HOMEWORK 4, DUE MARCH 19, 2019

ANALYSIS II

(1) Write the coordinates on $\mathbf{R}^{2n}$ as $(x_1, y_1, x_2, y_2, \cdots, x_n, y_n)$. Let ω be the two form defined by $\omega = dx_1 \wedge dy_1 + dx_2 \wedge dy2 + \cdots + dx_n \wedge dy_n = \sum_{i=1}^n dx_i \wedge dy_i$. Compute $\omega^n = \omega \wedge \cdots \wedge \omega$ (n-fold wedge product). (First work out the cases $n = 1, 2, 3$.)

(2) Write the coordinates on $\mathbf{R}^{2n+1}$ as $(x_1, y_1, x_2, y_2, \cdots, x_n, y_n, z)$. Let $\alpha = dz + \sum_{i=1}^n x_i dy_i$. Compute $\alpha \wedge (d\alpha)^n = \alpha \wedge d\alpha \wedge \cdots \wedge d\alpha$.

(3) Let $\omega^i = \sum_{j=1}^k A_j^i dy_j$ where $a_j^i \in \mathbf{R}$. Show that $\omega^1 \wedge \omega^2 \wedge \cdots \wedge \omega^k = det(A) dy_1 \wedge \cdots \wedge dy_k$.

(4) Let $T : \mathbf{R}^2 \mapsto \mathbf{R}^4$ defined by $T(y_1, y_2) = (y_1^3, y_1^2 y_2, y_1 y_2^2, y_2^3)$. Find the explicit formula for $T^*(dz_2 \wedge dz_3)$ and $T^*(dz_1 + dz_3)$.