Meeting Notebook

Ising Model

Meeting 1

- collaborate on the code
- data and report separate
- even could do individually

→ use fluctuations to determine the answer and uncertainty (error bars)

* numerically exact answer
* error bar → 1 st.d or 1%

* Blocking method

→ predict how error bar fluctuates

E.B $\propto \sqrt{N_{sim}}$ (controlled method)

↳ controlling statistical method, by waiting long enough

- Mean field solution for Ising model.

going through lecture notes

→ theory (can read handout)

→ developing code simultaneously.

→ literature review

- mimicing real research project

△ Future meetings - come up with questions and concepts

△ monitoring progress.

△ discussion / sharing ideas

Languages
- python (high perfomance python)
- object-oriented code
- Julia? Useful for optimization

For Meeting 2
→ understanding Monte-Carlo methods
  - Metropolitan algorithm
→ detailed balance condition
→ read through lecture notes (theory / general).

Financial modelling for Ising.

$$M(t) = \frac{1}{k^2} \sum_{j=1}^{k} S_j(t)$$ → total magnetisation

local field h_i (energy):

$$h_i(t) = \underbrace{\boxed{\sum_{j=1}^{k} J_{ij} S_i(t)}}_{\text{classical Ising}} - \boxed{\alpha S_i(t) \, |M(t)|}$$

$\alpha > 0$ - global coupling constant

1st term aligns S_i with its neighbours

2nd term attempts to change value of the spin (anti-ferromagnetic behaviour) based on total magnetisation, M

- Herding behaviour
- Minority Game

$$p = \frac{1}{1 + e^{-2\beta h_i(t)}}$$

where β represents how much the neighbours affect the trader's behavior

$\lim_{\beta \to 0}$ gives strongest correlation

→ Simple model with strategy spin

$$h_i(t) = \sum_{j=1}^{k} J_{ij} S_i(t) - \alpha C_i(t) M(t)$$

- if $C_i = +1$ - agent i always seeks to be a minority trader, limiting the dependance on neighbours. Fundamentalists
- if $C_i = -1$ - acting like majority of traders chartists (noise traders).
- more realistic to allow agents choose which strategy to follow in each round.

Transition Rule?

risky strategy to maximize returns:

$$C_i(t+1) = -C_i(t) \text{ if } S_i(t) C_i(t) M(t) < 0$$

- majority chooses $C_i(t) = 1$
- minority goes for $C_i(t) = -1$

→ Compare total magnetization & strategy of agents!

→ 3rd option : Stay Inactive

$$s_i(t+1) = \mathrm{sgn}_{\lambda|M(t)|}\left(\sum_{j=1}^{T} J_{ij} s_j(t) + \delta \nu_i(t)\right)$$

$$\mathrm{sgn}_q(x) = \begin{cases} -1 & \text{if } x < -q \\ 0 & \text{if } -q \le x \le q \\ 1 & \text{if } q \le x \end{cases}$$

$\lambda > 0$ - param. influences the width of range where agents stay inactive.

ν - normally distributed random variable, representing unpredictable behaviour of each agent.

δ - parametrizes the strength of variable.

→ autocorrelation ??

→ using anisotropy as agent's sensitivity to external news

- relative price change of return:

$$r_t = (p_t - p_{t-1})/p_{t-1}$$

$$H = \boxed{\sum_{\langle ij \rangle} J_{ij} S_i S_j} - \boxed{B \sum_i S_i} + \boxed{\Delta \sum_i S_i^2}$$

- influence of neighbouring particles
- external news, B - impact parameter
  S_i - relative sensitivity of the agents to the news. $[0, S_{max}]$
- $\sum_i S_i^2 = N$

$$Z = \int_{\sum_i S_i^2 = N} dS_1 \ldots dS_N \exp\left[\beta\left(B \sum_i S_i - \sum J_{ij} S_i S_j\right)\right]$$

$\beta = 1/T$

- method → see paper

→ <u>Ising model to predict financial crashes</u>

- using the Log Periodic Power Law model (LPPL)
- simplest form:

$$y_t = A + B(t_c - t)^\beta \left[1 + C\cos(\omega \log(t_c - t) + \varphi)\right]$$

$y_t > 0$ - price (index) / log (price)

$A > 0$ · value of y_t if bubble lasts until t_c

$B < 0$ - decrease of y_t over time until the crash if C is close to zero

C - magnitude of fluctuations around exp. growth

$t_c > 0$ - critical time

$t < t_c$ - time into the bubble

$\beta = 0.33 \pm 0.18$ - exponent of power law growth

$\omega = 6.36 \pm 1.56$ - freq. of oscillation during the bubble.

$0 \leq \varphi \leq 2\pi$ - shift parameter.

- derivation of LPPL
- search algorithm

→ Use worldquant article for descriptive text.

- learn to create log-periodic graphs
- $\log(pt) \sim A + B(t - tc)^a$
- 1929, 1987, 1997, 2008

Modelling:

- existing 2D model

$$H = \sum_{\langle ij \rangle} J_{ij} S_i S_j$$

measuring the local field. Neighbouring interactions only.

→ Metropolis algorithm: