Definitions

$$F\left(\omega\right)=\int\limits_{-\infty}^{\infty}f(t)\exp\left(-i\omega t\right)dt$$

$$f(t)=F^{-1}\left[F\left(\omega\right)\right]=\int\limits_{-\infty}^{\infty}F\left(\omega\right)\exp\left(i\omega t\right)\frac{d\omega}{2\pi}$$

Convolution theorem

$$f*g=\int\limits_{-\infty}^{\infty}g\left(t'\right)f\left(t-t'\right)dt'=\int\limits_{-\infty}^{\infty}g(t')\left[\int\limits_{-\infty}^{\infty}F(\omega)\exp\left(i\omega(t-t')\right)\frac{d\omega}{2\pi}\right]dt'=\int\limits_{-\infty}^{\infty}F(\omega)\left[\int\limits_{-\infty}^{\infty}g(t')\exp\left(i\omega(t-t')\right)dt'\right]\frac{d\omega}{2\pi}=$$

$$=\int\limits_{-\infty}^{\infty}F(\omega)G(\omega)\exp\left(i\omega t\right)\frac{d\omega}{2\pi}=F^{-1}\left[F(\omega)G(\omega)\right]$$

$$F\left[f*g\right]=F(\omega)G(\omega)$$

Correlation theorem

$$f\bigstar g=\int\limits_{-\infty}^{\infty}\bar{f}(\tau)g(t+\tau)d\tau=\int\limits_{-\infty}^{\infty}\left[\int\limits_{-\infty}^{\infty}\bar{F}(\omega)\exp(-i\omega\tau)\frac{d\omega}{2\pi}\right]\left[\int\limits_{-\infty}^{\infty}G(\omega')\exp\left(i\omega'(t+\tau)\right)\frac{d\omega'}{2\pi}\right]d\tau=$$

$$=\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\bar{F}(\omega)G(\omega')\exp\left(i\omega' t\right)\left[\int\limits_{-\infty}^{\infty}\exp\left(i\tau(\omega'-\omega)\right)\frac{d\tau}{2\pi}\right]\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}=\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\bar{F}(\omega)G(\omega')\exp\left(i\omega' t\right)\delta\left(\omega-\omega'\right)\frac{d\omega}{2\pi}d\omega'=$$

$$=\int\limits_{-\infty}^{\infty}\bar{F}(\omega)G(\omega)\exp\left(i\omega t\right)\frac{d\omega}{2\pi}=F^{-1}\left[\bar{F}(\omega)G(\omega)\right]$$

$$F\left[f\bigstar g\right]=\bar{F}(\omega)G(\omega)$$

Autocorrelation function and Wiener-Khintchine theorem

$$C(t)=f\bigstar f=\int\limits_{-\infty}^{\infty}\bar{f}(\tau)f(t+\tau)d\tau$$

$$F\left[C(t)\right]=\left|F(\omega)\right|^2\quad\longleftrightarrow\quad\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\bar{f}(\tau)f(t+\tau)\exp\left(-i\omega t\right)dt\,d\tau=\left|\int\limits_{-\infty}^{\infty}f(t)\exp\left(i\omega t\right)dt\right|^2$$