Lagrangian

$$\mathcal{L} = m\omega_x^2 r_0^2 \sin^2\left(\frac{1}{2}q(t)\right) + m\omega_y^2 r_0^2 + m\omega_z^2 r_0^2 \cos^2\left(\frac{1}{2}q(t)\right) + \frac{mr_0^2}{4}\frac{d}{dt}q(t)^2 \quad (1)$$

Hamiltonian

$$\mathcal{H} = \frac{J_x^2}{4mr_0^2 \sin^2\left(\frac{1}{2}q(t)\right)} + \frac{J_y^2}{4mr_0^2} + \frac{J_z^2}{4mr_0^2 \cos^2\left(\frac{1}{2}q(t)\right)} + \frac{p^2(t)}{mr_0^2} \quad (2)$$