Initial overview of the L-unification

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1 Disadvantages of the standard model

Non-sufficient self-consistency of the underlying version of quantum field theory.
Incapability of describing the gravitational sector at quantum level.
The standard model is too complicated to be the dreamlike distinguished fundamental theory.

2 Two basic ways to create a new theory

One can think that the particles of the standard model are not really elementary, but are made from a smaller number of simpler components – this leads to development of composite, e.g. technicolor models.
One can think that the particles of the standard model are only fragments of some bigger, more symmetric and universal entities – this is the idea behind unification models.

3 Unification models

The basic idea of all unification models is to enhance the symmetry, embedding the non-simple gauge group $G_{st} = SU(3) \times SU(2) \times U(1)$ of the standard model into a larger simple group G, which inevitably has larger dimension and thus introduces extra particles.

4 Main idea

The central idea of Lisi's approach is to consider a minimal modification – ideally just a reformulation – of the standard model, with as few additional fields as only possible.
No extra essences are introduced, like superpartners, extra dimensions, branes, strings etc.
Since the number of fields in the standard model is of the order of 200, the number of different generators of "unification algebra" should exceed this number. If Lie algebra is classical, like $SU(N)$, $SO(N)$ or $Sp(N)$, this requires it to have a large rank $= \sqrt{200} > 14$. If instead the group is exceptional, then the smallest group with dimension > 200 is actually the largest: E_8 with $dim(E_8) = 248$ and its rank is as low as $rank(E_8) = 8$ (dimension of the next smaller exceptional group $dim(E7) = 133 < 200$).

5 L and G - unification

Furthermore, while in G-unification models, in order to minimize the number of 3 additional gauge bosons, one attempts to find a group of the lowest dimension with a given rank, in L-unification for the same purpose one tries to minimize the rank for a given dimension, which is now defined by the number of standard-model particles, predominantly fermions.

6 The number of known elementary particles

Standard model fermions numbers $n_F = 96$ (degrees of freedom) and $N_F = 192$ (fields).
Standard model bosons numbers $n_B = 30$ and $N_B = 92$.

7 Without extra particles and fields?

Extra particles, are, however, necessary in any case: because of the rank mismatch: there is simply no Lie group which could both match the strict lower limit $dim(G) \geq n_B + n_F > n_F = 90$ and saturate the rank condition $rank(G) = rank(U(1)) + rank(SU(2)) + rank(SU(3)) + rank(SO(3,1)) = 1+1+2+2 = 6$.

8 The main claim

The action of the standard model coupled to Einstein gravity,

$$\int \sqrt{\det g}\, d^4x \{ M_{\rm Pl}^2 R(g) - \Lambda + \frac{1}{4g_3^2} {\rm Tr}_{SU(3)} G_{\mu\nu} G^{\mu\nu} + \frac{1}{4g_2^2} {\rm Tr}_{SU(2)} W_{\mu\nu} W^{\mu\nu} + \frac{1}{4g_1^2} V_{\mu\nu} V^{\mu\nu} +$$
$$+ \sum_{p=1}^{n_g=3} \left(\bar{l}_L^{(p)} \hat{D} l_L^{(p)} + \bar{l}_R^{(p)} \hat{D} l_R^{(p)} + \bar{q}_L^{(p)} \hat{D} q_L^{(p)} + \bar{q}_R^{(p)} \hat{D} q_R^{(p)} \right) + \frac{1}{2} D_\mu \phi^+ D^\mu \phi + V(\phi) +$$
$$+ \sum_{P,Q=1}^{2n_g=6} \left(M_l^{(PQ)} \bar{l}_R^{(P)} \phi l_L^{(Q)} + M_q^{(PQ)} \bar{q}_R^{(P)} \phi q_L^{(Q)} + \text{c.c.} \right) \} \quad (1)$$

can be rewritten in a compact form as

$$\int {\rm Tr}_{E_8} B \wedge F + \int Q(B) \quad (2)$$

where

$$F = dA + A \wedge A \quad (3)$$

is a curvature of the "generalized-connection" field $A(x)$, which is a linear combination of the standard-model fields, distributed over elements of the E_8-algebra matrix (adjoint representation of E_8), and $Q(B)$ is a quadratic function of the auxiliary field B, which is also an adjoint E_8 matrix.

In a little more detail:

$$A = \sum_{\alpha \in G}^{56} A^\alpha(x) T_\alpha \oplus \sum_{a \in E/G}^{192} \psi^a(x) T_a, \tag{4}$$

$$B = \sum_{\alpha \in G}^{56} B^\alpha(x) T_\alpha \oplus \sum_{a \in E/G}^{192} \chi^a(x) T_a \tag{5}$$

and

$$Q(B) = \sum_{\alpha, \alpha' \in G}^{56} \left(Q_{\text{grav}}^{\alpha,\alpha'} B^\alpha \wedge B^{\alpha'} + Q_{\text{YM}}^{\alpha,\alpha'} B^\alpha \wedge * B^{\alpha'} \right) \tag{6}$$

Here the sums are over 248 generators of E_8, which are divided into two different groups of 56 and 192, labeled by Greek and small Latin letters respectively, which correspond to decomposition of adjoint (the minimal possible) representation of E_8 into a subalgebra $G = SO(7,1) \times SO(8)$ and its representation $R = E/G$. Following, k underlines label k-forms. Bosonic 1-forms $A^\alpha = A_\mu^\alpha dx^\mu$ include the gauge fields of the standard model, the gravitational spin-connection and a mixture of Higgs scalars and gravitational tetrades. One of the two tetrade indices is included into the set of E_8-indices $\{\alpha\}$, the remaining vector index, as well as the one of the gauge vectors is the 1-form index of A. Fermionic (Grassmann-valued) 0-forms ψ^a and χ^a are made from the fermions of the standard model, with spinor indices included into the set of $\{a\}$. Auxiliary fields $B^\alpha = B_{\mu\nu}^\alpha dx^\mu \wedge dx^\nu$ and $\chi^a = \chi_{\mu\nu\lambda}^a dx^\mu \wedge dx^\nu \wedge dx^\lambda$ are bosonic 2-forms and fermionic 3-forms respectively. No 4-form component is included into B, therefore the ψ^2 term in $F = F^\alpha T_\alpha + (D\psi)^a T_a + \psi^a \psi^b [T_a, T_b]$ does not contribute to the action.

The action of the standard model coupled to Einstein gravity

$$\int \mathrm{Tr}_{E_8} B \wedge F + \int Q(B) \tag{7}$$

The main disadvantage is, however, more serious: the claim of [1] is not fully justified and, most probably, is simply wrong. Desired distribution of all the fields of the standard model among the generators of E8 is not actually found. One can even argue that it can not be found, at least without some additional modifications of the claim [16]. Still, the attempt deserves attention and it is too close to being true to be simply ignored: some new piece of reality seems to be captured in this way.

9 Main drawbacks

Only unification (symmetry) issues are addressed: other serious problems – of hierarchy and of quantum gravity – remain to be somehow resolved.

Cosmological constant is non-vanishing in BF -version of the Palatini formalism. Of course, this is only a classical-level statement, while cosmological-constant problem is essentially quantum – and its resolution is far beyond the scope of L-unification.
G-projector and the Hodge star, which explicitly depends on det(e), appear in the action, what makes it not fully topological as is ideally required for a full unification with gravity.
Higher generations are not adequately described. Neither universality of gravitational interaction nor the proper chirality properties are fully established in the fermionic sector.???
Introduction of new fields beyond the standard model is still unavoidable. The rank of the standard-model gauge group, even after gravitational sector is included, Gst+gr = SU (3) × SU (2) × U (1) × SO(3, 1) is still 6, i.e. less than the rank 8 of the minimal possible "unification group" E8. In [1] the deficit rank 2 is placed into additional gauge group – the right-hand copy of the Weinberg-Salam group, giving rise to the right-hand copies

10 Some phenomenological features of L-unification

Unification algebra $G = SO(7,1) \times SO(8)$ contains gravitational $SO(3,1) = SL(2,C)$ and Yang-Mills $G_{YM} = SU(3)(SU(2) \times U(1)) \times 2$.
This is a vector-like theory, with right-hand copies of W±, Z bosons and second photon – which should somehow decouple at low energies: along with other gauge bosons from G/Gst should acquire large masses.???
Instead, since unification group G = SO(7, 1) × SO(8) is not simple, with strong and electroweak groups belonging to different factors – SO(8) and SO(7, 1) respectively, – there is no danger of proton decay, which is a usual problem for simple-group unifications.
The same non-simplicity implies that unification of coupling constants gstr and gweak does not need to take place in this model.
Flavor-changing neutral currents ???

11 From Palatini action to Einstein equations

In the case of arbitrary space-time dimension d Palatini action involves the curvature of SO(d) connection

$$S_p = \epsilon_{a_1 \ldots a_d} \int \exp a_1 \wedge \ldots \wedge \exp a_{d-2} \wedge R^{a_{d-1} a_d} \tag{8}$$

$$\ldots \tag{9}$$

$$R^a_\mu + \frac{1}{2} e^a_\mu R \sim \det(e) T^a_\mu - e^a_\mu T \tag{10}$$

It is easy to deduce that $F = -T$ and therefore previous equation is actually equivalent to the proper Einstein equation

12 E_8 and its subalgebras

$$... \tag{11}$$

13 Decompositions of E_8 algebra

$$E_8 = SO(16) + R_{128} \tag{12}$$

where R_{128} is spinor representation of $SO(16)$.

14 Decomposition of connection A

$$E_8 = D_8 + R_{128} = D_4 + D_4 + (8 \times 8 + R_{128}) \tag{13}$$

Gauge fields are distributed among the two D_4 factors, with gravity and two weak groups (left and right) belonging to one of them and with strong and two abelian groups belonging to another.
Fermionic fields are distributed among the spinor representation $R_1 28$ of maximal subgroup $SO(16) \subset E8$ and the 8×8 generators of the factor $D_8/D_4 \times D_4$. The three 64-plet constituents of the fermionic sector are related by discrete triality symmetry.