Design of the stillin basin

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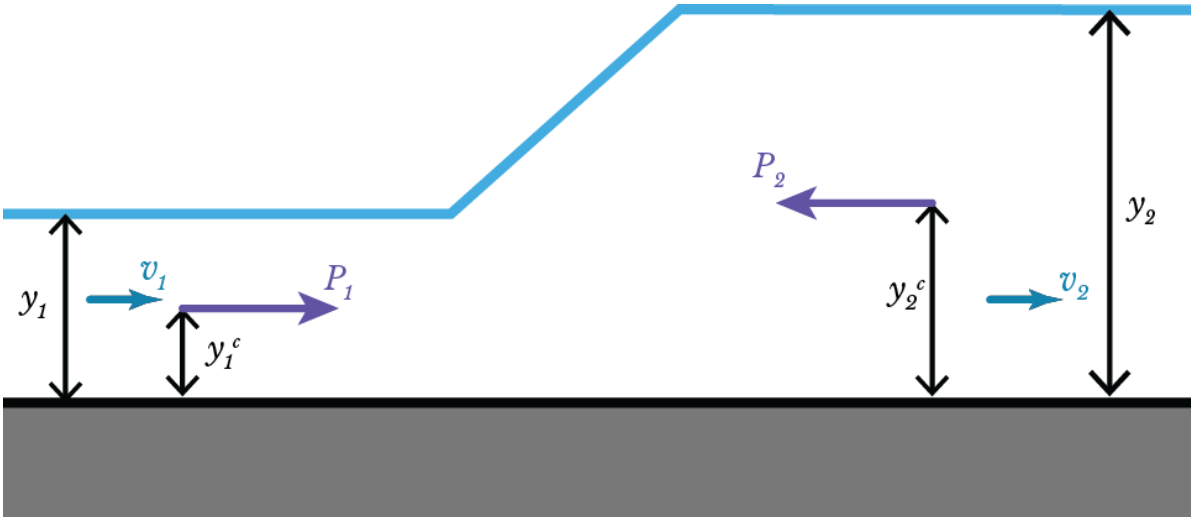
Find the hydraulic jump equation



<u>We define :</u>

- y_i^c, the centre of gravity
- v_i, the velocity of the profile
- P_i, the external forces
- ρ, the density
- Q, the discharge
- b, the width of the basin

<u>We can write the momentum equation :</u> $\rho.Q(v_2 - v_1) = P_1 - P_2 - P_{friction}$, and we set $P_{friction} = 0$

- $v_1 = \frac{Q}{A_1}$ and $v_2 = \frac{Q}{A_2}$
- $P_1 = \rho g A_1 y_1^c$ and $P_2 = \rho g A_2 y_2^c$

<u>Then :</u>

$$\cancel{\rho} Q^2(\frac{1}{A_2} - \frac{1}{A_1}) = \cancel{\rho} g(A_1 y_1^c - A_2 y_2^c)$$

$$A_i = y_i.b \text{ and } y_i^c = \frac{y_i}{2}$$

$$\frac{Q^2}{b.y_2} + gb\frac{y_2^2}{2} = \frac{Q^2}{b.y_1} + gb\frac{y_1^2}{2}$$

$$\text{we note : } \frac{Q}{b} = Q_s \text{ ,the specific discharge}$$

$$(y_2^2 - y_1^2) = \frac{2Q_s^2}{g}(\frac{1}{y_1} - \frac{1}{y_2})$$

$$(y_2 + y_1)\cancel{(y_2 - y_1)} = \frac{2Q_s^2}{g}\frac{\cancel{(y_2 - y_1)}}{y_1.y_2}$$

$$y_1.y_2^2 + y_1^2.y_2 - \frac{2Q_s^2}{g} = 0 \text{ ,a quadratic equation}$$

$$y_2 = \frac{-y_1^2 + \sqrt{y_1^4 - \frac{8Q_s^2 y_1}{g}}}{2y_1} = \frac{y_1}{2}(\sqrt{1 - \frac{8Q_s^2}{gy_1}} - 1)$$

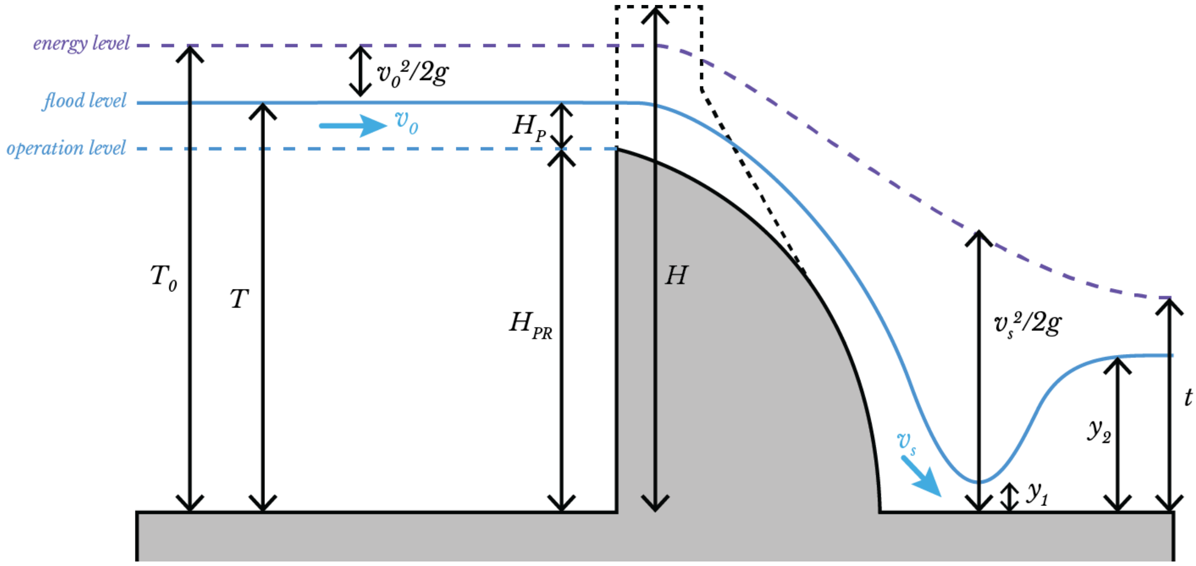
<u>We note :</u> $\frac{Q_s}{y_1} = v$, so $\frac{Q_s}{gy_1^3} = \frac{v^2}{gh} = F_r^2$, the Froude number

<u>Conclusion :</u> we have the following hydraulic jump equation $y_2 = \frac{y_1}{2}(\sqrt{1 + 8F_r^2} - 1)$

Specific energy loss

- $\Delta E = E_2 - E_1 = (y_2 - \frac{Q_s^2}{2gy_2^2}) - (y_1 - \frac{Q_s^2}{2gy_1^2}) = \frac{y_2 - y_1}{4y_2y_1}$
- $\frac{E_2}{E_1} = \frac{(\sqrt{1+8F_r^2})^3 - 4F_r^2 + 1}{8F_r^2(2+F_r^2)}$

Calculation for the design

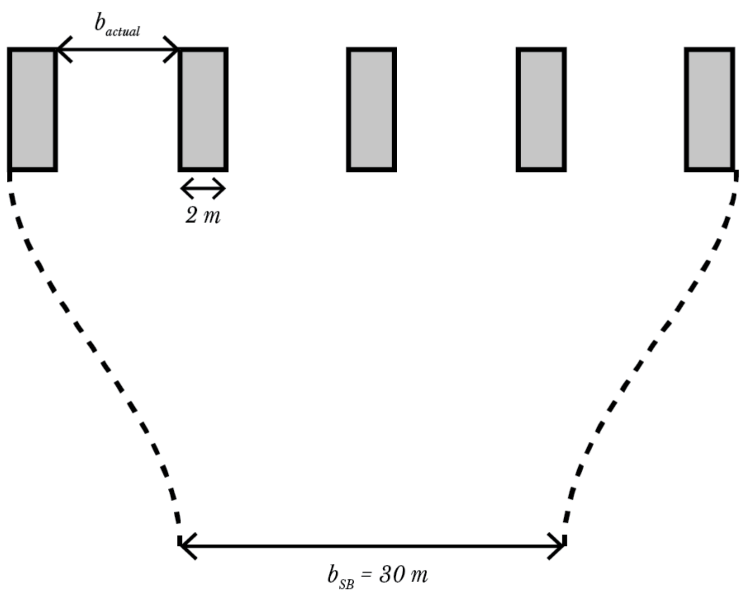


We have : $T_0 = H_{PR} + H_P + \frac{\cancel{v_0^2}}{2g} = 65 + 2 = 67$ m

We use the Bernoulli equations between the T and t :

$$T_0 = y_1 + \frac{v_s^2}{2g} + \sum \xi . \frac{v_s^2}{2g}$$
$$v_s = \frac{1}{\sqrt{1 + \sum \xi}} . \sqrt{2g(T_0 - y_1)}$$

We need to find v_s and y_1.



The specific discharge : $Q_s = \frac{Q}{b_{SB}} = y_1 . v_s = \frac{400}{30} = 13,33$ m

We also note : $\varphi = \frac{1}{\sqrt{1+\sum \xi}} = 0,9$ the hydraulic loss coefficient.

So : $y_1 = \frac{Q_s}{v_s} = \frac{Q_s}{\sqrt{2g(T_0 - y_1)}}$

By iteration :

- $y_{1,0} = 0$
- $y_{1,1} = \frac{Q_s}{\varphi\sqrt{2g(T_0 - y_1)}} = \frac{13{,}33}{0{,}9\sqrt{2.9{,}81(67-0)}} = 0,4085$
- $y_{1,2} = \frac{13{,}33}{0{,}9\sqrt{2.9{,}81(67-0{,}4085)}} = 0,4097$
- $y_{1,3} = 0,4097$

So : $v_s = \varphi\sqrt{2g(T_0 - y_1)} = 0,9\sqrt{2.9,81(67 - 0,4097)}$

$v_s = 32,53$ m.s-1

With the Froude number : $F_r = \frac{v_s}{\sqrt{g \cdot y_1}} = \frac{32{,}53}{\sqrt{9{,}81.0{,}097}} = 16,22$

$y_2 = \frac{y_1}{2}(\sqrt{1 + 8F_r^2} - 1) = \frac{0{,}4097}{2}(\sqrt{1 + 8.16,22^2} - 1)$

$y_2 = 9,19$ m