ФИНАНСОВЫЙ УНИВЕРСИТЕТ
ПРИ ПРАВИТЕЛЬСТВЕ РОССИЙСКОЙ ФЕДЕРАЦИИ

Современные нейросетевые технологии

Лекция 3. Обучение линейного классификатора изображений

Учебные вопросы

1) Стохастический градиентный спуск (SGD)
2) Регуляризация параметров W
3) Производные функции нескольких переменных

Материалы курса:
github.com/balezz/modern_dl
Срок сдачи А2 – 17.09.2022 г.

Источники:
- dlcourse.ai
- cs231n.stanford.edu
- cs230.stanford.edu

Функция потерь

Softmax Classifier (Multinomial Logistic Regression)

cat	**3.2**
car	5.1
frog	-1.7

scores = unnormalized log probabilities of the classes.

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$ where $$s = f(x_i; W)$$

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$L_i = -\log P(Y = y_i|X = x_i)$$

---

in summary: $$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

	unnormalized log probabilities		unnormalized probabilities		probabilities	
cat	**3.2**	exp →	**24.5**	normalize →	**0.13**	→ L_i = -log(0.13) = **0.89**
car	5.1		164.0		0.87	
frog	-1.7		0.18		0.00	

unnormalized log probabilities

probabilities

Поиск оптимальных параметров

Поиск оптимальных параметров

current W:	W + h (first dim):	gradient dW:
[0.34,	[0.34 + **0.0001**,	[-2.5,
-1.11,	-1.11,	?,
0.78,	0.78,	?,
0.12,	0.12,	
0.55,	0.55,	
2.81,	2.81,	
-3.1,	-3.1,	
-1.5,	-1.5,	?,
0.33,…]	0.33,…]	?,…]
loss 1.25347	**loss 1.25322**	

(1.25322 - 1.25347)/0.0001
= -2.5

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

ФИНАНСОВЫЙ УНИВЕРСИТЕТ

current W:

[0.34,
-1.11,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,…]

loss 1.25347

W + h (second dim):

[0.34,
-1.11 + **0.0001**,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,…]

loss 1.25353

gradient dW:

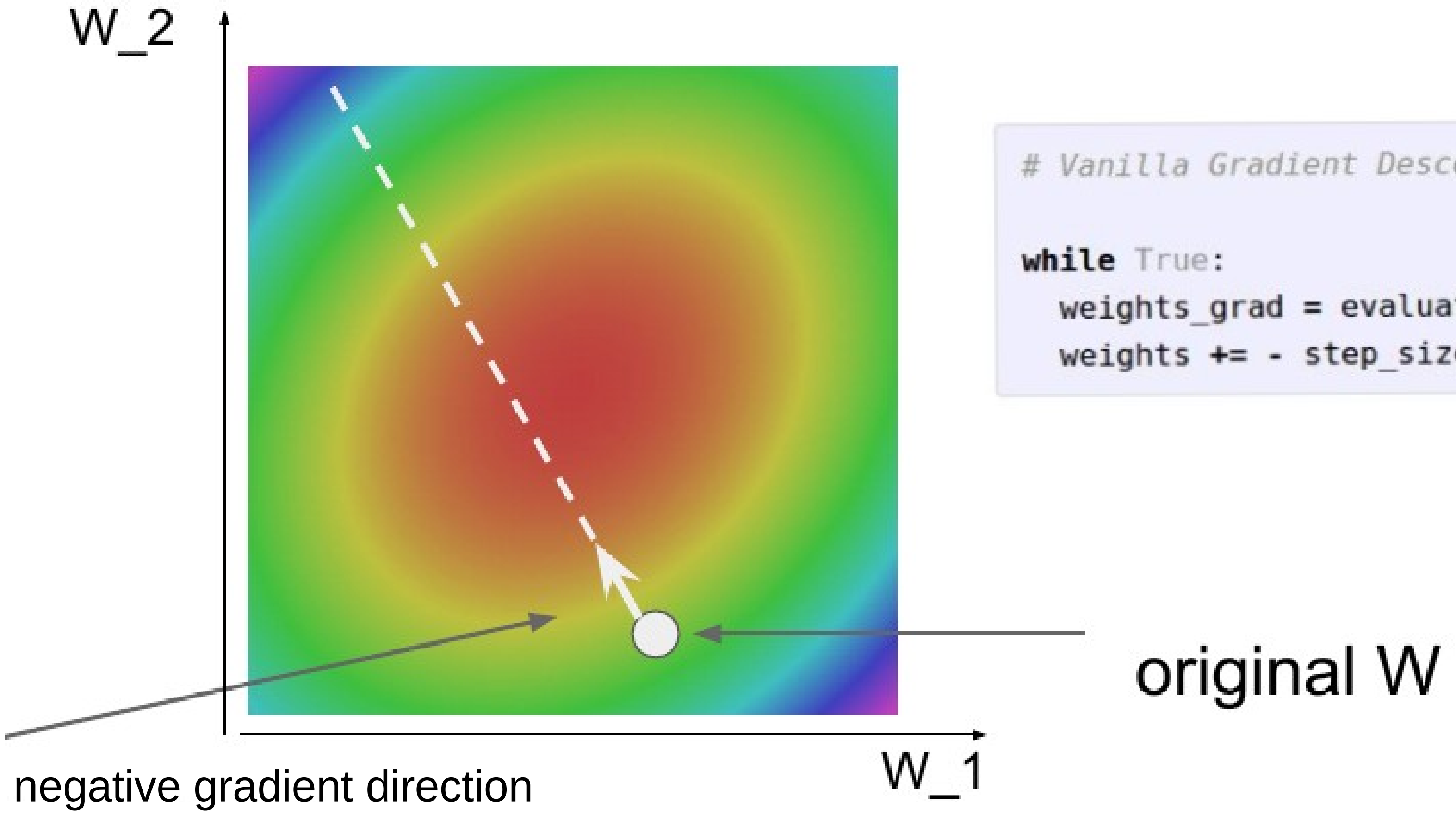
[-2.5,
0.6,
?,
?,

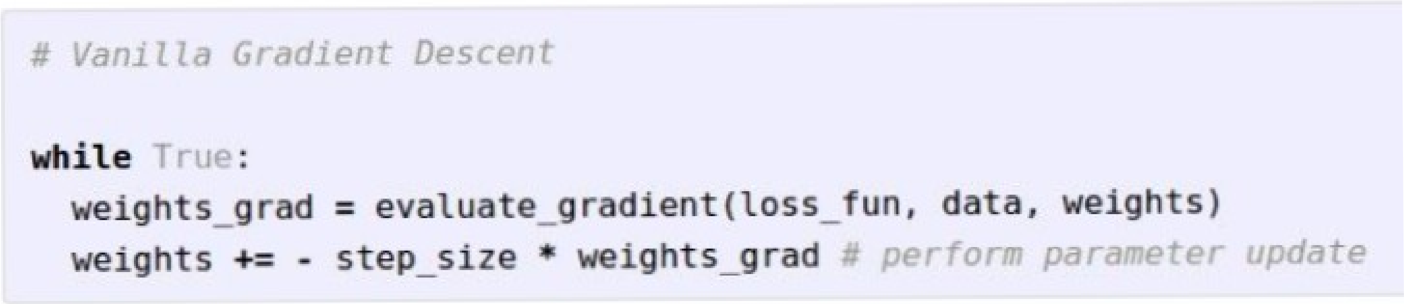
(1.25353 - 1.25347)/0.0001
= 0.6

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

?,…]

Поиск оптимальных параметров




```
# Vanilla Gradient Descent

while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```

Поиск оптимальных параметров

$$L(W) = \frac{1}{N} \sum_{i=1}^{N} L_i(x_i, y_i, W) + \lambda R(W)$$

$$\nabla_W L(W) = \frac{1}{N} \sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W) + \lambda \nabla_W R(W)$$

Full sum expensive when N is large!

Approximate sum using a **minibatch** of examples
32 / 64 / 128 common

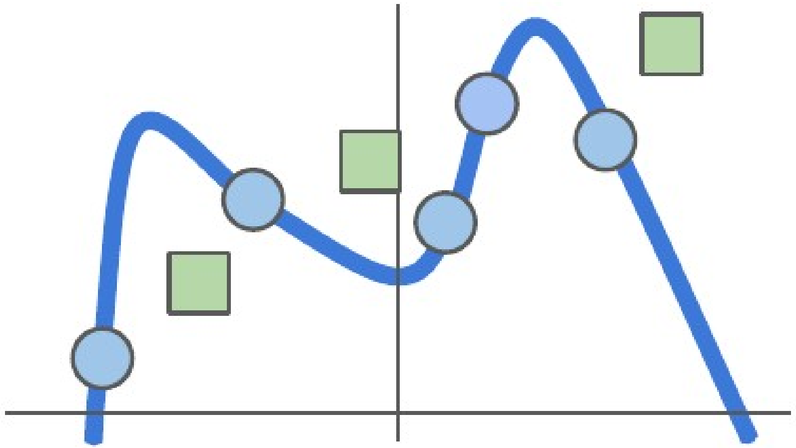
```
# Vanilla Minibatch Gradient Descent

while True:
  data_batch = sample_training_data(data, 256) # sample 256 examples
  weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
  weights += - step_size * weights_grad # perform parameter update
```

Регуляризация параметров W

$$\underbrace{L(W) = \frac{1}{N}\sum_{i=1}^{N} L_i(f(x_i, W), y_i)}_{}$$

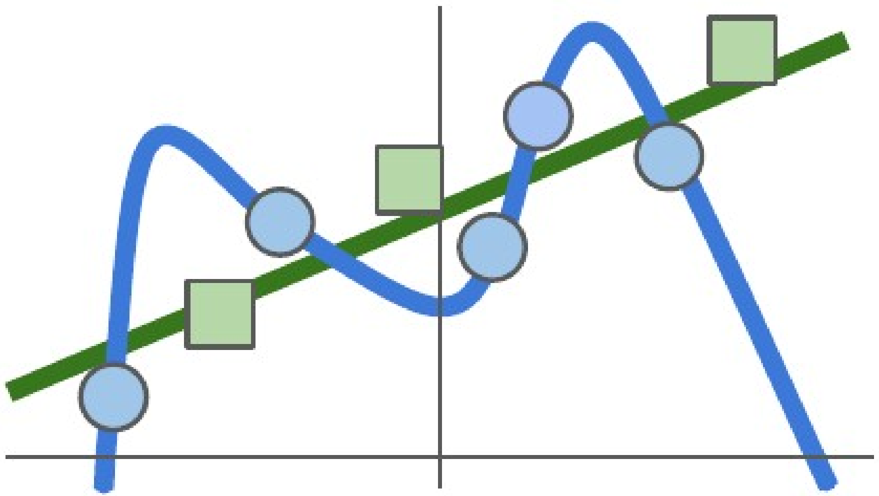
Data loss: Model predictions should match training data



Регуляризация параметров W

$$L(W) = \underbrace{\frac{1}{N}\sum_{i=1}^{N} L_i(f(x_i, W), y_i)}_{} + \underbrace{\lambda R(W)}_{}$$

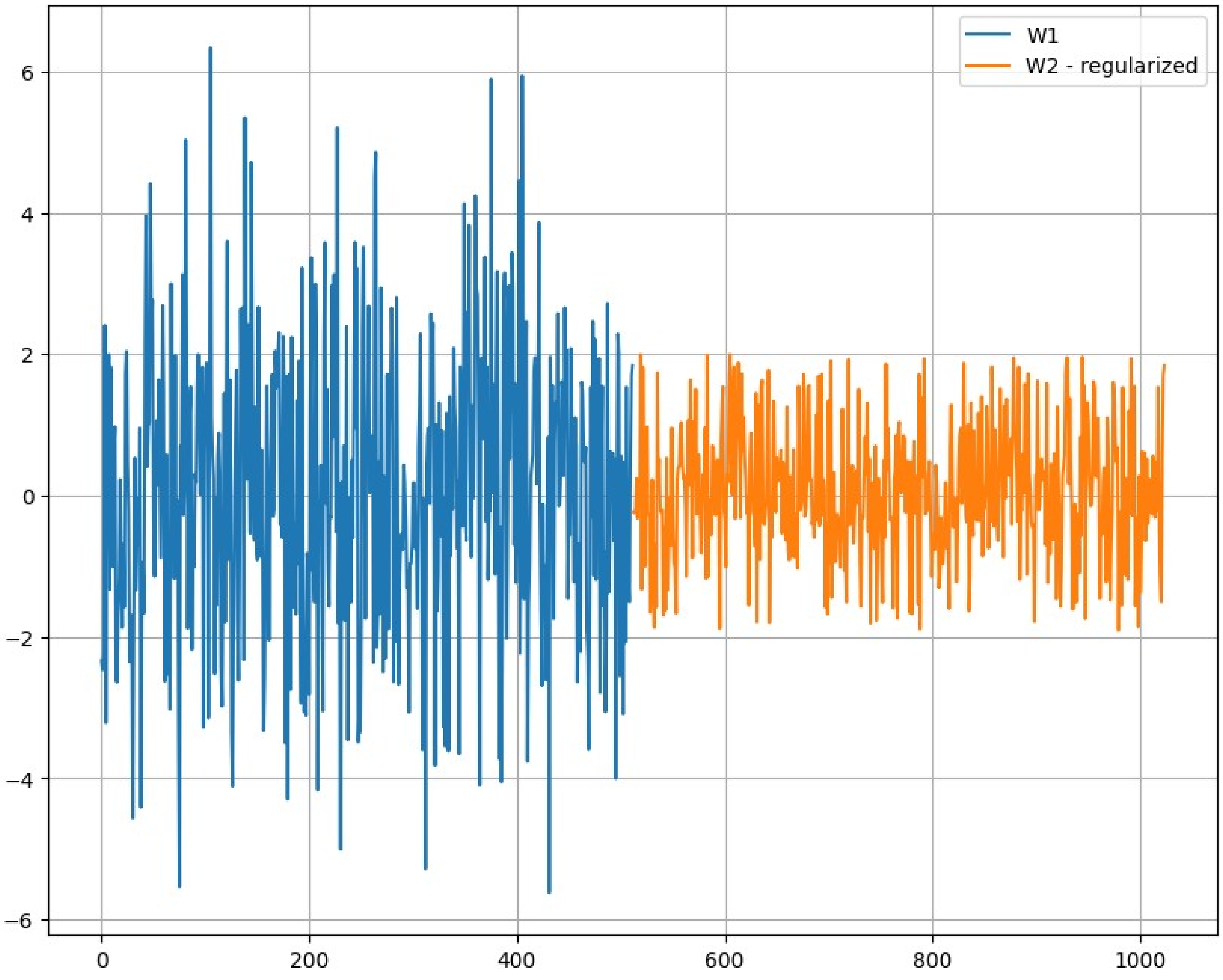
Data loss: Model predictions should match training data



Regularization: Model should be “simple”, so it works on test data

Occam’s Razor:
“Among competing hypotheses, the simplest is the best”
William of Ockham, 1285 - 1347

Регуляризация параметров W



$$R(W) = \sum_k \sum_l W_{k,l}^2$$

$$R(W) = \sum_k \sum_l |W_{k,l}|$$

$$R(W) = \sum_k \sum_l \beta W_{k,l}^2 + |W_{k,l}|$$

L2 Regularization (Weight Decay)

$$x = [1, 1, 1, 1]$$

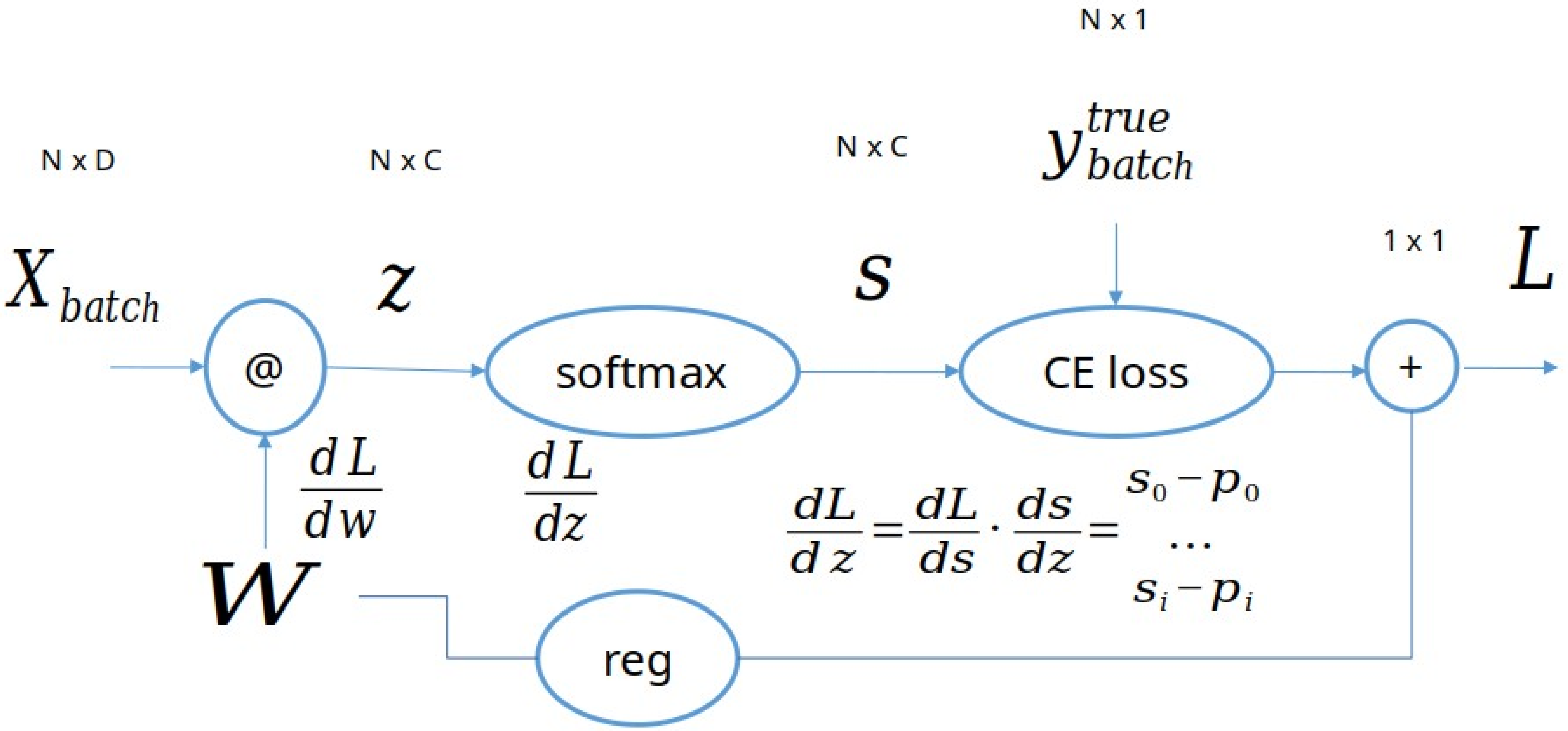
$$R(W) = \sum_k \sum_l W_{k,l}^2$$

$$w_1 = [1, 0, 0, 0]$$

$$w_2 = [0.25, 0.25, 0.25, 0.25]$$

$$w_1^T x = w_2^T x = 1$$

Регуляризация параметров W



N – number of batch samples
D – dimension of features
C – class

$$s = \frac{e^z}{\sum e^z}$$

$$L = -\frac{1}{N} \sum p \cdot \log(s) + \|W\| \cdot reg$$

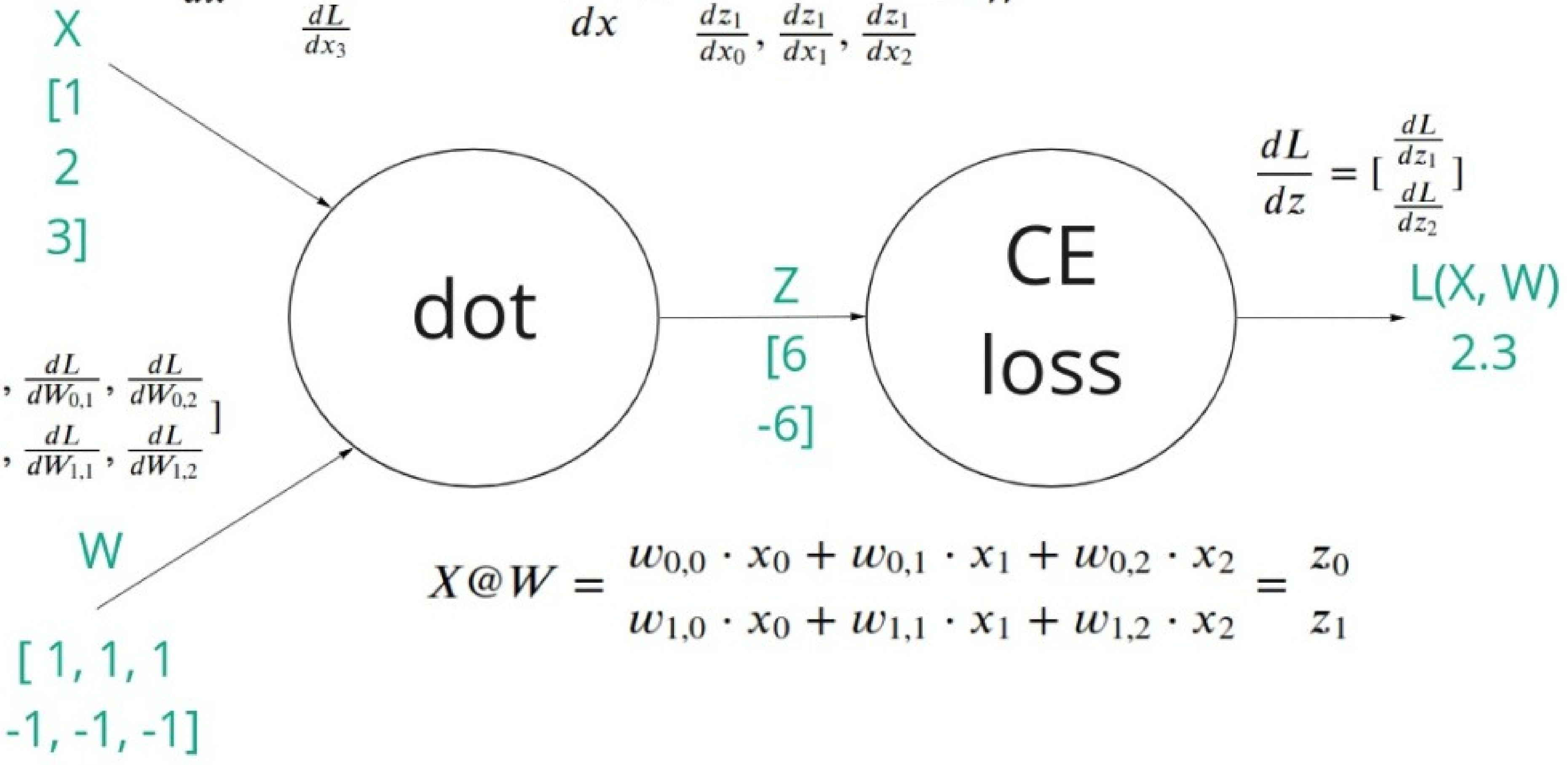
Gradient descent

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Numerical gradient: slow :(, approximate :(, easy to write :)
Analytic gradient: fast :), exact :), error-prone :(

In practice: Derive analytic gradient, check your implementation with numerical gradient

Производная функции нескольких переменных



$$\frac{dL}{dx} = \begin{matrix} \frac{dL}{dx_1} \\ \frac{dL}{dx_2} \\ \frac{dL}{dx_3} \end{matrix}$$

$$\frac{dz}{dx} = \begin{matrix} \frac{dz_0}{dx_0}, \frac{dz_0}{dx_1}, \frac{dz_0}{dx_2} \\ \frac{dz_1}{dx_0}, \frac{dz_1}{dx_1}, \frac{dz_1}{dx_2} \end{matrix} = W$$

$$\frac{dL}{dz} = [\begin{matrix} \frac{dL}{dz_1} \\ \frac{dL}{dz_2} \end{matrix}]$$

$$\frac{dL}{dW} = [\begin{matrix} \frac{dL}{dW_{0,0}}, \frac{dL}{dW_{0,1}}, \frac{dL}{dW_{0,2}} \\ \frac{dL}{dW_{1,0}}, \frac{dL}{dW_{1,1}}, \frac{dL}{dW_{1,2}} \end{matrix}]$$

$$X@W = \begin{matrix} w_{0,0} \cdot x_0 + w_{0,1} \cdot x_1 + w_{0,2} \cdot x_2 \\ w_{1,0} \cdot x_0 + w_{1,1} \cdot x_1 + w_{1,2} \cdot x_2 \end{matrix} = \begin{matrix} z_0 \\ z_1 \end{matrix}$$