Artemis Pados
Dr. Walker-Dale
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In Support of Mathematical Platonism

Whereas Philosophy is the broad study of the fundamental nature of knowledge, reality, and existence, mathematics falls within that scope as the organized study of numbers, shapes, and patterns in an attempt to describe the natural world (Lewis). Putting these together, the philosophy *of* mathematics is involved with the assumptions, foundations, and implications of mathematics and aims to understand its nature and methods. That is, it "is concerned with two major questions: one concerning the meanings of ordinary mathematical sentences and the other concerning the issue of whether abstract objects exist" (Balaguer). I will address only the second question in this paper, as its implications are profound for one's larger metaphysics. "Different interpretations of mathematics seem to produce different metaphysical views about the nature of reality" (Balaguer) and it is useful to agree upon one single interpretation such that there is a consistency among claims. In the pages that follow, I will argue that a Platonist view toward mathematics is appropriate and favored because it presents a clear and thorough argument for the existence of abstract objects (the second fundamental question of the philosophy of mathematics) and puts forth an interpretation of mathematics that provides a plausible, metaphysical account of reality. In constructing my argument, I will begin by defining Mathematical Platonism against its close alternatives and then analyze the underlying theses of its definition. I will look at what it means for objects to be abstract and attempt to justify their existence. I also will investigate the views that counter Platonism, canvassing Intuitionism as it relates to the discussion, and finally attempt to refute the counterclaims using a Fregean manner, affirming Mathematical Platonism as an appropriate view. I will close with a description of what it means in practice to adopt a Platonist view toward Mathematics and discuss its practical utility in understanding the foundations and evolution of present-day pure mathematics and the translation from said pure, fundamental laws to applied mathematical models of our natural world.

There are three main views one can adopt in responding to ‘Do abstract objects exist?’ – the view of Mathematical Platonism, Realistic Anti-Platonism, and Mathematical Nominalism. Simply put, Mathematical Platonism is the position that “an abstract object is both nonphysical and nonmental. According to Platonists, abstract objects exist but not anywhere in the physical world or in people’s minds. In fact, they do not exist in space and time at all” (Balaguer). Mathematical Platonism is a metaphysical view that abstract entities exist and are independent of all our rational activities, our language, thought, and practices (Linnebo and Cole). Realistic Anti-Platonists, on the other hand, believe that objects such as numbers and sets do exist (and theorems/laws provide true descriptions of them), but they deny that these entities are abstract objects (Balaguer). On the complete contrary, the view of Mathematical Nominalism rejects the belief in the existence of numbers, sets, etc. and therefore rejects that theorems/laws describe parts of reality. Put more explicitly against its alternatives, Mathematical Platonism can be defined by the conjunction of three theses. The first two theses are reasonably clear:

“[1] **Existence**

There are mathematical objects.

[2] **Abstractness**

Mathematical objects are abstract” (Cole).

Theses One and Two imply that, per Platonists, mathematical objects are abstract existing things. Thesis Three implies that they are *independent* abstract existing things, which can be particularly controversial:

“[3] **Independence**.

Mathematical objects are independent of intelligent agents and their language, thought, and practices” (Cole).

Thesis three is essentially a response to a question asked originally when defining mathematics – “Is math a language we created in order to explain phenomena of nature or is it a preexisting truth among nature which we discovered?” (Lewis). Thesis Three, and therefore Mathematical Platonists, claim the latter.

In an attempt to digest the theses of Mathematical Platonism, I will begin by revisiting Thesis One. Say that you draw a circle in the sand of diameter one meter. To start building Thesis One about the existence of mathematical objects, one might consider as an example the circle and its diameter (a line segment), both mathematical entities. You may think that you see these entities as physical illustrations in the sand rather than as abstract objects, but nonetheless, abstract or not – in one way or another, the circle and its line segment exist in front of you. Now, to elaborate on the *abstractness* of their existence, we must first familiarize ourselves with what it means for an object to be abstract by moving our focus to Thesis Two. Let us shift attention from the circle and segment to the length of the circle, the circumference which is π. π is nowhere to be seen in the sand, yet it exists in the circle, seemingly abstractly. To generalize, the predominant trait of abstract objects is nonspatiotemporality. “According to Platonists, [abstract objects are] unchanging and entirely noncausal. Because abstract objects are not extended in space and not made of physical matter, it follows that they cannot enter into cause-and-effect relationships with other objects” (Balaguer). This is observed as one cannot directly see ‘the number six’ in practice but can see ‘six of something’ – six pencils for example. Sets, functions, shapes, and numbers, mathematical entities that is, are not observable unequivocally, but their properties can be detected either through physicals means (you can see a representation of the diameter, circle, and π in the sand) or their properties can be detected metaphysically. Under this argumentation, Thesis Two holds: mathematical entities are abstract objects. Thesis Three, independence from agents, language, thought, practices, is more nuanced. Support of its validity can come, however, from two different directions. In the small history of humanity, the same mathematical objects (say for example circle, or π, or ‘six’) came up from different agents (Eastern and Western for example) presumably with no contact and hence independent. In a broader context, “abstract objects are not located anywhere in the physical universe, and they are also entirely nonmental, yet they have always existed and they always will exist” (Balaguer). If this is in fact the case, debate of Thesis Three should be settled because if said objects have always existed and will continue to always exist, they are, in fact, independent of us, our thoughts, and practices.

Looking back now at the three theses of Mathematical Platonism, one may summarize that there exist mathematical, abstract objects independent of intelligence. Modern day pure mathematics uses exactly this language of existence to present theorems. Interestingly, a significant, equally-valued bulk of work in mathematics is directed toward proofs of non-existence for hypothesized objects. Along these lines, the philosophy of mathematics is also concerned with mathematical sentences and laws and whether they provide true descriptions of mathematical objects. Consider the sentence ‘six is even’. We already established above that the number ‘six’ is an abstract object, and so is ‘evenness’. This sentence, about abstract objects, is evidently true by basic mathematical principles rooted on the definition of ‘evenness’. By accepting the overall truth of the statement, we are also acknowledging the true existence of the constituent parts of the sentence – ‘six’ and ‘evenness’ – as Aristotle first grappled with when discussing Holism in his book *Metaphysics*. By reaching this point, it should seem almost natural to endorse a Platonist view since simply the agreement with a short phrase such as ‘six is even’ more or less requires one to conform with the underlying three theses of Mathematical Platonism.

Platonism finds also support from other philosophical methods such as Intuitionism; the view that non-constructive mathematical proofs/arguments are unacceptable. “Intuitionists embrace the nonstandard view that mathematical sentences of the form ‘The object *O* has the property *P*’ really mean that there is a proof that the object *O* has the property *P*, and they also embrace the view that mathematical sentences of the form ‘not-*P*’ mean that a contradiction can be proven from *P*” (Balaguer). In terms of our recurring example, an Intuitionist would interpret the statement ‘six is even’ to really mean that there is a complete, step-by-step procedure to establish that the number 6 has the property of evenness, which, there is.

Let x be any real number. By definition, for x to be even, it must be divisible by 2.

Therefore $x/2 = n$ where $n \in \mathbf{Z} = \{0,-1,1,-2,2,\ldots\}$.

$6/2 = 3 \in \mathbf{Z}$. Therefore 6 is even.

A treatment of Mathematical Platonism would not be complete without acknowledgment and refutation of opposing views. In answering the age-old question “Do abstract objects exist?”, which Mathematical Platonism answers in the affirmative, the two main views against, as mentioned earlier, are Realistic Anti-Platonism and Mathematical Nominalism. These two views put forth three main arguments: (1) Psychologism, (2) Physicalism, and (3) Deductivism. I will attempt to refute these arguments using techniques Gottlob Frege introduced.

Psychologism can be understood as the union of two claims: (1) number-ideas exist only in people's minds and (2) common mathematical sentences are interpreted as describing these ideas. Psychologism then implies that if all humans spontaneously died, number-ideas would cease to exist and therefore ordinary mathematical sentences, such as 'six is even', would suddenly become void of meaning. Secondly, finitely many humans with finite brains over finite time can only hold a finite number of distinct ideas. If number-concepts existed in the human mind only, infinitely many numbers would be impossible to exist there. The natural conclusion is that numbers cannot reside solely in the human mind.

Physicalism, the view that everything is physical, asserts that sets are simply collections of physical objects. Physicalist beliefs find resistance when trying to account for the sheer size of the infinities involved in set theory (Balaguer). There exist not only infinitely large sets, but infinitely many sizes of infinitely large sets. There is simply no tenable way to conceive of this in terms of the physical world. Another problem with Physicalism from the perspective of a Platonist is that it seems to imply that "mathematics is an empirical science, contingent on physical facts and susceptible to empirical falsification" (Balaguer). However, mathematics, as we organize and teach it today, is not an experimental science and despises everything experimental as being patently non-provably true or false. As such, 'six is even' for example, is not susceptible to empirical falsification by discoveries of the physical world.

Deductivism claims that mathematical sentences should not be interpreted literally, which is the opposite of an Intuitionist view. Reverting to the example of the sentence 'six is even' yet again, a deductivist would declare that the statement in truth means that, if numbers existed, then six would be even.

> There is no evidence, however, that people use mathematical sentences nonliterally. It seems that the best interpretation of mathematical discourse takes it to be about (or at any rate, to purport to be about) certain kinds of objects. Furthermore, as has already been shown, there are good reasons to think that the objects in question could not be physical or mental objects. (Balaguer)

In summary, the philosophy of mathematics is concerned with finding an interpretation of mathematics that produces an accurate metaphysical view of reality by asking questions such as "Do abstract objects exist?". In this paper, I argued in accordance with a Platonist view of Mathematics. By methodically defining the theses of Mathematical Platonism, investigating what it means for an object to have independent abstract existence, and pointing out flaws in the three major counterviews, I created an argument for a Platonist view. Mathematical Platonism today is as useful as ever in understanding the abstract essence of modern pure mathematics, which generally involves thought-experiments with abstract objects, and its translation to what is known as applied mathematics for modeling nature. Applied mathematics, in turn, have been credited for the advancement of physical sciences, medicine, engineering, and technology.

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