

$$\mathcal{H} = \frac{p_R^2}{2\mu} + \frac{J_x^2}{2\mu R^2} + \frac{J_y^2}{2\mu R^2} + U$$

$$\begin{cases} x_1^2 = \frac{p_R^2}{2\mu kT} \\ x_2^2 = \frac{J_x^2}{2\mu R^2 kT} \\ x_3^2 = \frac{J_y^2}{2\mu R^2 kT} \end{cases}$$

$$dp_R dJ_x dJ_y = (2\mu kT)^{\frac{3}{2}} R^2 dx_1 dx_2 dx_3$$

$$\int_{\mathcal{H} < 0} \exp\left(-\frac{H}{kT}\right) dR dp_R dJ_x dJ_y = (2\mu kT)^{\frac{3}{2}} \int R^2 dR \exp\left(-\frac{U}{kT}\right) \int \exp(-x_1^2 - x_2^2 - x_3^2) dx_1 dx_2 dx_3$$

$$\int_{x_1^2 + x_2^2 + x_3^2 + \frac{U}{kT} < 0} \exp(-x_1^2 - x_2^2 - x_3^2) dx_1 dx_2 dx_3 = \frac{\pi^{\frac{3}{2}}}{\Gamma(\frac{3}{2})} \gamma\left(\frac{3}{2}, -\frac{U}{kT}\right)$$

$$Q_{pair}^{bound} = 4\pi \left(\frac{2\pi M kT}{h^2}\right)^{\frac{3}{2}} V \left(\frac{2\pi \mu kT}{h^2}\right)^{\frac{3}{2}} \int_R R^2 \frac{\gamma(\frac{3}{2}, -\frac{U}{kT})}{\Gamma(\frac{3}{2})} \exp\left(-\frac{U}{kT}\right) dR$$