Computer Science 220S2C (2018)

Assignment 3 (Graph Algorithms)

Due date October 20, 2018, 11pm

Automarker Submission

This assignment tests your understanding of graph optimization problems. To get marks for this assignment you need to submit three programs for the following tasks to `http://www.cs.auckland.ac.nz/automated-marker` twice. Note this marker runs on a linux box so no Microsoft specific code should be used; read the help/FAQ for more details. Each program will have an easy test case (2 marks) and harder test case (1 mark). Please use the suffix `E` of your program basename to denote 'E'asy (test data) and `H` to denote 'H' arder (test data).

For Task 1, name your source code `mstE.ext` and `mstH.ext`, where `ext` denotes one of {`java`, `cpp`, `py`, `cs`} to indicate a java/c++/python/c#(mono) program. For Task 2, name your source code named `maxpathE.ext` and `maxpathH.ext`, respectively. For Task 3, please submit source code named `snakeE.ext` and `snakeH.ext`, respectively.

An excessive number of submissions (over 10) for a particular problem will accrue a 20% penalty per that problem if you eventually solve it.

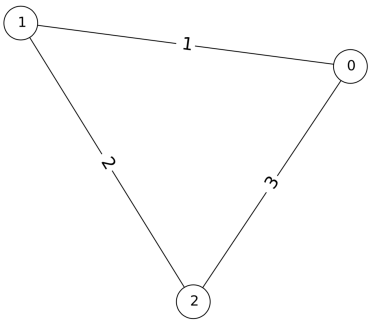
Task 1: Minimum Spanning Trees

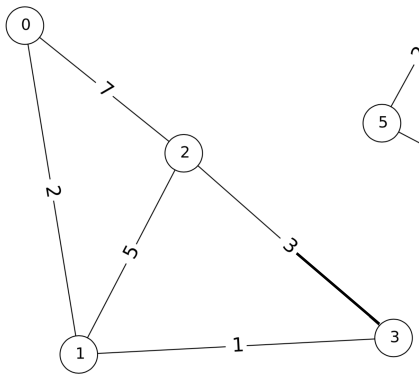
For this warm-up task you are to implement any efficient minimum spanning tree algorithm that takes a sequence of edge-weighted graphs and outputs the minimum cost weight of a spanning tree of each.

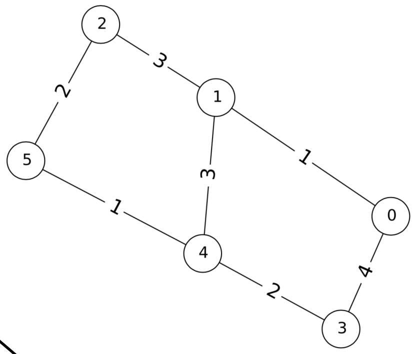
Input Format

For this assignment we use adjacency matrices with positive integer weights. Here a zero entry at row i and column j indicates that no edge ij exists in the graph. The first line consists of an integer $n \leq 1000$ denoting the order of the graph. This is then followed by n lines of n white-space separated integers denoting edge weights. The sequence of graphs is terminated by a value $n = 0$, which is not processed.

```
3
0 1 3
1 0 2
3 2 0
4
0 2 7 0
2 0 5 1
7 5 0 3
0 1 3 0
6
0 1 0 4 0 0
1 0 3 0 3 0
0 3 0 0 0 2
4 0 0 0 2 0
0 3 0 2 0 1
0 0 2 0 1 0
0
```







Output Format

Output should be one line for each input graph indicating the minimum cost weighted tree. The sample output for the previous input cases is as follows.

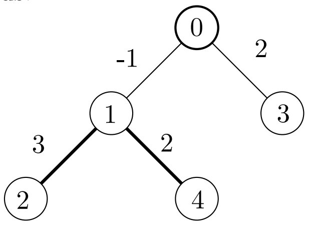
```
3
6
9
```

Task 2: A Paths Optimization Problem for Weighted Trees

A developer has designed a subdivision within a city such that all roads connect at intersections in a treelike design. This is to prevent all petrol-head hooligans from disturbing the residences by not having any road loops for races. Only the entering intersection is connected to the rest of the city. The developer is selling off land alongside roads between adjacent intersections. A real estate agent has produced a green book indicating the expected dollar profit (positive, zero or negative) that can be obtained by purchasing the land alongside each road.

Potential buyers want to maximize their profit, but prefer to buy a contiguous stretch of land alongside a simple road chain that connects two intersections of the subdivision. Your task is to write a program to determine the maximum non-negative profit that can be obtained this way.

As an example, consider the following representation of a subdivision, where road labels represent expected profits. In this scenario, the maximum non-negative profit is 5, and can be obtained alongside the emphasized roads:



Input Format

Input for this problem consists of a sequence of one or more scenarios, taken from the keyboard/stdin. Each scenario contains two or more lines.

- The first line contains an integer n, $1 \leq n \leq 500000$, indicating the number of intersections, including the entrance intersection, implicitly labelled 0.
- This is then followed by one or more lines, containing $n - 1$ pairs of integers. All integers are separated by single spaces or newlines. The y-th intersection is defined by the y-th pair of integers 'x p', where $1 \leq y < n$, $0 \leq x < y$, $-1000 \leq p \leq 1000$. This pair indicates that a road is added between y and a previously defined intersection, x, with a green book profit value given by p.

The input will be terminated by a line consisting of one zero (0). This line should not be processed. Some sample input scenarios follows.

```
5
0 -1 1 3 0 2 1 2
5
0 1 1 -3 0 -2 1 -2
5
0 -1 1 -3 0 -2 1 -2
10
0 -1 0 -1 0 0 1 3 1 4 2 4 2 2 3
3 3 3
0
```

Output Format

Output will be a sequence of lines, one for each input scenario. Each line will contain an integer, indicating the maximum non-negative profit sum, over all possible simple road chains connecting two intersections of the subdivision. Write zero (0) if no profit can be obtained. The output for the previous sample input follows.

```
5
1
0
7
```

Task 3: Snakes in a Graph

For a graph $G = (V, E)$, a **snake** (also called an *induced path*) is a path $v_1, v_2, \ldots, v_k$ such that for all $j - i > 1$ $(v_i, v_j) \notin E$. That is, it is a sequence of vertices in G such that each two adjacent vertices in the sequence are connected by an edge in G, and each two nonadjacent vertices in the sequence are not connected by any edge in G.

For this task, our goal is to determine the largest snake of a graph.

Input Format

Input for this problem consist of a sequence of one or more (undirected) graphs taken from the keyboard. Each graph is represented by an adjacency list. The first line is an integer n indicating the order of the graph. This is followed by n white space separated lists of adjacencies for nodes labeled 0 to $n - 1$. The input will be terminated by a line consisting of one zero (0). This line should not be processed. Two sample input graphs are listed below. The easy (harder) test cases are graphs of order at most 10 (50).

```
4
1
0 3

1
5
1 4
0 2
1 3 4
2 4
0 2 3
0
```

Output Format

Output will be just one integer per line sent to the console (e.g. `System.out`). For the above, input we would output the following two integers denoting the longest snake of the two graphs. Recall that the length of a path is the number of edges.

```
2
3
```