s)u_{ip} + s\tilde{u}_i + \alpha s(1-s)\frac{\partial}{\partial x_i}\left\{\tilde{\rho} - \sum_{k=1}^{N}\rho_k\right\} \Big/ \sum_{k=1}^{N}\rho_k \quad (1.1)$$

$$\frac{du_{jp}}{dt} = (1-s_p)F_{jp}^{MD}/\rho_p + s_p F_{jp}/\rho_p + \left[\frac{\partial}{\partial x_i}\left(\frac{\sum_{p=1}^{N}\left[\alpha s_p(1-s_p)u_{jp}\rho_p\left[\frac{\partial}{\partial x_i}\left\{\tilde{\rho} - \sum_{p=1}^{N}\rho_p\right\}\right]_p\right]}{\sum_{p=1}^{N}\rho_p}\right)\right]_p \Bigg/ \sum_{p=1}^{N}\rho_p + \quad (1.2)$$

$$+\left[\beta s(1-s)\left\{\tilde{u}_j\tilde{\rho} - \sum_{p=1}^{N}u_{jp}\rho_p\right\}\right]_p \Big/ \sum_{p=1}^{N}\rho_p - \left[\frac{\partial}{\partial x_i}\left(\tilde{u}_i\sum_{p=1}^{N}s_p\rho_p u_{jp} - \sum_{p=1}^{N}\left(\tilde{u}_{ip}s_p\rho_p u_{jp}\right)\right)\right]_p \Big/ \sum_{p=1}^{N}\rho_p$$

we rewrite the equations in the form of 1.3

$$\dot{\mathbf{q}}_i = \frac{\mathbf{p}_i}{m_i} + \mathbf{C}_i \bullet \mathbf{F}(t),$$
$$\dot{\mathbf{p}}_i = \mathbf{F}_i - \mathbf{D}_i \bullet \mathbf{F}(t) - \lambda \mathbf{p}_i, \quad (1.3)$$

$$\frac{dx_{ip}}{dt} = u_{ip} + s_p(\tilde{u}_i - u_{ip}) + \alpha s_p(1-s_p)\frac{\partial}{\partial x_i}\left\{\tilde{\rho} - \sum_{k=1}^{N}\rho_k\right\} \Big/ \sum_{k=1}^{N}\rho_k \quad (1.4)$$

$$\mathbf{C}_{ip} \bullet \mathbf{F}(t) = s_p(\tilde{u}_i - u_{ip}) + \alpha s_p(1-s_p)\frac{\partial}{\partial x_i}\left\{\tilde{\rho} - \sum_{k=1}^{N}\rho_k\right\} \Big/ \sum_{k=1}^{N}\rho_k$$

$$\frac{dm_p u_{jp}}{dt} = F_{jp}^{MD} - s_p F_{jp}^{MD} + s_p F_{jp} +$$

$$+m_p \left[\frac{\partial}{\partial x_i} \left(\frac{\sum_{p=1}^{N} \left[\alpha s_p \left(1 - s_p\right) u_{jp} \rho_p \left[\frac{\partial}{\partial x_i} \left\{ \tilde{\rho} - \sum_{p=1}^{N} \rho_p \right\} \right]_p \right]}{\sum_{p=1}^{N} \rho_p} \right) \right]_p \Bigg/ \sum_{p=1}^{N} \rho_p +$$

$$+m_p \left[\left(s\left(1-s\right) \beta \left\{ \tilde{u}_j \tilde{\rho} - \sum_{p=1}^{N} u_{jp} \rho_p \right\} \right) \right]_p \Bigg/ \sum_{p=1}^{N} \rho_p =$$

$$= F_{jp}^{MD} - s_p F_{jp}^{MD} + s_p F_{jp} +$$

$$+ \frac{m_p}{\sum_{p=1}^{N} \rho_p} \left(\frac{\sum_{k=1, k \neq p}^{N} \left[\alpha s_k \left(1 - s_k\right) u_{jk} \rho_k \left[\frac{\partial^2}{\partial {x_i}^2} \left\{ \tilde{\rho} - \sum_{k=1}^{N} \rho_k \right\} \right]_k \right]}{\sum_{p=1}^{N} \rho_p} \right) +$$

$$+ \frac{m_p}{\sum_{p=1}^{N} \rho_p} \frac{\sum_{p}^{N} \left[\alpha s_p \left(1 - s_p\right) \rho_p \left[\frac{\partial}{\partial x_i} \left\{ \tilde{\rho} - \sum_{p=1}^{N} \rho_p \right\} \right]_p \frac{\partial u_{jp}}{\partial x_i} \right]}{\sum_{p=1}^{N} \rho_p} +$$

$$+m_p \left[\left(s\left(1-s\right) \beta \tilde{u}_j \tilde{\rho} \right) \right]_p \Big/ \sum_{p=1}^{N} \rho_p -$$

$$-m_p \left[\left(s\left(1-s\right) \beta \left\{ \sum_{\substack{k=1, \\ k \neq p}}^{N} u_{jk} \rho_k \right\} \right) \right]_p \Bigg/ \sum_{p=1}^{N} \rho_p$$

$$+ \frac{m_p}{\sum_{p=1}^{N} \rho_p} \left(\frac{\alpha s_p \left(1 - s_p\right) u_{jp} \rho_p \left[\frac{\partial^2}{\partial {x_i}^2} \left\{ \tilde{\rho} - \sum_{k=1}^{N} \rho_k \right\} \right]_p - s(1-s) \beta u_{jp} \rho_p}{\sum_{p=1}^{N} \rho_p} \right)_p \qquad (1.5)$$

From 1.5 we can see that

$$\lambda_i = -\frac{1}{\sum_{p=1}^{N} \rho_p} \frac{\alpha s_p \left(1 - s_p\right) \rho_p \left[\frac{\partial^2}{\partial {x_i}^2} \left\{ \tilde{\rho} - \sum_{p=1}^{N} \rho_p \right\} \right]_p - s(1-s) \beta \rho_p}{\sum_{p=1}^{N} \rho_p} \qquad (1.6)$$

$$
\begin{aligned}
&\mathbf{D}_{jp}\bullet\mathbf{F}(t)=s_pF_{jp}^{MD}-s_pF_{jp}-\\
&-\frac{m_p}{\sum_{p=1}^{N}\rho_p}\left(\frac{\sum_{k=1,k\neq p}^{N}\left[\alpha s_k\left(1-s_k\right)u_{jk}\rho_k\left[\frac{\partial^2}{\partial x_i{}^2}\left\{\tilde{\rho}-\sum_{k=1}^{N}\rho_k\right\}\right]_k\right]}{\sum_{p=1}^{N}\rho_p}\right)-\\
&-\frac{m_p}{\sum_{p=1}^{N}\rho_p}\frac{\sum_{k=p}^{N}\left[\alpha s_p\left(1-s_p\right)\rho_p\left[\frac{\partial}{\partial x_i}\left\{\tilde{\rho}-\sum_{p=1}^{N}\rho_p\right\}\right]_p\frac{\partial u_{jp}}{\partial x_i}\right]}{\sum_{p=1}^{N}\rho_p}-\\
&-m_p\left[\left(s\left(1-s\right)\beta\tilde{u}_j\tilde{\rho}\right)\right]_p\Big/\sum_{p=1}^{N}\rho_p+\\
&+m_p\left[\left(s\left(1-s\right)\beta\left\{\sum_{\substack{k=1,\\k\neq p}}^{N}u_{jk}\rho_k\right\}\right)\right]_p\Bigg/\sum_{p=1}^{N}\rho_p
\end{aligned}
\qquad (1.7)
$$

From the formula for $\lambda(t)$ 1.8

$$
\lambda(t)=-\frac{1}{\sum_i\frac{\mathbf{p}_i^2}{m_i}}\sum_i\mathbf{F}_i\bullet\mathbf{C}_i\bullet\mathbf{F}(t)-\frac{1}{\sum_i\frac{\mathbf{p}_i^2}{m_i}}\sum_i\frac{\mathbf{p}_i}{m_i}\bullet\mathbf{D}_i\bullet\mathbf{F}(t). \qquad (1.8)
$$

we get the following equation:

$$\lambda_i(t) = -\frac{1}{\sum_p \frac{\mathbf{p}_p^2}{m_p}} \sum_p \left[F_{ip}^{MD} \bullet \left(s_p\left(\tilde{u}_i - u_{ip}\right) + \alpha s_p\left(1-s_p\right)\frac{\partial}{\partial x_i}\left\{\tilde{\rho} - \sum_{k=1}^{N}\rho_k\right\} \Big/ \sum_{k=1}^{N}\rho_k \right) \right] -$$

$$-\frac{1}{\sum_p \frac{\mathbf{p}_p^2}{m_p}} \sum_p \frac{\mathbf{p}_p}{m_p} \bullet \left\{ \begin{array}{l} s_p F_{jp}^{MD} - s_p F_{jp} - \\ -\frac{m_p}{\sum_{p=1}^{N}\rho_p}\left(\frac{\sum_{k=1,k\neq p}^{N}\left[\alpha s_k\left(1-s_k\right) u_{jk}\rho_k \left[\frac{\partial^2}{\partial x_i{}^2}\left\{\tilde{\rho} - \sum_{k=1}^{N}\rho_k\right\}\right]_k \right]}{\sum_{p=1}^{N}\rho_p} \right) - \\ -\frac{m_p}{\sum_{p=1}^{N}\rho_p} \frac{\sum_{k=p}^{N}\left[\alpha s_p\left(1-s_p\right)\rho_p \left[\frac{\partial}{\partial x_i}\left\{\tilde{\rho} - \sum_{p=1}^{N}\rho_p\right\}\right]_p \frac{\partial u_{jp}}{\partial x_i} \right]}{\sum_{p=1}^{N}\rho_p} - \\ -m_p\left[\left(s\left(1-s\right)\beta\tilde{u}_j\tilde{\rho}\right)\right]_p \Big/ \sum_{p=1}^{N}\rho_p + \\ +m_p\left[\left(s\left(1-s\right)\beta\left\{\sum_{\substack{k=1,\\ k\neq p}}^{N} u_{jk}\rho_k\right\}\right)\right]_p \Big/ \sum_{p=1}^{N}\rho_p \end{array} \right\} \qquad (1.9)$$

From 1.6 and 1.9 we have

$$-\frac{1}{\sum_{p=1}^{N}\rho_p}\frac{\alpha s_p\left(1-s_p\right)\rho_p\left[\frac{\partial^2}{\partial x_i^{\,2}}\left\{\tilde{\rho}-\sum_{p=1}^{N}\rho_p\right\}\right]_p-s(1-s)\beta\rho_p}{\sum_{p=1}^{N}\rho_p}=$$

$$=-\frac{1}{\sum_p \frac{\mathbf{p}_p^2}{m_p}}\sum_p\left[F_{ip}^{MD}\bullet\left(s_p\left(\tilde{u}_i-u_{ip}\right)+\alpha s_p\left(1-s_p\right)\frac{\partial}{\partial x_i}\left\{\tilde{\rho}-\sum_{k=1}^{N}\rho_k\right\}\Big/\sum_{k=1}^{N}\rho_k\right)\right]-$$

$$-\frac{1}{\sum_p \frac{\mathbf{p}_p^2}{m_p}}\sum_p\frac{\mathbf{p}_p}{m_p}\bullet\left\{\begin{array}{l} s_pF_{jp}^{MD}-s_pF_{jp}- \\ -\frac{m_p}{\sum_{p=1}^{N}\rho_p}\left(\frac{\sum_{k=1,k\neq p}^{N}\left[\alpha s_k\left(1-s_k\right)u_{jk}\rho_k\left[\frac{\partial^2}{\partial x_i^{\,2}}\left\{\tilde{\rho}-\sum_{k=1}^{N}\rho_k\right\}\right]_k\right]}{\sum_{p=1}^{N}\rho_p}\right)- \\ -\frac{m_p}{\sum_{p=1}^{N}\rho_p}\frac{\sum_{k=p}^{N}\left[\alpha s_p\left(1-s_p\right)\rho_p\left[\frac{\partial}{\partial x_i}\left\{\tilde{\rho}-\sum_{p=1}^{N}\rho_p\right\}\right]_p\frac{\partial u_{jp}}{\partial x_i}\right]}{\sum_{p=1}^{N}\rho_p}- \\ -m_p\left[\left(s\left(1-s\right)\beta\tilde{u}_j\tilde{\rho}\right)\right]_p\Big/\sum_{p=1}^{N}\rho_p+ \\ +m_p\left[\left(s\left(1-s\right)\beta\left\{\sum_{\substack{k=1,\\k\neq p}}^{N}u_{jk}\rho_k\right\}\right)\right]_p\Big/\sum_{p=1}^{N}\rho_p \end{array}\right\} \qquad (1.10)$$

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$$-\frac{1}{\sum_{p=1}^{N}\rho_p}\frac{\alpha s_p\left(1-s_p\right)\rho_p\left[\frac{\partial^2}{\partial x_i^{\,2}}\left\{\tilde{\rho}-\sum_{p=1}^{N}\rho_p\right\}\right]_p}{\sum_{p=1}^{N}\rho_p}+\frac{1}{\sum_{p=1}^{N}\rho_p}\frac{s(1-s)\beta\rho_p}{\sum_{p=1}^{N}\rho_p}=$$

$$=-\frac{1}{\sum_p\frac{\mathbf{p}_p^2}{m_p}}\sum_p\left[F_{ip}^{MD}\bullet\left(s_p\left(\tilde{u}_i-u_{ip}\right)+\alpha s_p\left(1-s_p\right)\frac{\partial}{\partial x_i}\left\{\tilde{\rho}-\sum_{k=1}^{N}\rho_k\right\}\Big/\sum_{k=1}^{N}\rho_k\right)\right]-$$

$$-\frac{1}{\sum_p\frac{\mathbf{p}_p^2}{m_p}}\sum_p\frac{\mathbf{p}_p}{m_p}\bullet\left\{\begin{array}{l}s_pF_{jp}^{MD}-s_pF_{jp}-\\ -\frac{m_p}{\sum_{p=1}^{N}\rho_p}\left(\frac{\sum_{k=1,k\neq p}^{N}\left[\alpha s_k\left(1-s_k\right)u_{jk}\rho_k\left[\frac{\partial^2}{\partial x_i^{\,2}}\left\{\tilde{\rho}-\sum_{k=1}^{N}\rho_k\right\}\right]_k\right]}{\sum_{p=1}^{N}\rho_p}\right)-\\ -\frac{m_p}{\sum_{p=1}^{N}\rho_p}\frac{\sum_{k=p}^{N}\left[\alpha s_p\left(1-s_p\right)\rho_p\left[\frac{\partial}{\partial x_i}\left\{\tilde{\rho}-\sum_{p=1}^{N}\rho_p\right\}\right]_p\frac{\partial u_{jp}}{\partial x_i}\right]}{\sum_{p=1}^{N}\rho_p}\end{array}\right\}-$$

$$-\frac{1}{\sum_p\frac{\mathbf{p}_p^2}{m_p}}\beta\sum_p\frac{\mathbf{p}_p}{m_p}\bullet\left(\frac{-m_p\left[\left(s\left(1-s\right)\tilde{u}_j\tilde{\rho}\right)\right]_p}{\sum_p\rho_p}+\frac{m_p\left[\left(s\left(1-s\right)\left\{\sum_{\substack{k=1,\\k\neq p}}^{N}u_{jk}\rho_k\right\}\right)\right]_p}{\sum_p\rho_p}\right) \qquad (1.11)$$

$$
\begin{aligned}
&\beta\left\{\frac{1}{\sum\limits_{p=1}^{N}\rho_p}\frac{s(1-s)\rho_p}{\sum\limits_{p=1}^{N}\rho_p}+\frac{1}{\sum\limits_{p}\frac{\mathbf{p}_p^2}{m_p}}\sum_p\frac{\mathbf{p}_p}{m_p}\bullet\left(\frac{-m_p\left[\left(s\left(1-s\right)\tilde{u}_j\tilde{\rho}\right)\right]_p}{\sum\limits_p\rho_p}+\frac{m_p\left[\left(s\left(1-s\right)\left\{\sum\limits_{\substack{k=1,\\k\neq p}}^{N}u_{jk}\rho_k\right\}\right)\right]_p}{\sum\limits_p\rho_p}\right)\right\}=\\
&=\frac{1}{\sum\limits_{p=1}^{N}\rho_p}\frac{\alpha s_p\left(1-s_p\right)\rho_p\left[\frac{\partial^2}{\partial x_i{}^2}\left\{\tilde{\rho}-\sum\limits_{p=1}^{N}\rho_p\right\}\right]_p}{\sum\limits_{p=1}^{N}\rho_p}-\\
&-\frac{1}{\sum\limits_p\frac{\mathbf{p}_p^2}{m_p}}\sum_p\left[F_{ip}^{MD}\bullet\left(s_p\left(\tilde{u}_i-u_{ip}\right)+\alpha s_p\left(1-s_p\right)\frac{\partial}{\partial x_i}\left\{\tilde{\rho}-\sum_{k=1}^{N}\rho_k\right\}\Big/\sum_{k=1}^{N}\rho_k\right)\right]-\\
&-\frac{1}{\sum\limits_p\frac{\mathbf{p}_p^2}{m_p}}\sum_p\frac{\mathbf{p}_p}{m_p}\bullet\left\{\begin{array}{l}s_pF_{jp}^{MD}-s_pF_{jp}-\\[1ex]
-\frac{m_p}{\sum\limits_{p=1}^{N}\rho_p}\left(\frac{\sum\limits_{k=1,k\neq p}^{N}\left[\alpha s_k\left(1-s_k\right)u_{jk}\rho_k\left[\frac{\partial^2}{\partial x_i{}^2}\left\{\tilde{\rho}-\sum\limits_{k=1}^{N}\rho_k\right\}\right]_k\right]}{\sum\limits_{p=1}^{N}\rho_p}\right)-\\[1ex]
-\frac{m_p}{\sum\limits_{p=1}^{N}\rho_p}\frac{\sum\limits_{k=p}^{N}\left[\alpha s_p\left(1-s_p\right)\rho_p\left[\frac{\partial}{\partial x_i}\left\{\tilde{\rho}-\sum\limits_{p=1}^{N}\rho_p\right\}\right]_p\frac{\partial u_{jp}}{\partial x_i}\right]}{\sum\limits_{p=1}^{N}\rho_p}\end{array}\right\}
\end{aligned}
\tag{1.12}
$$

$$\beta =$$

$$= \frac{1}{\left\{ \frac{1}{\sum\limits_{p=1}^{N} \rho_p} \frac{s(1-s)\rho_p}{\sum\limits_{p=1}^{N} \rho_p} + \frac{1}{\sum\limits_{p} \frac{\mathbf{p}_p^2}{m_p}} \sum_p \frac{\mathbf{p}_p}{m_p} \bullet \left(\frac{-m_p \left[\left(s\left(1-s\right) \tilde{u}_j \tilde{\rho} \right) \right]_p}{\sum\limits_p \rho_p} + \frac{m_p \left[\left(s\left(1-s\right) \left\{ \sum\limits_{\substack{k=1, \\ k \neq p}}^{N} u_{jk} \rho_k \right\} \right) \right]_p}{\sum\limits_p \rho_p} \right) \right\}} \cdot$$

$$\cdot \left(\begin{array}{l} \frac{1}{\sum\limits_{p=1}^{N} \rho_p} \frac{\alpha s_p \left(1-s_p\right) \rho_p \left[\frac{\partial^2}{\partial x_i{}^2} \left\{ \tilde{\rho} - \sum\limits_{p=1}^{N} \rho_p \right\} \right]_p}{\sum\limits_{p=1}^{N} \rho_p} - \\ - \frac{1}{\sum\limits_p \frac{\mathbf{p}_p^2}{m_p}} \sum_p \left[F_{ip}^{MD} \bullet \left(s_p \left(\tilde{u}_i - u_{ip} \right) + \alpha s_p \left(1-s_p\right) \frac{\partial}{\partial x_i} \left\{ \tilde{\rho} - \sum\limits_{k=1}^{N} \rho_k \right\} \Big/ \sum\limits_{k=1}^{N} \rho_k \right) \right] - \\ - \frac{1}{\sum\limits_p \frac{\mathbf{p}_p^2}{m_p}} \sum_p \frac{\mathbf{p}_p}{m_p} \bullet \left\{ \begin{array}{l} s_p F_{jp}^{MD} - s_p F_{jp} - \\ - \frac{m_p}{\sum\limits_{p=1}^{N} \rho_p} \left(\frac{\sum\limits_{k=1, k \neq p}^{N} \left[\alpha s_k \left(1-s_k\right) u_{jk} \rho_k \left[\frac{\partial^2}{\partial x_i{}^2} \left\{ \tilde{\rho} - \sum\limits_{k=1}^{N} \rho_k \right\} \right]_k \right]}{\sum\limits_{p=1}^{N} \rho_p} \right) - \\ - \frac{m_p}{\sum\limits_{p=1}^{N} \rho_p} \frac{\sum\limits_{k=p}^{N} \left[\alpha s_p \left(1-s_p\right) \rho_p \left[\frac{\partial}{\partial x_i} \left\{ \tilde{\rho} - \sum\limits_{p=1}^{N} \rho_p \right\} \right]_p \frac{\partial u_{jp}}{\partial x_i} \right]}{\sum\limits_{p=1}^{N} \rho_p} \end{array} \right\} \end{array} \right) \qquad (1.13)$$

Exponential form (lamba from one term)

Equation of motion for the particles:

$$\frac{dx_{ip}}{dt} = (1-s)u_{ip} + s\tilde{u}_i + \alpha s(1-s)\frac{\partial}{\partial x_i}\left\{\tilde{\rho} - \sum_{k=1}^{N}\rho_k\right\} \Big/ \sum_{k=1}^{N}\rho_k \quad (1.14)$$

$$\frac{du_{jp}}{dt} = (1-s_p)F_{jp}^{MD}/\rho_p + s_p F_{jp}/\rho_p + \left[\frac{\partial}{\partial x_i}\left(\frac{\sum_{p=1}^{N}\left[\alpha s_p(1-s_p)u_{jp}\rho_p\left[\frac{\partial}{\partial x_i}\left\{\tilde{\rho} - \sum_{p=1}^{N}\rho_p\right\}\right]_p\right]}{\sum_{p=1}^{N}\rho_p}\right)\right]_p \Bigg/ \sum_{p=1}^{N}\rho_p +$$

$$+\left[\beta s(1-s)\left\{\tilde{u}_j\tilde{\rho} - \sum_{p=1}^{N}u_{jp}\rho_p\right\}\right]_p \Big/ \sum_{p=1}^{N}\rho_p - \left[\frac{\partial}{\partial x_i}\left(\tilde{u}_i\sum_{p=1}^{N}s_p\rho_p u_{jp} - \sum_{p=1}^{N}\left(\tilde{u}_{ip}s_p\rho_p u_{jp}\right)\right)\right]_p \Big/ \sum_{p=1}^{N}\rho_p$$

(1.15)

we rewrite the equations in the form of 1.3

$$\dot{\mathbf{q}}_i = \frac{\mathbf{p}_i}{m_i} + \mathbf{C}_i \bullet \mathbf{F}(t),$$

$$\dot{\mathbf{p}}_i = \mathbf{F}_i - \mathbf{D}_i \bullet \mathbf{F}(t) - \lambda \mathbf{p}_i, \quad (1.16)$$

$$\frac{dx_{ip}}{dt} = u_{ip} + s_p(\tilde{u}_i - u_{ip}) + \alpha s_p(1-s_p)\frac{\partial}{\partial x_i}\left\{\tilde{\rho} - \sum_{k=1}^{N}\rho_k\right\} \Big/ \sum_{k=1}^{N}\rho_k \quad (1.17)$$

$$\mathbf{C}_{ip} \bullet \mathbf{F}(t) = s_p(\tilde{u}_i - u_{ip}) + \alpha s_p(1-s_p)\frac{\partial}{\partial x_i}\left\{\tilde{\rho} - \sum_{k=1}^{N}\rho_k\right\} \Big/ \sum_{k=1}^{N}\rho_k$$

$$\frac{dm_p u_{jp}}{dt} = F_{jp}^{MD} - s_p F_{jp}^{MD} + s_p F_{jp} +$$

$$+m_p \left[\frac{\partial}{\partial x_i} \left(\frac{\sum_{p=1}^{N} \left[\alpha s_p \left(1 - s_p\right) u_{jp} \rho_p \left[\frac{\partial}{\partial x_i} \left\{ \tilde{\rho} - \sum_{p=1}^{N} \rho_p \right\} \right]_p \right]}{\sum_{p=1}^{N} \rho_p} \right) \right]_p \Bigg/ \sum_{p=1}^{N} \rho_p +$$

$$+m_p \left[\left(s\left(1-s\right) \beta \left\{ \tilde{u}_j \tilde{\rho} - \sum_{p=1}^{N} u_{jp} \rho_p \right\} \right) \right]_p \Bigg/ \sum_{p=1}^{N} \rho_p =$$

$$= F_{jp}^{MD} - s_p F_{jp}^{MD} + s_p F_{jp} +$$

$$+m_p \left[\frac{\partial}{\partial x_i} \left(\frac{\sum_{p=1}^{N} \left[\alpha s_p \left(1 - s_p\right) u_{jp} \rho_p \left[\frac{\partial}{\partial x_i} \left\{ \tilde{\rho} - \sum_{p=1}^{N} \rho_p \right\} \right]_p \right]}{\sum_{p=1}^{N} \rho_p} \right) \right]_p \Bigg/ \sum_{p=1}^{N} \rho_p +$$

$$+m_p \left[\left(s\left(1-s\right) \beta \left\{ \tilde{u}_j \tilde{\rho} - \sum_{\substack{k=1 \\ k \neq p}}^{N} u_{jk} \rho_k \right\} \right) \right]_p \Bigg/ \sum_{p=1}^{N} \rho_p - \quad (1.18)$$

$$-m_p\, s(1-s) \beta u_{jp} \rho_p \Bigg/ \sum_{p=1}^{N} \rho_p$$

(1.19)

$$-\mathbf{D}_i \bullet \mathbf{F}(t) - \lambda \mathbf{p}_i, = -s_p F_{jp}^{MD} + s_p F_{jp} +$$

$$+m_p \left[\frac{\partial}{\partial x_i} \left(\frac{\sum_{p=1}^{N} \left[\alpha s_p (1-s_p) u_{jp} \rho_p \left[\frac{\partial}{\partial x_i} \left\{ \tilde{\rho} - \sum_{p=1}^{N} \rho_p \right\} \right]_p \right]}{\sum_{p=1}^{N} \rho_p} \right) \right]_p \Bigg/ \sum_{p=1}^{N} \rho_p +$$

$$+m_p \left[\left(s(1-s) \beta \left\{ \tilde{u}_j \tilde{\rho} - \sum_{\substack{k=1 \\ k \neq p}}^{N} u_{jk} \rho_k \right\} \right) \right]_p \Bigg/ \sum_{p=1}^{N} \rho_p -$$

$$-m_p \, s(1-s) \beta u_{jp} \rho_p \Big/ \sum_{p=1}^{N} \rho_p$$

$$\mathbf{D}_i \bullet \mathbf{F}(t) = s_p F_{jp}^{MD} - s_p F_{jp} -$$

$$-m_p \left[\frac{\partial}{\partial x_i} \left(\frac{\sum_{p=1}^{N} \left[\alpha s_p (1-s_p) u_{jp} \rho_p \left[\frac{\partial}{\partial x_i} \left\{ \tilde{\rho} - \sum_{p=1}^{N} \rho_p \right\} \right]_p \right]}{\sum_{p=1}^{N} \rho_p} \right) \right]_p \Bigg/ \sum_{p=1}^{N} \rho_p -$$

(1.20)

$$-m_p \left[\left(s(1-s) \beta \left\{ \tilde{u}_j \tilde{\rho} - \sum_{\substack{k=1 \\ k \neq p}}^{N} u_{jk} \rho_k \right\} \right) \right]_p \Bigg/ \sum_{p=1}^{N} \rho_p$$

$$\lambda \mathbf{p}_i = m_p \, s(1-s) \beta u_{jp} \rho_p \Big/ \sum_{p=1}^{N} \rho_p$$

$$\lambda\left(t\right)=-\frac{1}{\sum_i \frac{\mathbf{p}_i^2}{m_i}}\sum_i \mathbf{F}_i\bullet\mathbf{C}_i\bullet\mathbf{F}\left(t\right)-\frac{1}{\sum_i \frac{\mathbf{p}_i^2}{m_i}}\sum_i \frac{\mathbf{p}_i}{m_i}\bullet\mathbf{D}_i\bullet\mathbf{F}\left(t\right).$$

$$\lambda=-\frac{1}{\sum_i \frac{\mathbf{p}_i^2}{m_i}}\sum_p F_{jp}^{MD}\bullet\left(s_p\left(\tilde{u}_i-u_{ip}\right)+\alpha s_p\left(1-s_p\right)\frac{\partial}{\partial x_i}\left\{\tilde{\rho}-\sum_{k=1}^{N}\rho_k\right\}\Big/\sum_{k=1}^{N}\rho_k\right)-$$

$$-\frac{1}{\sum_i \frac{\mathbf{p}_i^2}{m_i}}\sum_{\substack{p\\ i}}\frac{\mathbf{p}_i}{m_i}\bullet\left\{\begin{array}{l} s_pF_{jp}^{MD}-s_pF_{jp}- \\ -m_p\left[\frac{\partial}{\partial x_i}\left(\frac{\sum_{p=1}^{N}\left[\alpha s_p\left(1-s_p\right)u_{jp}\rho_p\left[\frac{\partial}{\partial x_i}\left\{\tilde{\rho}-\sum_{p=1}^{N}\rho_p\right\}\right]_p\right]}{\sum_{p=1}^{N}\rho_p}\right)\right]_p\Big/\sum_{p=1}^{N}\rho_p- \\ -m_p\left[\left(s\left(1-s\right)\beta\left\{\tilde{u}_j\tilde{\rho}-\sum_{\substack{k=1\\ k\neq p}}^{N}u_{jk}\rho_k\right\}\right)\right]_p\Big/\sum_{p=1}^{N}\rho_p \end{array}\right\}= \tag{1.21}$$

$$=s(1-s)\beta\rho_p\Big/\sum_{p=1}^{N}\rho_p$$

$$-\frac{1}{\sum_i \frac{\mathbf{p}_i^2}{m_i}} \sum_p F_{jp}^{MD} \bullet \left(s_p \left(\tilde{u}_i - u_{ip} \right) + \alpha s_p \left(1 - s_p \right) \frac{\partial}{\partial x_i} \left\{ \tilde{\rho} - \sum_{k=1}^{N} \rho_k \right\} \Big/ \sum_{k=1}^{N} \rho_k \right) -$$

$$-\frac{1}{\sum_i \frac{\mathbf{p}_i^2}{m_i}} \sum_{\substack{p \\ i}} \frac{\mathbf{p}_i}{m_i} \bullet \left\{ \begin{matrix} s_p F_{jp}^{MD} - s_p F_{jp} - \\ -m_p \left[\frac{\partial}{\partial x_i} \left(\frac{\sum_{p=1}^{N} \left[\alpha s_p \left(1 - s_p \right) u_{jp} \rho_p \left[\frac{\partial}{\partial x_i} \left\{ \tilde{\rho} - \sum_{p=1}^{N} \rho_p \right\} \right]_p \right]}{\sum_{p=1}^{N} \rho_p} \right) \right]_p \end{matrix} \Big/ \sum_{p=1}^{N} \rho_p \right\} =$$

$$= s(1-s)\beta\rho_p \Big/ \sum_{p=1}^{N} \rho_p + \frac{1}{\sum_i \frac{\mathbf{p}_i^2}{m_i}} \sum \frac{\mathbf{p}_i}{m_i} \bullet \left(-m_p \left[\left(s\left(1-s\right) \beta \left\{ \tilde{u}_j \tilde{\rho} - \sum_{\substack{k=1 \\ k \neq p}}^{N} u_{jk} \rho_k \right\} \right) \right]_p \Big/ \sum_{p=1}^{N} \rho_p \right)$$

$$\beta \left\{ s(1-s)\rho_p \Big/ \sum_{p=1}^{N} \rho_p + \frac{1}{\sum_i \frac{\mathbf{p}_i^2}{m_i}} \sum_{\substack{i \\ p}} \frac{\mathbf{p}_i}{m_i} \bullet \left(-m_p \left[\left(s\left(1-s\right) \left\{ \tilde{u}_j \tilde{\rho} - \sum_{\substack{k=1 \\ k \neq p}}^{N} u_{jk} \rho_k \right\} \right) \right]_p \Big/ \sum_{p=1}^{N} \rho_p \right) \right\} =$$

$$= -\frac{1}{\sum_i \frac{\mathbf{p}_i^2}{m_i}} \sum_p F_{jp}^{MD} \bullet \left(s_p \left(\tilde{u}_i - u_{ip} \right) + \alpha s_p \left(1 - s_p \right) \frac{\partial}{\partial x_i} \left\{ \tilde{\rho} - \sum_{k=1}^{N} \rho_k \right\} \Big/ \sum_{k=1}^{N} \rho_k \right) -$$

(1.22)

$$-\frac{1}{\sum_i \frac{\mathbf{p}_i^2}{m_i}} \sum_{\substack{p \\ i}} \frac{\mathbf{p}_i}{m_i} \bullet \left\{ \begin{matrix} s_p F_{jp}^{MD} - s_p F_{jp} - \\ -m_p \left[\frac{\partial}{\partial x_i} \left(\frac{\sum_{p=1}^{N} \left[\alpha s_p \left(1 - s_p \right) u_{jp} \rho_p \left[\frac{\partial}{\partial x_i} \left\{ \tilde{\rho} - \sum_{p=1}^{N} \rho_p \right\} \right]_p \right]}{\sum_{p=1}^{N} \rho_p} \right) \right]_p \end{matrix} \Big/ \sum_{p=1}^{N} \rho_p \right\}$$

(1.23)

$$\beta =$$

$$
= \frac{\left[\begin{array}{l}
-\dfrac{1}{\sum_i \dfrac{\mathbf{p}_i^2}{m_i}} \sum_p F_{jp}^{MD} \bullet \left(s_p\left(\tilde{u}_i - u_{ip}\right) + \alpha s_p \left(1-s_p\right) \dfrac{\partial}{\partial x_i}\left\{\tilde{\rho} - \sum_{k=1}^{N} \rho_k\right\} \Big/ \sum_{k=1}^{N} \rho_k \right) - \\
-\dfrac{1}{\sum_i \dfrac{\mathbf{p}_i^2}{m_i}} \sum_{\substack{p \\ i}} \dfrac{\mathbf{p}_i}{m_i} \bullet \left\{ \begin{array}{l}
s_p F_{jp}^{MD} - s_p F_{jp} - \\
-m_p \left[\dfrac{\partial}{\partial x_i} \left(\dfrac{\sum_{p=1}^{N} \left[\alpha s_p \left(1-s_p\right) u_{jp} \rho_p \left[\dfrac{\partial}{\partial x_i}\left\{\tilde{\rho} - \sum_{p=1}^{N} \rho_p \right\}\right]_p \right]}{\sum_{p=1}^{N} \rho_p} \right) \right]_p \Big/ \sum_{p=1}^{N} \rho_p
\end{array} \right\}
\end{array}\right]}{\left\{ s(1-s)\rho_p \Big/ \sum_{p=1}^{N} \rho_p + \dfrac{1}{\sum_i \dfrac{\mathbf{p}_i^2}{m_i}} \sum_{\substack{i \\ p}} \dfrac{\mathbf{p}_i}{m_i} \bullet \left(-m_p \left[\left(s\left(1-s\right) \left\{ \tilde{u}_j \tilde{\rho} - \sum_{\substack{k=1 \\ k \neq p}}^{N} u_{jk} \rho_k \right\} \right) \right]_p \Big/ \sum_{p=1}^{N} \rho_p \right) \right\}} =
$$

$$
= \frac{\left[\begin{array}{l}
-\sum_p \mathbf{F}_p^{MD} \bullet \left(s_p\left(\tilde{\mathbf{u}} - \mathbf{u_p}\right) + \alpha s_p \left(1-s_p\right) \dfrac{\partial}{\partial x_i}\left\{\tilde{\rho} - \sum_{k=1}^{N} \rho_k\right\} \Big/ \sum_{k=1}^{N} \rho_k \right) - \\
-\sum_p \dfrac{\mathbf{p}_p}{m_p} \bullet \left\{ \begin{array}{l}
s_p \mathbf{F}_p^{MD} - s_p \mathbf{F}_p - \\
-m_p \left[\nabla \left(\dfrac{\sum_{p=1}^{N} \left[\alpha s_p \left(1-s_p\right) \mathbf{u}_p \rho_p \left[\nabla \left\{\tilde{\rho} - \sum_{p=1}^{N} \rho_p \right\}\right]_p \right]}{\sum_{p=1}^{N} \rho_p} \right) \right]_p \Big/ \sum_{p=1}^{N} \rho_p
\end{array} \right\}
\end{array}\right]}{s(1-s)\left\{ \rho_p \sum_p \dfrac{\mathbf{p}_p^2}{m_p} \Big/ \sum_{p=1}^{N} \rho_p - \sum_p \mathbf{p}_p \left(\left[\tilde{\mathbf{u}} \tilde{\rho} - \sum_{\substack{k=1 \\ k \neq p}}^{N} \mathbf{u}_k \rho_k \right]_p \Big/ \sum_{p=1}^{N} \rho_p \right) \right\}}
$$

$$(1.24)\ \beta_p = \frac{\left[\begin{array}{l} -\sum_p \mathbf{F}_p^{MD} \bullet \left(s_p\left(\tilde{\mathbf{u}} - \mathbf{u}_\mathbf{p}\right) + \alpha s_p\left(1 - s_p\right)\frac{\partial}{\partial x_i}\left\{\tilde{\rho} - \sum_{k=1}^{N}\rho_k\right\} \Big/ \sum_{k=1}^{N}\rho_k \right) - \\ -\sum_p \frac{\mathbf{p}_p}{m_p} \bullet \left\{ \begin{array}{l} s_p\mathbf{F}_p^{MD} - s_p\mathbf{F}_p - \\ -m_p\left[\nabla\left(\dfrac{\sum_{p=1}^{N}\left[\alpha s_p\left(1 - s_p\right)\mathbf{u}_p\rho_p\left[\nabla\left\{\tilde{\rho} - \sum_{p=1}^{N}\rho_p\right\}\right]_p\right]}{\sum_{p=1}^{N}\rho_p}\right)\right]_p \Big/ \sum_{p=1}^{N}\rho_p \end{array} \right\} \end{array}\right]}{s(1-s)\left\{\rho_p\sum_p \dfrac{\mathbf{p}_p^2}{m_p} \Big/ \sum_{p=1}^{N}\rho_p - \sum_p \mathbf{p}_p\left(\left[\tilde{\mathbf{u}}\tilde{\rho} - \sum_{\substack{k=1\\k\neq p}}\mathbf{u}_k\rho_k\right]_p \Big/ \sum_{p=1}^{N}\rho_p\right)\right\}}$$