

## Lagrangian

$$\begin{aligned}
\mathcal{L} = & \frac{m\omega_y}{2} (-r_1^2(t) + r_2^2(t)) \frac{d}{dt}q(t) + \frac{m}{8} (r_1^2(t) + r_2^2(t)) \frac{d}{dt}q(t)^2 + \frac{m}{2} \frac{d}{dt}r_1(t)^2 \\
& + \frac{m}{2} \frac{d}{dt}r_2(t)^2 + \omega_x \left( \frac{\omega_x}{2} \left( mr_1^2(t) \sin^2 \left( \frac{1}{2}q(t) \right) + mr_2^2(t) \sin^2 \left( \frac{1}{2}q(t) \right) \right) \right. \\
& \quad \left. + \frac{\omega_z}{2} \left( -mr_1^2(t) \sin \left( \frac{1}{2}q(t) \right) \cos \left( \frac{1}{2}q(t) \right) \right. \right. \\
& \quad \left. \left. + mr_2^2(t) \sin \left( \frac{1}{2}q(t) \right) \cos \left( \frac{1}{2}q(t) \right) \right) \right) \\
& + \frac{\omega_y^2}{2} \left( m \left( r_1^2(t) \sin^2 \left( \frac{1}{2}q(t) \right) + r_1^2(t) \cos^2 \left( \frac{1}{2}q(t) \right) \right) \right. \\
& \quad \left. + m \left( r_2^2(t) \sin^2 \left( \frac{1}{2}q(t) \right) + r_2^2(t) \cos^2 \left( \frac{1}{2}q(t) \right) \right) \right) \\
& + \omega_z \left( \frac{\omega_x}{2} \left( -mr_1^2(t) \sin \left( \frac{1}{2}q(t) \right) \cos \left( \frac{1}{2}q(t) \right) \right. \right. \\
& \quad \left. \left. + mr_2^2(t) \sin \left( \frac{1}{2}q(t) \right) \cos \left( \frac{1}{2}q(t) \right) \right) \right. \\
& \quad \left. + \frac{\omega_z}{2} \left( mr_1^2(t) \cos^2 \left( \frac{1}{2}q(t) \right) + mr_2^2(t) \cos^2 \left( \frac{1}{2}q(t) \right) \right) \right) \\
& \quad \quad \quad (1)
\end{aligned}$$

## Hamiltonian

$$\begin{aligned}
\mathcal{H} = & \frac{J_y p(t)}{2mr_2^2(t)} - \frac{J_y p(t)}{2mr_1^2(t)} + \frac{p^2(t)}{2mr_2^2(t)} + \frac{p^2(t)}{2mr_1^2(t)} + \frac{p_1^2(t)}{2m} + \frac{p_2^2(t)}{2m} \\
& + \frac{1}{2m(-\cos(q(t)) + 1)(\cos(q(t)) + 1)r_1^2(t)r_2^2(t)} \left( J_x^2 r_1^2(t) \cos^2 \left( \frac{1}{2}q(t) \right) \right. \\
& \quad \left. + J_x^2 r_2^2(t) \cos^2 \left( \frac{1}{2}q(t) \right) + J_x J_z r_1^2(t) \sin(q(t)) - J_x J_z r_2^2(t) \sin(q(t)) \right. \\
& \quad \left. + \frac{J_y^2}{8} (-\cos(2q(t)) + 1) r_1^2(t) + \frac{J_y^2}{8} (-\cos(2q(t)) + 1) r_2^2(t) \right. \\
& \quad \left. + J_z^2 r_1^2(t) \sin^2 \left( \frac{1}{2}q(t) \right) + J_z^2 r_2^2(t) \sin^2 \left( \frac{1}{2}q(t) \right) \right) \\
& \quad \quad \quad (2)
\end{aligned}$$