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Large financial crashes

Didier Sornette [a,b,*], Anders Johansen [c]

[a] *Department of Earth and Space Science and Institute of Geophysics and Planetary Physics, University of California, Los Angeles, CA 90095, USA*

[b] *Laboratoire de Physique de la Matière Condensée, CNRS URA 190, Université de Nice-Sophia Antipolis, B.P. 71, Parc Valrose, 06108 Nice Cedex 2, France*

[c] *CATS, Niels Bohr Institute, Blegdamsvej 17, Copenhagen 2100, Denmark*

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Abstract

We propose that large stock market crashes are analogous to critical points studied in statistical physics with log-periodic correction to scaling. We extend our previous renormalization group model of stock market prices prior to and after crashes (D. Sornette, A. Johansen, J.P. Bouchaud, J. Phys. I France 6 (1996) 167) by including the first non-linear correction. This predicts the existence of a log-frequency shift over time in the log-periodic oscillations prior to a crash. This is tested on the two largest historical crashes of the century, the October 1929 and October 1987 crashes, by fitting the stock market index over an interval of 8 yr prior to the crashes. The good quality of the fits, as well as the consistency of the parameter values obtained from the two crashes, promote the theory that crashes have their origin in the collective "crowd" behavior of many interacting agents.

0. Introduction

Stock markets are fascinating structures with analogies with arguably the most complex dynamical system found in the Natural Sciences [1], i.e., the human mind. Stock market prices weave patterns on many different scales, luring traders to believe (may be correctly) that some predictability is possible [2,3]. Many attempts have been made to model stock markets and recently a wealth of works from the physical community has pointed at similarities between stock markets and dynamical out-of-equilibrium systems [4–31].

In this work, we will focus on the most extreme behavior of stock markets, namely the two largest financial crashes in this century. Our Ariadne's thread is that complex systems often reveal more of their structure and organization in highly stressed situations than in equilibrium. Hence, our hope is that the study of these two large

* Corresponding author.

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crashes will enable us to extract important new information about the dynamics of stock markets. Specifically, we are interested in describing the stock market behavior before and after a crash and our problem thus belongs to the more general problem of describing the transient behavior preceding a final equilibrium state assuming it exists. Our point of view is influenced by the concept of criticality developed in statistical physics in the last 30 years in order to describe a class of cooperative phenomena, such as magnetism and melting, and our hypothesis is that the stock market behaves as a driven out-of-equilibrium many-body system [1,32].

From the open on Wednesday 23 October 1929 to the close of Tuesday 29 October 1929, the New York Stock Exchange lost almost 30% of its value. An often quoted origin of the crash is that traders thought that the bullish trend was due to continue, while the efficiency eventually brought back the market to its fundamentals. In a similar fashion, major indexes of market valuation in the United States declined by 30 percent or more from the opening on October 14 1987 to the market close on October 19 and in addition all major world markets declined substantially in the following month, in contrast with the usual modest correlations of returns across countries. A lot of work has been carried out to unravel the origin(s) of the 1987 crash, notably in the properties of trading and the structure of markets; however, no clear cause has been singled out [33]. Maybe the most quoted scenario involves the role of portfolio insurance strategies in amplifying the descent. In the present work, we would like to defend the thesis that these two crashes have fundamentally similar origins, which must be found in the collective organization of the market traders leading to a regime known as a "critical" point.

In a previous paper [34], we have identified precursory patterns, as well as aftershock signatures and characteristic oscillations of relaxation, associated with the October 1987 crash up to 2.5 yr in advance. Very similar results were obtained [35] for both the 1929 and 1987 crashes and here it was pointed out that the log-frequency of the observed log-periodic oscillations seems to decrease as the time of crash is approached.

In the present work, we will further substantiate that the concept of criticality can be applied to stock market crashes. We generalize our previous work [34] and show how the first generic non-linear correction to the renormalization group equation previously proposed accounts quantitatively for the behavior of the Dow Jones and S&P500 (Standard and Poor) indices up to 8 yr prior to the crashes of 1929 and 1987. The conclusion is that the qualitative observation of [35] of the log-frequency shift is rationalized by the non-linear correction.

1. Model and non-linear generalization

Using the renormalization group (RG) formalism on the stock market index I amounts to assuming that the index at a given time t is related to that at another time t' by the transformations [34]

$$x' = \phi(x), \tag{1}$$

$$F(x) = g(x) + \frac{1}{\mu} F(\phi(x)), \tag{2}$$

where $x = t_c - t$. t_c is the time of global crash and ϕ is called the RG flow map. Here,

$$F(x) = I(t_c) - I(t),$$

such that $F = 0$ at the critical point and μ is a constant describing the scaling of the index evolution upon a rescaling of time (Eq. (1)). The function $g(x)$ represents the non-singular part of the function $F(x)$. We assume as usual [36] that the function $F(x)$ is continuous and that $\phi(x)$ is differentiable.

As the simplest non-trivial solution beyond the pure power-law solution to Eq. (2), we have proposed [34]

$$I(t) = A + B(t_c - t)^{\alpha}[1 + C\cos(\omega \log(t_c - t) - \phi)]. \tag{3}$$

It includes the first Fourier component of a general log-periodic correction to a pure power law behavior of an observable (here, the stock market index) exhibiting a singular behavior at the time t_c of the crash, i.e., which becomes scale-invariant at the critical point t_c. We have found Eq. (3) to fit the S&P500 data prior to the 1987 crash very well over a period of approximately two years before the crash. We have also tested Eq. (3) on a few other minor crashes after 1987. [1] Expression (3) accounts very well for the two most important structures that are visible with the naked eye, i.e., the overall accelerated increase before the crash as much as two years before it and its decoration by large scale oscillations whose linear frequency increases on the approach to the crash.

We now proceed to extend these results in order to show that precursors can be identified over a much longer time interval and that their structure can be deduced from a general renormalization group approach that we now present. In this goal, we notice that the solution (3) of the RG Eq. (2) together with Eq. (1) and the linear approximation $\phi(x) = \lambda x$ can be rewritten as

$$\frac{dF(x)}{d \log x} = \alpha F(x), \tag{4}$$

stating simply that a power law is nothing but a linear relationship when expressed in the variables $\log F(x)$ and $\log x$. A critical point is characterized by observables which have an invariant description with respect to scale transformations on x. We can exploit this and the expression (4) to propose the structure of the leading corrections to the power law with log-periodicity. Hence, we notice that Eq. (4) can be interpreted as a

[1] These smaller crashes do not have a very long build-up time as the 1929 and 1987 crashes and the number of oscillations observed prior to these crashes are only one or two. Hence, the need for a log-periodic correction is not as obvious as with the two large crashes and we have not included the analysis of these smaller crashes here. In analogy with usual critical phenomena, we expect that the time interval over which the precursors of a crash are detectable (corresponding to the so-called width of the critical region) increases with the size of the crash. This rationalizes the strength of the log-periodic corrections for the two large crashes of the century.

bifurcation equation for the variable F as a function of a fictitious "time" ($\log x$) as a function of the "control parameter" α. When $\alpha > 0$, $F(x)$ increases with $\log x$ while it decreases for $\alpha < 0$. The special value $\alpha = 0$ separating the two regimes corresponds to a bifurcation [37]. Once we have recognized the structure of the expression (4) in terms of a bifurcation, we can use the general reduction theorem [37] telling us that the structure of the equation for F close to the bifurcation can only take a universal *non-linear* form given symmetries. Introducing the amplitude B and phase ψ of $F(x) = Be^{i\psi(x)}$, the only symmetry we can use is the fact that a global shift of the phase should keep the observable constant under a global change of units. This implies the following expansion:

$$\frac{dF(x)}{d\log x} = (\alpha + i\omega)F(x) + (\eta + i\kappa)|F(x)|^2 F(x) + \mathcal{O}(F^5)\,, \tag{5}$$

where $\alpha > 0, \omega, \eta$ and κ are real coefficients and $\mathcal{O}(F^3)$ means that higher-order terms are neglected. Such expansions are known in the physics literature as Landau expansions [36]. We stress that this expression represents a non-trivial addition to the theory, constrained uniquely by symmetry laws.

The interesting situation is the one where $x = 0$ corresponds to a critical or singular point (characterized by an unstable fixed point) and the other fixed point (appearing due to the non-linear correction) is stable. This occurs for $\eta/\alpha < 0$. Then, small x corresponds to being close to the critical point, whereas large x corresponds to the stable regime. In terms of the amplitude B and phase ψ of $F(x) = Be^{i\psi(x)}$, Eq. (5) yields

$$\frac{\partial B}{\partial \log x} = \alpha B + \eta B^3 + \cdots\,, \tag{6}$$

$$\frac{\partial \psi}{\partial \log x} = \omega + \kappa B^2 + \cdots\,, \tag{7}$$

whose solution reads

$$B^2 = B_\infty^2 \frac{(x/x_0)^{2\alpha}}{1 + (x/x_0)^{2\alpha}}\,, \tag{8}$$

$$\psi = \omega \log \frac{x}{x_0} + B_\infty^2 \frac{\kappa}{2\alpha} \log\left(1 + \left(\frac{x}{x_0}\right)^{2\alpha}\right)\,, \tag{9}$$

where $B_\infty^2 = \alpha/|\eta|$ and x_0 is an arbitrary factor, fixing the time scale. [2]

Two regimes can be identified:

(1) for small x close to the singular point, the solution is of the form

$$B = B_\infty \left(\frac{x}{x_0}\right)^{\alpha}\,, \tag{10}$$

[2] These equations correspond to $F(x)$ describing a spiral in the complex plane, while physical values occur only at intersections of the spiral with the real axis. This is obvious in the linear approximation and remains true qualitatively in presence of the non-linear corrections. Thus, this suggests to view the discrete renormalization group equation studied above as analogous to discrete maps obtained from Poincaré sections [37] of a continuous renormalization group flow.

$$\psi = \omega \log \frac{x}{x_0} . \tag{11}$$

The log-periodic oscillations have a frequency equal to $\omega/2\pi$ in $\log x$.

(2) For large x, far from the singular point, the interesting situation is that of a stable phase at large x ($\eta/\alpha < 0$) and we get

$$B \to B_\infty , \tag{12}$$

$$\psi = (\omega + B_\infty^2 \kappa) \log \frac{x}{x_0} . \tag{13}$$

The log-periodic oscillations have a frequency equal to $(\omega + B_\infty^2 \kappa)/2\pi$ in $\log x$.

For $\kappa > 0$, log-periodic oscillations, which can be detected far from the singular point, must have a larger frequency than close to the critical point. When going from outside the critical regime to inside the critical regime, the frequency decreases. The reverse holds for $\kappa < 0$.

The general form of Eqs. (8) and (9) of the solutions of the non-linear renormalization group Eq. (5) lead to the following modification of Eq. (3):

$$I(\tau) = A + B \frac{(\tau_c - \tau)^\alpha}{\sqrt{1 + \left(\frac{\tau_c - \tau}{\Delta t}\right)^{2\alpha}}} \times \left[1 + C \cos \left(\omega \log(\tau_c - \tau) + \frac{\Delta \omega}{2\alpha} \log \left(1 + \left(\frac{\tau_c - \tau}{\Delta t} \right)^{2\alpha} \right) \right) \right] , \tag{14}$$

where $\tau = t/\phi$ and $\Delta\omega = B_\infty^2 \kappa$.

Two new effects are predicted by Eq. (14).

- There is a saturation of the function $I(\tau)$ far from the critical point;
- the log-frequency shifts from $(\omega + \Delta\omega)/2\pi$ to $\omega/2\pi$, when approaching the time of the crash.

An interesting observation is that both effects are linked and controlled by the same parameter $\Delta t = x_0$, which measures the characteristic time-scale controlling both the saturation and the log-frequency cross over.

2. Analysis of the 1929 and 1987 crashes

We have used Eq. (14) to fit the Dow Jones index prior to the 1929 crash and the S&P500 index prior to the 1987 crash, both starting approximately 8 yr prior to the crash.

It is not clear a priori what is a good measure of the "state" of the market. In [34], we used the simplest and most straightforward parameter, namely the market index itself. Here, we take a slightly more sophisticated approach and test for other possibilities. The logarithm of the index can be argued to be a better choice. The reason is that the average growth of the index over the century is well-captured by an exponential rise with a typical rate of about 7% per year. Over 8 yr, this gives an increase by a factor ≈ 1.7, which is not negligible. This long-term average exponential growth is not the phenomenon that we are trying to detect as a signature of cooperative behavior but rather reflects the global price index variation, as well as the global economic behavior. Instead of detrending the index by an average exponential growth with the caveat that it might distort the signal, we propose to use the logarithm of the index as a *non-parametric* proxy for the possible cooperative market behavior. Notice that our previous fit over 2 yr has been done directly on the index as the exponential drift has only a minor influence over this restricted time scale. Pushing this line of reasoning further, we have also tested whether the subtraction of the average 7% yearly growth to the logarithm of the index modified the results. We now present the results obtained for the index I, for $\ln I$ and for $\ln I - r\tau$, where r is the average yearly growth rate.

The fitting was done as a minimization of the root-mean-square (rms) as cost-function, assuming Gaussian distributed fluctuations, and using Downhill Simplex as minimization algorithm [38]. Since we are fitting a highly non-linear function with a priori 9 parameters to noisy data, many local minima exist for the cost-function. Hence, the fitting was done in a rather elaborate way. First, the effective number of parameters was reduced by minimizing the cost-function with respect to the three linear variables A, B, C, thus determining A, B, C explicitly as a function of the six non-linear variables $\alpha, t_c, \Delta t, \omega, \Delta\omega$ and ϕ. This procedure thus reduces the number of free variables in the fit from 9 to 6. Due to the risk of the minimization algorithm getting trapped in one of the many potential local minima, a preliminary scan, or a so-called Taboo search [39], was made using a range of physically reasonable values for $t_c, \Delta t, \omega$ and $\Delta\omega$ and fitting only α and ϕ. To be specific, this means that Δt cannot have a value that is much larger or much smaller than the time interval of the data, since it measures the characteristic time-scale controlling both the saturation and the log-frequency cross over. Also, very large values for ω cannot be accepted either, since it means that we are fitting fluctuations on very short time scales, i.e., "noise".

After the scan, all minima satisfying $0 < \alpha < 1$ was selected and are taken as the starting values of fits with Eq. (14) to the data with all 6 non-linear variables free. However, since ϕ is just a time unit and only depends on whether we count in days, months or years, the fit is essentially controlled by the five parameters $\alpha, t_c, \Delta t, \omega, \Delta\omega$ and only those minima with financially reasonable parameter values for those parameters have been taken into account. For the 1929 crash, this means that between the 3 solutions with similar rms (within $\approx 0.5\%$) the solution having Δt closest to the time-interval of the date has been chosen as the best fit.

In Figs. 1a and 2a, the best fits to the logarithm of the S&P500 index prior to the 1987 crash and to the logarithm of the Dow Jones index prior to the 1929 crash are shown. We see that the general trend of the data is well-captured by the proposed relation over 8 yr. In order to quantify this statement, the relative error of the fit to the data has been calculated, see Figs. 1b and 2b, and the error is $\lesssim 10\%$ on the entire time interval. In Fig. 1a, the thin line represents the best fit with Eq. (14) over the whole time interval, while the thick line is the fit by Eq. (3) on the subinterval

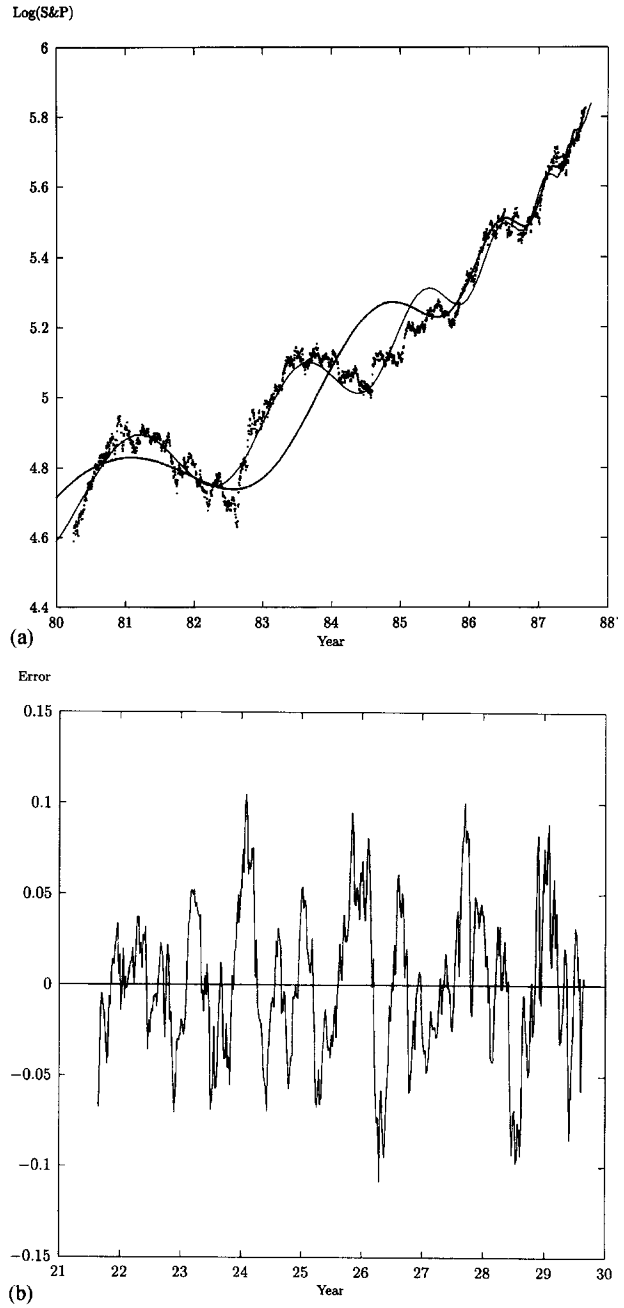

Fig. 1. (a) Time dependence of the logarithm of the New York stock exchange index S&P500 from January 1980 to September 1987 and best fit by (14) (thin line). The crash of 14 October 1987 corresponds to 1987.78 decimal years. The thin line represents the best fit with equation parameters of the fit are: rms= 0.043, t_c = 1987.81 year, $\alpha = 0.68$, $\omega = 8.9$, $\Delta\omega = 18$, $\Delta t = 11$ years, $A = 5.9$, $B = -0.38$, $C = 0.043$. The thick line is the fit by (3) on the subinterval from July 1985 to the end of 1987 and is represented on the full time interval starting in 1980. The parameters of this fit with (3) are rms = 6.2, t_c = 1987.74 year, $\alpha = 0.33$, $\omega = 7.4$, $A = 412$, $B = -165$, $C = 0.07$. The comparison with the thin line allows one to visualize the frequency shift described by (14). (b) The relative error of the fit by (14) to the data.

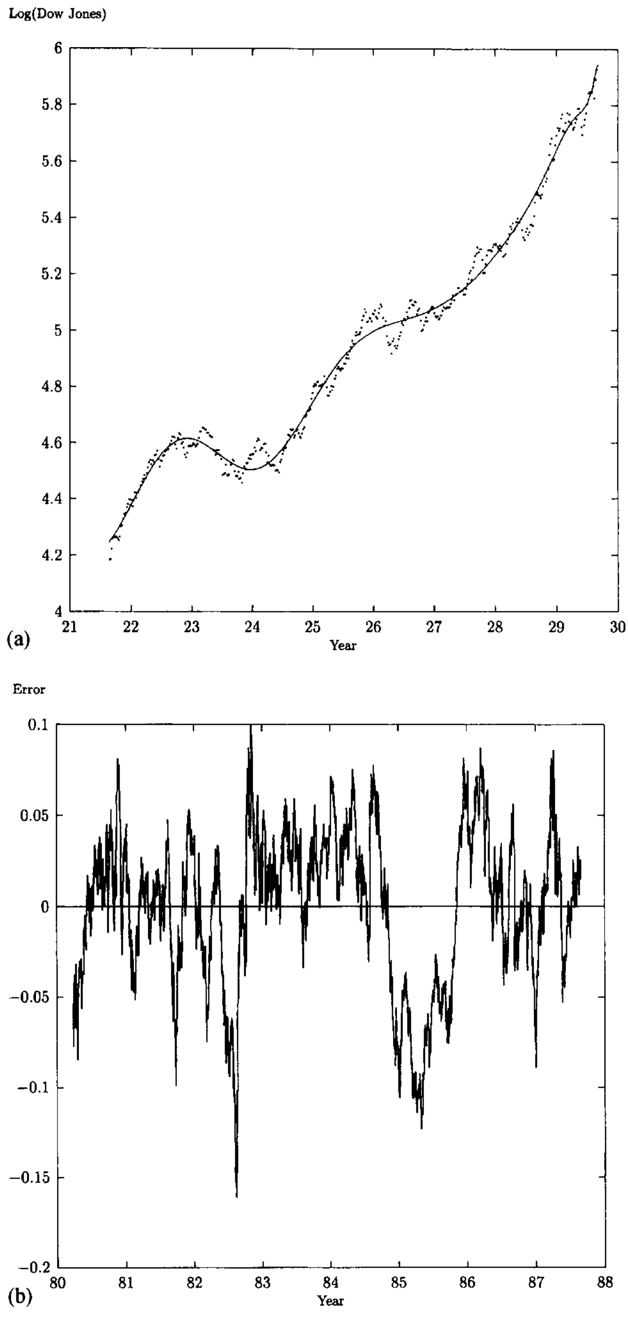

Fig. 2. (a) Time dependence of the logarithm of the Dow Jones stock exchange index from June 1921 to September 1929 and best fit by (14). The crash of October 23, 1929 corresponds to 1929.81 decimal years. The parameters of the fit are: rms= 0.041, $t_c = 1929.84$ year, $\alpha = 0.63$, $\omega = 5.0$, $\Delta\omega = -70$, $\Delta t = 14$ years, $A = 61$, $B = -0.56$, $C = 0.08$. (b) The relative error of the fit by Eq. (14) to the data.

from July 1985 to the end of 1987 as done in [34] but is represented on the full time interval starting in 1980. The comparison with the thin line allows one to visualize the frequency shift described by Eq. (14).

Using the indexes I themselves gave similar solutions, but the fitting were quite unstable and with a lot of degeneracy. We attribute this to the superposition of the average exponential trend. Also, the fits of $\ln I - rt$ gave results very similar to those shown in Figs. 1 and 2. In general, the time of crash τ_c changes very little ($< 2\%$) and the log-frequency ω only moves by $\approx 25\%$ for the best fits using the three different measures. This is to be expected as the time scale is not modified. However, the modulation of the log-frequency, determined by Δt and $\Delta\omega$, is less constrained than τ_c and ω and changes significantly as we go from I to $\ln I$ or to $\ln I - r\tau$. Furthermore, the exponent α increases (by a factor of ≈ 3) for the best fits as the I-axis is compressed by the detrending (when going from I to $\ln I$).

Observe that on relatively short time scales, we see many jumps not accounted for by Eq. (14) and obviously other processes than the ones considered here are influencing the stock market behavior on these time scales. Also, higher-order terms of the solution of the renormalization group equation are not included in Eq. (14). They could play an important role at shorter time scales as the complete solution to all orders is expected to lead to a "fractal" structure with wavy patterns at all scales.

3. Discussion

To validate the proposed model requires that one obtains a good fit in several data sets with approximately the same parameter values. Stated somewhat pointedly: one good fit make a data description; two good fits make a system description. In both cases, the fit has an overall error of less than 10% over a time interval extending up to 8 yr before the crashes. There are presumably other sorts of fits that would work, although the fact that 8 yr of data can be adjusted with an error of less than 10% using only 5 parameters (not counting the arbitrary time unit ϕ) is rather remarkable. It is surely irrational to infer the validity of a description based on a single fit. If it works for many fits, however, and if there is a reasonable theory for it, it should have some truth in it. In order to qualify such a statement, we observe that the value of the exponent α and the log-frequency ω for the two great crashes are quite close to each other. We find $\alpha_{1929} = 0.63$ and $\alpha_{1987} = 0.68$. This is in agreement with the universality of the exponent α predicted from the renormalization group theory. A similar universality is also expected for the log-frequency, albeit with a weaker strength as it has been shown [40] that fluctuations and noise will modify ω differently depending on their nature. We find from the fits that $\omega_{1929} = 5.0$ and $\omega_{1987} = 8.9$. These values are not unexpected and correspond to what has been found for other crashes as well as for earthquakes [41,42] and for related rupture and growth phenomena [43–46]. If we were fitting random fluctuations around some average power-law rise, then the obtained value of ω would generally be higher and fluctuate more due to the noise-fitting we would be performing. Furthermore, the values obtained for the amplitude C shows that a log-periodic correction to a pure power law is not insignificant. The analysis of the two great crashes in this century presented here and supplementing [41,43,47,34,44] suggests a very coherent picture, namely that *complex* critical exponents are a general phenomenon in irreversible self-organizing systems. It is interesting that the similarity between the two situations in 1929 and 1987 has in fact been noticed qualitatively in an article in the *Wall Street Journal* on 19 October 1987, the very morning of the day of the stock market crash (with a plot of stock prices in the 1920s and the 1980s). See the discussion in [48].

The stock market provides a remarkable realization of a complex self-organizing system and the log-periodic structure found prior to crashes implies the existence of a hierarchy of characteristic time scales, corresponding to the time intervals $t_c - t_n$, determined from the equation $\omega \log(t_c - t_n) + \phi = n2\pi$ for which the cosine in Eq. (14) is largest. These time scales could reflect the characteristic relaxation times associated with the coupling between traders and the fundamentals of the economy. The larger value of ω (smaller ratio λ) for the more recent crash could reflect the faster nature of the fluctuations resulting from more efficient computerized trading systems. While α is expected to remain robust, future crashes should have a similar or even larger value ω.

The main point of this paper is that the market anticipates the crash in a subtle self-organized and cooperative fashion, hence releasing precursory "fingerprints" observable in the stock market prices. In other words, this implies that market prices contain information on impending crashes. If the traders were to learn how to decipher and use this information, they would act on it and on the knowledge that others act on it and the crashes would probably not happen. Our results suggest a weaker form of the "weak efficient market hypothesis" [49], according to which the market prices contain, in addition to the information generally available to all, subtle information formed by the global market that most or all individual traders have not yet learned to decipher and use. Instead of the usual interpretation of the efficient market hypothesis in which traders extract and incorporate consciously (by their action) all information contained in the market prices, we propose that the market as a whole can exhibit an "emergent" behavior not shared by any of its constituent. In other words, we have in mind the process of the emergence of intelligent behavior at a macroscopic scale that individuals at the microscopic scale have no idea of. This process has been discussed in biology for instance in animal populations such as ant colonies or in connection with the emergence of consciousness [1,50]. The usual efficient market hypothesis will be recovered in this context when the traders learn how to extract this novel collective information and act on it.

Most previous models proposed for crashes have pondered the possible mechanisms to explain the collapse of the price at very short time scales. Here in contrast, we propose that the underlying cause of the crash must be searched years before it in the progressive accelerating ascent of the market price, reflecting an increasing built-up of the market cooperativity. From that point of view, the specific manner by which prices collapsed is not of real importance since, according to the concept of the critical point, any small disturbance or process may have triggered the instability. The intrinsic divergence of the sensitivity and the growing instability of the market close to a critical point might explain why attempts to unravel the local origin of the crash have been so diverse. Essentially anything would work once the system is ripe. Our view is that the crash has an endogenous origin and that exogenous shocks only serve as triggering factors. We propose that the origin of crashes is much more subtle and is constructed progressively by the market as a whole. In this sense, this could be termed a systemic instability.

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References

[1] P.W. Anderson, K.J. Arrow, D. Pines (eds.), The Economy as an Evolving Complex System, Addison–Wesley, New York, 1988.
[2] A.J. Prost, R. Prechter, Elliott Waves Principle, New Classic Library, 1985.
[3] T. Béchu, E. Bertrand, L'analyse Technique, Collection Gestion, Economica, 1992.
[4] B.B. Mandelbrot, J. Bus. Univ. Chicago 39 (1966) 242.
[5] B.B. Mandelbrot, J. Bus. Univ. Chicago 40 (1967) 393.
[6] A. Orlean, La Recherche 22 (1991) 668.
[7] A. Orlean, Theory Decision 27 (1989) 63.
[8] L. Ingber, Phys. Rev. A 42 (1990) 705.
[9] L. Ingber, M.F. Wehner, G.M. Jabbour, T.M. Barnhill, Math. Comput. Modelling 15 (1991) 77.
[10] L. Ingber, Math. Comput. Modelling 23 (1996) 101.
[11] R.N. Mantegna, Physica A 179 (1991) 232.
[12] B. Barral, H. Chate, P. Manneville, Phys. Lett. A 163 (1992) 279.
[13] H. Chate, P. Manneville, Progr. Theoret. Phys. 87 (1992) 1.
[14] H. Takayasu, H. Miura, T. Hirabayashi, K. Hamada, Physica A 184 (1992) 127.
[15] P. Bak, K. Chen, J.A. Scheinkman, M. Woodford, Ric. Economiche 47 (1993) 3.
[16] L.L. Ghezzi, Y.A. Kuznetsov, Int. J. Systems Sci. 25 (1994) 1941.
[17] L.L. Ghezzi, L. Peccati, Appl. Math. Comput. 63 (1994) 123.
[18] N.S. Glance, B.A. Huberman, Phys. Lett. A 165 (1992) 432.
[19] R.G. Palmer, W.B. Arthur, J.H. Holland, B. LeBaron, P. Tayler, Physica D 75 (1994) 264.
[20] P. Bak, M. Paczuski, M. Shubik, Price variations in a stock market with many agents, Physica A, in press, preprint 9609144.
[21] T. Hirabayashi, H. Takayasu, H. Miura, K. Hamada, Fractals 1 (1993) 29.
[22] T. Hellthaler, Int. J. Mod. Phys. C 6 (1995) 845.
[23] M. Levy, H. Levy, S. Solomon, Economic Lett. 45 (1995) 103.
[24] M. Levy, H. Levy, S. Solomon, J. Phys. I France 5 (1995) 1087.
[25] M. Levy, S. Solomon, Int. J. Mod. Phys. C 7 (1996) 65.
[26] R.P.J. Perazzo, S.L. Reich, J. Schvarzer, M.A. Virasoro, Chaos, Solitons & Fractals 6 (1995) 455.
[27] E. Tostesen, Dynamics of hierarchically clustured cooperative agents Cand. Scient., Thesis, University of Copenhagen.
[28] C. Tsallis, A.M.C. Desouza, E.M.F. Curado, Chaos, Solitons & Fractals 6 (1995) 561.
[29] R. Mantegna, H.E. Stanley, Nature 376 (1995) 46.

[30] S. Ghashghaie, W. Breymann, J. Peinke, P. Talkner, Y. Dodge, Nature 381 (1996) 767.
[31] A. Arneodo, J.P. Bouchaud, R. Cont, J.F. Muzy, M. Potter, D. Sornette, Comment on Turbulent cascades in foreign exchange markets, cond-mat/9607120, 1996.
[32] C. Nicola, Imperfect General Equilibrium: The Economy as an Evolutionary Process: Individualistic, Discrete, Deterministic, Springer, Berlin, 1994.
[33] R.J. Barro, E.F. Fama, D.R. Fischel, A.H. Meltzer, R. Roll, L.G. Telser, R.W. Kamphuis, Jr, R.C. Kormendi, J.W.H. Watson (Eds.), Black Monday and the Future of Financial Markets, Mid American Institute for Public Policy Research, Inc. and Dow Jones-Irwin, Inc., 1989.
[34] D. Sornette, A. Johansen, J.P. Bouchaud, J. Phys. I France 6 (1996) 167.
[35] J.A. Feigenbaum, P.G.O. Freund, Int. J. Mod. Phys. B 10 (1996) 3737.
[36] N. Goldenfeld, Lectures on Phase Transitions and the Renormalization Group, Addison-Wesley, Advanced Book Program, Reading, MA, 1992.
[37] P. Bergé, Y. Pomeau, C. Vidal, Order Within Chaos: Towards a Deterministic Approach to Turbulence, Wiley, New York, 1996.
[38] Numerical Recipes, Cambridge University Press, Cambridge, 1992.
[39] D. Cvijović, J. Klinowski, Science 267 (1995) 664.
[40] H. Saleur, D. Sornette, J. Phys. I France 6 (1996) 327.
[41] D. Sornette, C.G. Sammis, J. Phys. I France 5 (1995) 607.
[42] D.J. Varnes, C.G. Bufe, Geophys. J. Int. 124 (1996) 149.
[43] J.C. Anifrani, C. Le Floc'h, D. Sornette, B. Souillard, J. Phys. I France 5 (1995) 631.
[44] D. Sornette, A. Johansen, A. Arnéodo, J.F. Muzy, H. Saleur, Phys. Rev. Lett. 76 (1996) 251.
[45] H. Saleur, C.G. Sammis, D. Sornette, Nonlinear Process. Geophy. 3 (1996) 102.
[46] H. Saleur, C.G. Sammis, D. Sornette, J. Geophys. Res. 101 (1996) 17661.
[47] A. Johansen, D. Sornette, H. Wakita, U. Tsunogai, W.I. Newman, H. Saleur, J. Phys. I France 6 (1996) 1391.
[48] R.J. Shiller, Market Volatility, The MIT Press, Cambridge, MA, 1989.
[49] E.F. Fama, J. Finance 46 (1991) 1575.
[50] J.H. Holland, Daedalus 121 (1992) 17.