Computation concerning a water hammer

Arthur Guillot - Le Goff

Autumn semester 2021 - Hydraulic modelling

Introduction

This exercise focus on the calculus of the water pressure and the velocity at three different locations (point A, B and C) on a 2040 m horizontal pipe.

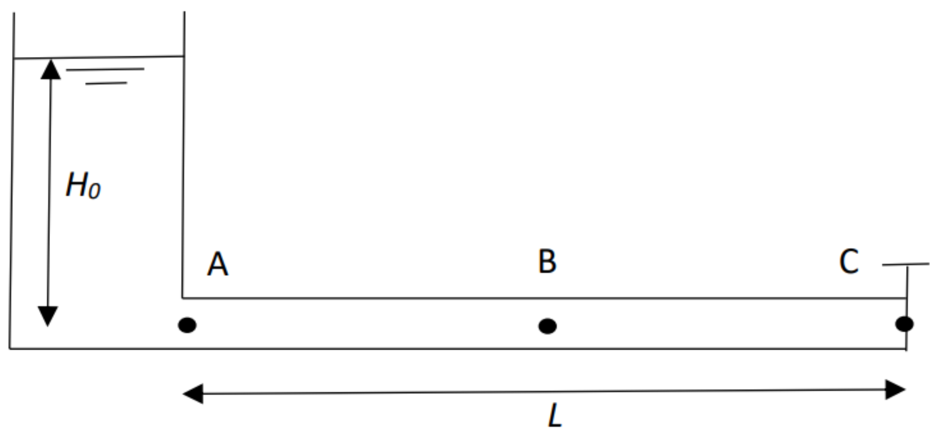


Figure 1: Scheme of the problem

Our starting point is the following equations :

- **the continuity equation for flow under pressure :**
  $\frac{a^2}{g} \cdot \frac{\partial v}{\partial x} + v\frac{\partial H}{\partial x} + \frac{\partial H}{\partial t} + v \cdot \sin\theta = 0$
- **and the momentum equation for flow under pressure :**
  $\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + g\frac{\partial H}{\partial x} + \frac{\lambda v|v|}{2D} = 0$

As we are trying to calculate H and v at any time t we are using the method of characteristics to get an accurate estimates of these values. We are therefore going to described $H(t,x)$ and $v(t,x)$ such as : $dH = \frac{\partial H}{\partial x}dx + \frac{\partial H}{\partial t}dt$ and $dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial t}dt$.

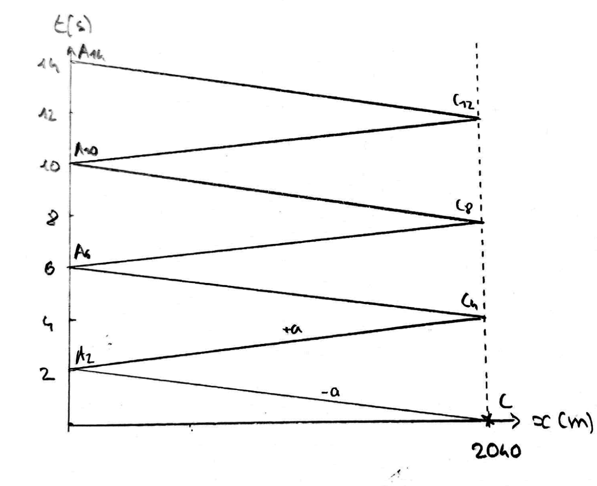
Our exercise focus on three different scenario that we need to analyse :

1. instantaneous closure in point C, friction neglected,
2. instantaneous closure in point C, friction coefficient $\lambda = 0,032$,
3. gradual closure in 8 seconds in point C, friction coefficient $\lambda = 0,032$.

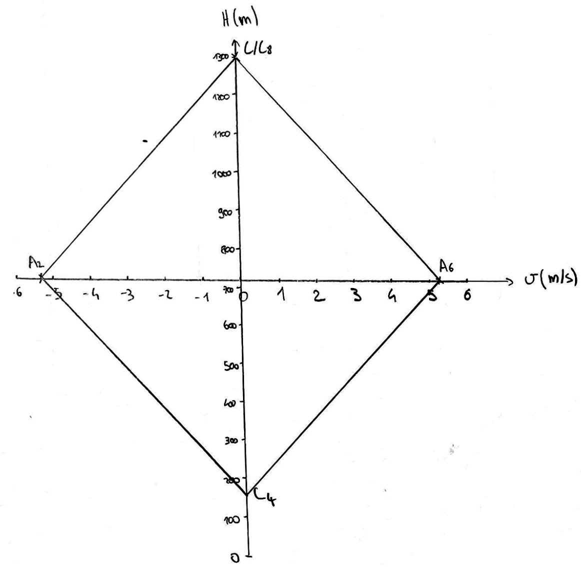
6) Graphs x-t, v-H and H(t)

Before going to our calculation we drew few graphs representing the results we are expecting.

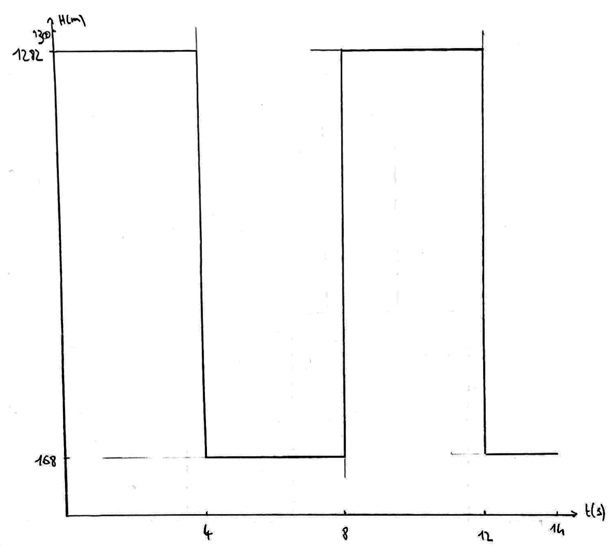
6.1) Graph x-t (line of characteristics)



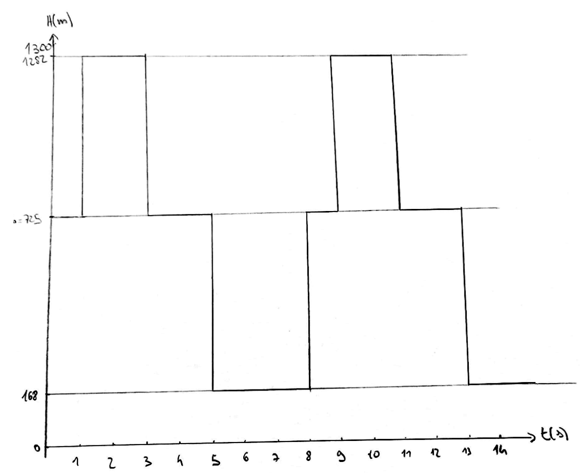
6.2) Graph v-H (line of characteristics)



6.3) Graph H(t) at the point C



6.3) Graph H(t) at the point B



7) Calculation of the water hammer

Methodology

To get a glimpse at the methodology we can detailed my excel calculation sheet as it follow. First I have clarified the different constants useful for our calculations.

	Entry		
L	2040	*m*	Lenght of the pipe
Q	30	*m³/s*	Initial discharge
D	2.67	*m*	Diameter of the pipe
H0	725	*m*	Reservoir level
e	0.03	*m*	Pipe wall thickness
E	1.96E+11	*N/m²*	Pipe elasticity module
K	1.96E+11	*N/m²*	Water compressiibility
θ	0	*rad*	Pipe inclination
N	10		Number of pipe divisions
Δx	204	*m*	Lenght of a pipe division
S	5.59902497	*m²*	Pipe section
v0	5.35807577	*m/s*	Velocity in the pipe at t=0
Δt	0.1	*s*	Temporal calculation step
a	1020	*m/s*	The velocity of water hammer propagation
g	9.81	*m/s²*	Gravity constant
2*g	19.62	*m/s²*	
λ	0		Friction coefficient

Figure 2: Calculation constants used

For our calculation we have to divide the equations used in four parts:

- the $t = 0$ equations (identified as blue backgroud in the next figure)
- the left boundary of the pipe (green)
- the right boundary of the pipe (yellow)
- the internal points (orange)

VITESSE										
i / t	1 (A)	2	3	4	5 (B)	6	7	8	9	10 (C)
0	5	5	5	5	5	5	5	5	5	5
0.1	5	5	5	5	5	5	5	5	5	0
0.2	5	5	5	5	5	5	5	5	3	0
0.3	5	5	5	5	5	5	5	3	1	0
0.4	5	5	5	5	5	5	3	1	1	0
0.5	5	5	5	5	5	3	1	1	0	0
0.6	5	5	5	5	3	1	1	0	0	0
0.7	5	5	5	3	1	1	0	0	0	0
0.8	5	5	3	1	1	0	0	0	0	0
0.9	5	3	1	1	0	0	0	0	0	0

HEAD										
i / t	1 (A)	2	3	4	5 (B)	6	7	8	9	10 (C)
0	724	724	724	724	724	724	724	724	724	724
0.1	724	724	724	724	724	724	724	724	724	724
0.2	724	724	724	724	724	724	724	724	1002	1002
0.3	724	724	724	724	724	724	724	1002	1141	1141
0.4	724	724	724	724	724	724	1002	1141	1211	1211
0.5	724	724	724	724	724	1002	1141	1211	1246	1246
0.6	724	724	724	724	1002	1141	1211	1246	1263	1263
0.7	724	724	724	1002	1141	1211	1246	1263	1272	1272
0.8	724	724	1002	1141	1211	1246	1263	1272	1276	1276
0.9	724	1002	1141	1211	1246	1263	1272	1276	1278	1278

Figure 3: Extract from the calculation table

For the internal points we use :

- $H_{Pi} = 0,5\left[H_{i-1} + H_{i+1} + \frac{a}{g}\left(v_{i-1} - v_{i+1}\right) - \sin\theta\Delta t\left(v_{i-1} + v_{i+1}\right) - \frac{a\lambda\Delta t}{2gD}\left(v_{i-1}\left|v_{i-1}\right| - v_{i+1}\left|v_{i+1}\right|\right)\right]$
- $v_{Pi} = 0,5\left[v_{i-1} + v_{i+1} + \frac{g}{a}\left(H_{i-1} - H_{i+1}\right) - \frac{g}{a}\sin\theta\Delta t\left(v_{i-1} - v_{i+1}\right) - \frac{\lambda\Delta t}{2D}\left(v_{i-1}\left|v_{i-1}\right| + v_{i+1}\left|v_{i+1}\right|\right)\right]$

For the left boundary :

- $H_{Pi} = H_0 - \frac{v_0^2}{2g}$
- $v_{Pi,t=T-\Delta t} = 0,5\left[v_1 + v_2 + \frac{g}{a}\left(H_1 - H_2\right) - \frac{g}{a}\sin\theta\Delta t\left(v_1 - v_2\right) - \frac{\lambda\Delta t}{2D}\left(v_1\left|v_1\right| + v_2\left|v_2\right|\right)\right]$

For the right boundary :

- $H_{Pi,t=T-\Delta t} = 0,5\left[H_{10} + H_9 + \frac{a}{g}\left(v_9 - v_{10}\right) - \sin\theta\Delta t\left(v_9 + v_{10}\right) - \frac{a\lambda\Delta t}{2gD}\left(v_9\left|v_9\right| - v_{10}\left|v_{10}\right|\right)\right]$
- $v_{Pi} = \tau . v_0\sqrt{\frac{H_{Pi}}{H_0}}$

For $t = 0$:

- $H_{Pi} = H_0 - \frac{v_0^2}{2g}$
- $v_{Pi} = v_0$

Results for scenario 1

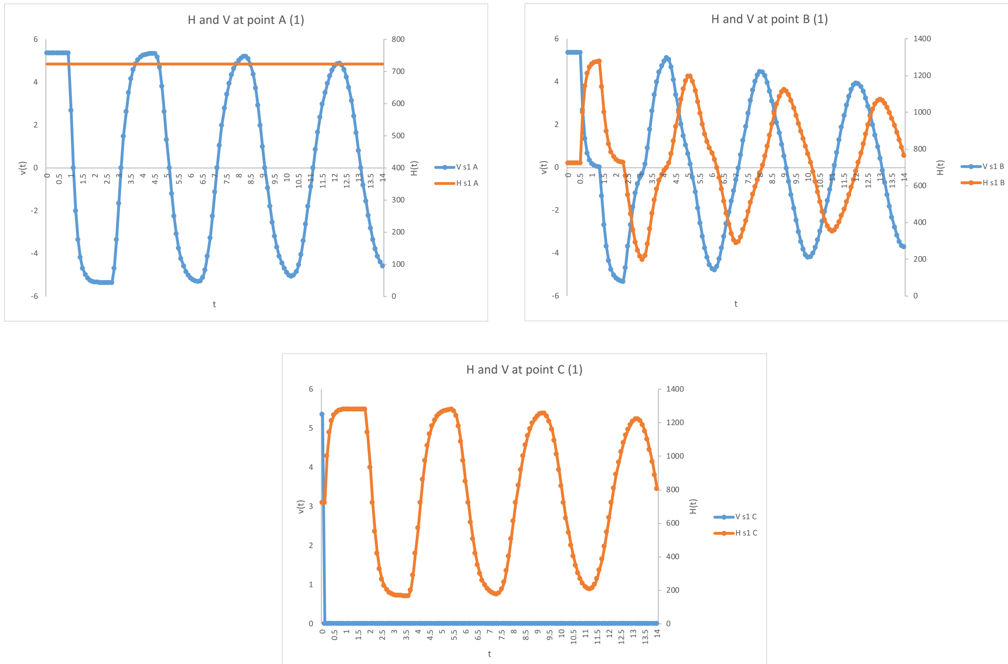


Figure 4: $H(t)$ and $v(t)$ at point A,B and C

Results for scenario 2

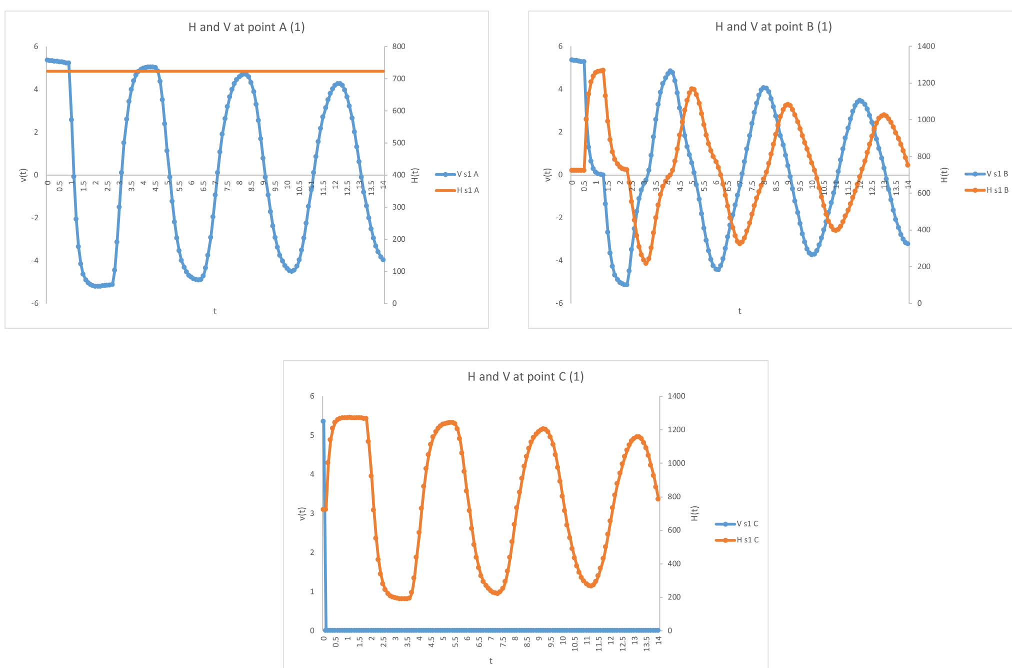


Figure 5: $H(t)$ and $v(t)$ at point A,B and C

Results for scenario 3

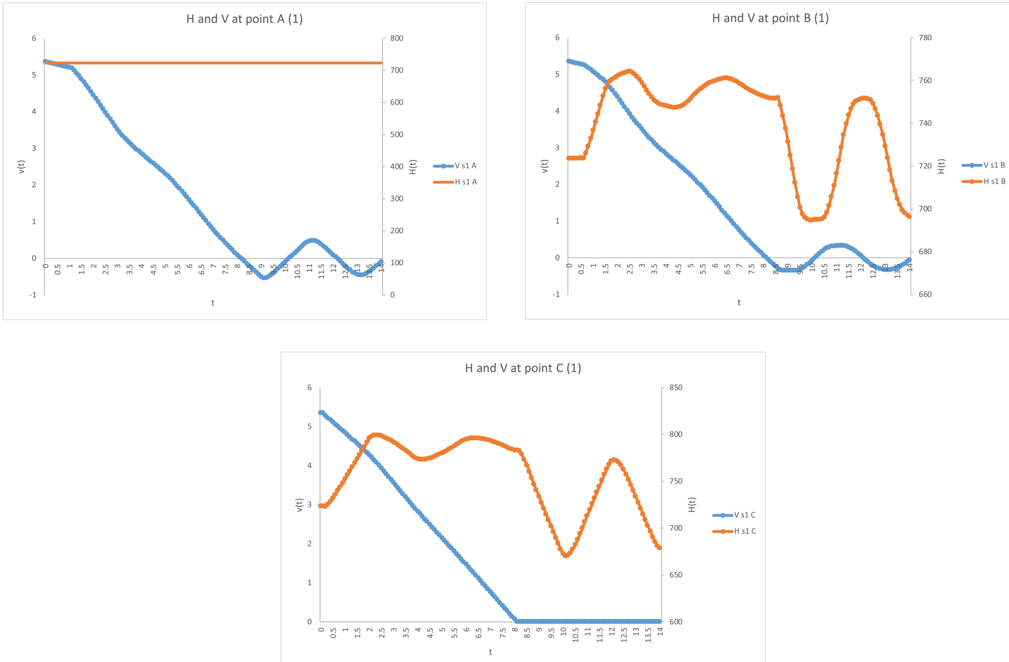


Figure 6: $H(t)$ and $v(t)$ at point A,B and C

Conlusion

We kind of found the result we were expecting as the Head of point B is oscillating arround 1280 and 160 $\mathrm{N/m^2}$ with a period of 4s. The behaviour we excepted for point B is slightly the one at the beggining of the curve we have calculated but we lost it as it become more like a sinusoidal curve. The second scenario including the friction coefficient λ,is decreasing the global amplitude of $H(t)$ and $v(t)$.
Working with the characteristics method makes sense when we calculate scenario 3. We can study the non-trivial behaviour of water in pipes without using our calculation set. As a result, we can see that the velocity gradually decreases as the valve closes at each point. The head at point B and point C follow a similar pattern, oscillating differently than in the first scenario and appearing to adopt a more regular behaviour as time progresses.