

## AI HW4

### Problem 1

Assumption: all the objects in this problem are people.

(1) People who got an S passed the class

Objects: People (X1, X2, X3, ..., Y1, Y2, Y3...)

Relation: GetS(X): A person gets S in the class; ClassPassed(X): A person who can pass the class. X is an object.

Final Solution:  $\forall X (GetS(X) \Rightarrow ClassPassed(X))$

(2) There are at least two people who passed the class.

Objects: People (X1, X2, X3, ..., Y1, Y2, Y3...)

Relation: ClassPassed(X): A person who can pass the class. X is an object.

Final Solution:  $\exists X \exists Y (X \neq Y \wedge ClassPassed(X) \wedge ClassPassed(Y))$

(3) There is exactly one person who passed the class.

Objects: People (X1, X2, X3, ..., Y1, Y2, Y3...)

Relation: ClassPassed(X): A person who can pass the class. X is an object.

Final Solution:  $\exists X \forall Y (X \neq Y \wedge ClassPassed(X) \wedge (\neg ClassPassed(Y)))$

(4) You can only get one grade, either S or N.

Objects: People (X1, X2, X3, ..., Y1, Y2, Y3...)

Relation: GetS(X): A person gets S in the class; GetN(X): A person gets N in the class. X is an object.

Final Solution:  $\forall X (GetS(X) \wedge \neg GetN(X)) \vee (\neg GetS(X) \wedge GetN(X))$

## Problem 2

$$A \wedge (B \vee C) \wedge (C \Rightarrow B)$$

(1) What is the model of these sentences?

KB = A; (B  $\vee$  C); (C  $\Rightarrow$  B)

The truth table:

Since A is a single sentence, so it must be true.

	A	B	C
I1	True	True	True
I2	True	True	False
I3	True	False	True
I4	True	False	False
I5	False	True	True
I6	False	True	False
I7	False	False	True
I8	False	False	False

Since a model of a set of clauses is an interpretation in which all the clauses are true, so from the truth table, I can conclude I1 and I2 are models of these sentences

(2) What is the model for the sentence: B?

	B
I1	True
I2	False

By definition in lecture slides, m satisfies  $\alpha$  means  $\alpha$  is true in m (also said as "m models  $\alpha$ "). So, B must be true for sentence B. Therefore, I can conclude I1 is model for sentence B.

(3) Show whether or not “(1) entails (2)”

1 entails 2 because all the models that evaluate (1) to true also evaluate sentence B to true.

(4) What does “X entails Y” and “Y entails X” mean in terms of the models of X and Y?

X and Y should be the same model. Since X is a subset of Y and Y is a subset of X implies they should be the same.

### Problem 3

$$\begin{aligned} & \forall x \text{ Hot}(x) \wedge \text{Large}(x) \Rightarrow \text{Star}(x) \wedge \text{Fusion}(x) \\ & \text{Hot}(\text{Sun}) \vee \text{Hot}(\text{Jupiter}) \\ & \text{Large}(\text{Jupiter}) \\ & \text{Large}(\text{Sun}) \wedge \text{Hot}(\text{Sun}) \end{aligned}$$

Assume our objects are: {Sun, Jupiter}

1: Convert these sentences to propositional logic

(1):  $\text{Hot}(\text{Sun}) \wedge \text{Large}(\text{Sun}) \Rightarrow \text{Star}(\text{Sun}) \wedge \text{Fusion}(\text{Sun})$ ;

(2):  $\text{Hot}(\text{Jupiter}) \wedge \text{Large}(\text{Jupiter}) \Rightarrow \text{Star}(\text{Jupiter}) \wedge \text{Fusion}(\text{Jupiter})$ ;

(3):  $\text{Hot}(\text{Sun}) \vee \text{Hot}(\text{Jupiter})$

(4):  $\text{Large}(\text{Jupiter})$

(5):  $\text{Large}(\text{Sun}) \wedge \text{Hot}(\text{Sun})$

Next, we can simplify these propositional phase.

1.  $\text{HS} \wedge \text{LS} \Rightarrow \text{SS} \wedge \text{FS}$

2.  $\text{HJ} \wedge \text{LJ} \Rightarrow \text{SJ} \wedge \text{FJ}$

3.  $\text{HS} \vee \text{HJ}$

4.  $\text{LJ}$

5.  $\text{LS} \wedge \text{HS}$

(2) Convert part (1) to CNF form.

1:  $\text{HS} \wedge \text{LS} \Rightarrow \text{SS} \wedge \text{FS} = \neg(\text{HS} \wedge \text{LS}) \vee (\text{SS} \wedge \text{FS}) = (\neg\text{HS} \vee \neg\text{LS}) \vee (\text{SS} \wedge \text{FS})$   
 $= (\neg\text{HS} \vee \neg\text{LS} \vee \text{SS}) \wedge (\neg\text{HS} \vee \neg\text{LS} \vee \text{FS})$

2:  $\text{HJ} \wedge \text{LJ} \Rightarrow \text{SJ} \wedge \text{FJ} = \neg(\text{HJ} \wedge \text{LJ}) \vee (\text{SJ} \wedge \text{FJ}) = (\neg\text{HJ} \vee \neg\text{LJ}) \vee (\text{SJ} \wedge \text{FJ})$   
 $= (\neg\text{HJ} \vee \neg\text{LJ} \vee \text{SJ}) \wedge (\neg\text{HJ} \vee \neg\text{LJ} \vee \text{FJ})$

3:  $\text{HS} \vee \text{HJ}$

4: LJ

5: LS  $\wedge$  HS

## Problem 4

$A$   
 $\neg A \vee B$   
 $C \vee \neg D$   
 $D \vee C$   
 $\neg B \vee \neg D \vee E$

(1) Use resolution to determine whether these sentences entail C

A

$\neg A \vee B$

By using resolution rule, we can eliminate A. Thus, we will have B.

$C \vee \neg D$

$D \vee C$

By using resolution rule, we can eliminate D. Then, we will have C.

Therefore, we can know these sentences entail C

(2) Use resolution to determine whether these sentences entail E

From (1) we can get B and C. Then using the following sentence.

B

C

$\neg B \vee \neg D \vee E$

By using resolution rule, we can eliminate B. Then, we will have:  $\neg D \vee E$ .

However, it seems we can't eliminate D in these sentences. So, I can conclude we can't entail E in these sentences.

## Problem 5

We have the following English sentences:

"Youtube videos are free and either about cats or dogs" //sentence is so confusing.

"Super secret website is a youtube video"

"Cat youtubes lose money"

“Dog youtubes lose money”

(1) Convert this into first order logic

Objects: Video ( $X_1, X_2, X_3, \dots$ ); Super secret website ( $S_1, S_2, S_3, \dots$ )

Relation: Free( $X$ ): A video is free; Cats( $X$ ): A video about cats; Dogs( $X$ ): A video about dogs;

LoseM( $X$ ): A video lose money; Youtube( $X$ ): a video is a youtube video; Youtube( $S$ ): a super secret website is a youtube video

First Order Logic:

1.  $\forall X(Youtube(X) \Rightarrow Free(X) \wedge (Cats(X) \vee Dogs(X)))$
2.  $\exists S(Youtube(S))$
3.  $\forall X(Cats(X) \wedge Youtube(X) \Rightarrow LoseM(X))$
4.  $\forall X(Dogs(X) \wedge Youtube(X) \Rightarrow LoseM(X))$

(2) Use only AND elimination and Modus Ponens to answer whether or not “Super secret website is free”

Step1: From 2, we can get  $\exists S(Youtube(S))$

Step2: using AND elimination on  $\forall X(Youtube(X) \Rightarrow Free(X) \wedge (Cats(X) \vee Dogs(X)))$  to get  $\forall X(Youtube(X) \Rightarrow Free(X))$

Step3: From step1 and step2 we can get the following relation:

$\exists S(Youtube(S)); \forall X(Youtube(X) \Rightarrow Free(X))$  .

Then, we can conclude  $Free(S)$  by using Modus Ponens.

**Therefore, I can conclude Super secret website is free.**

(3) Use only AND elimination and Modus Ponens to answer whether or not “Super secret website is losing money”

Step 1: From 1 and 2, we can get  $\exists S(Youtube(S)) \Rightarrow Free(S) \wedge (Cats(S) \vee Dogs(S))$  because we know for all videos if a video is a youtube video then it is free and either about cats or dogs. So, we can get there exist a super secret website which is a youtube video and it is free and either about cats or dogs.

Step 2: From step1, we can get  $\exists S(Youtube(S) \Rightarrow Cats(S) \vee Dogs(S))$  by using AND elimination.

Step 3: From step2, we got  $\exists S(Youtube(S) \Rightarrow Cats(S) \vee Dogs(S))$  and from 2 we know

$\exists S(Youtube(S))$  . Then, we can plug those 2 logic phase to 3 or 4 to conclude that

$\exists S(Dogs(S) \wedge Youtube(S) \Rightarrow LoseM(S))$  or  $\exists S(Cats(S) \wedge Youtube(S) \Rightarrow LoseM(S))$  by using Modus Ponens.

**Therefore, I can conclude Super secret website is losing money.**

## Problem 6

```
[3]> (setf kb (make-prop-kb))
```

```
#<a PROP-KB>
```

```
[4]> (tell kb "HS or HJ")
```

```
T
```

```
[5]> (tell kb "LJ")
```

```
T
```

```
[6]> (tell kb "LS and HS")
```

```
T
```

```
[7]> (tell kb "(HS^LS)=>(SS^FS)")
```

```
T
```

```
[8]> (tell kb "(HJ^LJ)=>(SJ^FJ)")
```

```
T
```

```
[9]> (ask kb "SS")
```

```
T
```

```
[10]> (ask kb "SJ")
```

```
NIL
```