Lagrangian

$$\begin{aligned}\mathcal{L} = & \frac{m\omega_y}{2}\left(-{r_1}^2(t)+{r_2}^2(t)\right)\frac{d}{dt}q(t)+\frac{m}{8}\left({r_1}^2(t)+{r_2}^2(t)\right)\left(\frac{d}{dt}q(t)\right)^2 \\ & +\frac{m}{2}\left(\frac{d}{dt}r_1(t)\right)^2+\frac{m}{2}\left(\frac{d}{dt}r_2(t)\right)^2 \\ & +\omega_x\left(\frac{\omega_x}{2}\left(m{r_1}^2(t)\sin^2\left(\frac{1}{2}q(t)\right)+m{r_2}^2(t)\sin^2\left(\frac{1}{2}q(t)\right)\right)\right. \\ & \qquad +\frac{\omega_z}{2}\left(-m{r_1}^2(t)\sin\left(\frac{1}{2}q(t)\right)\cos\left(\frac{1}{2}q(t)\right)\right. \\ & \qquad \left.\left.+m{r_2}^2(t)\sin\left(\frac{1}{2}q(t)\right)\cos\left(\frac{1}{2}q(t)\right)\right)\right) \\ & +\frac{\omega_y^2}{2}\left(m\left({r_1}^2(t)\sin^2\left(\frac{1}{2}q(t)\right)+{r_1}^2(t)\cos^2\left(\frac{1}{2}q(t)\right)\right)\right. \\ & \qquad \left.+m\left({r_2}^2(t)\sin^2\left(\frac{1}{2}q(t)\right)+{r_2}^2(t)\cos^2\left(\frac{1}{2}q(t)\right)\right)\right) \\ & +\omega_z\left(\frac{\omega_x}{2}\left(-m{r_1}^2(t)\sin\left(\frac{1}{2}q(t)\right)\cos\left(\frac{1}{2}q(t)\right)\right.\right. \\ & \qquad \left.+m{r_2}^2(t)\sin\left(\frac{1}{2}q(t)\right)\cos\left(\frac{1}{2}q(t)\right)\right) \\ & \quad +\frac{\omega_z}{2}\left(m{r_1}^2(t)\cos^2\left(\frac{1}{2}q(t)\right)+m{r_2}^2(t)\cos^2\left(\frac{1}{2}q(t)\right)\right)\Bigg)\end{aligned} \tag{1}$$

Hamiltonian

$$\begin{aligned}\mathcal{H} = & \frac{J_x^2\left(\frac{1}{4}{r_1}^2(t)+\frac{1}{4}{r_2}^2(t)\right)}{m\left(-\cos\left(q(t)\right)+1\right){r_1}^2(t){r_2}^2(t)}+\frac{J_xJ_z\left(r_1(t)-r_2(t)\right)\left(r_1(t)+r_2(t)\right)}{2m{r_1}^2(t){r_2}^2(t)\sin\left(q(t)\right)} \\ & +J_y^2\left(\frac{1}{8m{r_2}^2(t)}+\frac{1}{8m{r_1}^2(t)}\right)-\frac{J_y\left(-{r_1}^2(t)+{r_2}^2(t)\right)p(t)}{2m{r_1}^2(t){r_2}^2(t)} \\ & +\frac{J_z^2\left(\frac{1}{4}{r_1}^2(t)+\frac{1}{4}{r_2}^2(t)\right)}{m\left(\cos\left(q(t)\right)+1\right){r_1}^2(t){r_2}^2(t)}+\frac{\left({r_1}^2(t)+{r_2}^2(t)\right)p^2(t)}{2m{r_1}^2(t){r_2}^2(t)}+\frac{{p_1}^2(t)}{2m}+\frac{{p_2}^2(t)}{2m}\end{aligned} \tag{2}$$