Lagrangian

$$
\begin{aligned}
\mathcal{L} = {} & \frac{m\omega_y^2}{2}\left({r_1}^2(t) + {r_2}^2(t)\right) + \frac{m\omega_y}{2}\left(-{r_1}^2(t) + {r_2}^2(t)\right)\frac{d}{dt}q(t) \\
& + \frac{m}{8}\left({r_1}^2(t) + {r_2}^2(t)\right)\frac{d}{dt}q(t)^2 + \frac{m}{2}\frac{d}{dt}r_1(t)^2 \\
& + \frac{m}{2}\frac{d}{dt}r_2(t)^2 + \omega_x\left(\frac{m\omega_x}{2}\left({r_1}^2(t) + {r_2}^2(t)\right)\sin^2\left(\frac{1}{2}q(t)\right)\right. \\
& \left. + \frac{m\omega_z}{4}\left(-{r_1}^2(t) + {r_2}^2(t)\right)\sin\left(q(t)\right)\right) + \omega_z\left(\frac{m\omega_x}{4}\left(-{r_1}^2(t) + {r_2}^2(t)\right)\sin\left(q(t)\right)\right. \\
& \left. + \frac{m\omega_z}{2}\left({r_1}^2(t) + {r_2}^2(t)\right)\cos^2\left(\frac{1}{2}q(t)\right)\right)
\end{aligned}
\tag{1}
$$

Hamiltonian

$$
\begin{aligned}
& \mathcal{H} \\
& = \frac{J_x^2 \Bigl(-\left(\cos\left(q(t)\right) - 1\right)^2 {r_1}^4(t) + 2\left(\cos\left(q(t)\right) - 1\right)^2 {r_1}^2(t){r_2}^2(t) - \left(\cos\left(q(t)\right) - 1\right)^2 {r_2}^4(t) - 16{r_1}^2(t){r_2}^2(t)\sin}{16m\left({r_1}^2(t) + {r_2}^2(t)\right)\left(\cos\left(q(t)\right) - 1\right){r_1}^2(t){r_2}^2(t)\sin^4\left(\frac{1}{2}q(t)\right)} \\
& \quad + \frac{J_x J_z\left({r_1}^2(t) - {r_2}^2(t)\right)}{2m{r_1}^2(t){r_2}^2(t)\sin\left(q(t)\right)} + J_y^2\left(\frac{1}{8m{r_2}^2(t)} + \frac{1}{8m{r_1}^2(t)}\right) \\
& \quad + J_y\left(\frac{1}{2m{r_2}^2(t)} - \frac{1}{2m{r_1}^2(t)}\right)p(t) \\
& \quad + \frac{J_z^2\left({r_1}^2(t) + {r_2}^2(t)\right)\left(-\cos\left(q(t)\right) + 1\right)}{4m{r_1}^2(t){r_2}^2(t)\sin^2\left(q(t)\right)} \\
& \quad + \left(\frac{1}{2m{r_2}^2(t)} + \frac{1}{2m{r_1}^2(t)}\right)p^2(t) + \frac{{p_1}^2(t)}{2m} + \frac{{p_2}^2(t)}{2m}
\end{aligned}
\tag{2}
$$