*Flux Package

PARTICLE FLUX ACROSS A CYLINDER – THE DIFFICULT QUESTION

Problem 1 – Interpreting the Flux

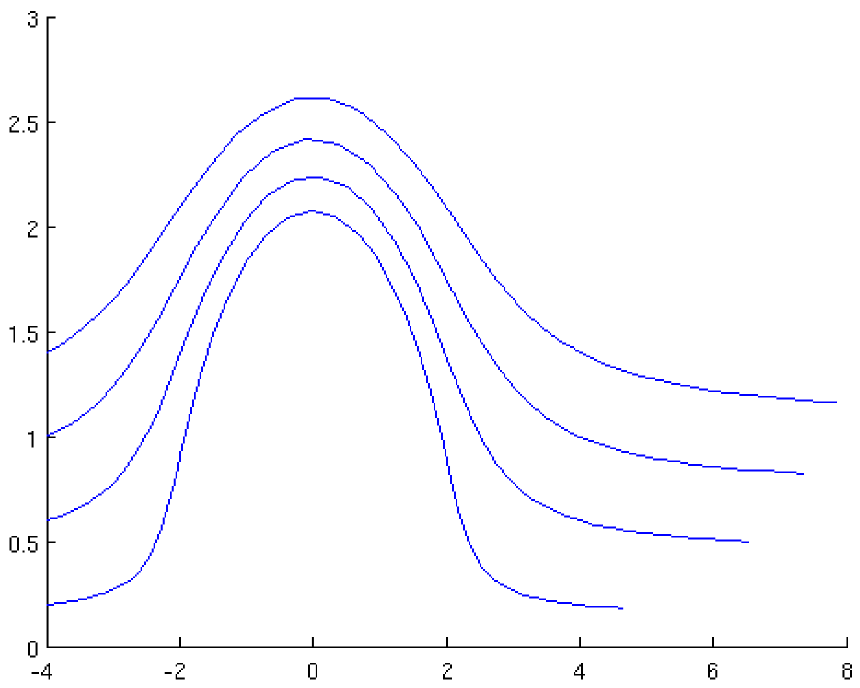
A cylinder with radius $R = 2$ is perpendicular to the direction of the flux (of particles). We are given that the position of a particle at any given time is determined by its initial position and the system of differential equations:

$$\frac{dx}{dt} = 1 - \frac{R^2(x^2 - y^2)}{(x^2 + y^2)^2}, \qquad \frac{dy}{dt} = -\frac{2xyR^2}{(x^2 + y^2)^2}$$

Initially, we have 4 particles at $t = 0$ having the same x value but different y values. We want to:

1. Find and draw the movement of the particles (find the flux) for $t = [0,12]$
2. The bottom particle is too slow. Find t when the bottom particle achieves the same y position as that of the top particle.

Solution 1 – Interpreting the Flux



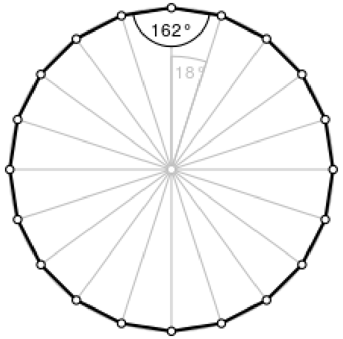
To solve the differential equation, we use **ode45**. The figure on LHS clearly depicts the presence of a cylinder.

In order to find the time t when the bottom particle gets the same y coordinate as the top one at $t = 12$, we continue solving the differential equation for a larger time interval. We tried $t = [0,16]$ with 0.01 as step length.

The time turns out be $t = 15.62$

Problem 2 – The Big Picture

1. We want to examine the way a set of particles will be affected by the cylinder. This is achieved by setting up particles in a familiar shape, in our case, a polygon with 20 edges. Our aim is to observe the position of the particles at different instances in time.

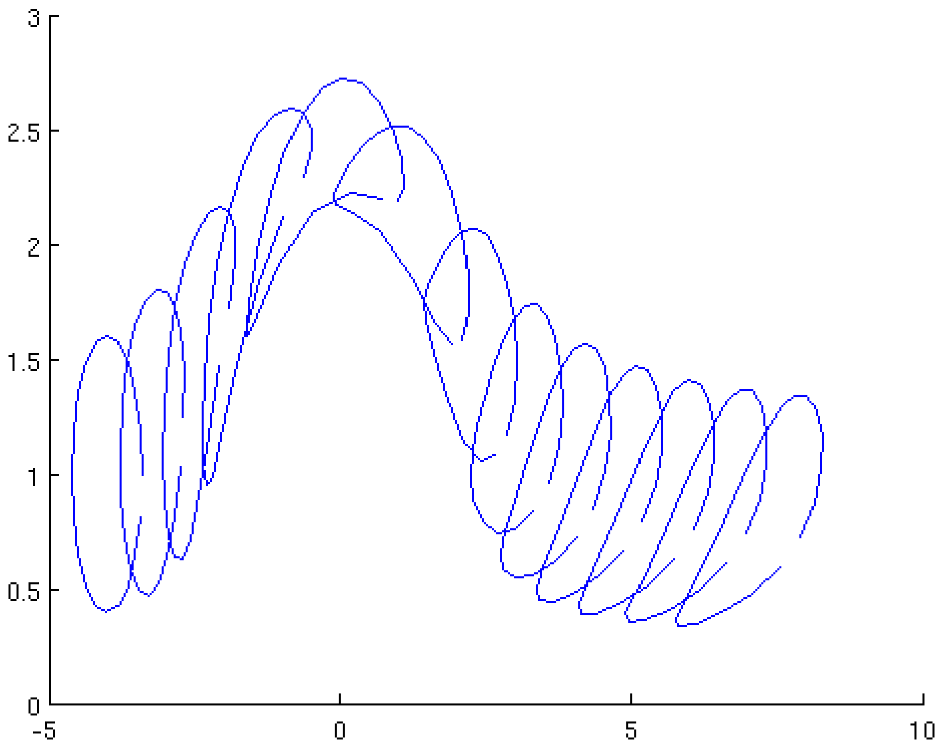


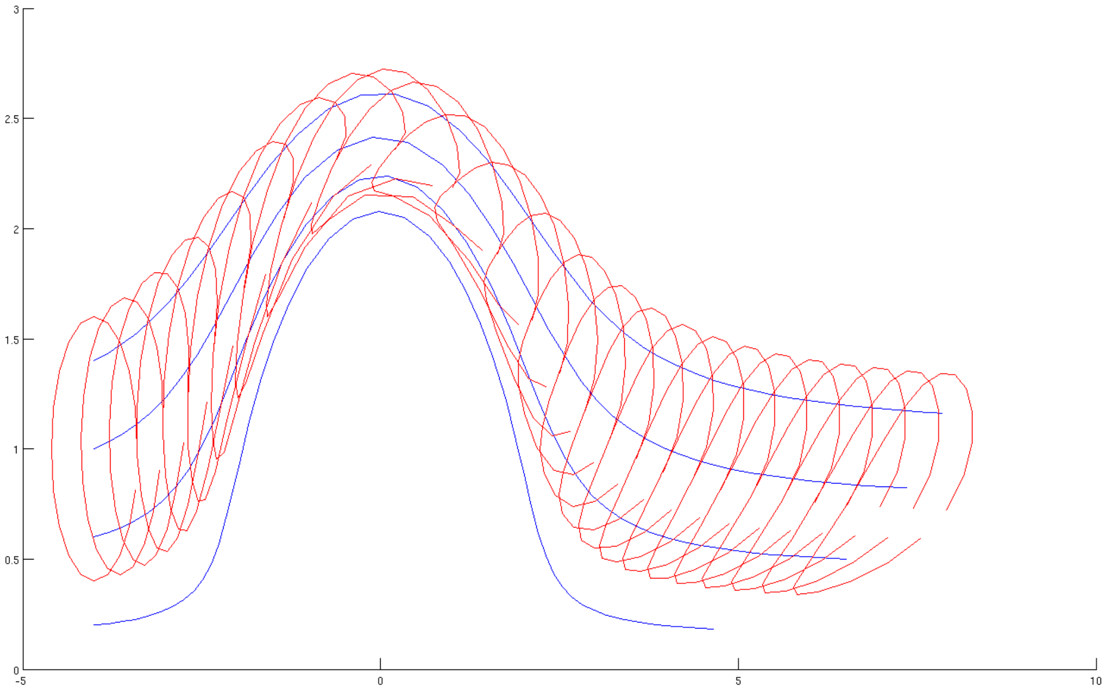
2. Later, we should examine the area of each polygon at these instances. Area is given by:

$$A_{polygon} = (x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + \cdots + x_ny_1 - x_1y_n)/2$$

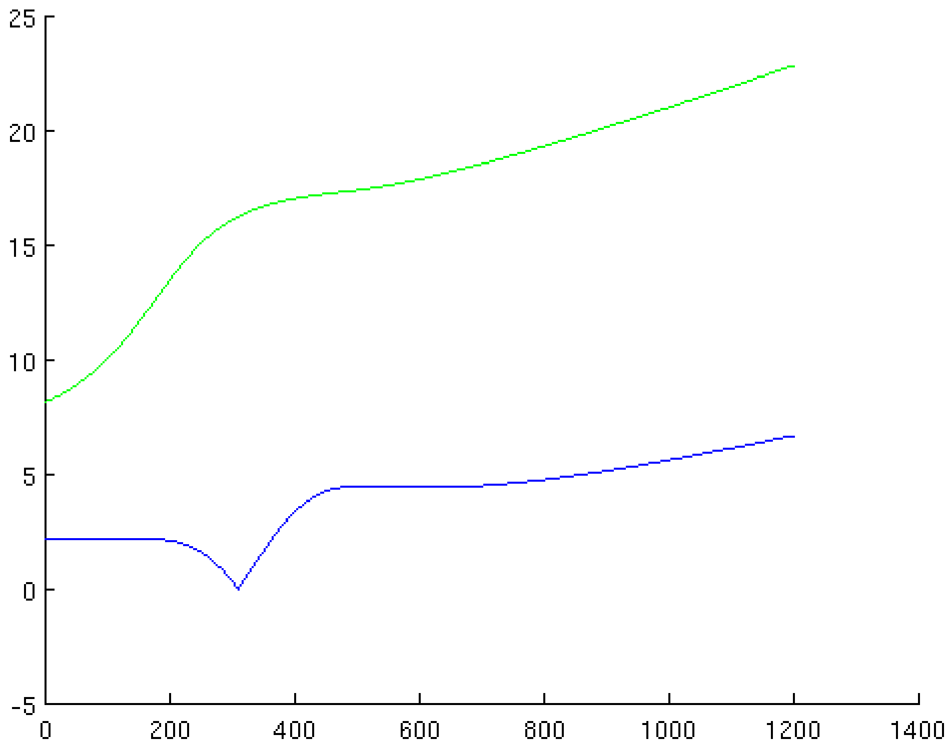
3. Later, we want to customize the configuration of the particles as well as changing the no. of edges in the polygon from 20 to 40.

Solution 2.1 – The Big Picture





Solution 2.2 – The Big Picture



Blue line shows how the area
changes over time (1/100 of a second) (extrapolating
the area of polygons with 20 edges and 10 edges).

The green line shows the area of a polygon with 10 edges
using our own pre-configuration.

The blue line was achieved using Richard Extrapolation.

Problem – The "Not So Easy" Question

Here we were supposed to replace the cylinder with a sphere centred at origin with $R = 2$. The sphere has a positive dipole in the direction of the positive x-axis. The position of any particle is given by the following set of differential equations:

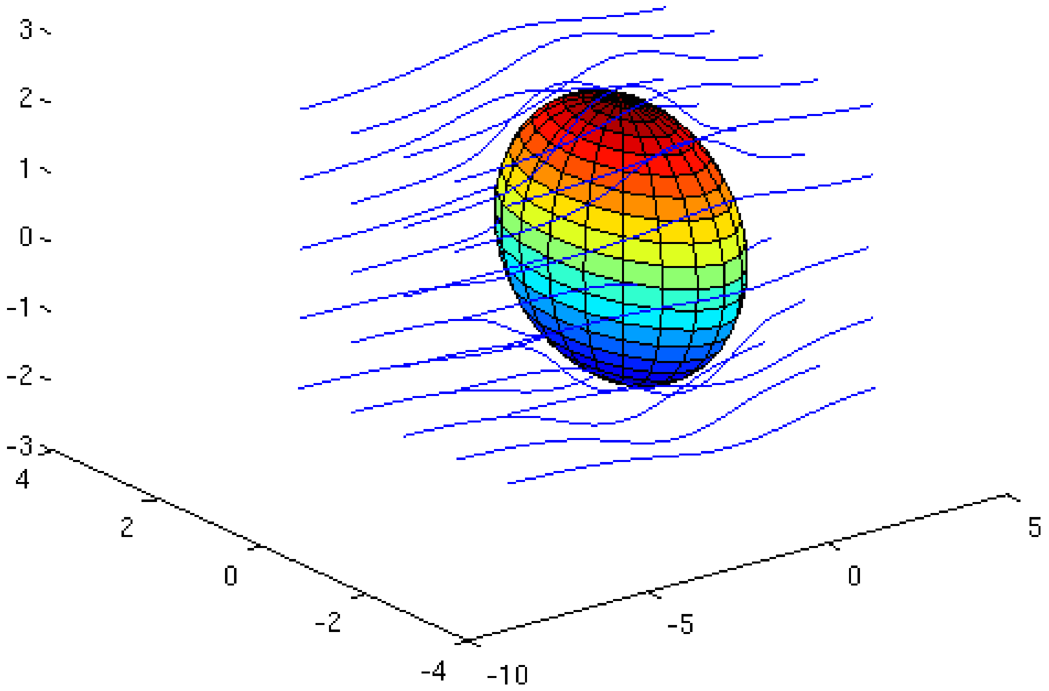
$$\frac{dx}{dt} = 1 - \frac{R^3(2x^2 - y^2 - z^2)}{2s^5}\,, \qquad \frac{dy}{dt} = -\frac{R^3xy}{2s^5}\,, \qquad \frac{dz}{dt} = \frac{-R^3xz}{2s^5}$$

$$s = \sqrt{x^2 + y^2 + z^2}$$

Our aim is to examine the way particles deform close to the sphere and later find the area of the deformed polyhedrons.

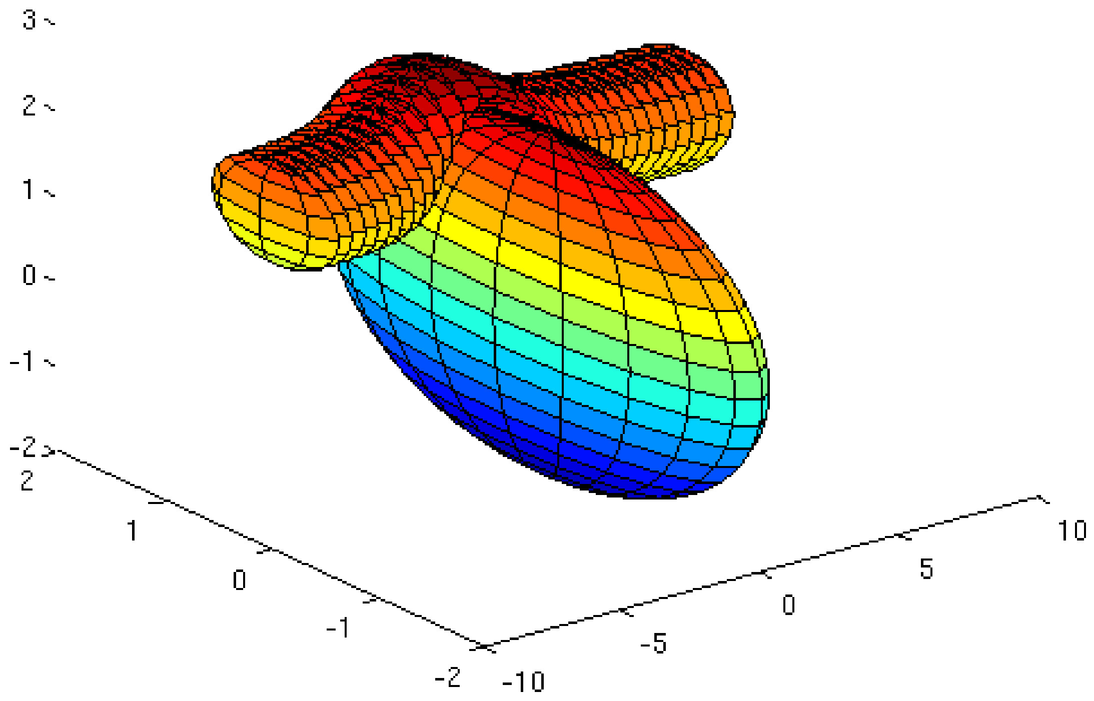
Solution – The "Not So Easy" Solution

This is an illustration of the flux for certain particles.



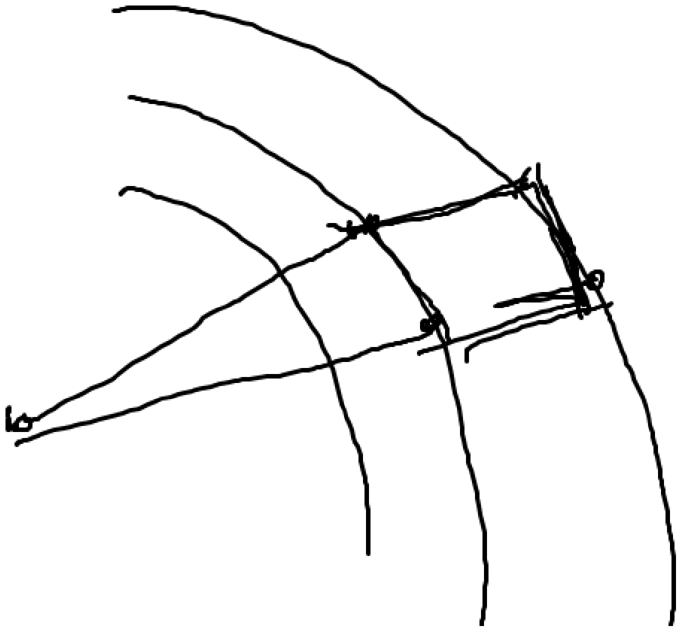
Solution – The "Not So Easy" Solution

This depicts the deformation of the polyhedron as it approaches the sphere.



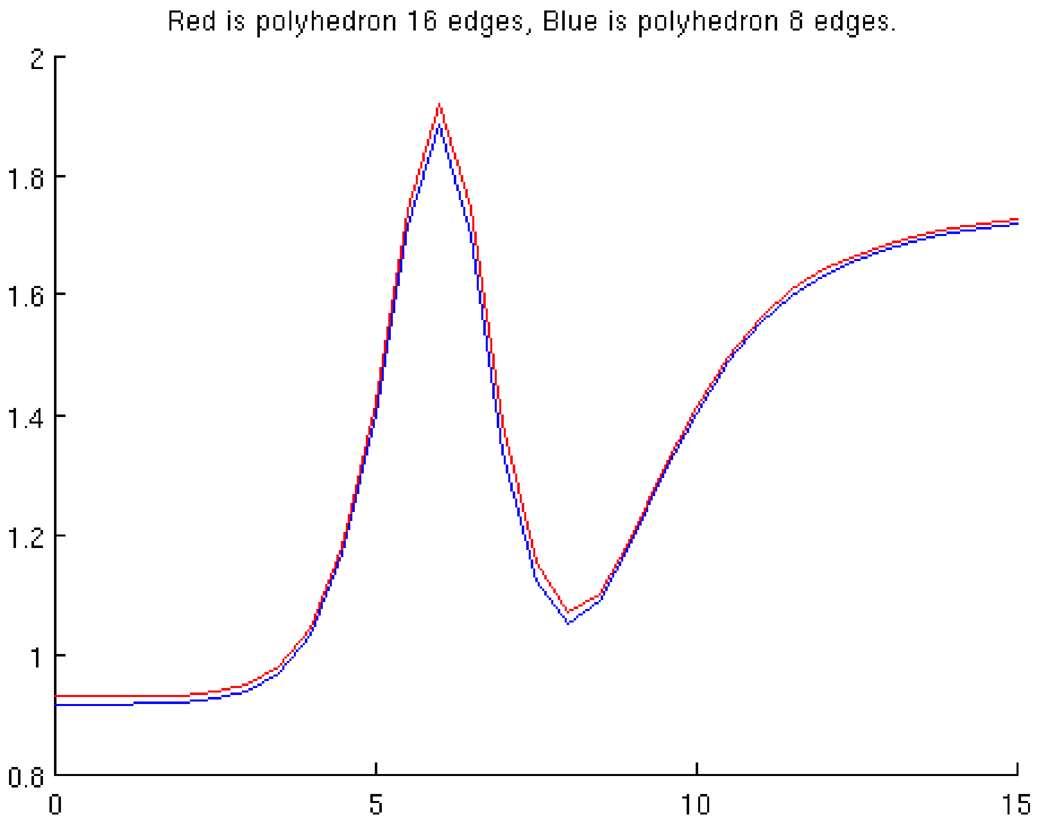
Finding the Volume of a Polyhedron

In order to find the volume of a polyhedron, we split it up into pyramids, whose area is common knowledge.



For this to work, we need points on the **boundary** of the polyhedron and one point **inside** the polyhedron. This can be an interior point and a boundary point (both types are inside the domain). We know boundary points, so we simply pick one of them.

Finding the Volume of a Polyhedron



Using the method we constructed to figure out the area of a polyhedron, we examined the way area changes as they approach the sphere.

The red line represents the area change of a polyhedron with 16 edges, while the blue line represents a polyhedron with 8 edges.

There's no significant deviation: both exhibit the same properties.