In [399…

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

In [416…

```
# alg for tasks with boundary conditions without derivatives
def alg(y0, yn):
    df = pd.DataFrame(data=np.zeros((3, N + 1)), columns=[i for i in range(N + 1)],

    df[N]['y'] = yn

    df[0]['y'] = y0

    for i in range(1, N+1):
        x = ax + i*h

        # V1
        n = (1 - p(x)/2*h)/(1 + p(x)/2*h)
        m = -(2-q(x)*h**2)/(1+p(x)/2*h)
        df[i]['c'] = 1/(m-n*df[i-1]['c'])
        df[i]['d'] = (f(x)/(1+p(x)/2*h))*h**2 - n*df[i-1]['c']*df[i-1]['d']

        # V2; shortened expression
        # df[i]['c']=1/(-2-h**2-df[i-1]['c'])
        # df[i]['d']=i*h**3-df[i-1]['c']*df[i-1]['d']

    for i in range(N-1, -1, -1):
        df[i]['y'] = df[i]['c']*(df[i]['d']-df[i+1]['y'])

    return df

def plot(x: list, y: list, name=None):
    for x_, y_ in zip(x, y):
        plt.plot(x_, y_, linewidth=1)
        plt.plot(x_, y_, marker='.')
    if name != False:
        plt.title(name)
        plt.savefig(f'{name}.png')
    plt.show()
```

Example from book

In [417…

```
# y’’ - y = x

def f(x):
    return x  # np.exp(-2*x)*np.log(x)

def p(x):
    return 0  # 4

def q(x):
    return -1  # 4

ax = 0
bx = 1
```

In [418…

```
N = 10
h = (bx - ax) / N

df_ex = alg(0, 0)
y_ex  = [[df_ex[i]['y']] for i in range(N+1)]
df_ex
```

Out[418...

	0	1	2	3	4	5	6	7	8	
c	0.0	-0.497512	-0.661162	-0.741379	-0.788258	-0.818503	-0.839280	-0.854176	-0.865183	-0.8735
d	0.0	0.001000	0.002498	0.004651	0.007448	0.010871	0.014898	0.019504	0.024660	0.0303
y	0.0	-0.014755	-0.028658	-0.040848	-0.050446	-0.056548	-0.058216	-0.054466	-0.044260	-0.0264

In [419...

```
def target_y(x):
    return 2*np.e / (np.e**2 - 1) * np.sinh(x) - x

arr = []
for i in range(N+1):
    x = ax + i*h
    arr += [[target_y(x)]]
```

In [420...

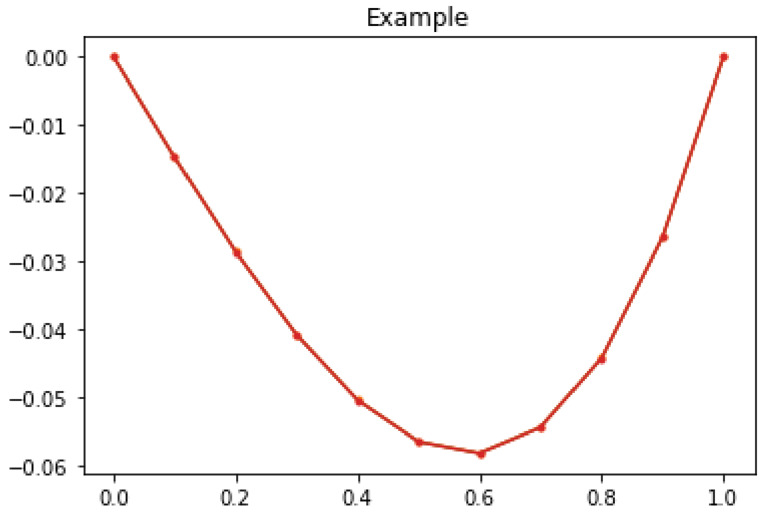
```
pd.DataFrame(np.concatenate((y_ex, arr), axis=1), columns=['Method', 'Target'], inde
```

Out[420...

	Method	Target
0.0	0.000000	0.000000
0.1	-0.014755	-0.014766
0.2	-0.028658	-0.028680
0.3	-0.040848	-0.040878
0.4	-0.050446	-0.050483
0.5	-0.056548	-0.056591
0.6	-0.058216	-0.058260
0.7	-0.054466	-0.054507
0.8	-0.044260	-0.044295
0.9	-0.026498	-0.026518
1.0	0.000000	0.000000

In [421...

```
x = [ax + i*h for i in range(0, N + 1)]
plot([x, x], [y_ex, arr], 'Example')
```



My task (10 option)

In [422…

```
# y’’ + 4y’ + 4y = e-2x ln(x); y(1) = 0, y(2) = −1

def f(x):
    return np.exp(-2*x)*np.log(x)

def p(x):
    return 4

def q(x):
    return 4

ax = 1
bx = 2
```

In [423…

```
N = 10
h = (bx - ax) / N

df1 = alg(0, -1)
y1 = [[df1[i]['y']] for i in range(N+1)]

N = 2*N
h = (bx - ax) / N

df2 = alg(0, -1)
y2 = [[df2[i]['y']] for i in range(N+1)]
```

In [424…

```
df1
```

Out[424…

	0	1	2	3	4	5	6	7	8	
c	0.0	-0.612245	-0.816213	-0.918112	-0.979184	-1.019842	-1.048835	-1.070538	-1.087380	-1.1008
d	0.0	0.000088	0.000174	0.000257	0.000328	0.000382	0.000420	0.000441	0.000449	0.0004
y	0.0	-0.617320	-1.008201	-1.235044	-1.344942	-1.373205	-1.346106	-1.283010	-1.198031	-1.1013

In [425…

```
df2
```

Out[425…

	0	1	2	3	4	5	6	7	8	
c	0.0	-0.552764	-0.737012	-0.829132	-0.884400	-0.921242	-0.947555	-0.967288	-0.982633	-0.9949
d	0.0	0.000014	0.000030	0.000050	0.000072	0.000093	0.000115	0.000135	0.000153	0.0001
y	0.0	-0.336705	-0.609117	-0.826438	-0.996701	-1.126908	-1.223155	-1.290739	-1.334255	-1.3576

3 rows × 21 columns

In [426…

```
pd.DataFrame(np.concatenate((y1, y2[::2]), axis=1), columns=['First', 'Second'], ind
```

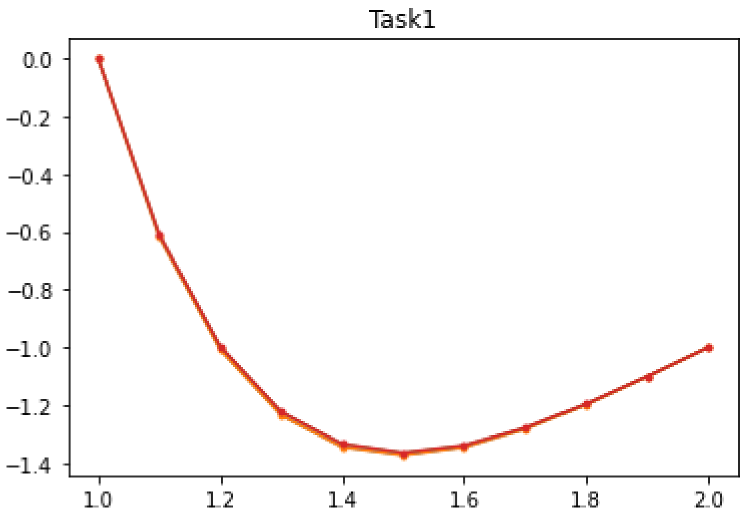
Out[426…

	First	Second
1.0	0.000000	0.000000
1.1	-0.617320	-0.609117
1.2	-1.008201	-0.996701
1.3	-1.235044	-1.223155

	First	Second
1.4	-1.344942	-1.334255
1.5	-1.373205	-1.364462
1.6	-1.346106	-1.339524
1.7	-1.283010	-1.278502
1.8	-1.198031	-1.195348
1.9	-1.101310	-1.100134
2.0	-1.000000	-1.000000

In [430…

```
x = [ax + i*h for i in range(0, N + 1, 2)]
plot([x, x], [y1, y2[::2]], 'Task1')
```



Addition task *

In [431…

```
# y’’ + y = 1; y(0) = 0, y(π) = 0

def f(x):
    return 1

def p(x):
    return 0

def q(x):
    return 1

ax = 0
bx = np.pi
```

In [432…

```
N = 10
h = (bx - ax) / N

df1 = alg(0, 0)
y1 = [[df1[i]['y']] for i in range(N+1)]

N = 2*N
h = (bx - ax) / N

df2 = alg(0, 0)
y2 = [[df2[i]['y']] for i in range(N+1)]
```

In [433...

```
df1
```

Out[433...

	0	1	2	3	4	5	6	7	8
c	0.0	-0.525955	-0.727088	-0.851632	-0.952679	-1.054157	-1.180433	-1.387211	-1.945172
d	0.0	0.098696	0.150606	0.208200	0.276006	0.361641	0.479922	0.665212	1.021485
y	-0.0	47.543862	90.494029	124.611489	146.528984	154.083344	146.528984	124.611489	90.494029

In [434...

```
df2
```

Out[434...

	0	1	2	3	4	5	6	7	
c	0.0	-0.506246	-0.680698	-0.772423	-0.831322	-0.874123	-0.908098	-0.937007	-0.9630
d	0.0	0.024674	0.037165	0.049972	0.063274	0.077275	0.092222	0.108420	0.1262
y	-0.0	96.710717	191.059866	280.719477	363.477286	437.291326	500.340309	551.068564	588.2244

3 rows × 21 columns

In [435...

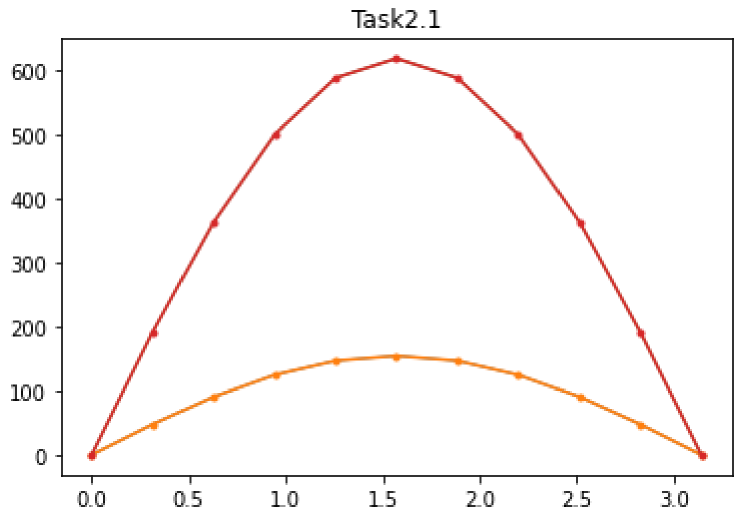
```
pd.DataFrame(np.concatenate((y1, y2[::2]), axis=1), columns=['First', 'Second'], ind
```

Out[435...

	First	Second
0.000000	-0.000000	-0.000000
0.314159	47.543862	191.059866
0.628319	90.494029	363.477286
0.942478	124.611489	500.340309
1.256637	146.528984	588.224422
1.570796	154.083344	618.509313
1.884956	146.528984	588.224422
2.199115	124.611489	500.340309
2.513274	90.494029	363.477286
2.827433	47.543862	191.059866
3.141593	0.000000	0.000000

In [436...

```
x = [ax + i*h for i in range(0, N + 1, 2)]
plot([x, x], [y1, y2[::2]], 'Task2.1')
```



Еще сильнее уменьшим шаг и сравним результат

In [437…

```
N = 10
h = (bx - ax) / N

df1 = alg(0, 0)
y1 = [[df1[i]['y']] for i in range(N+1)]

N = 2*N
h = (bx - ax) / N

df2 = alg(0, 0)
y2 = [[df2[i]['y']] for i in range(N+1)]

N = 2*N
h = (bx - ax) / N

df3 = alg(0, 0)
y3 = [[df3[i]['y']] for i in range(N+1)]

N = 2*N
h = (bx - ax) / N

df4 = alg(0, 0)
y4 = [[df4[i]['y']] for i in range(N+1)]

N = 2*N
h = (bx - ax) / N

df5 = alg(0, 0)
y5 = [[df5[i]['y']] for i in range(N+1)]
```

In [438…

```
pd.DataFrame(np.concatenate((y1, y2[::2], y3[::4], y4[::8], y5[::16]), axis=1),
             columns=['N=10', 'N=20', 'N=40', 'N=80', 'N=160'],
             index=[ax + i*h for i in range(0, N + 1, 16)])
```

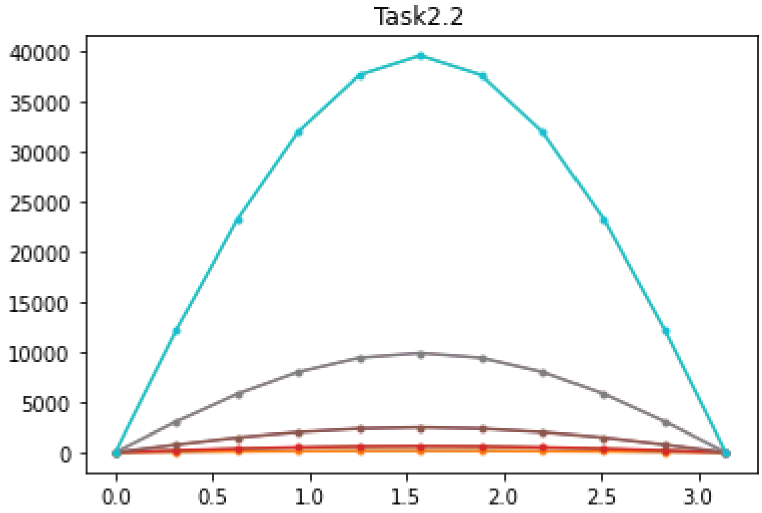
Out[438…

	N=10	N=20	N=40	N=80	N=160
0.000000	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000
0.314159	47.543862	191.059866	765.117563	3061.346790	12246.263204
0.628319	90.494029	363.477286	1455.399812	5823.087325	23293.836533
0.942478	124.611489	500.340309	2003.242635	8014.848737	32061.272078
1.256637	146.528984	588.224422	2354.991997	9422.058799	37690.324815

	N=10	N=20	N=40	N=80	N=160
1.570796	154.083344	618.509313	2476.198653	9906.952425	39629.966285
1.884956	146.528984	588.224422	2354.991997	9422.058799	37690.324815
2.199115	124.611489	500.340309	2003.242635	8014.848737	32061.272078
2.513274	90.494029	363.477286	1455.399812	5823.087325	23293.836533
2.827433	47.543862	191.059866	765.117563	3061.346790	12246.263204
3.141593	0.000000	0.000000	0.000000	0.000000	0.000000

In [439…

```
x = [ax + i*h for i in range(0, N + 1, 16)]
plot([x, x, x, x, x], [y1, y2[::2], y3[::4], y4[::8], y5[::16]], 'Task2.2')
```


Task2.2