Formulaire

Croissances comparées $\ln x \ll x \ll e^x$

Trigonométrie

- Pythagore : $\cos^2 + \sin^2 = 1$, $1 + \tan^2 = \dfrac{1}{\cos^2}$, $1 + \text{cotan}^2 = \dfrac{1}{\sin^2}$
- Addition d'arcs : $\cos(a \pm b)$, $\sin(a \pm b)$
- Linéarisation : $\cos(a)\cos(b)$, $\sin(a)\sin(b)$, $\sin(a)\cos(b)$
- Factorisation : $\cos(a) \pm \cos(b)$, $\sin(a) \pm \sin(b)$
- Arc moitié : $t = \tan\left(\dfrac{x}{2}\right)$: $\cos(x) = \dfrac{1-t^2}{1+t^2}$, $\sin(x) = \dfrac{2t}{1+t^2}$, $dx = \dfrac{2}{1+t^2}dt$
- Réciproque : $\text{asin} + \text{acos} = \dfrac{\pi}{2}$, $\text{atan}(x) + \text{atan}\left(\dfrac{1}{x}\right) = \text{sgn}(x)\dfrac{\pi}{2}$
- Hyperbolique : $\text{ch} + \text{sh} = \exp$ $\quad \text{ch}^2 - \text{sh}^2 = 1$
- Changements $\cos \leftrightarrow \text{ch}$, $\sin \leftrightarrow i\,\text{sh}$ pour le formulaire
- Hyperbolique réciproque : $\text{argsh}\, x = \ln(x + \sqrt{x^2+1})$

  $\text{argth}\, x = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$ pour $|x| < 1$

Primitives

- $x^m \rightsquigarrow \dfrac{x^{m+1}}{m+1}$ $(m \neq -1)$, $\dfrac{1}{x} \rightsquigarrow \ln|x|$
- $\exp \rightsquigarrow \exp$, $\ln \rightsquigarrow x(\ln x - 1)$
- $\sin \rightsquigarrow -\cos$, $\text{sh} \rightsquigarrow \text{ch}$
- $\cos \rightsquigarrow \sin$, $\text{ch} \rightsquigarrow \text{sh}$
- $\dfrac{1}{\sin} \rightsquigarrow \ln\left|\tan\dfrac{x}{2}\right|$, $\dfrac{1}{\cos} \rightsquigarrow \ln\left|\tan\left(\dfrac{x}{2} + \dfrac{\pi}{4}\right)\right|$
- $\dfrac{1}{\cos^2} \rightsquigarrow \tan$, $\dfrac{1}{\text{ch}^2} \rightsquigarrow \text{th}$
- $\dfrac{1}{1+x^2} \rightsquigarrow \text{atan}$, $\dfrac{1}{1-x^2} \rightsquigarrow \text{argth}$
- $\dfrac{1}{\sqrt{1-x^2}} \rightsquigarrow \text{asin}$, $\dfrac{1}{\sqrt{1+x^2}} \rightsquigarrow \text{argsh}$

Développements en série entière / limités

- $$e^x = \sum_{n=0}^{+\infty} \frac{1}{n!} x^n = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots \qquad (R = +\infty)$$

$$\mathrm{ch}(x) = \sum_{n=0}^{+\infty} \frac{1}{(2n)!} x^{2n} = 1 + \frac{x^2}{2} + \frac{x^4}{24} + \dots$$

$$\cos(x) = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2} + \frac{x^4}{24} + \dots$$

$$\mathrm{sh}(x) = \sum_{n=0}^{+\infty} \frac{1}{(2n+1)!} x^{2n+1} = x + \frac{x^3}{6} + \frac{x^5}{120} + \dots$$

$$\sin(x) = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{6} + \frac{x^5}{120} + \dots$$

- $$(1+x)^\alpha = \sum_{n=0}^{+\infty} \frac{\prod_{i=0}^{n-1} (\alpha - i)}{n!} x^n = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + \dots \qquad (R = 1 \ (\infty \text{ si } \alpha \in \mathbb{N}))$$

$$\frac{1}{1-x} = \sum_{n=0}^{+\infty} x^n = 1 + x + x^2 + \dots$$

$$-\ln(1-x) = \sum_{n=1}^{+\infty} \frac{x^n}{n} = x + \frac{x^2}{2} + \frac{x^3}{3} + \dots$$

$$\frac{1}{1+x} = \sum_{n=0}^{+\infty} (-1)^n x^n = 1 - x + x^2 + \dots$$

$$\ln(1+x) = \sum_{n=1}^{+\infty} (-1)^{n+1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots$$

$$\mathrm{atan}(x) = \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} + \dots$$

$$\mathrm{argth}(x) = \sum_{n=0}^{+\infty} \frac{x^{2n+1}}{2n+1} = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots$$

$$\mathrm{asin}(x) = \sum_{n=0}^{+\infty} \prod_{i=1}^{n} \left(\frac{2i-1}{2i}\right) \frac{x^{2n+1}}{2n+1} = x + \frac{x^3}{6} + \frac{3}{40} x^5 + \dots$$

$$\mathrm{argsh}(x) = \sum_{n=0}^{+\infty} (-1)^n \prod_{i=1}^{n} \left(\frac{2i-1}{2i}\right) \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{6} + \frac{3}{40} x^5 + \dots$$

- $$\tan(x) = x + \frac{x^3}{3} + \frac{2}{15} x^5 + o(x^6) \qquad (\text{DSE complexe})$$

$$\mathrm{th}(x) = x - \frac{x^3}{3} + \frac{2}{15} x^5 + o(x^6)$$