MEMS Based IMU for Tilting Measurement:

Comparison of Complementary and Kalman Filter Based Data Fusion

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***Abstract*—This research investigates real time tilting measurement using Micro-Electro-Mechanical-system (MEMS) based inertial measurement unit (IMU). Accelerometers suffer from errors caused by external accelerations that sums to gravity and make accelerometers based tilting sensing unreliable and inaccurate. Gyroscopes can offset such drawbacks but have data drifting problems. This paper presents a study on complementary and Kalman filter for tilting measurement using MEMS based IMU. The complementary filter algorithm uses low-pass filter and high-pass filter to deal with the data from accelerometer and gyroscope while Kalman filter takes the tilting angle and gyroscope bias as system states, combining the angle derived from the accelerometer to estimate the tilting angle. The study carried out both static and dynamic experiments. The results showed that both Complementary and Kalman filter were less sensitive to variations and almost no signal coupling phenomenon and able to obtain smooth and accurate results.**

Keywords—Complementary Filter; IMU; Tilt Measurement; Kalman Filter; Data Fusion

I. Introduction

The IMU, or inertial measurement unit, is an electronic device that measures accelerations, rotation rates and possibly earth magnetic field with the use of tri-axis accelerometer, tri-axis gyroscope, and sometimes tri-axis magnetometer to determine an object's attitude or orientation. Due to its unique characteristics, IMUs have long been the subject of extensive research in aerospace [1] and navigation [2] fields although its size was bulky initially. In recent years, with the advent of MEMS (Micro-Electro-Mechanical-system) based IMU, the size of IMU is dramatically reduced to chip size along with the reduction in cost and power consumption [3]. Such a leap in IMU significantly accelerated many research and development work. Robotics, human motion analysis and consumer handheld devices are the areas mostly benefited [4].

One of the applications of MEMS IMU is tilting measurement (or orientation sensing), which has a significant role in the fields of consumer electronics, robotics and navigation. A typical example is smart phones and handheld electronics. The control of menu options, image rotation or function selection all link to tilting measurement [5]. In robotics area, in terms of navigation robot, climbing robot, terrain robot etc., tilt sensing plays a key role in system balancing control. Michelle et al. in 2005 used a low-g MEMS accelerometer and trigonometric function relationship to measure the tilt of an object in a static environment [5]. The result was simple and straightforward but noisy and nonlinear. In 2013, Mark Pedley documented the mathematics of orientation using a MEMS three-axis accelerometer and also made an analysis on the regions of instability (gimbal lock) [6]. As the data from the accelerometer is in general very noisy and susceptible to external acceleration interference, when it is used to measure the gravitational acceleration, it is hard to obtain accurate result in vibrating environment such as in a car or an airplane. Hence tilting measurement only using accelerometer is not effective enough to get accurate data although the accelerometer data is stable and without drift in long term. A gyroscope offers angular velocities around the three axes and the signals are not susceptible to external forces comparing to accelerometers. Therefore, a tri-axis gyroscope and a tri-axis accelerometer (mutually orthogonal) based MEMS IMU seems to be a complete solution for tilt sensing. However, a gyroscope also has its own disadvantages that the data measured has a tendency to drift because of the angular velocity data bias accumulation over time. In short, the data from the gyroscope on a short term is more trustable. Taking the advantages and disadvantages of gyroscopes and accelerometers into consideration, the IMU data fusion is an essential for a reliable tilt sensing especially for real time applications.

In terms of IMU data fusion algorithm, the complementary filter [7]-[13] and Kalman filter [14]-[19] are the most widely used algorithms. Each has its unique advantages and disadvantages [20]. Kalman filter is an iterative filter, which is efficient but high computational complexity. The complementary filter uses relative easy algorithm, which only requires light computation and easy to implement. Such a feature makes it preferred for embedded systems. This paper presents a study of IMU data fusion for real time tilt sensing based on a 6 DOF IMU. The research first studied both complementary and Kalman filter in IMU data fusion under static scenario separately and repeated the same study for dynamic situation. A comparison was made on the experiment results obtained using the two algorithms. The analysis and outcome showed that there is a small difference between these two filters. However, complementary filter gives better result.

The body of paper is organized in four major parts. In section Ⅱ and Ⅲ, the two IMU data fusion algorithms are

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briefly introduced. The study, comparison and analysis of the data fusion using the two filters are presented in section Ⅳ together with the experiment results. Section Ⅴ is the discussions and conclusions.

II. Using Complementary Filter for IMU Data Fusion

A complementary filter was proposed by Shane Colton in 2007 [21]. For tilt sensing, the filter performs low-pass filtering on low-frequency tilt estimation and the data is from the accelerometer while the high-pass filtering on biased high-frequency tilt estimation is handled by directly integrating with the gyroscope output. The fusion of the two estimations gives an all-pass estimation of the orientation [7]. Obviously, the complementary filter takes the advantage of both accelerometer and gyroscope. On the short term, it uses the data from the gyroscope and the data is precise and not susceptible to external forces; on the long term, it relies on the data from the accelerometer to prevent data drift. The principle of the complementary filter is illustrated in Figure 1.

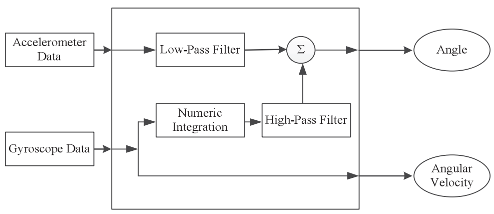

Fig. 1. Complementary Filter Algorithm

The function of the low-pass filter is to only let long-term changes pass through, filtering out short-term fluctuations. One way to achieve such a goal is to force the fluctuations building up little by little through subsequent time intervals. For example, if the tilting angle starts from zero, the accelerometer reading suddenly jumps to a specific degree, e.g. 10 degrees, and stays at that level. The angle estimation using complementary filter will smoothly rise to 10 degrees without spikes. The time it takes to reach the full value of 10 degrees depends on both the filter's parameters and the sample rate of the data acquisition. The high-pass filter does the same thing for long-term fluctuations. It allows short-duration signals to pass through while filtering out signals that are steady over time. Such a character is essential to cancel out the gyroscope data drifting in order to get an accurate estimated angle.

The mathematical model of the complementary filter can be represented as:

$$\theta_{Angle} = \alpha * (\theta_{Angle} + \omega_{Gyro} * dt) + (1-\alpha) * a_{Acc} \qquad (1)$$

θ_{Angle} is the tilting angle (pitch or roll), α is the filter coefficient, ω_{Gyro} represents the angular velocity from the gyroscope, and a_{Acc} is the angle obtained through the data from accelerometer. The data from the gyro and accelerometer must be zeroed and scaled before using Equation (1) to calculate the angle. The filter coefficient α is determined by Equation (2).

$$\alpha = \frac{\tau}{\tau + dt} \qquad (2)$$

where τ is the time constant of the filter.

For a low-pass filter, the signals that are much longer than the time constant can pass the filter unaltered while the signals shorter than the time constant are filtered out. The opposite is also true for a high-pass filter. For every time interval, the gyroscope data is first integrated with the current angle and then combined with the low-pass data from the accelerometer. The filter coefficients, α and 1- α, have to be added up to one, so that the output is an accurate and linear estimation in units that make sense [21].

The data processing procedure of the complementary filter in IMU data fusion in tilt sensing is shown in Figure 2.

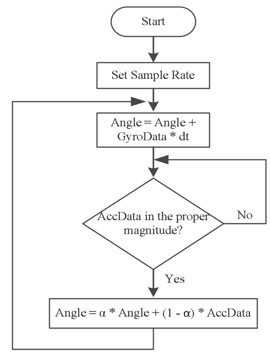

Fig. 2. Complementary Filter Flowchart

The complementary filter applied in the process of IMU data fusion is implemented within a loop. The angle values of the pitch or roll are updated with gyroscope output by means of integration over time. The filter then checks whether the magnitude of the force measured by the accelerometer has a reasonable value that could be a real g-force vector. If the value is too small or too big, it will not be taken into account as a disturbance. In this data fusion process, the current pitch or roll angle is determined by previous pitch or roll angle, present gyroscope output, filter coefficient α, and the percentage of the current data from the accelerometer.

From Equation (1), it is obvious that the complementary filter has low computation burden in terms of IMU data fusion. However, the coefficient of the filter depends on the tuning of the filter time constant τ, which is heavily relies on experience.

III. Data Fusion Using Kalman Filter

Since Kalman filter was first published by R.E. Kalman in 1960, due to its advances in digital computing, the Kalman filter has been the subject of extensive research and applications, particularly in the area of autonomous vehicles, navigation, robotic systems, and human motion control [22].

The Kalman filter, also known as linear quadratic estimation, is an iterative algorithm which relies on a series of measurements observed over time. Noises in measurement data contribute to the errors. In the IMU data fusion process, the Kalman filter takes the noises into account via covariance matrices and updates the matrices at each time interval. It estimates the state of the system based on the current and previous states. The precision of the data filtered is significantly improved. The key of Kalman filter based 6-DOF IMU data fusion algorithm is to find the weighted average (Kalman gain K), with more weight being given to the estimation with higher certainty. Such a process based on the accelerometer model and gyroscope model.

A. Accelerometer Model

An accelerometer measures all forces that are working on the object, which includes instantaneous linear accelerations as well as the gravitational acceleration plus some added biases and noises [3]. The accelerometer model is represented as:

$$\alpha_{Acc} = \alpha_{extra} - g + b_a + n_a \tag{3}$$

Where, α_{extra}, g, b_a, and n_a means external acceleration, gravitational acceleration, accelerometer bias and noise respectively.

B. Gyroscope Model

A gyroscope is used to measure the angular velocity around three mutually perpendicular axes. It is not easy to be affected by external interference but its data will suffer from drifting in long term.

$$\omega_{gyro} = \omega + b_g + n_g \tag{4}$$

Here, b_g and n_g are the gyroscope bias and noise.

The Kalman filter operates by producing a statistically optimal estimation of the system state relied upon the measurements. The filtering process is based on the noise of the input to the filter called the measurement noise and the noise of the system itself called the process noise. To make things easier, assume the noises are Gaussian distributed and have a mathematical expectation of zero.

C. Kalman Filter for IMU Data Fusion

Equation (5) and (6) are the standard Kalman filter expressions.

$$x_k = Fx_{k-1} + Bu_k + w_k \tag{5}$$

$$z_k = Hx_k + v_k \tag{6}$$

x_k is the system state matrix at time k, which is given by:

$$x_k = \begin{bmatrix} \theta \\ \dot{\theta}_b \end{bmatrix}_k \tag{7}$$

The outputs of the filter are the angle θ and the bias $\dot{\theta}_b$ based upon the measurements from the accelerometer and gyroscope. The bias is the amount that the gyroscope has drifted. F is the state transition matrix which is applied to the previous state x_{k-1}. For the 6-DOF IMU case that contains a three-axis accelerometer and a three-axis gyroscope, F is defined as:

$$F = \begin{bmatrix} 1 & -\Delta t \\ 0 & 1 \end{bmatrix} \tag{8}$$

u_k is the control input. For the IMU data fusion discussed, it is the gyroscope measurement in degrees per second at time k, which is also called the angular rate $\dot{\theta}$. Then the state Equation (5) can be rewritten as:

$$x_k = Fx_{k-1} + B\dot{\theta}_k + w_k \tag{9}$$

B is called the control matrix and is defined by:

$$B = \begin{bmatrix} \Delta t \\ 0 \end{bmatrix} \tag{10}$$

Angle θ is obtainable through Equation (10) when multiplying the rate $\dot{\theta}$ by time Δt. Since the bias cannot be calculated directly based on the angular velocity the bottom of the matrix is set to zero.

In equation (5) and (6), the variables w_k and v_k represent the process and measurement noise respectively. They are assumed to be independent of each other and with normal probability distributions (Gaussian white noise) [22].

$$w_k \sim N(0, \quad Q_k) \tag{11}$$

$$v_k \sim N(0, \quad R) \tag{12}$$

Q_k is the process noise covariance matrix which represents the state estimation of the accelerometer and bias. If consider the estimation of the bias and the accelerometer to be independent, then Q_k is equal to the variance of the estimation of the accelerometer and bias.

$$Q_k = \begin{bmatrix} Q_\theta & 0 \\ 0 & Q_{\dot{\theta}_b} \end{bmatrix} \Delta t \tag{13}$$

The Q_k covariance matrix depends on the current time k. So the accelerometer variance Q_θ and the variance of the bias $Q_{\dot{\theta}_b}$ are multiplied by time Δt. As the time goes, the process noise will be increased since the last update of the state.

In equation (6), z_k is the measured output. H is the measurement matrix and is used to map the true state space into the observed space. The true state cannot be observed since the measurement is just from the accelerometer. H is given by:

$$H = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{14}$$

The measurement noise covariance R is not a matrix. It is equal to the variance of the measurement noise since the covariance of the same variable is equal to its variance.

$$R = E\begin{bmatrix} v_k & v_k^T \end{bmatrix} = \mathrm{var}(v_k) \tag{15}$$

Assume that the measurement noise is the same and does not depend on the time k:

$$\mathrm{var}(v_k) = \mathrm{var}(v) \tag{16}$$

If the measurement noise variance $\mathrm{var}(v)$ is set too high, the filter will respond slowly as it trusts new measurements less. On the contrast, if the value is set too low the filter will overshoot and be noisy since it trusts the accelerometer measurement too much.

Therefore, the Kalman filter can be written as:

$$\begin{pmatrix}\theta \\ \dot{\theta}_b\end{pmatrix}_k = \begin{pmatrix}1 & -\Delta t \\ 0 & 1\end{pmatrix}\begin{pmatrix}\theta \\ \dot{\theta}_b\end{pmatrix}_{k-1} + \begin{pmatrix}\Delta t \\ 0\end{pmatrix}\dot{\theta}_k + w_k \quad (17)$$

$$z_k = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}\theta \\ \dot{\theta}_b\end{pmatrix}_k + v_k$$

D. Kalman Filter Implementation

The implementation of Kalman filter includes two main steps: predict process and update process.

In predict process, the filter will first estimate the current state and the error covariance matrix at time k. Equation (18) is for the estimation of the current state based on the previous state and the gyroscope measurement.

$$\hat{x}_{k|k-1} = F\,\hat{x}_{k-1|k-1} + B\,\dot{\theta}_k \quad (18)$$

Here, $\hat{x}_{k-1|k-1}$ is the previous estimated state based on the previous state and the estimated state before. $\hat{x}_{k|k-1}$ is priori state which is the estimation of the state matrix at the current time k based on the previous state of the system. $\hat{x}_{k|k}$ is posteriori state which represents the estimation of the state at time k given observations up to and including at time k. The next step for the filter is to estimate the priori error covariance matrix $P_{k|k-1}$ based on the previous error covariance matrix $P_{k-1|k-1}$, which is defined by:

$$P_{k|k-1} = FP_{k-1|k-1}F^T + Q_k \quad (19)$$

The matrix $P_{k|k-1}$ is used to estimate how much trust can be put on the current values of the estimated state. The smaller the values are, the more trust on the current estimated state. Therefore, for the 6-DOF IMU case discussed, the error covariance matrix P is a 2 x 2 matrix.

$$P = \begin{pmatrix}P_{00} & P_{01} \\ P_{10} & P_{11}\end{pmatrix} \quad (20)$$

In the update process, the filter first computes the difference between the measurement z_k and the priori state $\hat{x}_{k|k-1}$.

$$y_k = z_k - H\,\hat{x}_{k|k-1} \quad (21)$$

Then the filter calculates the innovation covariance:

$$S_k = HP_{k|k-1}H^T + R \quad (22)$$

The Kalman gain is therefore defined by:

$$K_k = P_{k|k-1}H^T S_k^{-1} \quad (23)$$

For the 6-DOF IMU case, the Kalman gain is a 2 x 1 matrix.

$$K = \begin{pmatrix}K_0 \\ K_1\end{pmatrix} \quad (24)$$

Updating the posteriori estimation of the current state gives:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k y_k \quad (25)$$

Finally update the posteriori error covariance matrix by:

$$P_{k|k} = (I - K_k H)\,P_{k|k-1} \quad (26)$$

IV. REAL TIME EXPERIMENT RESULT

To investigate the behavior of Complementary filter and Kalman filter in IMU data fusion, MATLAB and Microcontroller based hardware were employed. The experiment is to evaluate the two algorithms and to find out the best suit coefficients for the two filters in real time. Arduino Uno (ATMEGA 328p) is used as the MCU to acquire the 6-DOF IMU data and run the algorithms. The IMU used in the experiment is the InvenSense's MPU-9150 [23], which offers 9-DOF (3-axis MEMS gyroscope, 3-axis MEMS accelerometer and 3-axis MEMS magnetometer). Only 6-DOF (gyroscope and accelerometer) is used in the experiment since 6-DOF data is sufficient for tilting sensing. After the raw data is processed by Arduino board, MATLAB drew the filtering data and compared with the unfiltered data obtained from the accelerometer. The angles obtained from the accelerometer for pitch and roll are:

$$\theta_{AccPitch} = a\tan 2(AccData_z,\ AccData_x)$$
$$\theta_{AccRoll} = a\tan 2(AccData_z,\ AccData_y) \quad (27)$$

Where $AccData_x$, $AccData_y$ and $AccData_z$ are gravitational acceleration components projected on the three mutually perpendicular axes.

A. Complementary Filter Experiment Result

In the complementary filter experiment, the time constant τ and program loop-time is set at 0.75 and 0.03 (s) respectively. Then the complementary filter coefficient is calculated and α = 0.96. In the experiment, the output angle of the complementary filter are compared with angle $\theta_{AccPitch}$ and $\theta_{AccRoll}$. The experiment consisted of static and dynamic test. The dynamic test can be divided into three cases: single rotation around x-axis (roll), single rotation around y-axis (pitch) and rotation around x-axis and y-axis simultaneously (pitch and roll).

Figure 3 shows the static experiment result. It is obvious that the angle $\theta_{AccPitch}$ and $\theta_{AccRoll}$ (AccPitch and AccRoll) derived from accelerometer contain a lot of noises and disturbances. It is hard to use such angle signal in tilting orientation system as it is vibratory and noisy. However, the filtered angles (CFPitch and CFRoll) through the complementary filter are much more stable and with less sparks and drift.

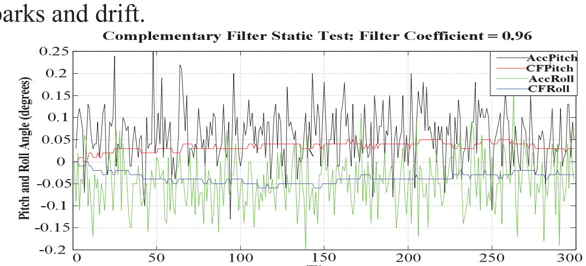

Fig. 3. Complementary Filter Static Test

Figure 4 to 6 show the outputs of the dynamic test of the complementary filter on IMU data fusion. Figure 4 and 5 is the outcome of the IMU rotates only around y-axis or x-axis. Figure 6 is the result that IMU rotates around x and y-axis simultaneously. Obviously, the filtered signal can nicely track the change trend of the signal derived from the accelerometer. Moreover, the signal filtered by complementary filter is smooth with less vibration, even some artificial shaking purposely added on the IMU board in real time. When the IMU rotates around one single axis, there is almost no vibration on the other axis. This means there is no signal coupling happening on both axes (pitch and roll). Therefore, the complementary filter is efficient and reliable in IMU data fusion once the filter coefficient is fine tuned.

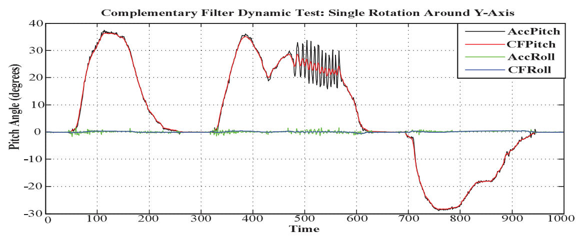

Fig. 4. Complementary Filter Dynamic Test: Pitch

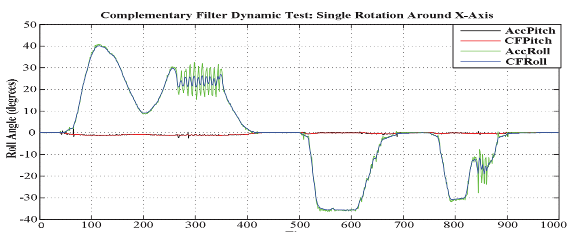

Fig. 5. Complementary Filter Dynamic Test: Roll

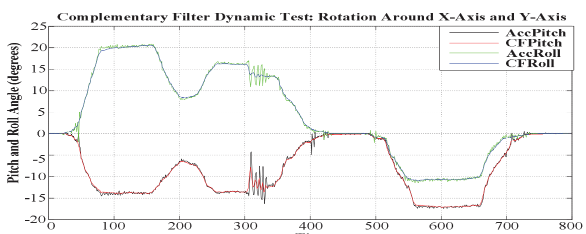

Fig. 6. Complementary Filter Dynamic Test: Pitch and Roll

B. Kalman Filter Experiment Result

In the Kalman Filter IMU data fusion experiment, the filter parameters Q_{θ} (Q_angle), $Q_{\dot{\theta}_b}$ (Q_gyrobias) and R are set at 0.001, 0.003 and 0.03 respectively. And the program loop-time is set to 0.033s. These parameters are chosen after many trials. The same as in the complementary filter experiment, the $\theta_{AccPitch}$ and $\theta_{AccRoll}$ (AccPitch and AccRoll) are used as the references to determine the effectiveness of the Kalman algorithm. The experiment also included static and dynamic tests. Figure 7 is the output of the static test while Figures 8 to 10 present the dynamic results.

Figure 7 shows that, in the static test, the signal filtered by Kalman filter is more stable than that derived from the accelerometer. However, the filtered roll signal is smoother than the filtered pitch signal.

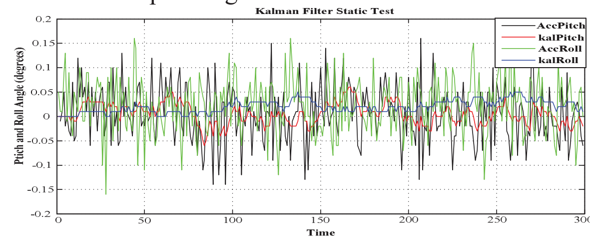

Fig. 7. Kalman Filter Static Test

Figure 8, 9 and 10 are the outcome of the Kalman filter dynamic test. Figure 8 and 9 shows the IMU board rotates only around one axis while figure 10 displays the IMU board rotates simultaneously around two axes in different directions. In figure 8 and 9, the filtered signal was also able to nicely and smoothly follow the AccPitch and AccRoll signal even adding some artificial vibrations. And there was almost no signal coupling phenomenon. However the result of “kalRoll” was better than the “kalPitch”. There were small overshoot and lagging in the filtered pitch signal. This may be caused by the IMU PCB board. When rotating around two axes simultaneously, the Kalman filter also gives smooth and fairly accurate tracking signal.

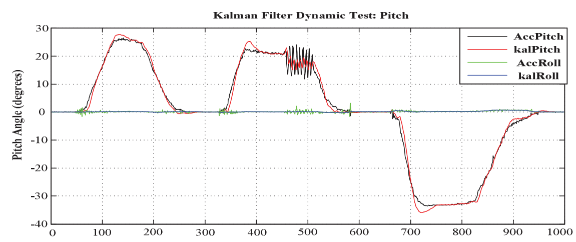

Fig. 8. Kalman Filter Dynamic Test: Pitch

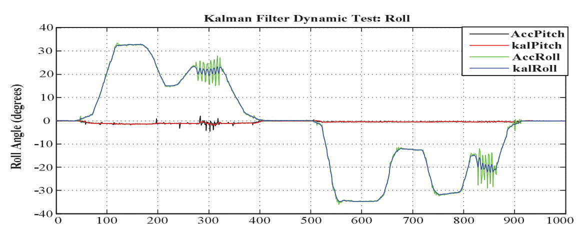

Fig. 9. Kalman Filter Dynamic Test: Roll

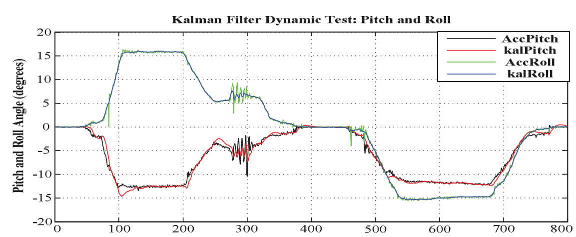

Fig. 10. Kalman Filter Dynamic Test: Pitch and Roll

Theoretically, the Kalman filter should have been more accurate than the complementary filter as it is a more complex and considerate algorithm that operates by producing a statistically optimal estimation of the system state based upon the measurements. It is an iterative process which always tries to find the statistically optimal value. However, the Kalman filter discussed in this paper has three parameters that need to be tuned. This heavily increases the difficulty of the Kalman filter to achieve a more accurate result. In addition, the effectiveness of a Kalman filter demands for computational complexity which requires more powerful microcontroller. In the experiments presented, the program loop-time is set to 0.033s, which is the highest sample rate the Arduino ATMEGA 328p can handle. If the sample rate could be improved, the Kalman filter should get a better result.

V. CONCLUSION

Two IMU data fusion algorithms for real time tilting sensing are studied. Clearly, it is difficult to get an accurate tilting orientation only using accelerometer, because the signal derived from inverse trigonometric function is sensitive to variation. Gyroscope data is necessary to offset the disadvantages of the data from the accelerometer. As gyroscope data suffers from drifting on long term, filter algorithms are needed to solve such problems.

The Kalman Filter is one of the most widely used methods for tracking and estimating orientation due to its optimality, tractability and robustness. However, under certain circumstances, it is difficult to apply it due to its computation complexity. Moreover, for multiple state-variable situations, more filter parameters need to be tuned. For such cases, complementary filter is more ideal than Kalman filter as it requires less computation and only one filter coefficient to be tuned, which greatly reduces the workload. In the experiments presented in this paper, both the complementary filter and Kalman filter can get smooth and accurate enough results either in the static or dynamic tests. Both algorithms are less sensitive to variations and almost no signal coupling phenomenon. Given the fine-tuned filter coefficients, the result of complementary filter can be more stable and accurate than that of Kalman filter. However, both techniques can be effectively used in tilt sensing as long as the proper filter parameters are found.

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