Quiz time: 17:40-18:15

Please also turn in the early feed back form (ignore the back)

My name is Oscar Chang

Hypothesis Testing

Lecture 5

NRE 538

Outline

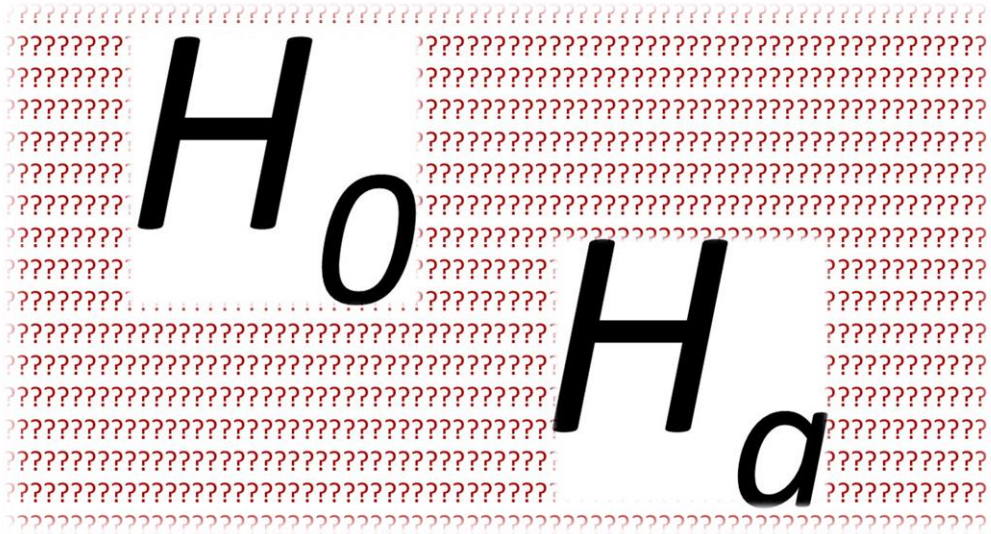
- Null VS Alternative Hypothesis
- One- VS two Tail Hypothesis Test
- Type I VS Type II Error
- Hands-on example

What is a “Hypothesis” ?

What is a “Hypothesis” ?

- An assumption about a parameter

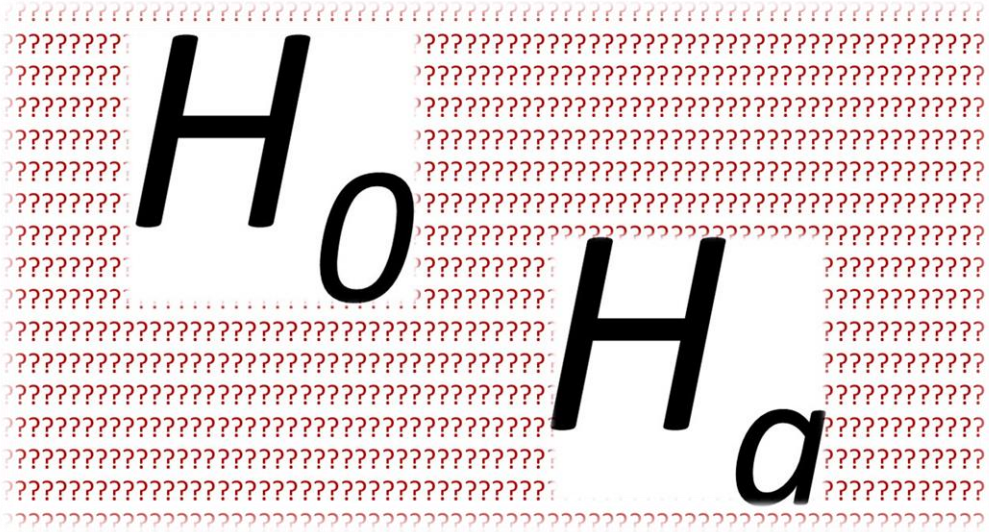
Null VS Alternative Hypothesis



Null VS Alternative Hypothesis

Null Hypothesis (H_0) :

- The belief that the parameter is ≥, =, or ≤ than a specific value
- A general statement or default position that there is no relationship between two measured phenomena, or no association among groups



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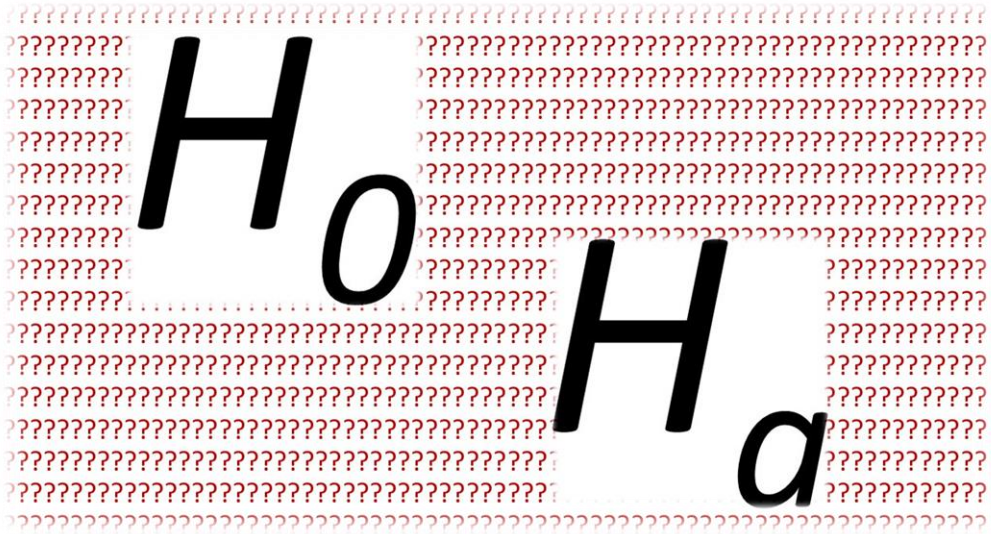
Alternative Hypothesis (H_a)

- The opposite of H_0



Null VS Alternative Hypothesis…examples

Null Hypothesis (H_0) :

My average 10K running time (50 min; μ_1) is not different from 55 min 52s (μ_2).

OR more statistically,

$\mu_1 = \mu_2$

, where μ_1 mean of population 1 and μ_2 mean of population 2

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Alternative Hypothesis (H_a) :

My average 10K running time (50 min; μ_1) is different from 55 min 52s (μ_2).

OR more statistically,

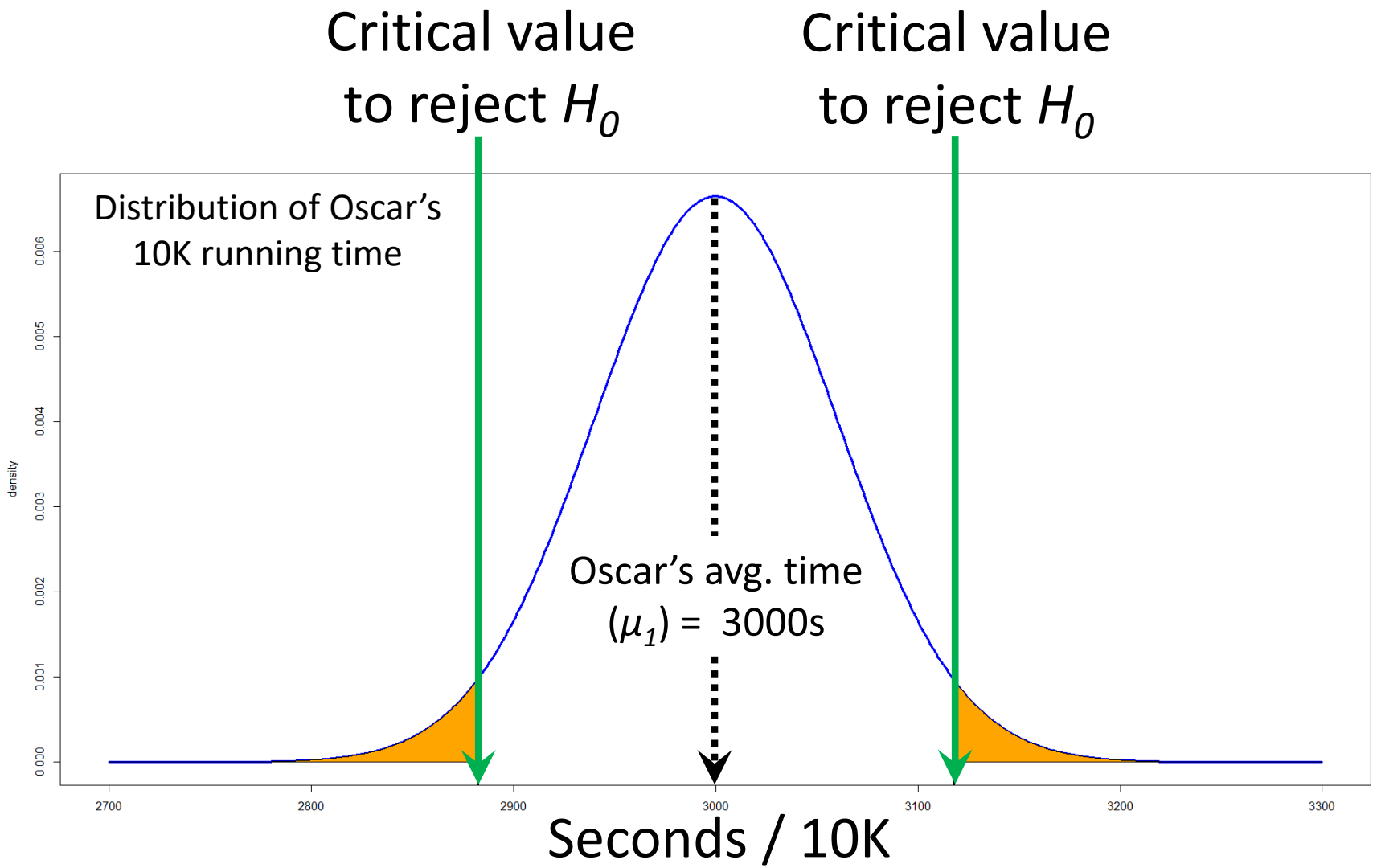
$\mu_1 \neq \mu_2$

, where μ_1 mean of population 1 and μ_2 mean of population 2

Two Tail Hypothesis Test

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$



Null VS Alternative Hypothesis…examples

Null Hypothesis (H_0) :

My average 10K running time (50 min; μ_1) is shorter than 55 min 52 s (μ_2).

OR more statistically,

$\mu_1 \leq \mu_2$

, where μ_1 mean of population 1 and μ_2 mean of population 2

Alternative Hypothesis (H_a) :

My average 10K running time (50 min; μ_1) is longer than 55 min 52s (μ_2).

OR more statistically,

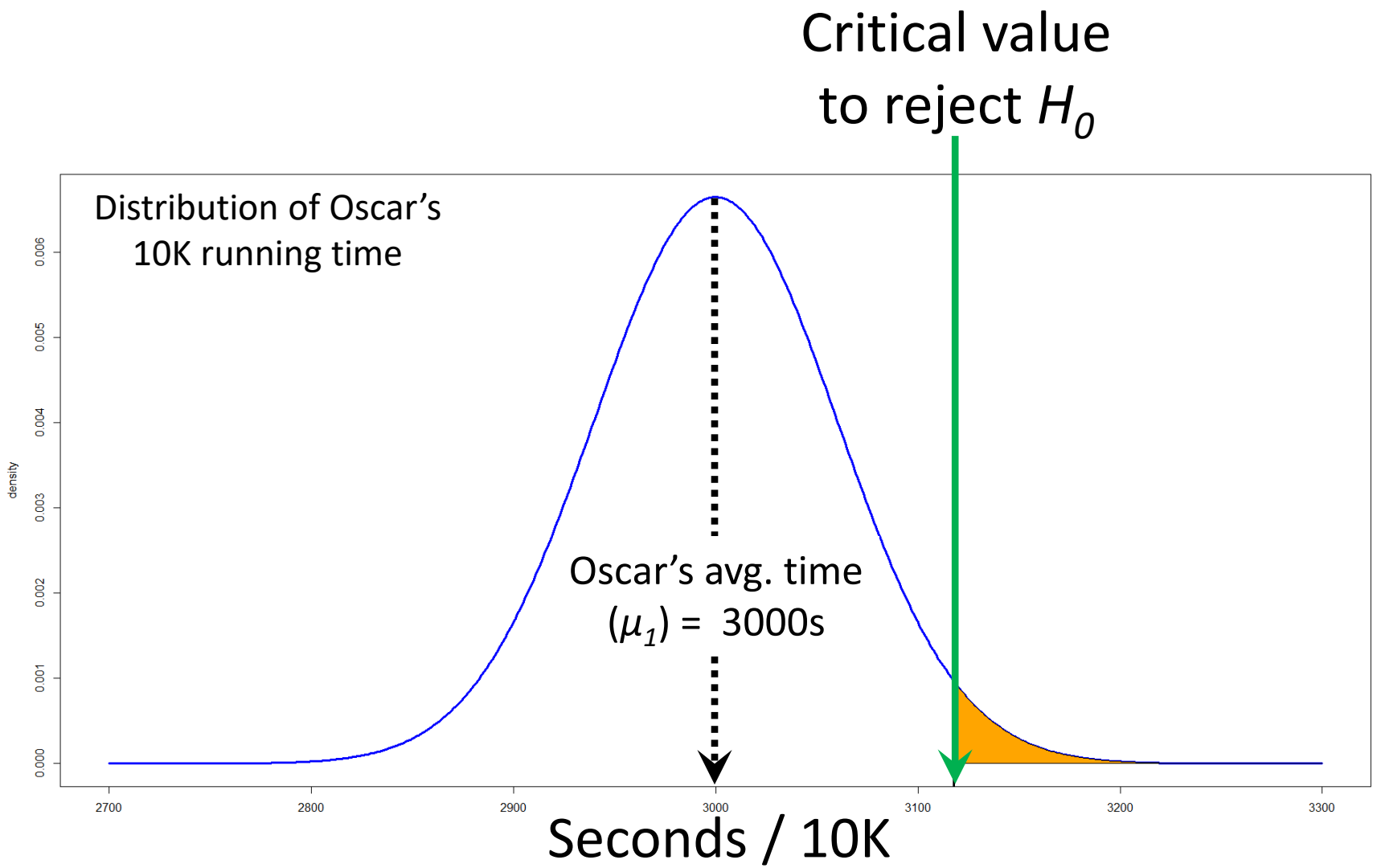
$\mu_1 > \mu_2$

, where μ_1 mean of population 1 and μ_2 mean of population 2

One Tail Hypothesis Test

$H_0: \mu_1 \leq \mu_2$

$H_a: \mu_1 > \mu_2$



Type I VS Type II Error

		H_0 is…	
		True	False
Judgement of H_0	Reject	Type I Error (probability=α)	True positive
	Fail to reject	True Negative	Type II Error (probability=β)

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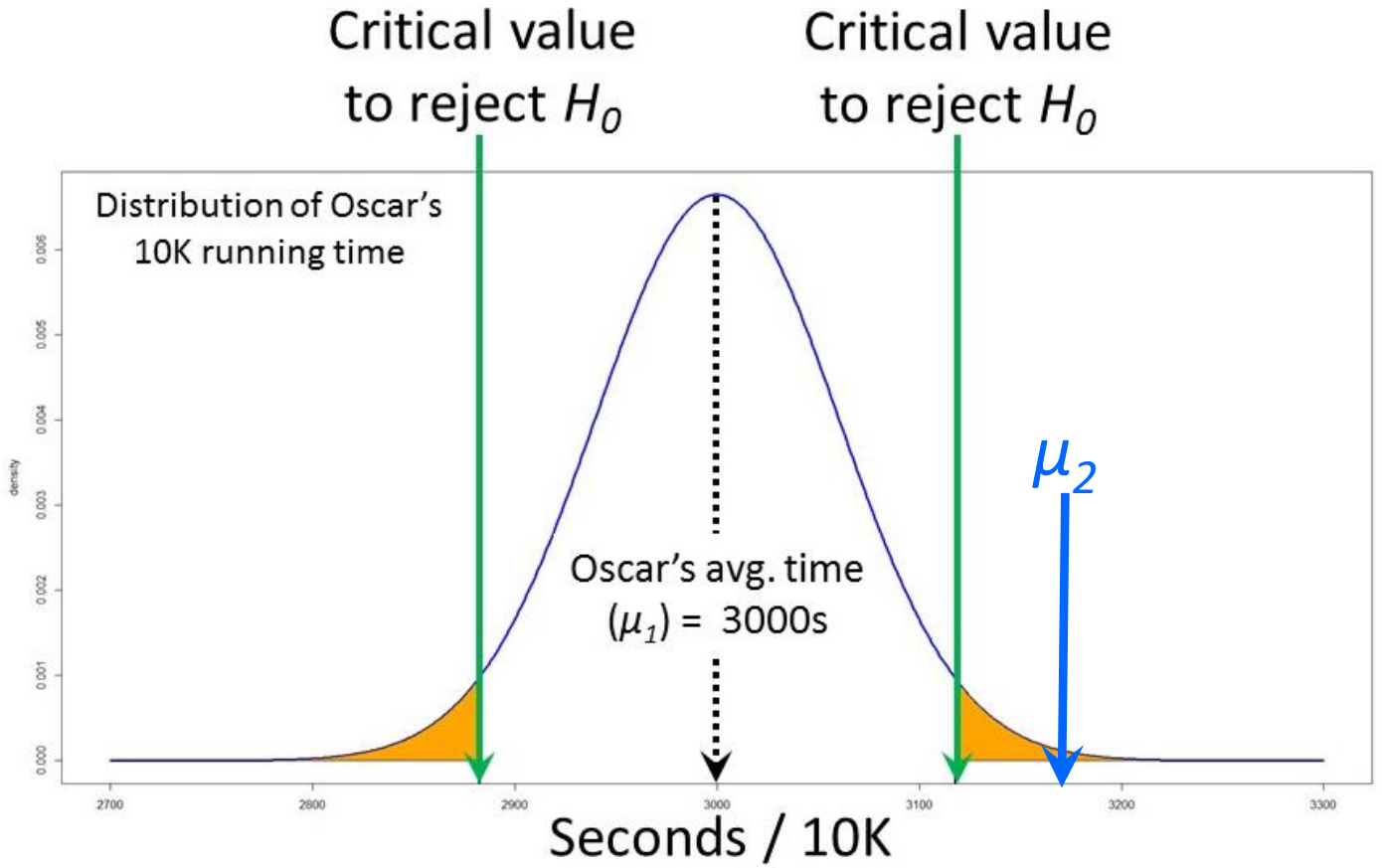

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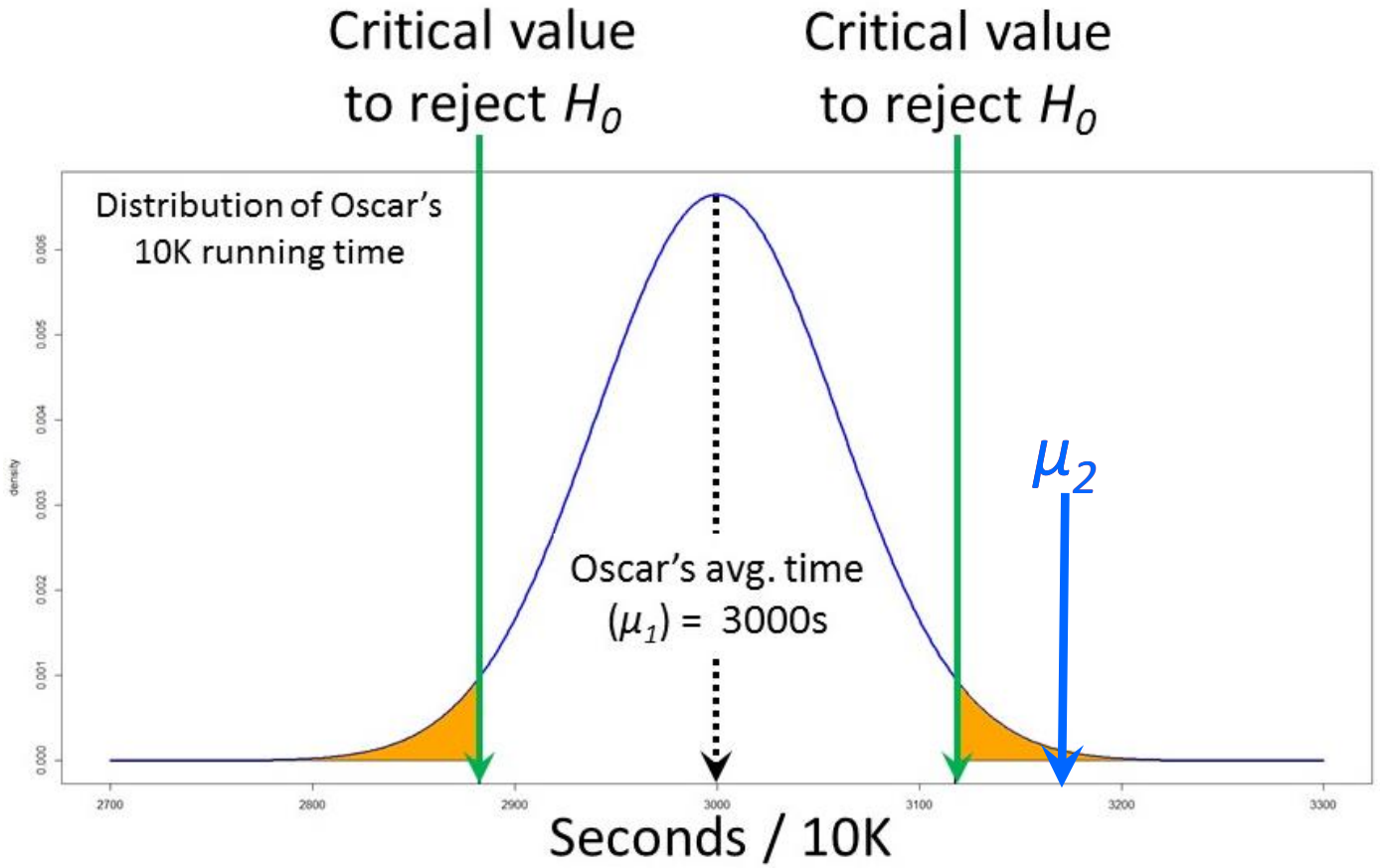

What if μ_2 is really one of my records?

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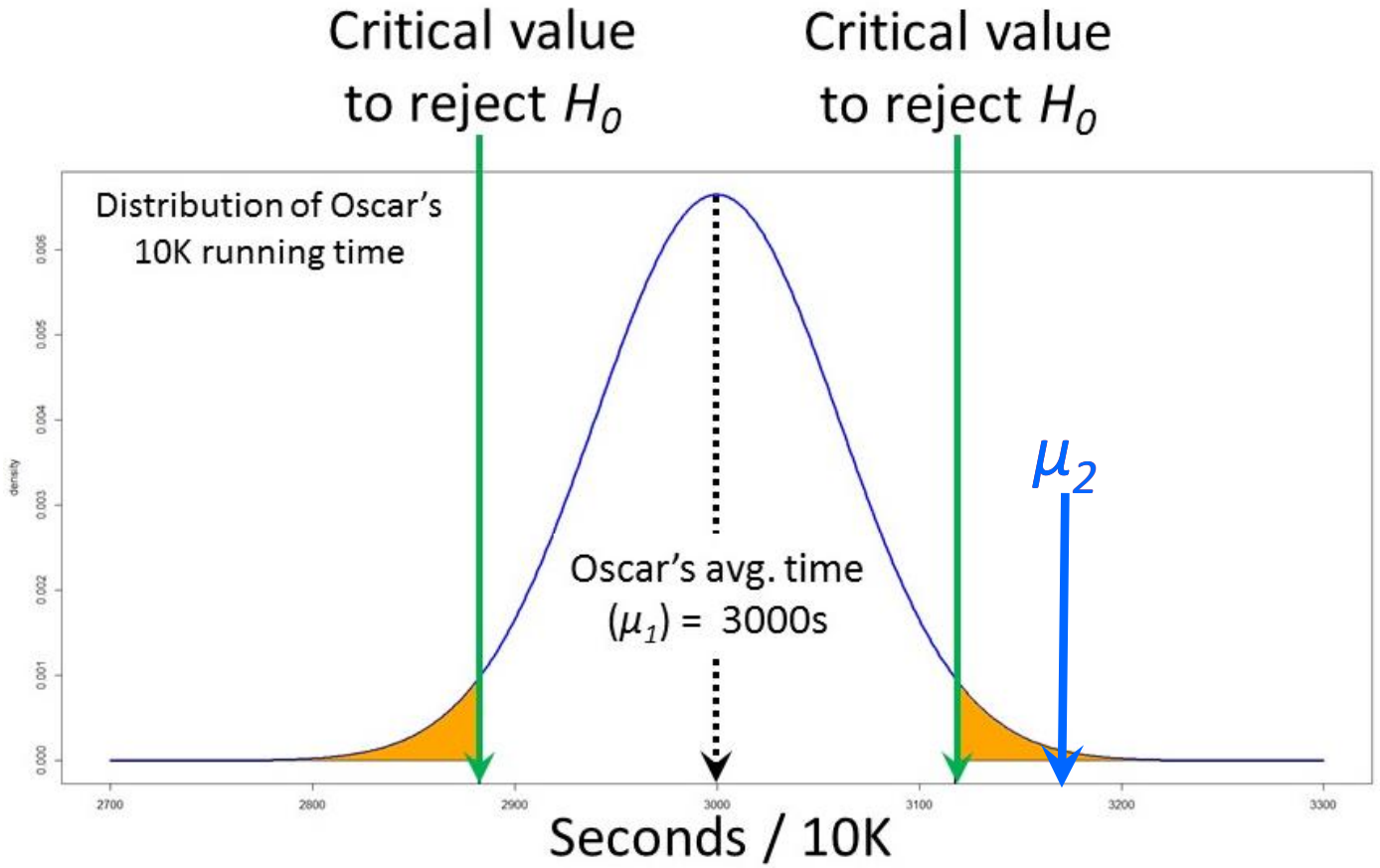

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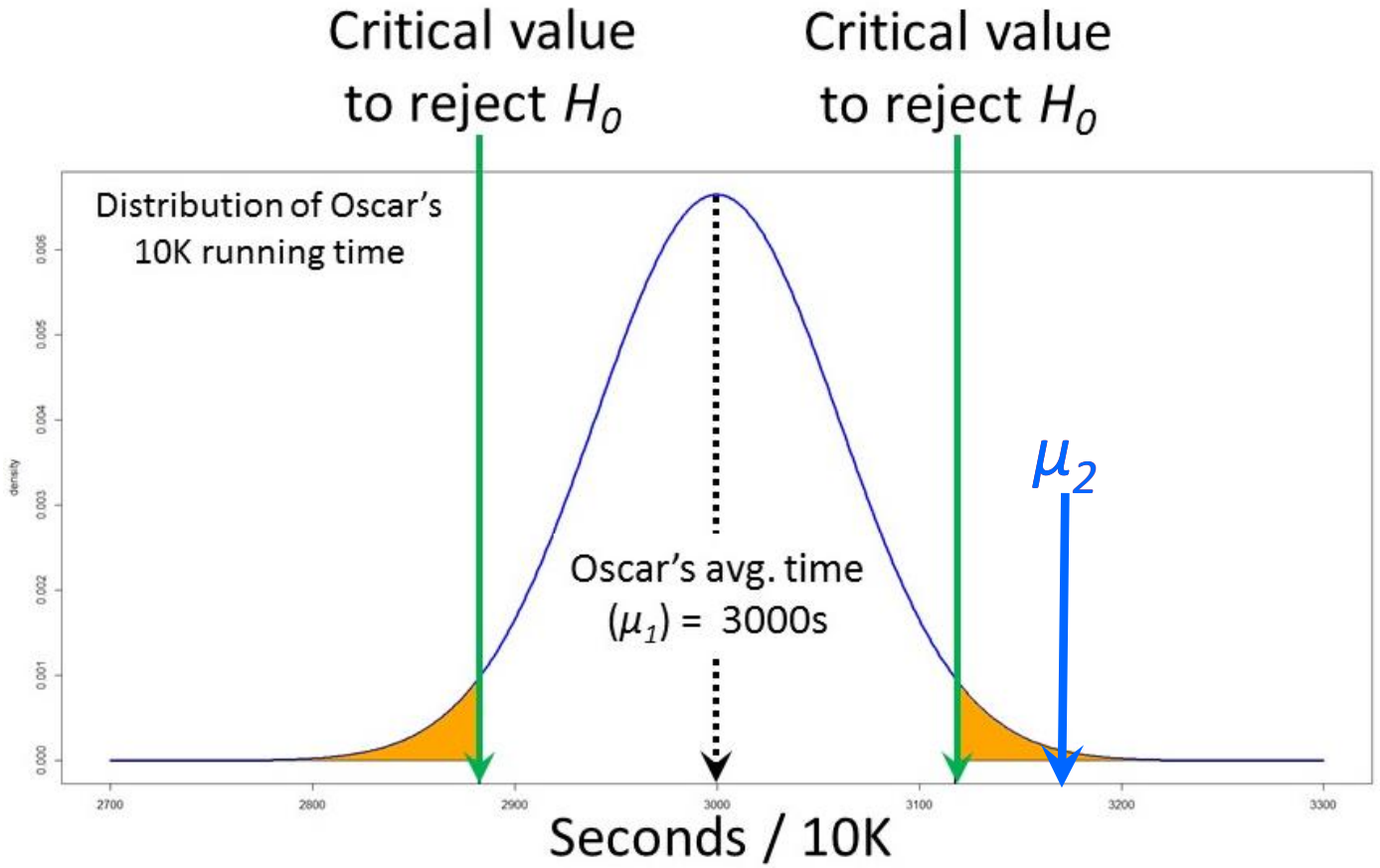

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=> **Orange area**

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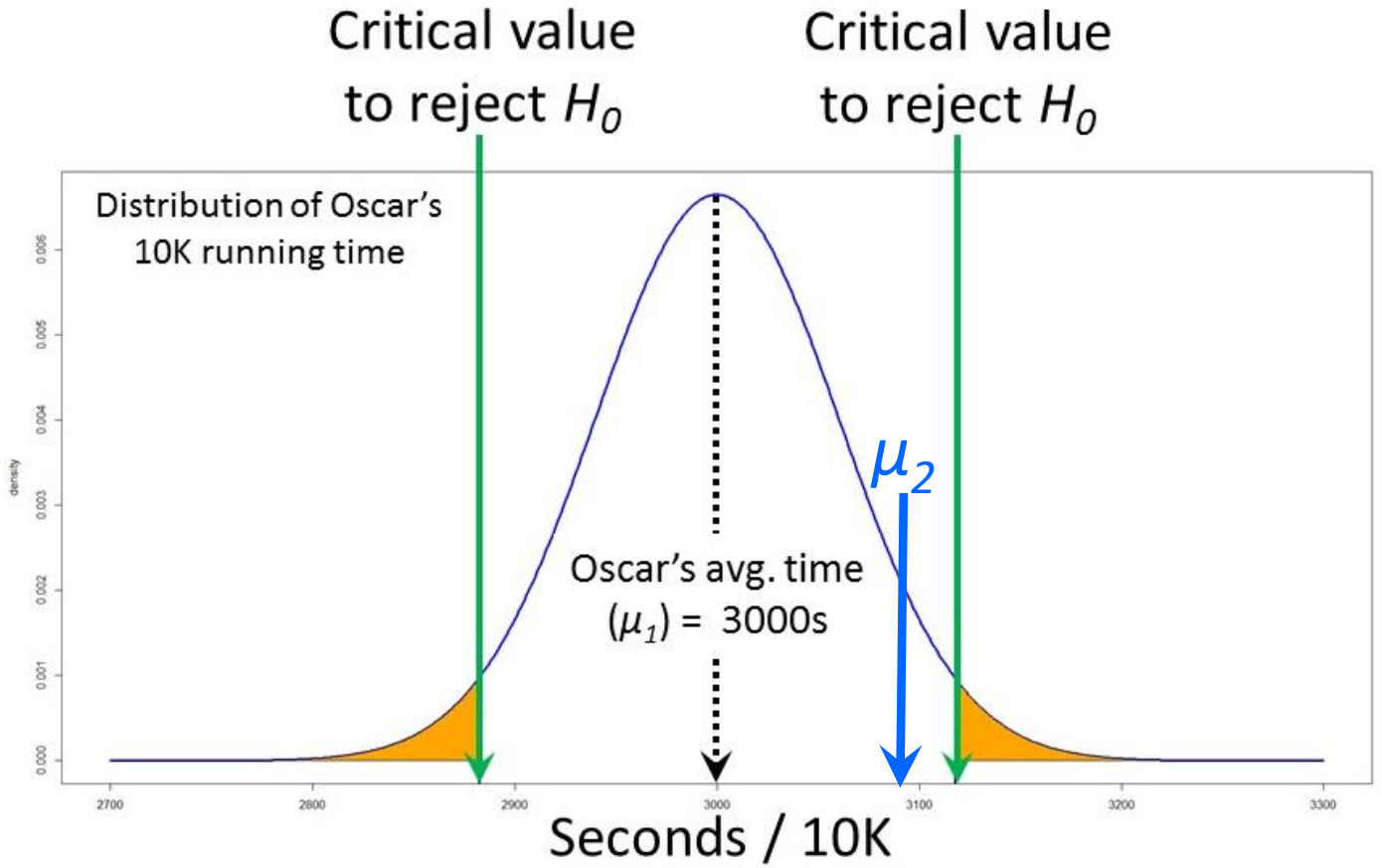

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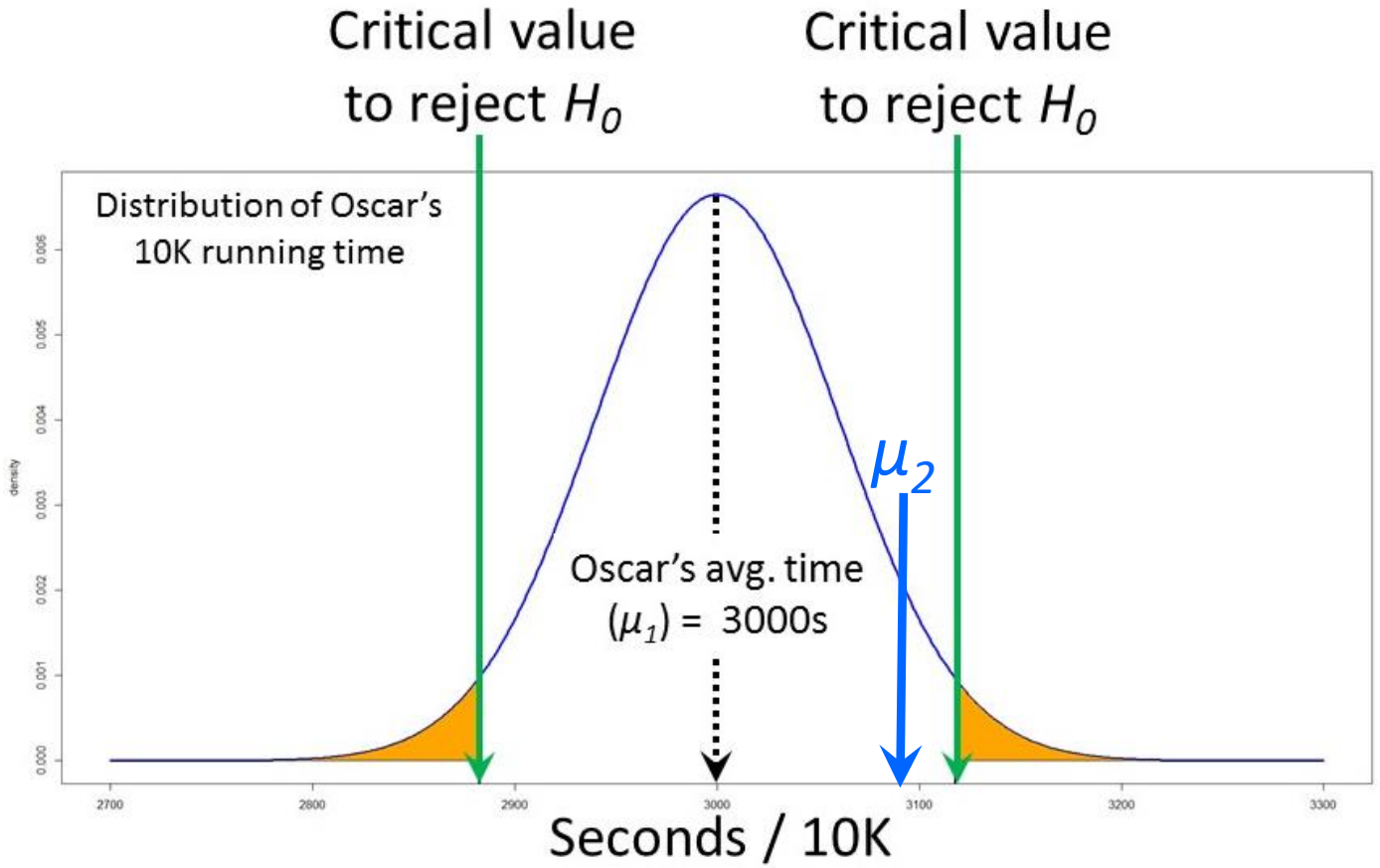

What if μ_2 is actually one of Usain Bolt's records?

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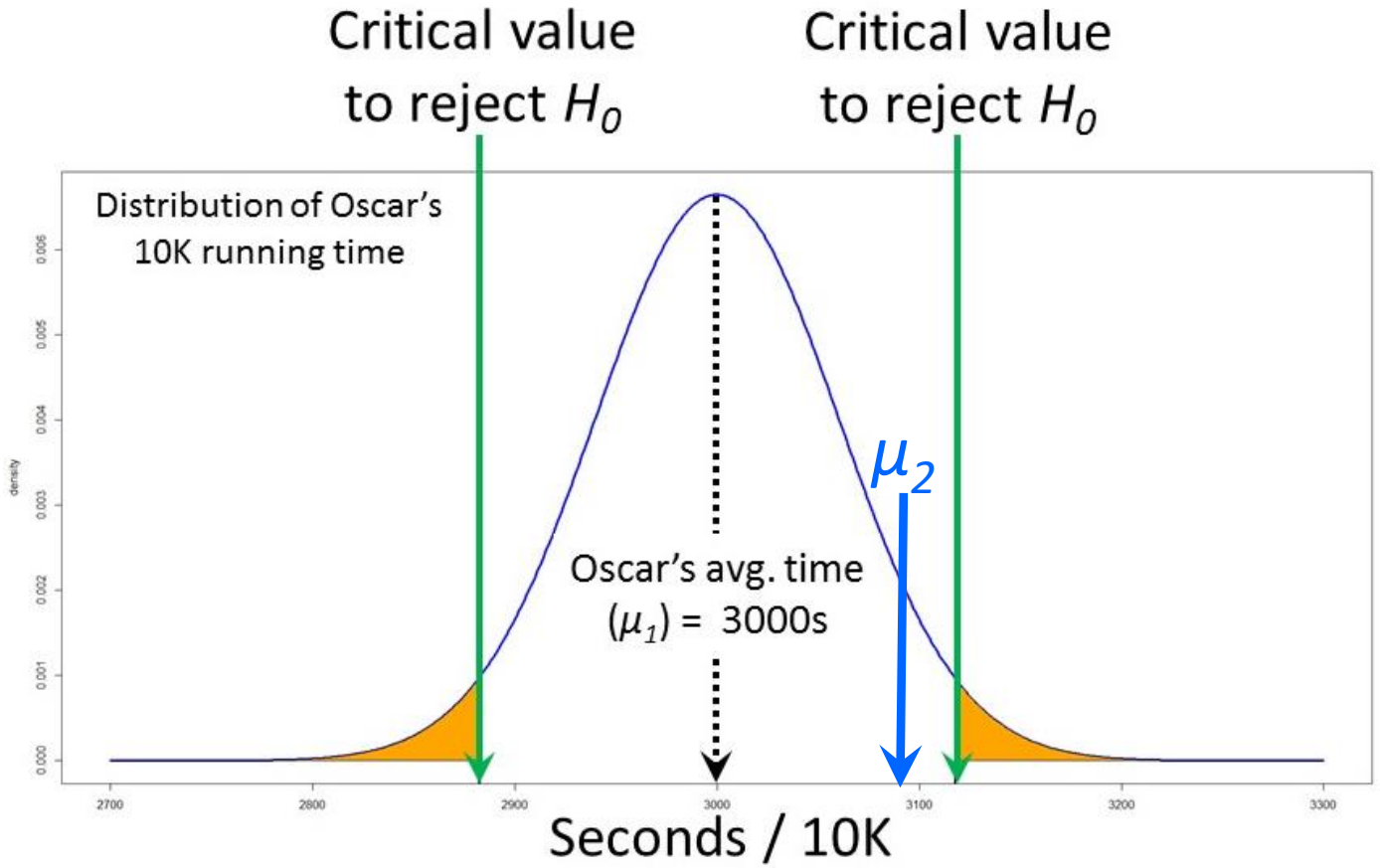

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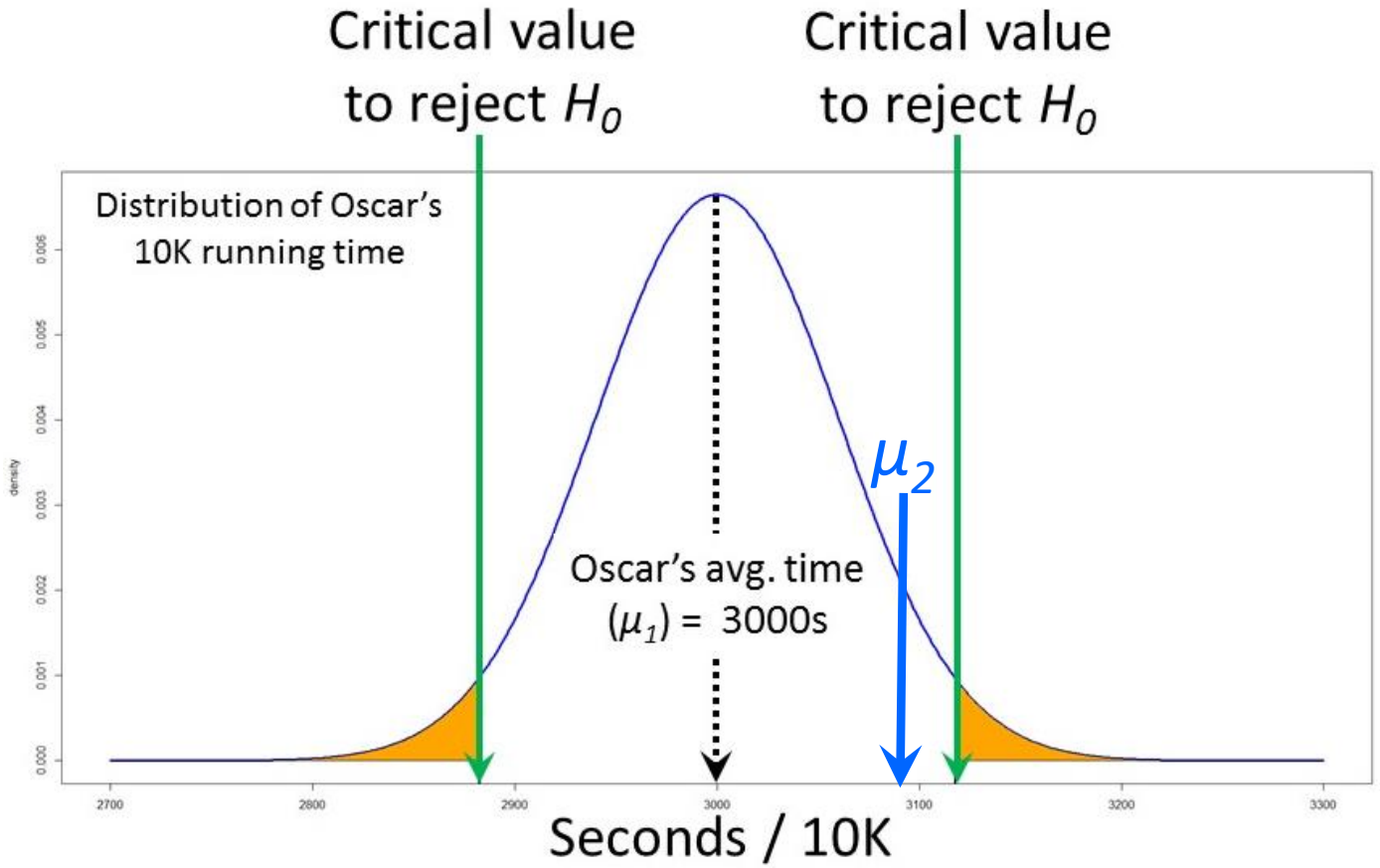

What is the probability of committing Type II Error?

=> 1 - **Orange area**

Type I VS Type II Error

		H_0 is…	
		True	False
Judgement of H_0	Reject	Type I Error (probability=α)	True positive
	Fail to reject	True Negative	Type II Error (probability=β)

- There is trade-off between α and β
- α is typically set to be 0.1-0.01 (1% - 10%)
  - If α is set to be 0.05, the critical value to reject the H_0 is the 95% CI.

Hands-on exercise

Am I really faster than average US men?

Null Hypothesis (H_0) :

$\mu_1 = \mu_2$

, μ_1= 2990.641 (s) and μ_2 = 3352 (s)

Alternative Hypothesis (H_a) :

$\mu_1 \neq \mu_2$

*Set α to be 0.05.

Should we Reject or NOT Reject the null hypothesis (H_0)?

What is the probability of committing Type I error?