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Кафедра теоретической физики

Конспект теорфиза

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Содержание

1 23.04.24: Дифференцирование тензоров

$$rot \frac{\overline{x}}{(\overline{k}, \overline{x})} =?,\ \overline{k} = \overline{const} \tag{1}$$

$$rot \frac{\overline{x}}{(\overline{k}, \overline{x})} = e_{ijk} d_j \frac{x_k}{k_l x_l} = e_{ijk} \frac{k_n x_n \cdot d_j x_k - x_k k_t d_j x_t}{(k_l x_l)^2} = e_{ijk} \frac{k_n x_n \delta_{jk} - x_k k_t \delta_{jt}}{(\overline{k}, \overline{x})^2}$$

$$e_{ijk} \delta_{jk} = 0\ (j = k, e_{ijj} = 0)$$

$$e_{ijk} \delta_{jt} = e_{itk}\ (j = t)$$

$$rot \frac{\overline{x}}{(\overline{k}, \overline{x})} = e_{itk} \frac{-x_k k_t}{(\overline{k}, \overline{x})^2} = e_{ikt} \frac{x_k k_t}{(\overline{k}, \overline{x})^2} = \frac{[x, k]}{(\overline{k}, \overline{x})^2}$$

$$rot \frac{\overline{x}}{(\overline{k}, \overline{x})} = \frac{[x, k]}{(\overline{k}, \overline{x})^2} \tag{2}$$

$$rotrot \frac{\overline{k}}{|\overline{r}|} =?,\ \overline{k} = \overline{const} \tag{3}$$

2 07.05.24: Квантовый осциллятор

$$\frac{1}{2}(x^2 - \frac{d^2}{dx^2})\Psi(x) = E\Psi(x),\ \int_{-\infty}^{+\infty} dx|\Psi(x)|^2 = 1,\ \Psi(x) =?,\ E =? \tag{4}$$

$$a = \frac{1}{\sqrt{2}}(x + \frac{d}{dx})$$

$$a^+ = \frac{1}{\sqrt{2}}(x - \frac{d}{dx})$$

$$[a^+, a] = a^+a - aa^+ = \frac{1}{2}((x - \frac{d}{dx})(x + \frac{d}{dx}) - (x + \frac{d}{dx})(x - \frac{d}{dx}))$$

$$[a^+, a] = \frac{1}{2}(xx + x\frac{d}{dx} - x\frac{d}{dx} - 1 - \frac{d^2}{dx^2} - xx + x\frac{d}{dx} - x\frac{d}{dx} - 1 + \frac{d^2}{dx^2})$$

$$[a^+, a] = -1$$

$$a^+a + aa^+ = \frac{1}{2}(xx + x\frac{d}{dx} - x\frac{d}{dx} - 1 - \frac{d^2}{dx^2} + xx - x\frac{d}{dx} + x\frac{d}{dx} + 1 - \frac{d^2}{dx^2})$$

$$a^+a + aa^+ = x^2 - \frac{d^2}{dx^2}$$

$$H = \frac{1}{2}(a^+a + aa^+) = \frac{1}{2}(2a^+a + 1) = a^+a + \frac{1}{2}$$

$$(a^+a + \frac{1}{2})\Psi(x) = E\Psi(x)$$

$$a\Psi_0(x) = 0$$

$$\frac{1}{2}\Psi(x) = E\Psi(x),\ E_0 = \frac{1}{2} \tag{5}$$

$$\frac{1}{\sqrt{2}}(x + \frac{d}{dx})\Psi_0(x) = 0$$

$$\Psi_0(x) = A \cdot exp(-\frac{x^2}{2}) \tag{6}$$

$$x \cdot exp(-\frac{x^2}{2}) + \frac{d}{dx}exp(-\frac{x^2}{2}) = 0$$

$$x \cdot exp(-\frac{x^2}{2}) - exp(-\frac{x^2}{2})x = 0$$

$$\int_{-\infty}^{+\infty} dx|A \cdot exp(-\frac{x^2}{2})|^2 = 1$$

$$\int_{-\infty}^{+\infty} dx|exp(-x^2)| = \frac{1}{A^2} = \sqrt{\pi},\ A_0 = \frac{1}{\pi^{1/4}} \tag{7}$$

$$\Psi_1(x) = a^+\Psi_0(x) = \frac{1}{\sqrt{2}}(x - \frac{d}{dx})\frac{1}{\pi^{1/4}} \cdot exp(-\frac{x^2}{2})$$

$$\Psi_1(x) = A_1 a^+\Psi_0(x) = A_1\frac{\sqrt{2}}{\pi^{1/4}} \cdot exp(-\frac{x^2}{2})x,\ A_1 =? \tag{8}$$

$$(a^+a + \frac{1}{2})\Psi_1(x) = E_1\Psi_1(x)$$

$$(a^+a + \frac{1}{2})a^+\Psi_0(x) = E_1 a^+\Psi_0(x)$$

$$a^+aa^+ = a^+(a^+a + 1)$$

$$a^+(a^+a + 1)\Psi_0(x) + \frac{1}{2}a^+\Psi_0(x) = E_1 a^+\Psi_0(x)$$

$$a\Psi_0(x) = 0 \Rightarrow E_1 = 1 + \frac{1}{2} = \frac{3}{2} \tag{9}$$

$$\Psi_n(x) = A_n(a^+)^n\Psi_0(x),\ E_n =? \tag{10}$$

$$(a^+a + \frac{1}{2})\Psi_{n+1}(x) = E_{n+1}\Psi_{n+1}(x),\ \Psi_{n+1}(x) = A_{n+1}a^+\Psi_n(x)$$

$$(a^+a + \frac{1}{2})a^+\Psi_n(x) = E_{n+1}a^+\Psi_n(x)$$

$$(a^+a + \frac{1}{2})\Psi_n(x) = E_n\Psi_n(x)$$

$$a^+(a^+a + 1)\Psi_n(x) + \frac{1}{2}a^+\Psi_n(x) = E_{n+1}a^+\Psi_n(x)$$

$$a^+(E_n + \frac{1}{2})\Psi_n(x) + \frac{1}{2}a^+\Psi_n(x) = E_{n+1}a^+\Psi_n(x)$$

$$E_n + 1 = E_{n+1}$$

$$E_n = E_0 + n$$

$$E_n = \frac{1}{2} + n \tag{11}$$