HOMEWORK 2, DUE MARCH 5, 2019

ANALYSIS II

(1) Suppose that in the square $S = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$ we define the set A as the set of points (x, y) such that x and y are rational and that, when they are represented in the form of $x = \frac{p_1}{q_1}, y = \frac{p_2}{q_2}$ (lowest terms), then $q_1 = q_2$. Suppose that $f : S \mapsto \mathbf{R}$ is given by

$$f(x, y) = \begin{cases} 0 & \text{f } (x, y) \in A \\ 1 & \text{if } (x, y) \in S \setminus A \end{cases}$$

(a) Prove that A is dense in S (that is, the closure of A contains S) but that any line parallel to the coordinate axes contains at most a finite subset of A.

(b) Show that $\int_0^1 (\int_0^1 f(x, y)dy)dx$ and $\int_0^1 (\int_0^1 f(x, y)dx)dy$, both exist and have the value 1.

(c) Show that $\int_S f$ does not exist.

(Hint: If p_k denotes the kth prime number, let $A_{p_k} = \{(\frac{n}{p_k}, \frac{m}{p_k}) | n = 1, 2, \cdots, p_k - 1, m = 1, 2 \cdots, p_k - 1\}$, then $A = \cup_{k=1}^{\infty} A_{p_k}$)

(2) Do problem 46 and 47 on p 375 from Chapter 5 of the textbook "Real Mathematical Analysis"/