

Lagrangian

$$\begin{aligned}\mathcal{L} = & m\omega_x^2 r_0^2 \sin^2\left(\frac{1}{2}q(t)\right) + m\omega_y^2 \left(r_0^2 \sin^2\left(\frac{1}{2}q(t)\right) + r_0^2 \cos^2\left(\frac{1}{2}q(t)\right)\right) \\ & + m\omega_z^2 r_0^2 \cos^2\left(\frac{1}{2}q(t)\right) + \frac{mr_0^2}{4} \left(\frac{d}{dt}q(t)\right)^2\end{aligned}\quad (1)$$

Hamiltonian

$$\mathcal{H} = \frac{J_x^2}{2mr_0^2(-\cos(q(t)) + 1)} + \frac{J_y^2}{4mr_0^2} + \frac{J_z^2}{2mr_0^2(\cos(q(t)) + 1)} + \frac{p^2(t)}{mr_0^2}\quad (2)$$