Application of the Regularized Optimization for Hyper-Spectral Analysis (ROHSA) Tool to Faraday Tomography Cubes

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Introduction

Magnetic fields are an unseen yet important contributor to the dynamics processes in our Galaxy like star formation. Although direct measurement of Galactic magnetic fields remains unfeasible, we are able to indirectly probe magnetic field structure through a variety of messengers. The modification of the polarization angle of polarized light as it passes through a magneto-ionic medium is known as the Faraday effect and is one of those messenger. Adopting the formalism of Brentjens and de Bruyn [1] I express the Faraday Depth as the integral along the line of sight:

$$\phi(\mathbf{r}) = 0.81 \int_{\text{there}}^{\text{here}} n_e \mathbf{B} \cdot \, d\mathbf{r} \text{ rad m}^{-2}$$

The polarization of light is modified by Δχ =φλ^2. The Stokes parameters are modelled according to the following integral:

$$P(\lambda^2) = \int_{-\infty}^{+\infty} F(\phi) e^{2i\phi\lambda^2} \, d\phi$$

The above relation resembles a Fourier transform and indeed can be inverted to solve for the Faraday Depth[1]. The resulting Faraday Depth cubes can be hard to interpret and component separation in Faraday space remains a challenge. The spectral line community developed a tool kit that shares a mathematical similarity to our own problem. Both data sets are cubes with two spatial dimension and a physical dimension:

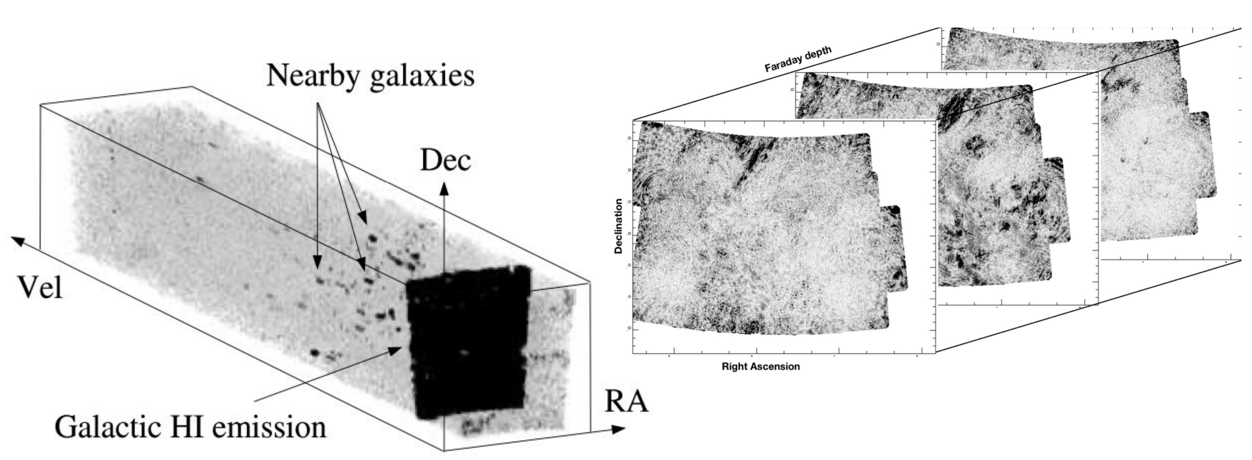

Figure 1: On the left is a cartoon of a 21cm PPV data cube (M. J. Meyer et al. 2004). On the right is cartoon of an RM cube PPF (Cameron Van Eck).

The Regularized Optimization for Hyper-Spectral Analysis (ROHSA) tool was developed by the Hyperstars collaboration at Paris-Saclay University to extract diffuse structures in hyper-spectral cubes. ROHSA attempts to decompose the spectral structure into a coherent Gaussian basis based on a multi-resolution process. ROHSA performs regression analysis in parameter space using a non-linear least squares criterion[2]. For an appropriate choice of wavelength band and in the case of Faraday thin sources the Gaussian waveform makes for an excellent approximation of the signal. I explore the application of the ROHSA tool to rotation measure data and will refer to the modified version as ROHSA-RM hereafter. Reliable recovery of Faraday Depth components is pivotal to spatial reconstruction of the parallel magnetic field along the line of sight.

Experimental Methods

ROHSA-RM: I modified ROHSA's FORTRAN code to eliminate the velocity dispersion, or the width parameter, of the gaussian model. The width parameter can now be passed through the user defined parameter file.

Numerical Simulations: I coded a Python numerical simulator that synthesizes stokes Q and U cubes given an arbitrary rotation measure, amplitude (A), intrinsic polarisation angle (χ), noise profiles (n), as well as a user passed frequency or wavelength (λ) observation window array. The statistical properties of interstellar gas can be captured by fractal Brownian motion (fBm). I generated 2D fBm amplitude and Faraday Depth (FD) maps to inform the simulated observations. A frequency band of 120MHz -178MHz with 488 equally wide bins centered at the midpoint was used in my simulation. Noise drawn from a normally distributed distribution was added to the Q and U observations per pixel and wavelength bin. The simulated observation cube was then fed to the sequential pipeline: RMsynthesis, RMclean, and finally ROHSA-RM. Where the rotation measure synthesis algorithm attempts to invert equation [2] for F(λ) and produces a Faraday Dispersion Function cube. ROHSA-RM is then applied to the RM cube to extract features in Faraday space and reconstruct the input amplitude and Faraday depth maps. The ROHSA-RM parameters were set to fit 2 gaussians with a fixed width informed by the RMSF main lobe of the RMTF. The ROHSA-RM hyper parameters were set to 1.0.

Results

I produced plots of the input signal to noise, amplitude, Faraday depths maps. Residual maps of the input and output ROHSA-RM maps were calculated and plotted. Spectra plots of the loudest and quietest pixels were plotted with the gaussians whose parameters are informed by the ROHSA-RM fit. The plots are presented next:

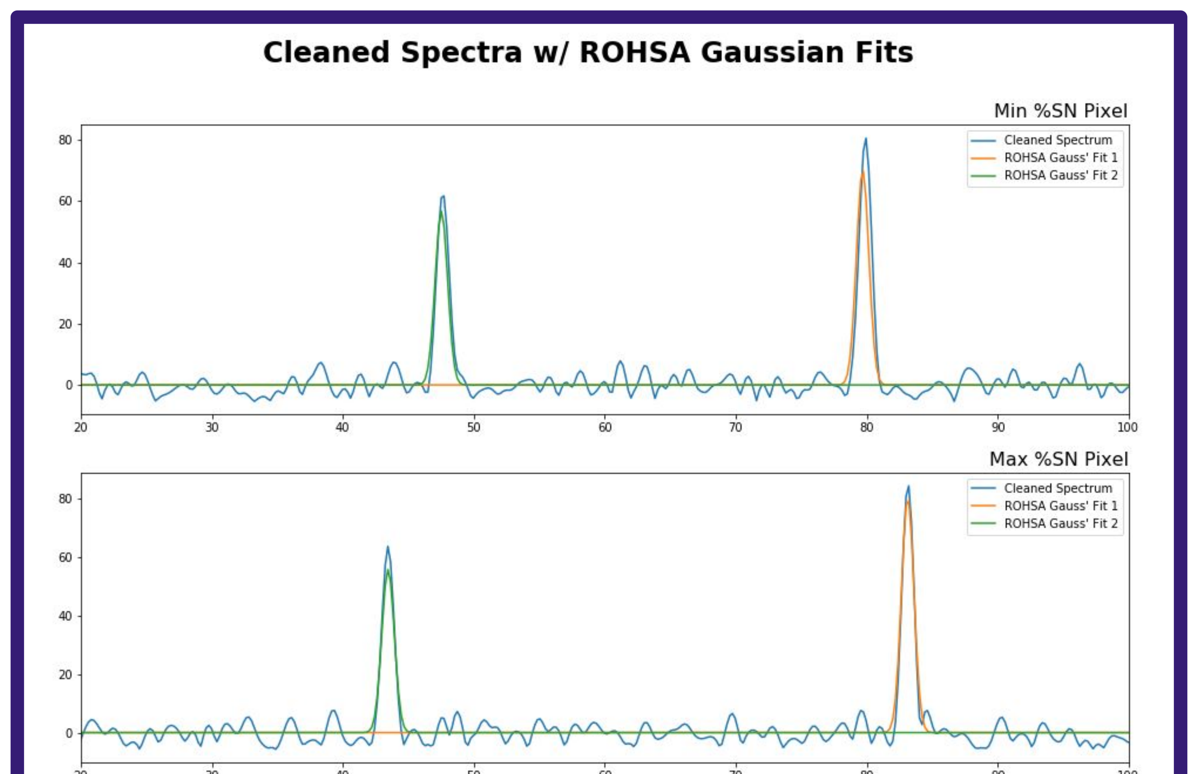

Figure 2: A plot of the spectrum corresponding to the pixel with the minimum signal to noise in the cube in the top panel and maximum signal to noise in the bottom panel. The blue curve is RMcleaned FDF, in orange and green are the ROHSA-RM gaussian fits. By visual inspection, the ROHSA-RM gaussian fits appear to successfully lock onto features in the spectrum and do fairly well at estimating the centroid. However it does appear that the Gaussian fits underestimate the amplitude of the features which suggests that a further inquiry into the model is necessary.

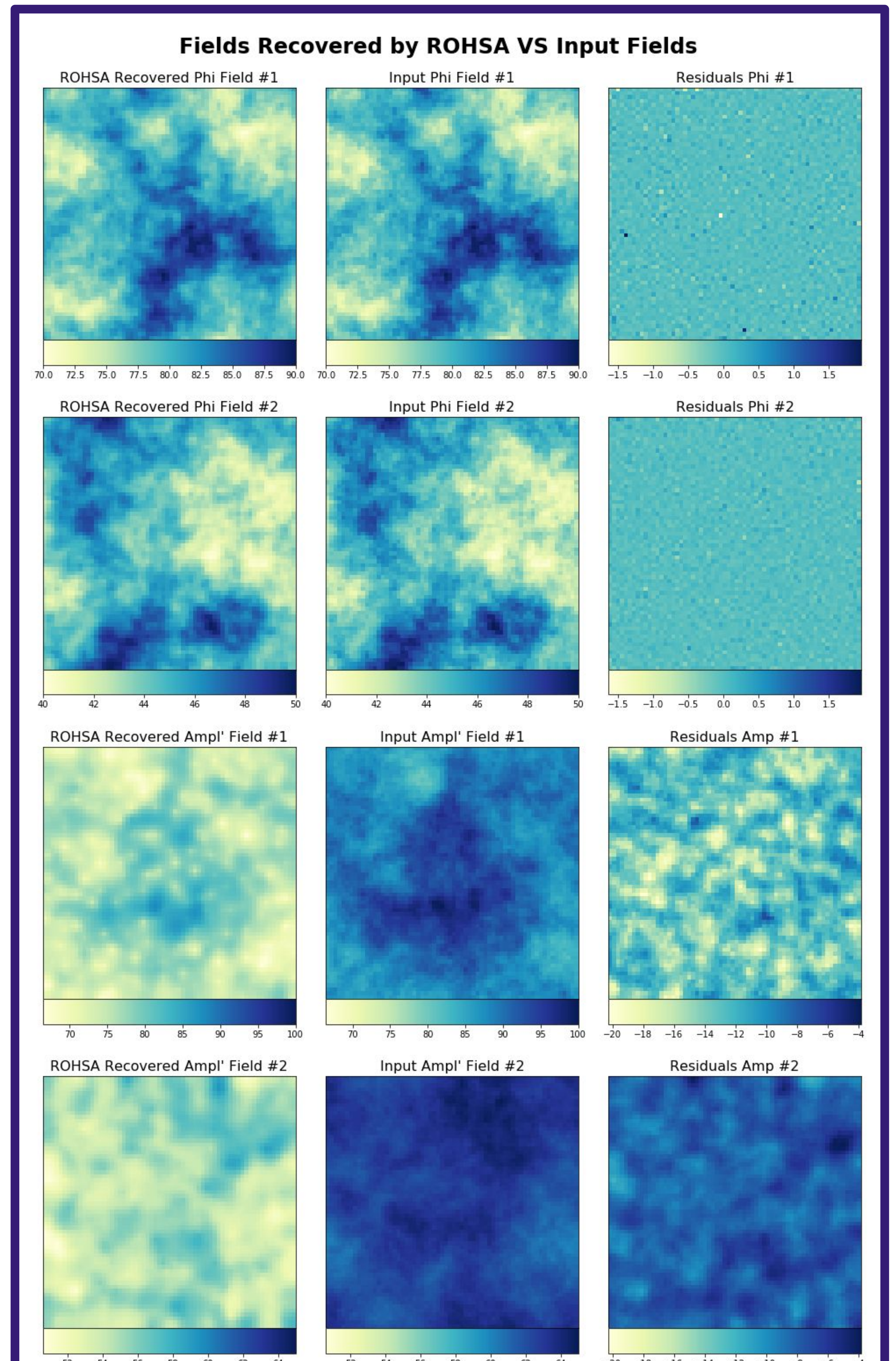

Figure 3: The left column corresponds to the ROHSA-RM recovered FD and amplitude fields and depicts 2D maps of each recovered feature. The middle column corresponds to the input maps. I used 2D fBm profiles for all input maps. Finally, the right column is a residual maps of the entries in the left and middle columns. The residual maps of the FD components show a uniformity within a tolerance of approximately +-0.2 units and appears to characterize the noise. There is no remarkable pattern in the FD residuals which suggests that the algorithm is successful at recovering the centroids. The amplitude map residual do have remarkable features and do no exhibit the same uniformity. ROHSA-RM's amplitude recovery performance for this test case is poor. However, the input and output amplitude maps do appear morphologically similar. The first amplitude input map (3rd row) is dominated by ~95 units central feature which gradually degrades towards the edges. This feature is visible in the output map however it is quieter by about ~10 units. The second map (4th row) also share a similarity between the output and input however singular components are harder to discern. There does appear to be a feature of about ~52 units on the right border of the input map and the output map where it is quieter by ~5 units. Variance and mean of all residual maps were calculated. The variance of the FD fits is at most 0.01 units, and the variance of the amplitude maps is at most ~3 units. There amplitude residuals appear to be dominated by a constant offset as suggested by the relatively large mean (~13 units for the first component and ~8 units for the second component).

Discussion

Understanding the limitations and behavior of our pipeline is necessary to interpret its output and using it to make physical insights. The numerical simulator provides us a controlled environment where we can attack the many parameters characteristic RM cubes.

The ROHSA-RM tool successfully recovered the centroids of features in Faraday Depth space for the case of two well separated fBm components as seen in figure 3. The residuals of the output and input phi fields are centered about 0 with variation characteristic of noise. The amplitude fits are unsatisfactory and poorly estimate the input amplitude fields. The amplitude field residuals for both FD components show discernible structure. Figure 2 reveals that the ROHSA-RM fit gaussians under-appreciate the amplitude of the features. But it also shows that ROHSA-RM performs well on pixels characterized by a low signal-to-noise. It is important to note that a fraction of the loss in power is accounted for by a necessary preprocessing procedure performed before ROHSA-RM. A baseline is subtracted from all spectra in the input cleaned cube and is not added back in my analysis. Further investigation is necessary to determine the factors affecting the quality of the amplitude parameter fits. A preliminary investigation direction can be the effect of hyperparameters of ROHSA which control the severity of smoothness imposed on the fit parameters. Another possible direction is the use of an alternate model. An model that may better the fit to data is an RMTF template.

I performed the same analysis on a combination of Faraday Depth profiles: one is a plane and the other is an intersecting gradient. The test revealed that ROHSA is unable to properly lock onto and describe the gradient.

Both the fBm and gradient profile cases provide valuable insight into the caveats of the tool before application to real world data. I applied the ROHSA-RM tool to LOFAR IC342 data. ROHSA-RM was able to recover mixed features in FD of components identified in literature [3].

References

[1] Brentjens, M. A., and A. G. De Bruyn. "Faraday Rotation Measure Synthesis." Astronomy & Astrophysics, vol. 441, no. 3, 2005, pp. 1217–1228., doi:10.1051/0004-6361:20052990.

[2] Marchal, Antoine, et al. "ROHSA: Regularized Optimization for Hyper-Spectral Analysis." Astronomy & Astrophysics, vol. 626, 2019, doi:10.1051/0004-6361/201935335.

[3] Van Eck, C. L., et al. "Faraday Tomography of the Local Interstellar Medium With LOFAR: Galactic FOREGROUNDS towards IC 342." Astronomy & Astrophysics, vol. 597, 2017, doi:10.1051/0004-6361/201629707.