ФИНАНСОВЫЙ УНИВЕРСИТЕТ
ПРИ ПРАВИТЕЛЬСТВЕ РОССИЙСКОЙ ФЕДЕРАЦИИ

Современные нейросетевые технологии

Лекция 2. Линейная классификация изображений

Учебные вопросы

1) Линейный классификатор изображений
2) Преобразование softmax
3) Функция потерь Cross Enthropy Loss

Материалы курса:
github.com/balezz/modern_dl
Срок сдачи A1 – 10.09.2022 г.

Источники:

- dlcourse.ai
- cs231n.stanford.edu
- cs230.stanford.edu

Линейный классификатор

Image Classification: A core task in Computer Vision

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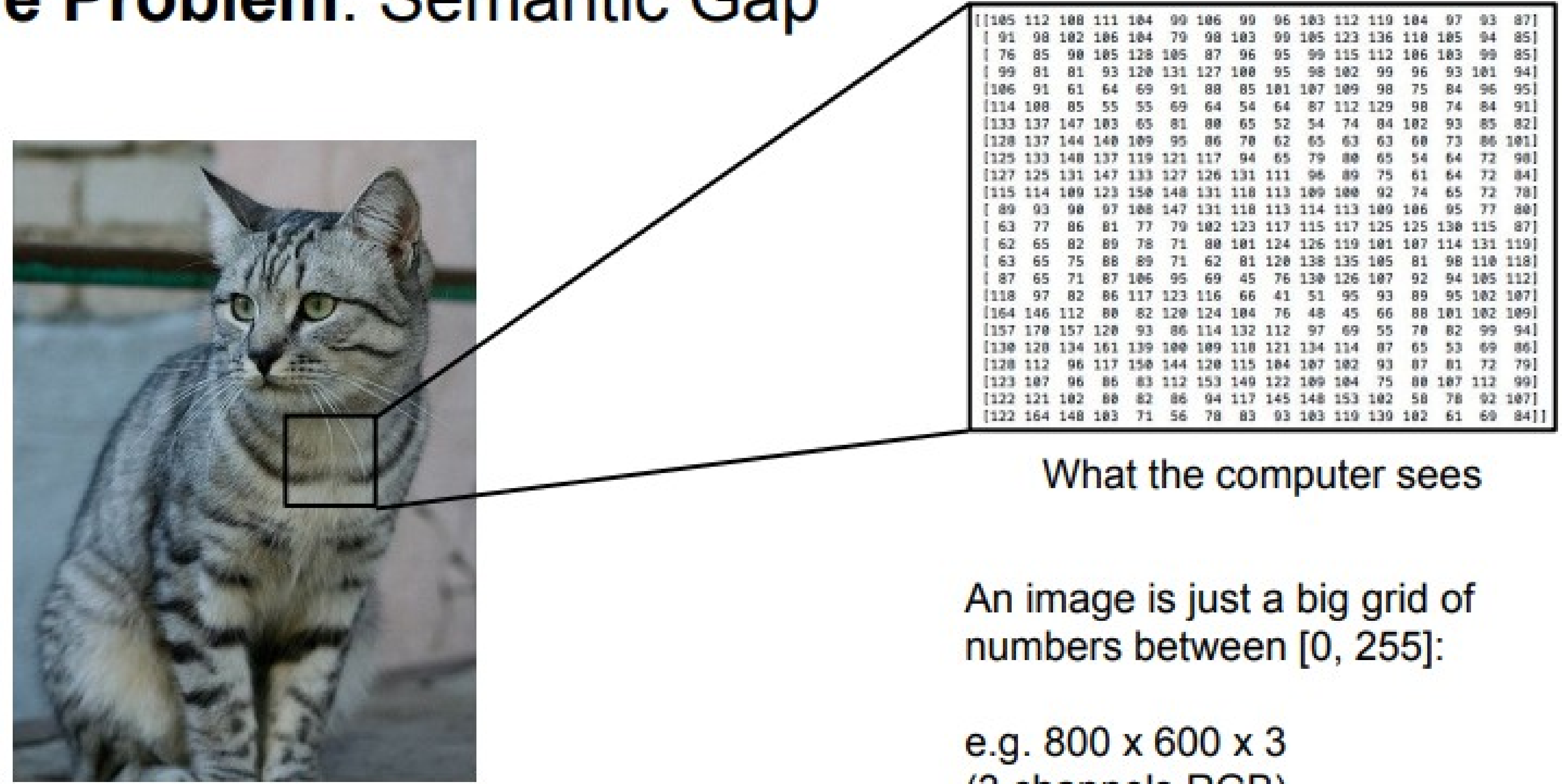
(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

→ cat

Линейный классификатор

The Problem: Semantic Gap



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 [ 99  81  81  93 120 131 127 100  95  98 102  99  96  93 101  94]
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 [122 164 148 103  71  56  78  83  93 103 119 139 102  61  69  84]]
```

What the computer sees

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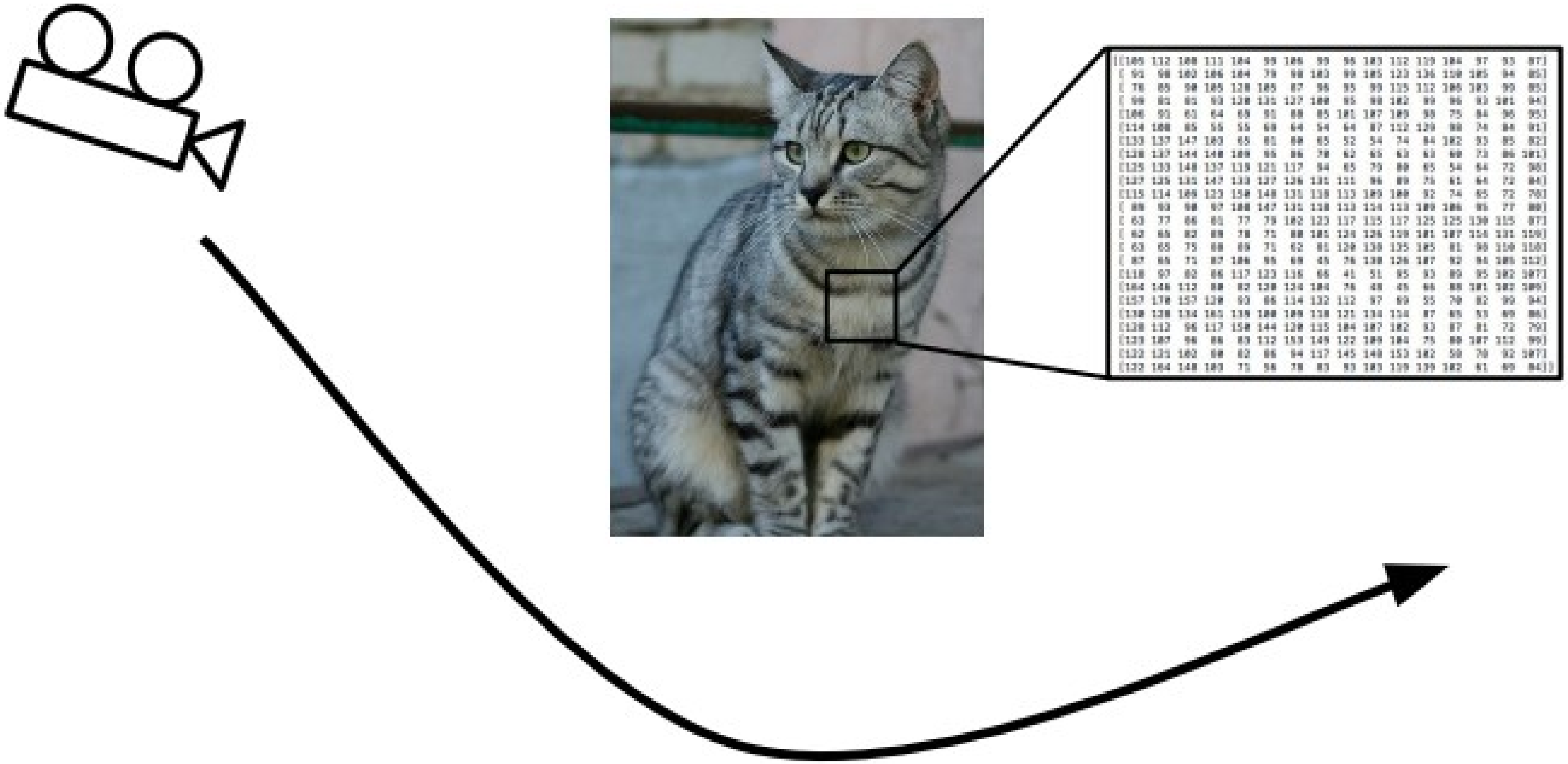
An image is just a big grid of
numbers between [0, 255]:

e.g. 800 x 600 x 3
(3 channels RGB)

Challenges: Viewpoint variation



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 [122 121 102  80  82  86  94 117 145 148 153 102  58  78  92 107]
 [122 164 148 103  71  56  78  83  93 103 119 139 102  61  69  84]]
```

All pixels change when the camera moves!

Проблемы классификации

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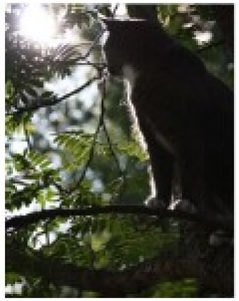

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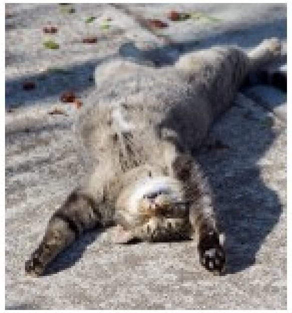

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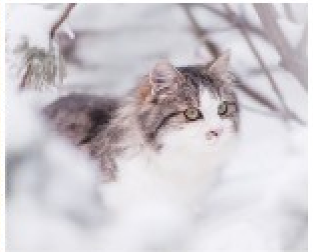

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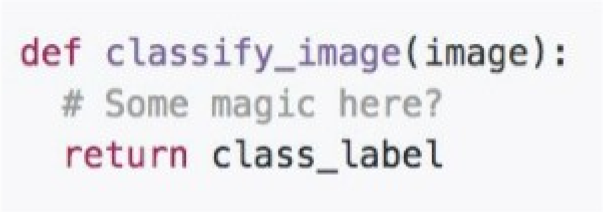
Линейный классификатор

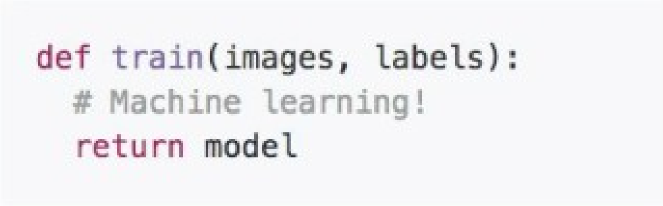
Naïve image classifier



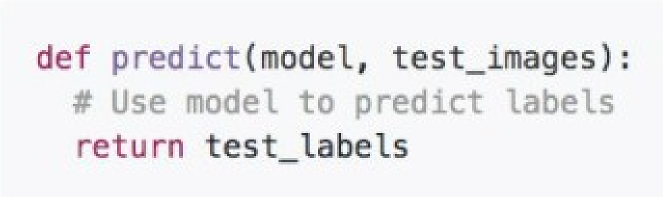
```
def classify_image(image):
  # Some magic here?
  return class_label
```

Unlike sorting, no obvious way to hard-cod the algorithm for recognizing a cat or other classes

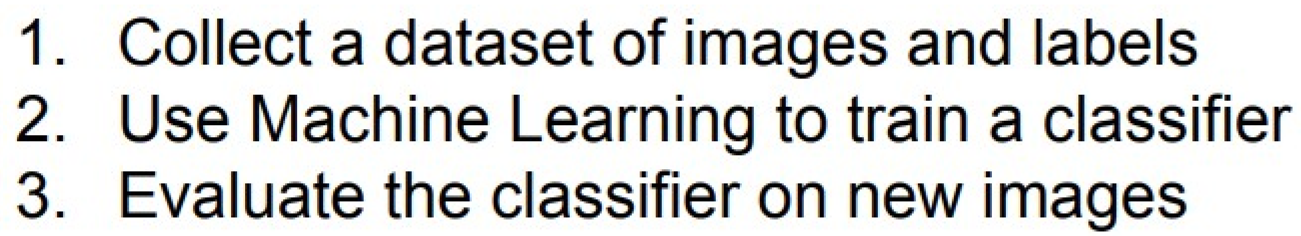
Data driven approach



```
def train(images, labels):
  # Machine learning!
  return model
```



```
def predict(model, test_images):
  # Use model to predict labels
  return test_labels
```

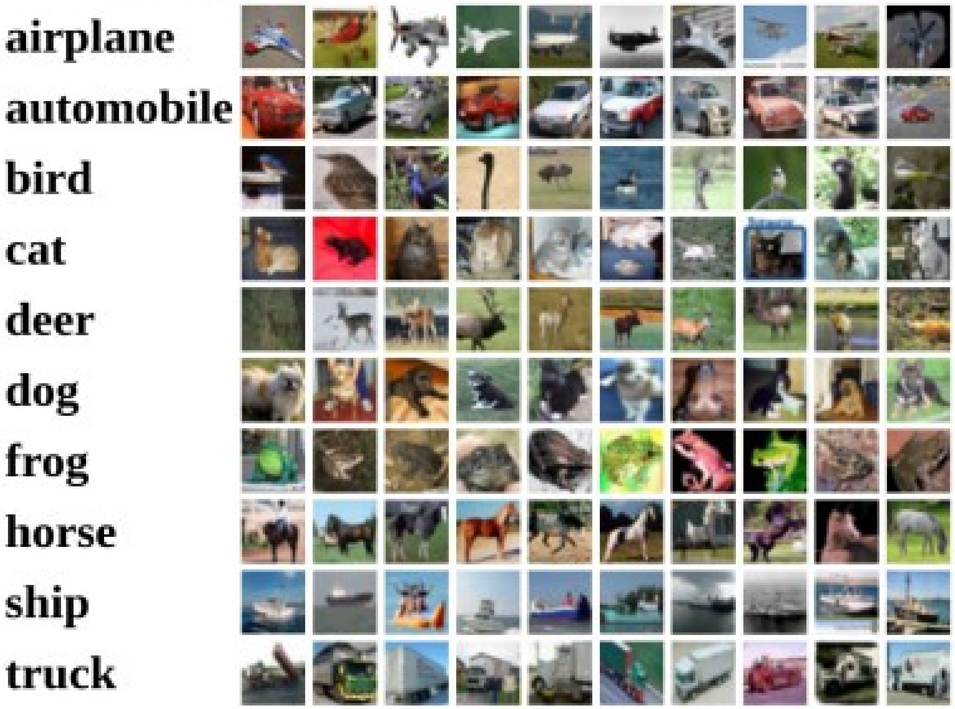


1. Collect a dataset of images and labels
2. Use Machine Learning to train a classifier
3. Evaluate the classifier on new images

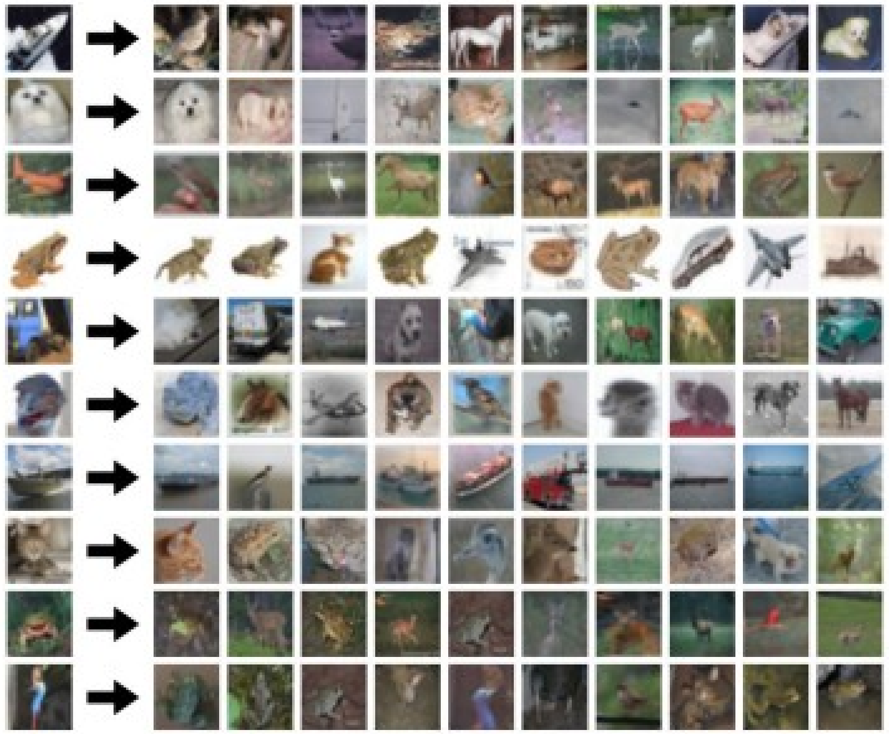
Линейный классификатор

Example Dataset: **CIFAR10**

10 classes
50,000 training images
10,000 testing images

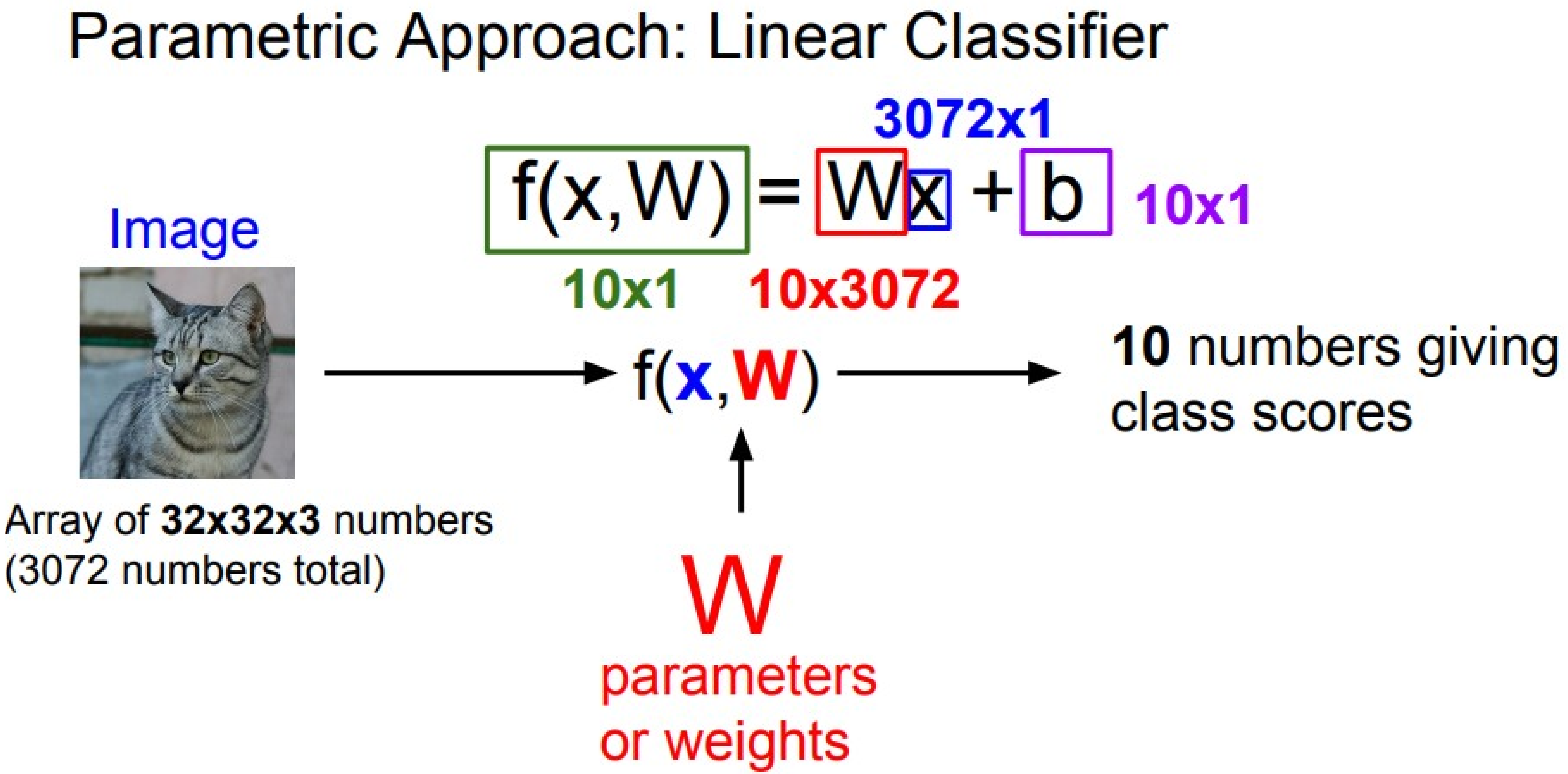


Test images and nearest neighbors



Alex Krizhevsky, "Learning Multiple Layers of Features from Tiny Images", Technical Report, 2009.

Линейный классификатор



Parametric Approach: Linear Classifier

$$\underbrace{\boxed{f(x,W)}}_{10x1} = \underbrace{\boxed{W}}_{10x3072}\overset{3072x1}{\boxed{x}} + \boxed{b}\quad 10x1$$

Image

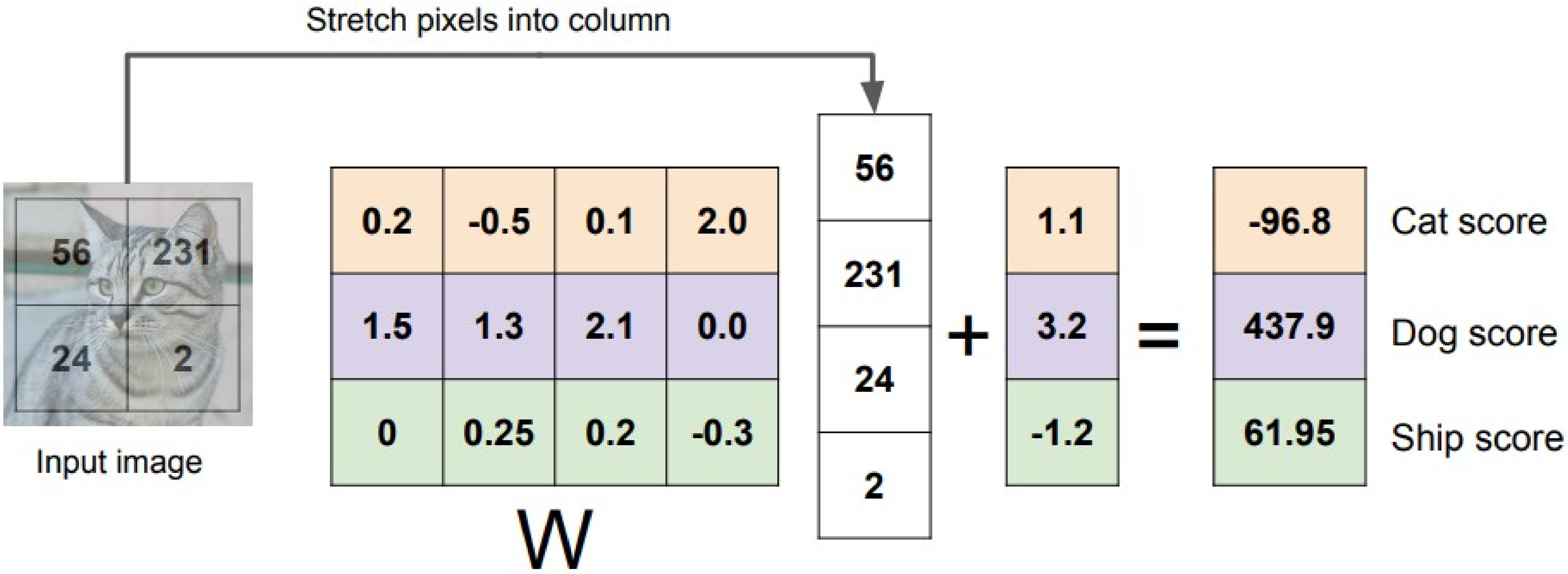
Array of **32x32x3** numbers
(3072 numbers total)

$\xrightarrow{\quad} f(x,W) \xrightarrow{\quad}$

10 numbers giving
class scores

W
parameters
or weights

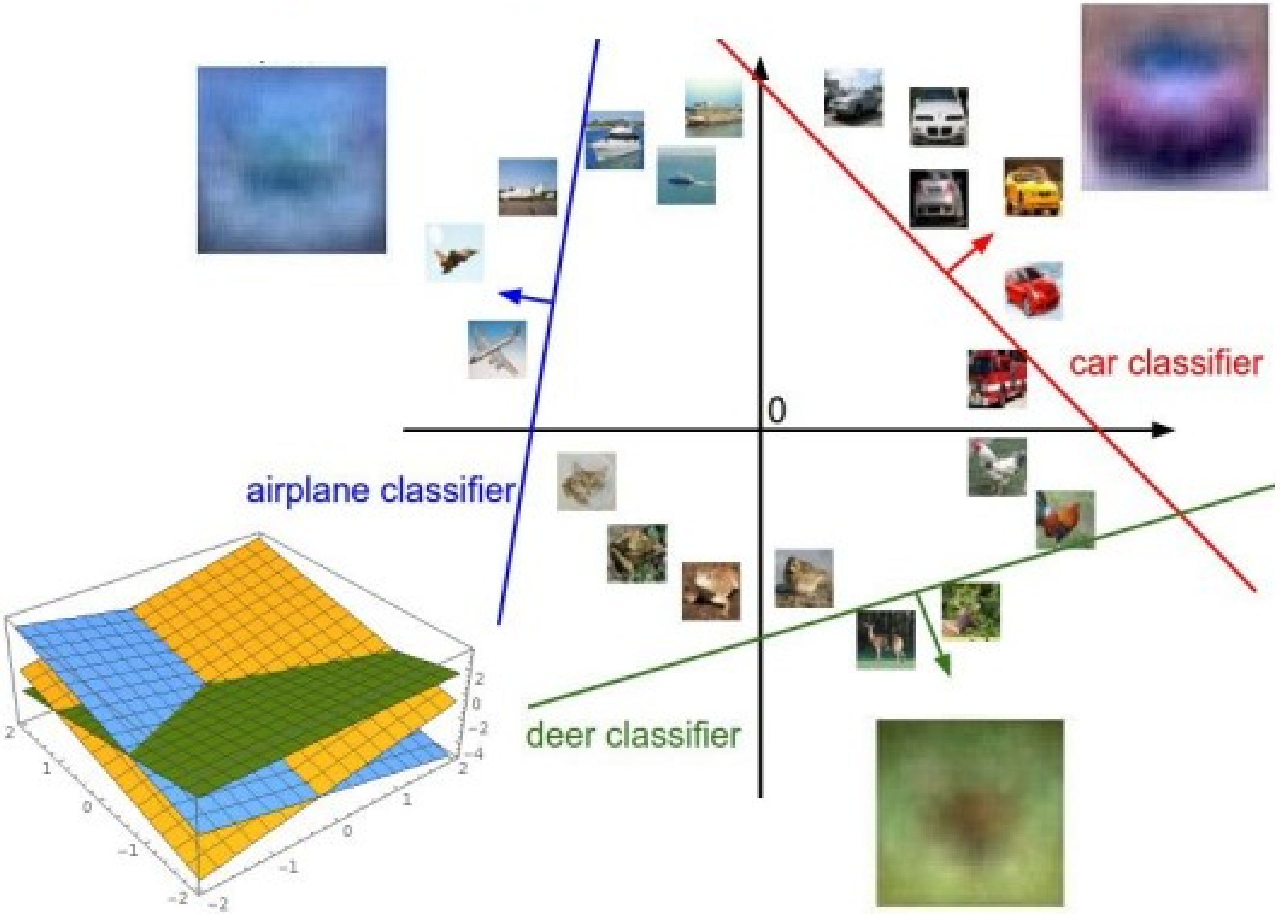
Линейный классификатор



$$\begin{bmatrix} 0.2 & -0.5 & 0.1 & 2.0 \\ 1.5 & 1.3 & 2.1 & 0.0 \\ 0 & 0.25 & 0.2 & -0.3 \end{bmatrix} \begin{bmatrix} 56 \\ 231 \\ 24 \\ 2 \end{bmatrix} + \begin{bmatrix} 1.1 \\ 3.2 \\ -1.2 \end{bmatrix} = \begin{bmatrix} -96.8 \\ 437.9 \\ 61.95 \end{bmatrix}$$

Линейный классификатор

Interpreting a Linear Classifier



$$f(x,W) = Wx + b$$

Array of **32x32x3** numbers
(3072 numbers total)

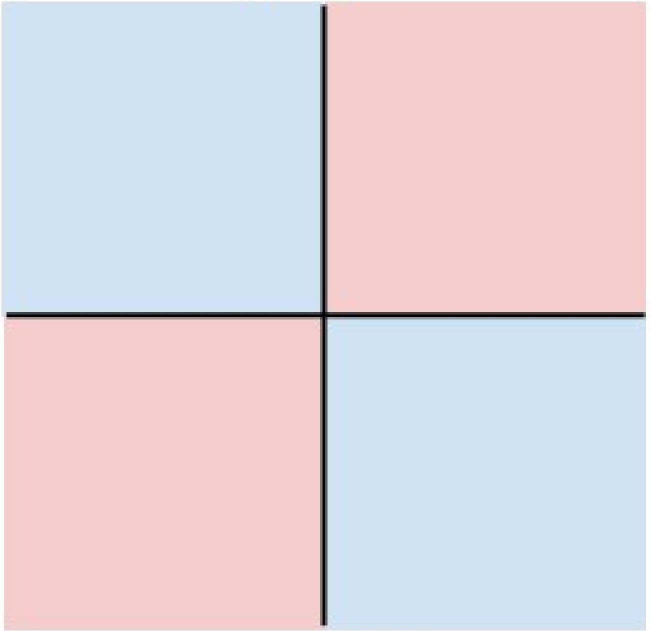
Plot created using Wolfram Cloud

Cat image by Nikita is licensed under CC-BY 2.0

Hard cases for a linear classifier

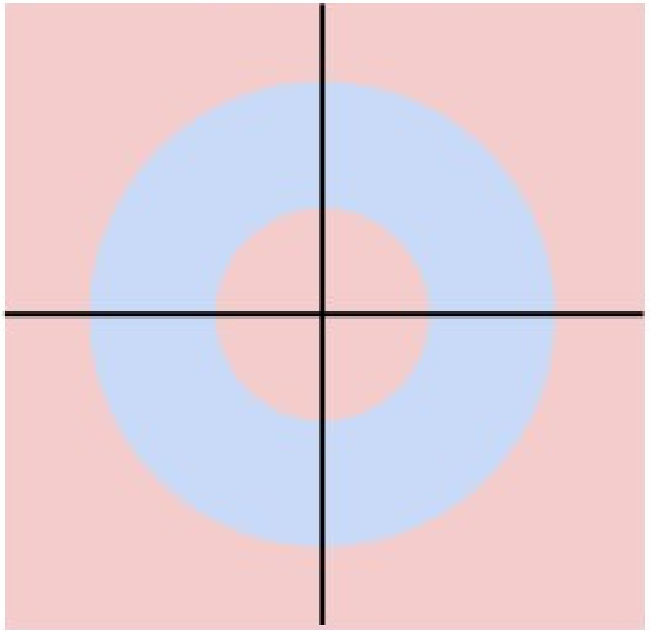
Class 1:
number of pixels > 0 odd

Class 2:
number of pixels > 0 even



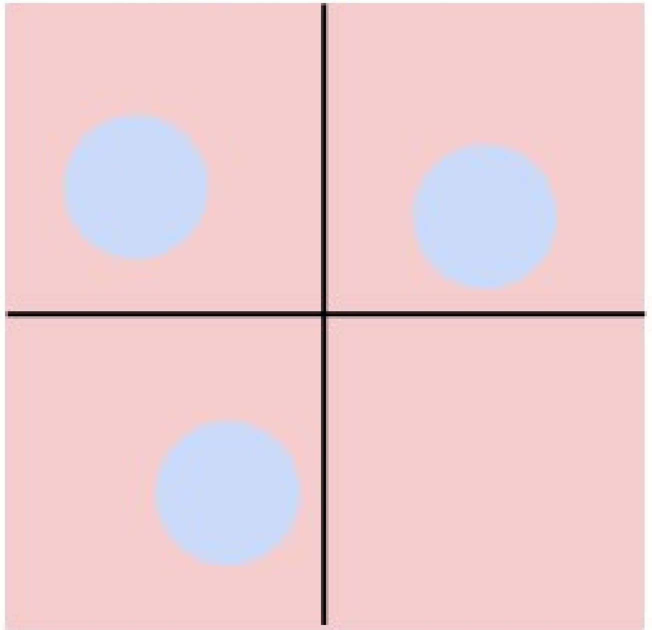
Class 1:
1 <= L2 norm <= 2

Class 2:
Everything else

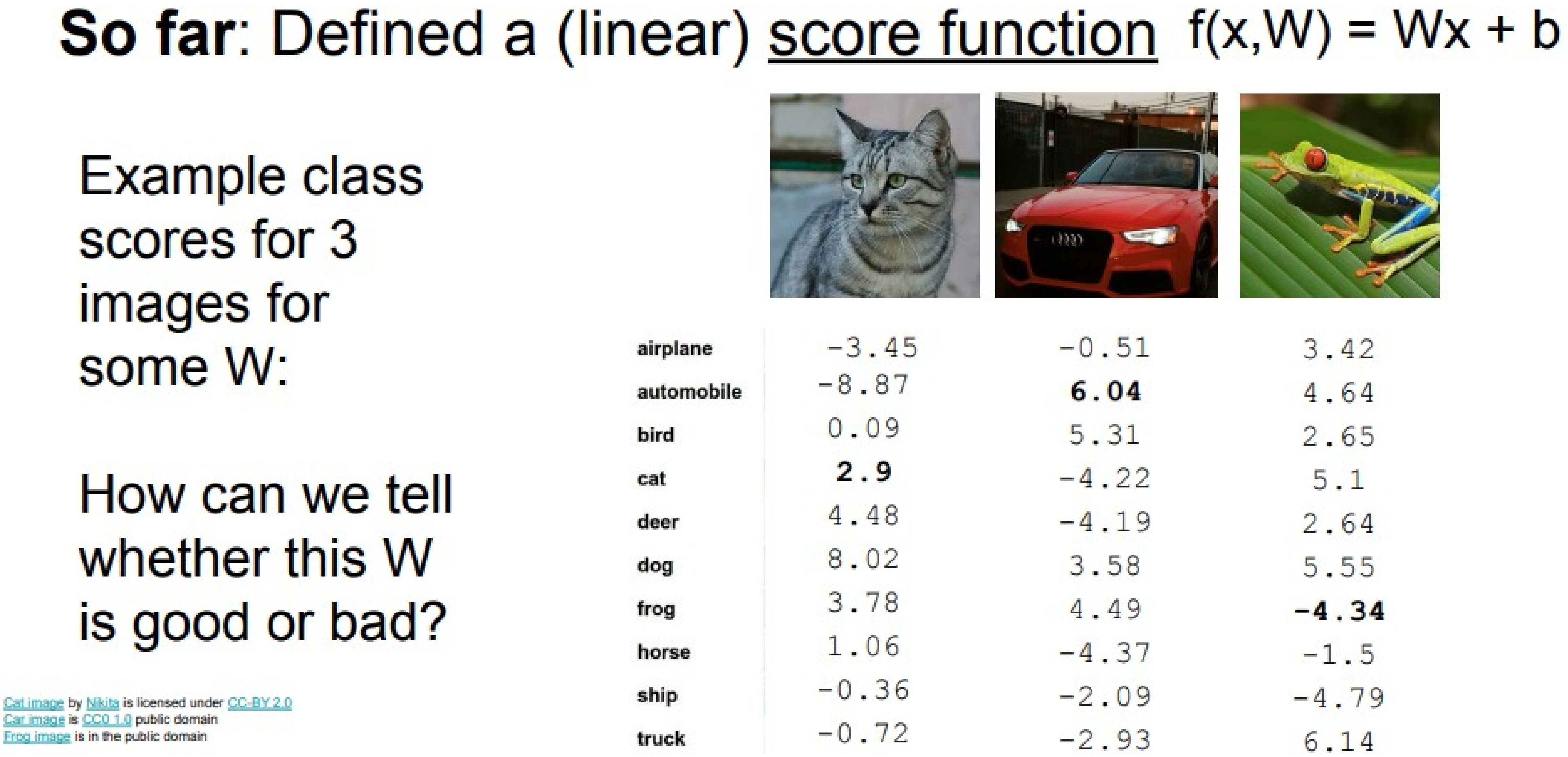


Class 1:
Three modes

Class 2:
Everything else



Функция потерь



So far: Defined a (linear) score function $f(x,W) = Wx + b$

Example class scores for 3 images for some W:

How can we tell whether this W is good or bad?

airplane	-3.45	-0.51	3.42
automobile	-8.87	**6.04**	4.64
bird	0.09	5.31	2.65
cat	**2.9**	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	**-4.34**
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

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Car image is CC0 1.0 public domain
Frog image is in the public domain

Функция потерь

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x,W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

A **loss function** tells how good our current classifier is

Given a dataset of examples

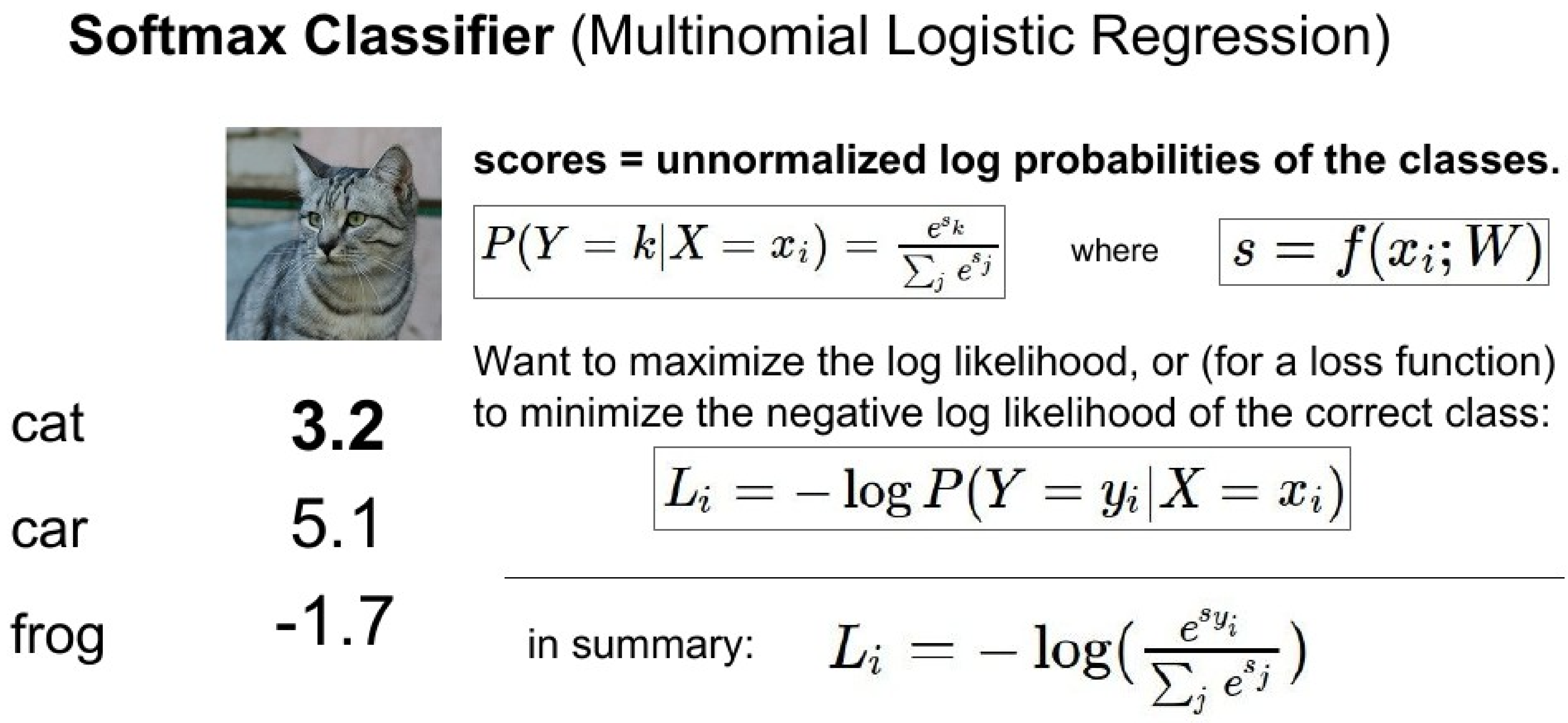
$$\{(x_i, y_i)\}_{i=1}^{N}$$

Where x_i is image and y_i is (integer) label

Loss over the dataset is a sum of loss over examples:

$$L = \frac{1}{N} \sum_i L_i(f(x_i, W), y_i)$$

Преобразование Softmax



Softmax Classifier (Multinomial Logistic Regression)

cat	**3.2**
car	5.1
frog	-1.7

scores = unnormalized log probabilities of the classes.

$$P(Y=k|X=x_i)=\frac{e^{s_k}}{\sum_j e^{s_j}}$$ where $$s=f(x_i;W)$$

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$L_i=-\log P(Y=y_i|X=x_i)$$

in summary: $$L_i=-\log(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}})$$

Функция потерь Cross Enthropy

$$\begin{array}{ccccccccc}
\text{X} & & & & \text{Z-scores} & & & \text{y\_prob} & \text{y\_true} \\
\begin{bmatrix}1\\56\\125\\4\\33\\42\\69\\167\\93\\66\end{bmatrix} & @ & \boxed{\quad W \quad} & = & \begin{bmatrix}10\\0\\-6\\25\\12\\16\\0\\-6\\-50\\11\end{bmatrix} & \dfrac{e^{z_i}}{\sum_{j=1}^{K} e^{z_j}} & = & \begin{bmatrix}0.1\\0\\0\\0.5\\0.1\\0.2\\0\\0\\0\\0.1\end{bmatrix} & \begin{bmatrix}0\\0\\0\\1\\0\\0\\0\\0\\0\\0\end{bmatrix}
\end{array}$$

Функция потерь Cross Enthropy

y_prob	y_true
0.1	0
0	0
0	0
0.5	1
0.1	0
0.2	0
0	0
0	0
0	0
0.1	0

$$H(p,q) = -\sum_{x \in \mathcal{X}} p(x)\, \log q(x)$$

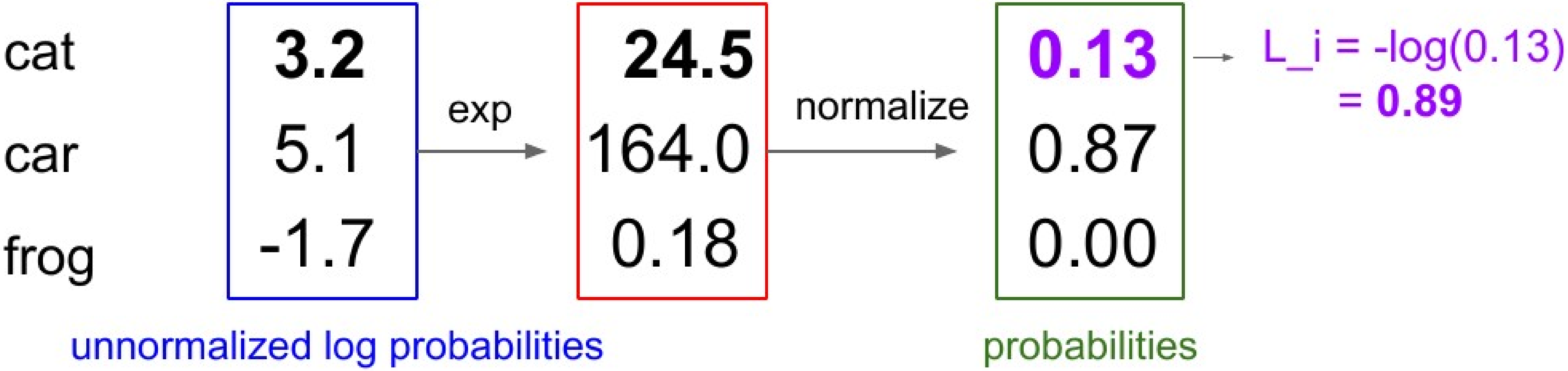
$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

Функция потерь Cross Enthropy

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities



cat 3.2 $\xrightarrow{\text{exp}}$ 24.5 $\xrightarrow{\text{normalize}}$ 0.13 → L_i = -log(0.13) = 0.89

car 5.1 → 164.0 → 0.87

frog -1.7 → 0.18 → 0.00

unnormalized log probabilities

probabilities