

Assignment 3 - Numerical Linear Algebra

Arthur Rabello Oliveira¹

06/06/2025

Abstract

We design and test a function `to_hessemberg(A)` that reduces an arbitrary square matrix to (upper) Hessenberg form with Householder reflectors, returns the reflector vectors, the compact Hessenberg matrix H , and the accumulated orthogonal factor Q , verifying numerically that $A = QHQ^*$ and $Q^*Q = I$ for symmetric and nonsymmetric inputs of orders $10 - 10000$. Timings confirm the expected $O(\text{something})$ cost and reveal the $2 \times$ speed-up attainable for symmetric matrices through trivial bandwidth savings. Leveraging this routine, we investigate the spectral structure of orthogonal matrices: we show that all eigenvalues lie on the unit circle, analyse the consequences for the power method and inverse iteration, and obtain a closed-form spectrum for generic 2×2 orthogonals. Random 4×4 orthogonal matrices generated via QR factorisation are then reduced to Hessenberg form; the eigenvalues of their trailing 2×2 blocks are computed analytically and reused as fixed shifts in the QR iteration, where experiments demonstrate markedly faster convergence. Throughout, every algorithm is documented and supported by commented plots that corroborate the theoretical claims.

Contents

1. Introduction	2
2. Hessemberg Reduction (Problem 1)	3
2.1. Calculating the Householder Reflectors (a)	3
2.2. Evaluating the Function (b), (c), (d)	6
2.2.1. Complexity (c)	14
2.2.2. The Symmetric Case (d)	15
3. Eigenvalues and Iterative Methods	15
3.1. Power iteration	15
3.2. Inverse Iteration	16
4. Orthogonal Matrices (Problem 2) (a)	16
4.1. Orthogonal Matrices and the Power Iteration	16
4.2. Orthogonal Matrices and Inverse Iteration	17
4.3. The 2×2 Case (b)	17
4.4. Random Orthogonal Matrices (c)	17
4.5. Shift With an Eigenvalue (d)	34
Bibliography	105

¹Escola de Matemática Aplicada, Fundação Getúlio Vargas (FGV/EMAp), email: arthur.oliveira.1@fgv.edu.br

1. Introduction

One could calculate the eigenvalues of a square matrix using the following algorithm:

1. Compute the n -th degree polynomial $\det(A - \lambda I) = 0$,
2. Solve for λ (somehow).

On step 2, the eigenvalue problem would have been reduced to a polynomial root-finding problem, which is awful and extremely ill-conditioned. From the [previous assignment](#) we know that in the denominator of the relative condition number $\kappa(x)$ there's a $|x - n|$. So $\kappa(x) \rightarrow \infty$ when $x \rightarrow 0$. As an example, consider the polynomial

$$p(x) = (x - 2)^9 = x^9 - 18x^8 + 144x^7 - 672x^6 + 2016x^5 - 4032x^4 + 5376x^3 - 4608x^2 + 2304x - 512 \quad (1)$$

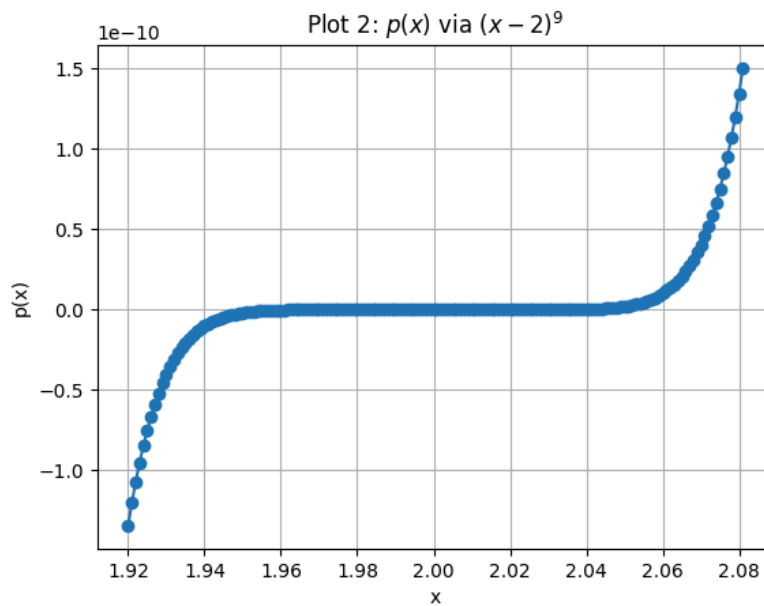


Figure 1: $p(x)$ via the coefficients in [eq. \(1\)](#)

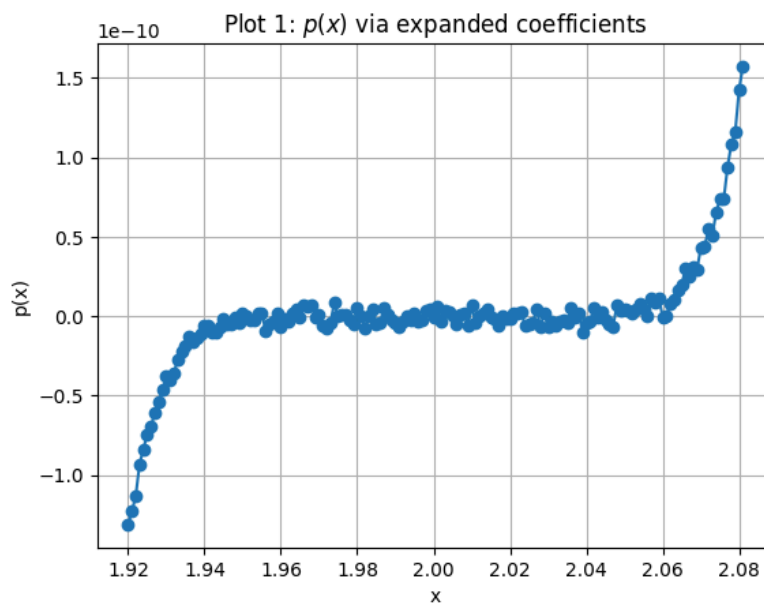


Figure 2: $p(x)$ via $(x - 2)^9$

Figure 2 shows a smooth curve, while Figure 1 shows a weird oscillation around $x = 0$ (And pretty much everywhere else if the reader is sufficiently persistent).

This is due to the round-off errors when $x \approx 0$ and the big coefficients of the polynomial. In general, polynomial are very sensitive to perturbations in the coefficients, which is why rootfinding is a bad idea to find eigenvalues.

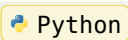
Here we discuss aspects of some iterative eigenvalue algorithms, such as power iteration, inverse iteration, and QR iteration.

2. Hessenberg Reduction (Problem 1)

2.1. Calculating the Householder Reflectors (a)

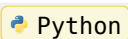
The following packages will be used in the next functions:

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3 from scipy.linalg import hessenberg, qr, eig
4 import time
5 from typing import List, Tuple
6 import pandas as pd
7 import math
8 from IPython.display import display, Markdown
9 from ast import literal_eval
10 import re
```



The following function calculates the Householder reflectors that reduce a matrix to Hessenberg form. It returns the reflector vectors, the compact Hessenberg matrix H , and the accumulated orthogonal factor Q .

```
1 def build_householder_unit_vector(
2     target_vector: np.ndarray
3 ) -> np.ndarray:
4
5     """
6     Builds a Householder unit vector
7
8     Args:
9         1. target_vector (np.ndarray): Column vector that we want to annihilate
          (size ≥ 1).
10
11     Returns:
12         np.ndarray:
13             The normalised Householder vector ( $\|v\|_2 = 1$ ) with a real first
            component.
14
15     Raises:
16         1. ValueError: If 'target_vector' has zero length.
17     """
```



```

18
19     if target_vector.size == 0:
20         raise ValueError("The target vector is empty; no reflector needed.")
21
22     vector_norm: float = np.linalg.norm(target_vector)
23
24     if vector_norm == 0.0: #nothing to annihilate – return canonical basis
        vector
25         householder_vector: np.ndarray = np.zeros_like(target_vector)
26         householder_vector[0] = 1.0
27         return householder_vector
28
29     sign_correction: float = (
30         1.0 if target_vector[0].real >= 0.0 else -1.0
31     )
32     copy_of_target_vector: np.ndarray = target_vector.copy()
33     copy_of_target_vector[0] += sign_correction * vector_norm
34     householder_vector: np.ndarray = (
35         copy_of_target_vector / np.linalg.norm(copy_of_target_vector)
36     )
37     return householder_vector
38
39
40 def to_hessenberg(
41     original_matrix: np.ndarray,
42 ) -> Tuple[List[np.ndarray], np.ndarray, np.ndarray]:
43
44     """
45     Reduce 'original_matrix' to upper Hessenberg form by Householder
        reflections.
46
47     Args
48         1. original_matrix (np.ndarray): Real or complex square matrix of order
            'matrix_order'.
49
50     Returns
51         Tuple consisting of:
52
53         1. householder_reflectors_list (List[np.ndarray])
54         2. hessenberg_matrix (np.ndarray)
55         3. accumulated_orthogonal_matrix (np.ndarray) s.t.
56              $original\_matrix = Q \cdot H \cdot Q^H$ 
57
58     Raises
59         1. ValueError: If 'original_matrix' is not square.
60     """

```

```

61
62     working_matrix: np.ndarray = np.asarray(original_matrix).copy()
63
64     if working_matrix.shape[0] != working_matrix.shape[1]:
65         raise ValueError("Input matrix must be square.")
66
67     matrix_order: int = working_matrix.shape[0]
68     accumulated_orthogonal_matrix: np.ndarray = np.eye(
69         matrix_order, dtype=working_matrix.dtype
70     )
71     householder_reflectors_list: List[np.ndarray] = []
72
73     for column_index in range(matrix_order - 2): #extract the part of column
74         'column_index' that we want to zero out
75         target_column_segment: np.ndarray = working_matrix[
76             column_index + 1 :, column_index
77         ]
78         householder_vector: np.ndarray = build_householder_unit_vector(
79             target_column_segment
80         ) #build Householder vector for this segment
81         householder_reflectors_list.append(householder_vector)
82
83         #expand it to the full matrix dimension
84         expanded_householder_vector: np.ndarray = np.zeros(
85             matrix_order, dtype=working_matrix.dtype
86         )
87         expanded_householder_vector[column_index + 1 :] = householder_vector
88
89
90         working_matrix -= 2.0 * np.outer(
91             expanded_householder_vector,
92             expanded_householder_vector.conj().T @ working_matrix,
93         ) #apply reflector from BOTH sides
94         working_matrix -= 2.0 * np.outer(
95             working_matrix @ expanded_householder_vector,
96             expanded_householder_vector.conj().T,
97         )
98
99         #accumulate Q
100         accumulated_orthogonal_matrix -= 2.0 * np.outer(
101             accumulated_orthogonal_matrix @ expanded_householder_vector,
102             expanded_householder_vector.conj().T,
103         )
104
105     hessenberg_matrix: np.ndarray = working_matrix

```

```

106     return (
107         householder_reflectors_list,
108         hessenberg_matrix,
109         accumulated_orthogonal_matrix,
110     )

```

We will evaluate this function in [Section 2.2](#).

2.2. Evaluating the Function (b), (c), (d)

We present another algorithm for evaluating the function `to_hessenberg(A)` for random matrices of various sizes, inputed by the user, which also gets to choose if symmetric matrices will be generated or not.

```

1  #RANDOM MATRIX GENERATOR Python
2  def generate_random_matrix(n:int, distribution:str="normal",
3                             symmetric:bool=False, seed:int|None=None):
4      rng = np.random.default_rng(seed)
5      if distribution == "normal":
6          A = rng.standard_normal((n, n))
7      elif distribution == "uniform":
8          A = rng.uniform(-1.0, 1.0, size=(n, n))
9      else:
10         raise ValueError("distribution must be 'normal' or 'uniform'")
11     return (A + A.T) / 2.0 if symmetric else A
12
13
14 #REFLECTOR CALCULATOR
15 def _house_vec(x:np.ndarray) -> np.ndarray:
16
17     """
18     Builds a Householder reflector for a given column vector x.
19     Args:
20         x (np.ndarray): Column vector to be transformed.
21     Returns:
22         np.ndarray: Normalised Householder vector with a real first component.
23     Raises:
24         None
25     """
26
27     sigma = np.linalg.norm(x)
28     if sigma == 0.0:
29         e1 = np.zeros_like(x)
30         e1[0] = 1.0
31         return e1
32     sign = 1.0 if x[0].real >= 0.0 else -1.0
33     v = x.copy()
34     v[0] += sign * sigma

```

```

35     return v / np.linalg.norm(v)
36
37 def hessenberg_reduction(A_in:np.ndarray, symmetric:bool=False,
38     accumulate_q:bool=True):
39     """
40     Reduces a matrix to upper Hessenberg form using Householder reflections.
41     Args:
42         A_in (np.ndarray): Input matrix to be reduced.
43         symmetric (bool): If True, treat the matrix as symmetric and reduce to
44             tridiagonal form.
45         accumulate_q (bool): If True, accumulate the orthogonal matrix Q.
46     Returns:
47         Tuple[np.ndarray, np.ndarray]: The reduced matrix in Hessenberg form
48             and the orthogonal matrix Q.
49     Raises:
50         None
51     """
52     A = A_in.copy()
53     n = A.shape[0]
54     Q = np.eye(n, dtype=A.dtype)
55
56     if not symmetric:    #GENERAL caSe
57         for k in range(n-2):
58             v = _house_vec(A[k+1:, k])
59             w = np.zeros(n, dtype=A.dtype)
60             w[k+1:] = v
61             A -= 2.0 * np.outer(w, w.conj().T @ A)
62             A -= 2.0 * np.outer(A @ w, w.conj().T)
63             if accumulate_q:
64                 Q -= 2.0 * np.outer(Q @ w, w.conj().T)
65         return A, Q
66
67     #SYMMETRIC TRIDIAGONAL CASE
68     for k in range(n-2):
69         x = A[k+1:, k]
70         v = _house_vec(x)
71         beta = 2.0
72
73         w = A[k+1:, k+1:] @ v    #trailing submatrix rank-2 update ( $A \leftarrow A - v w^T - w v^T$ )
74         tau = beta * 0.5 * (v @ w)
75         w -= tau * v
76         A[k+1:, k+1:] -= beta * np.outer(v, w) + beta * np.outer(w, v)

```

```

77         new_val = -np.sign(x[0]) * np.linalg.norm(x) #store the single sub-
           diagonal element, zero the rest
78         A[k+1, k] = new_val
79         A[k, k+1] = new_val
80         A[k+2:, k] = 0.0
81         A[k, k+2:] = 0.0
82
83         if accumulate_q: #accumulate Q if requested
84             Q[:, k+1:] -= beta * np.outer(Q[:, k+1:] @ v, v)
85
86         A = np.triu(A) + np.triu(A, 1).T #force symmetry
87         return A, Q
88
89
90 #VERIFYING PART
91 def verify_factorisation_once(n:int, dist:str, symmetric:bool, seed:int|None):
92
93     """
94     Verifies the factorisation of a random matrix of size n.
95     Args:
96         n (int): Size of the matrix.
97         dist (str): Distribution type ('normal' or 'uniform').
98         symmetric (bool): Whether the matrix is symmetric.
99         seed (int | None): Random seed for reproducibility.
100     Returns:
101         None
102     Raises:
103         None
104     """
105
106     A = generate_random_matrix(n, dist, symmetric, seed)
107     T, Q = hessenberg_reduction(A, symmetric=symmetric)
108     res_fact = np.linalg.norm(A - Q @ T @ Q.T)
109     res_orth = np.linalg.norm(Q.T @ Q - np.eye(n))
110     colour = "green" if res_fact < 1e-11 else "red"
111     typ = "symmetric" if symmetric else "general"
112     display(Markdown(
113         f"***{n}x{n} {typ}** \n"
114         f"<span style='color:{colour}>||A - Q T Q^T|| = {res_fact:.2e}</span> \n"
115         f"||Q^T Q - I|| = {res_orth:.2e}"
116     ))
117
118
119 def benchmark_hessenberg(size_list, dist:str, mode:str, seed:int|None,
           reps_small:int=5):

```



```

120
121     """
122     Benchmark the Hessenberg reduction for various matrix sizes and types.
123     Args:
124         size_list (list of int): List of matrix sizes to test.
125         dist (str): Distribution type ('normal' or 'uniform').
126         mode (str): Matrix type ('general', 'symmetric', or 'both').
127         seed (int | None): Random seed for reproducibility.
128         reps_small (int): Number of repetitions for small matrices.
129     Returns:
130         pd.DataFrame: DataFrame containing the benchmark results.
131     Raises:
132         None
133     """
134
135     records = []
136     for n in size_list:
137         for sym in ([False, True] if mode=="both" else [mode=="symmetric"]):
138             A = generate_random_matrix(n, dist, sym, seed)
139
140             t0 = time.perf_counter()
141             hessenberg_reduction(A, symmetric=sym, accumulate_q=False)
142             probe = time.perf_counter() - t0
143             reps = reps_small if probe*reps_small >= 1.0 else math.ceil(1.0 /
144                                 probe)
145
146             times = []
147             for _ in range(reps):
148                 start = time.perf_counter()
149                 hessenberg_reduction(A, symmetric=sym, accumulate_q=False)
150                 times.append(time.perf_counter() - start)
151
152             records.append(dict(size=n,
153                                type="symmetric" if sym else "general",
154                                reps=reps,
155                                avg=np.mean(times)))
156
157     df = pd.DataFrame(records)
158     display(df.style.format({"avg": "{:.3e}"}).hide(axis="index"))
159
160     plt.figure(figsize=(7, 5))
161
162     mark = {"general": "o", "symmetric": "s"}
163     for label, sub in df.groupby("type"):
164         # simple scatter/line plot – no regression curves
165         plt.plot(

```

```

165         sub["size"],          # x-axis: matrix order
166         sub["avg"],           # y-axis: average runtime
167         marker=mark[label],
168         ls="-",
169         label=label,
170     )
171
172     plt.xlabel("matrix size (linear)")
173     plt.ylabel("runtime [s] (linear)")      # keep or change to log scale as
174     you prefer
175     plt.title("Hessenberg (general) vs Tridiagonal (symmetric)")
176     plt.grid(True, which="both", ls=":")
177     plt.legend()
178     plt.tight_layout()
179     plt.show()
180
181
182 # === INTERACTIVE PART =====
183
184
185 def parse_size_spec(spec: str):
186     """
187     Accepts either '[64,128,256,512]' or '64:1024:64'
188     Returns a sorted list of unique integers ≥ 2
189     """
190     spec = spec.strip()
191     interval = re.fullmatch(r"\s*(\d+)\s*:\s*(\d+)\s*:\s*(\d+)\s*", spec)
192     if interval:
193         # range syntax
194         lo, hi, step = map(int, interval.groups())
195         if step <= 0 or lo < 2 or hi < lo:
196             raise ValueError
197         return list(range(lo, hi + 1, step))
198     # otherwise fall back to literal - must be a list
199     sizes = literal_eval(spec)
200     if (not isinstance(sizes, (list, tuple)) or
201         any((not isinstance(k, int)) or k < 2 for k in sizes)):
202         raise ValueError
203     return sorted(set(sizes))
204
205 try:
206     raw = input(
207         "\nMatrix sizes – list '[64,128,256]' or interval '64:1024:64' "
208         "(default 64:1024:64): "
209     )
210     sizes = parse_size_spec(raw) if raw else list(range(64, 1025, 64))

```

```

210 except Exception:
211     print("Bad specification → using default 64:1024:64.")
212     sizes = list(range(64, 1025, 64))
213
214 dist = input("Distribution ('normal'/'uniform') [normal]: ").strip().lower()
215     or "normal"
216 mode_txt = input("Matrix type g=general, s=symmetric, b=both [g]:
217 ").strip().lower() or "g"
218 mode = "symmetric" if mode_txt == "s" else "both" if mode_txt == "b" else
219     "general"
220 seed_txt = input("Random seed (None/int) [None]: ").strip()
221 seed_val = None if seed_txt.lower() in {"", "none"} else int(seed_txt)
222
223 # accuracy check
224 for n in sizes:
225     for sym in ([False, True] if mode == "both" else [mode == "symmetric"]):
226         verify_factorisation_once(n, dist, sym, seed_val)
227
228 benchmark_hessenberg(sizes, dist, mode, seed_val) # timings

```

The reader should be aware that my poor Dell Inspiron 5590 has crashed precisely 5 times while i was writing this (i might have tried with matrices of order $10^6 \times 10^6$). Unfortunately the runtime was around 4 minutes for a matrix $A \approx 10^3 \times 10^3$.

An expected output is:

	Python
1	64×64 general
2	$\ A - Q T Q^T\ = 8.03e-14$
3	$\ Q^T Q - I\ = 7.67e-15$
4	
5	64×64 symmetric
6	$\ A - Q T Q^T\ = 4.78e-14$
7	$\ Q^T Q - I\ = 7.32e-15$
8	
9	128×128 general
10	$\ A - Q T Q^T\ = 1.80e-13$
11	$\ Q^T Q - I\ = 1.24e-14$
12	
13	128×128 symmetric
14	$\ A - Q T Q^T\ = 1.13e-13$
15	$\ Q^T Q - I\ = 1.25e-14$
16	
17	192×192 general
18	$\ A - Q T Q^T\ = 3.09e-13$
19	$\ Q^T Q - I\ = 1.76e-14$
20	
21	192×192 symmetric
22	$\ A - Q T Q^T\ = 1.91e-13$

```

23 ||QTQ - I|| = 1.75e-14
24
25 256×256 general
26 ||A - Q T QT|| = 4.53e-13
27 ||QTQ - I|| = 2.27e-14
28
29 256×256 symmetric
30 ||A - Q T QT|| = 2.97e-13
31 ||QTQ - I|| = 2.38e-14
32
33 320×320 general
34 ||A - Q T QT|| = 6.07e-13
35 ||QTQ - I|| = 2.72e-14
36
37 320×320 symmetric
38 ||A - Q T QT|| = 3.76e-13
39 ||QTQ - I|| = 2.71e-14
40
41 384×384 general
42 ||A - Q T QT|| = 7.78e-13
43 ||QTQ - I|| = 3.11e-14
44
45 384×384 symmetric
46 ||A - Q T QT|| = 4.71e-13
47 ||QTQ - I|| = 3.16e-14
48
49 448×448 general
50 ||A - Q T QT|| = 9.35e-13
51 ||QTQ - I|| = 3.50e-14
52
53 448×448 symmetric
54 ||A - Q T QT|| = 5.93e-13
55 ||QTQ - I|| = 3.72e-14
56
57 512×512 general
58 ||A - Q T QT|| = 1.16e-12
59 ||QTQ - I|| = 4.13e-14
60
61 512×512 symmetric
62 ||A - Q T QT|| = 7.14e-13
63 ||QTQ - I|| = 4.13e-14
64
65 576×576 general
66 ||A - Q T QT|| = 1.38e-12
67 ||QTQ - I|| = 4.54e-14
68

```

```

69 576×576 symmetric
70 ||A - Q T QT|| = 8.39e-13
71 ||QTQ - I|| = 4.56e-14
72
73 640×640 general
74 ||A - Q T QT|| = 1.58e-12
75 ||QTQ - I|| = 4.93e-14
76
77 640×640 symmetric
78 ||A - Q T QT|| = 9.77e-13
79 ||QTQ - I|| = 5.07e-14
80
81 704×704 general
82 ||A - Q T QT|| = 1.81e-12
83 ||QTQ - I|| = 5.41e-14
84
85 704×704 symmetric
86 ||A - Q T QT|| = 1.08e-12
87 ||QTQ - I|| = 5.35e-14
88
89 768×768 general
90 ||A - Q T QT|| = 2.05e-12
91 ||QTQ - I|| = 5.92e-14
92
93 768×768 symmetric
94 ||A - Q T QT|| = 1.25e-12
95 ||QTQ - I|| = 5.98e-14
96
97 832×832 general
98 ||A - Q T QT|| = 2.29e-12
99 ||QTQ - I|| = 6.32e-14
100
101 832×832 symmetric
102 ||A - Q T QT|| = 1.38e-12
103 ||QTQ - I|| = 6.28e-14
104
105 896×896 general
106 ||A - Q T QT|| = 2.53e-12
107 ||QTQ - I|| = 6.71e-14
108
109 896×896 symmetric
110 ||A - Q T QT|| = 1.50e-12
111 ||QTQ - I|| = 6.65e-14
112
113 960×960 general
114 ||A - Q T QT|| = 2.78e-12

```

```

115 ||QTQ - I|| = 7.14e-14
116
117 960×960 symmetric
118 ||A - Q T QT|| = 1.68e-12
119 ||QTQ - I|| = 7.19e-14
120
121 1024×1024 general
122 ||A - Q T QT|| = 3.09e-12
123 ||QTQ - I|| = 7.71e-14
124
125 1024×1024 symmetric
126 ||A - Q T QT|| = 1.84e-12
127 ||QTQ - I|| = 7.53e-14
128

```

As n grows, we observe that the residuals also grow, but still in machine precision. The difference between the symmetric and nonsymmetric cases are more pronounced in larger matrices.

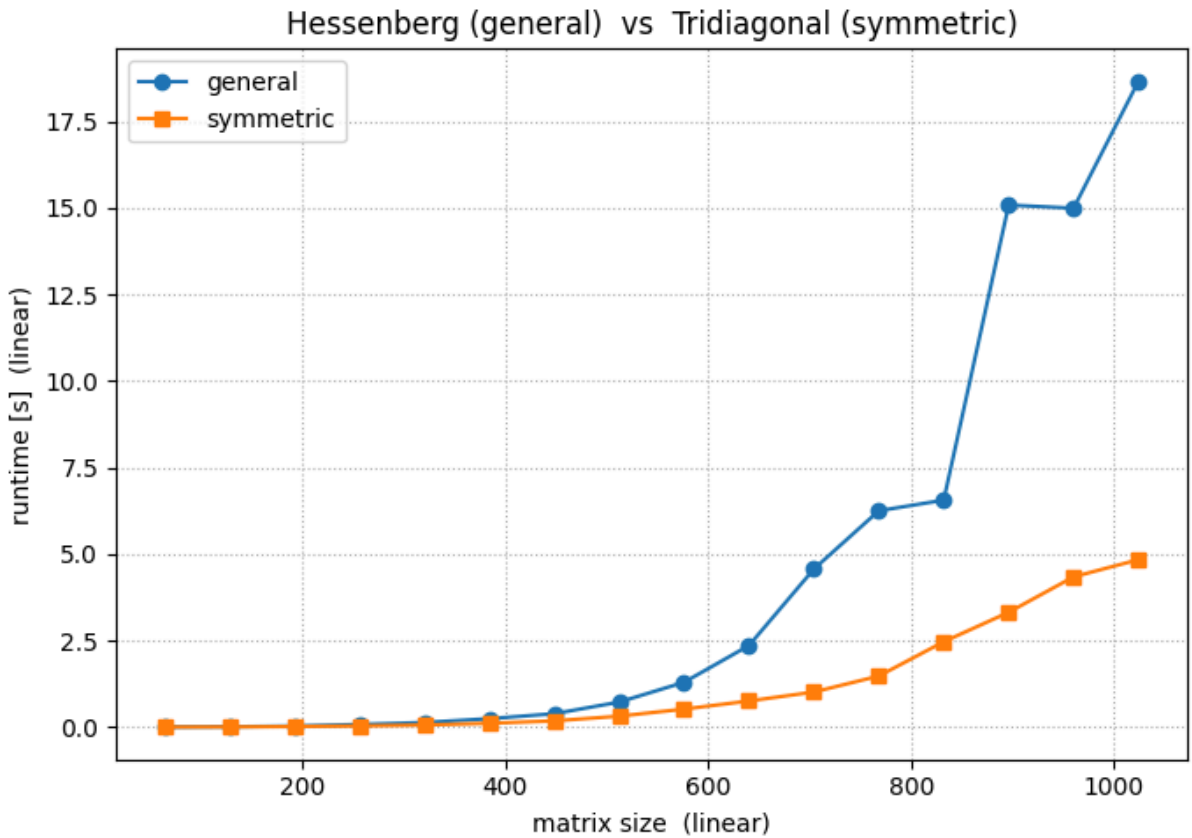


Figure 3: Runtime of the Hessenberg reduction for ordinary and symmetric matrices

2.2.1. Complexity (c)

Figure 3 shows the expected $O(n^3)$ complexity for the general case and usually half of it for the symmetric case. The latter is better discussed in Section 2.2.2.

The weird behavior of the general case after $n \approx 760$ is due purely to hardware restrictions, not mathematical ones. NumPy delegates the heavy multiply-add to OpenBLAS / MKL. Those libraries switch to different blocking sizes and sometimes to multi-threaded kernels at dimension

“milestones” (often multiples of 64 or 128). The change in algorithmic constant shows up as local bumps or dips. Residual noise comes from thread scheduling and page-fault variability in long runs.

To understand why the complexity is $O(n^3)$ in the general case, we can look at the algorithm. The outer loop runs $n - 2$ times, and inside it, we have two matrix-vector products and two outer products, which are all $O(n^2)$. Thus, the total complexity is $O(n^3)$.

2.2.2. The Symmetric Case (d)

On the symmetric case we know that reflectors will be applied in only one side of the matrix, since $v^T A = A v^T$. That is precisely what the function `generate_random_matrix` does. Which cuts complexity from the expected $O(n^3)$ seen in the previous section to a $O(n^2)$. [1]

3. Eigenvalues and Iterative Methods

3.1. Power iteration

The power iteration consists on computing large powers of the sequence:

$$\frac{x}{\|x\|}, \frac{Ax}{\|Ax\|}, \frac{A^2x}{\|A^2x\|}, \dots, A \in \mathbb{C}^{m \times m} \quad (2)$$

To see why this sequence converges (under good assumptions), let A be diagonalizable. And write:

$$x = \sum_{i=1}^m \varphi_i v_i \quad (3)$$

In a basis of eigenvectors v_i with respective eigenvalues λ_i . Then for $x \in \mathbb{C}^m$ we have:

$$Ax = \sum_{i=1}^m \lambda_i \varphi_i v_i \quad (4)$$

Or even better:

$$A^n x = \sum_{i=1}^m \lambda_i^n \varphi_i v_i \quad (5)$$

Let v_j be the eigenvector associated to the biggest eigenvalue λ_j , then we have:

$$A^n x = \frac{1}{\lambda_j^n} \cdot \sum_{i=1}^m \lambda_i^n \varphi_i v_i = \frac{\lambda_1^n}{\lambda_j^n} \varphi_1 v_1 + \dots + \varphi_j v_j + \dots + \frac{\lambda_m^n}{\lambda_j^n} \varphi_m v_m \quad (6)$$

When $n \rightarrow \infty$ all of the smaller $\frac{\lambda_k}{\lambda_j}$ will approach 0, so we have:

$$\lim_{n \rightarrow \infty} A^n x = \varphi_j v_j \quad (7)$$

So the denominator on the original expression becomes

$$\|A^n x\| = \|\varphi_j v_j\| = |\varphi_j| \|v_j\| \quad (8)$$

And the limit is:

$$\lim_{n \rightarrow \infty} \frac{A^n x}{\|A^n x\|} = \frac{\varphi_j v_j}{|\varphi_j| \|v_j\|} \quad (9)$$

²See page 194 of [Trefethen & Bau's Numerical Linear Algebra book](#)

Since $\frac{\varphi_j}{|\varphi_j|} = \pm 1$, the sequence converges to the eigenvector v_j associated to the eigenvalue λ_j .

3.2. Inverse Iteration

We know that if $A \in \mathbb{C}^{m \times m}$, $\det(A) \neq 0$ has $\Lambda \subset \mathbb{R} := \{\lambda_j\}$ as eigenvalues, then the eigenvalues of A^{-1} are λ_j^{-1} . Similarly the eigenvalues of $A + \varphi I$ are $\lambda_j + \varphi$. So the eigenvalues of $(A - \mu I)^{-1}$, $\mu \in \mathbb{R} \setminus \Lambda$ are:

$$\hat{\lambda}_j = \frac{1}{\lambda_j - \mu} \quad (10)$$

Where λ_j are the eigenvalues of A . So if μ is close to an eigenvalue, then $\hat{\lambda}$ will be large. Power iteration seems interesting here, so the sequence:

$$\frac{x}{\|x\|}, \frac{(A - \mu I)^{-1}x}{\|(A - \mu I)^{-1}x\|}, \frac{(A - \mu I)^{-2}x}{\|(A - \mu I)^{-2}x\|}, \dots \quad (11)$$

Converges to the eigenvector associated to the eigenvalue $\hat{\lambda}$.

4. Orthogonal Matrices (Problem 2) (a)

Here we will discuss how orthogonal matrices behave when we apply the iterations discussed in [Section 3.1](#), and [Section 3.2](#).

So let $Q \in \mathbb{C}^{m \times m}$ be an orthogonal matrix. We are interested in its eigenvalues λ . We know that:

$$\begin{aligned} Qx = \lambda x &\Leftrightarrow x^T Qx = \lambda x^T x \\ &\Leftrightarrow Q\langle x, x \rangle = \lambda \langle x, x \rangle \end{aligned} \quad (12)$$

Since Q preserves inner product, we have:

$$\begin{aligned} Q\langle x, x \rangle = \lambda \langle x, x \rangle &\Leftrightarrow \langle x, x \rangle = \lambda \langle x, x \rangle \\ &\Leftrightarrow |\lambda| = 1 \end{aligned} \quad (13)$$

So λ lies in the unit circle, i.e $\lambda = e^{i\varphi}$, $\varphi \in \mathbb{R}$. We now discuss how this affects efficiency of some iterative methods

4.1. Orthogonal Matrices and the Power Iteration

The power method is better discussed in [Section 3.1](#). Here we will write straight forward the result:

$$Q^n x = \frac{1}{\lambda_j^n} \cdot \sum_{i=1}^m \lambda_i^n \varphi_i v_i \quad (14)$$

Where λ_i are the eigenvalues of $Q \in \mathbb{C}^{m \times m}$, φ_i are the coefficients of the expansion of x in the basis of eigenvectors v_i . Since we have that $|\lambda_i| = 1$, we have:

The fact that $|\lambda_i| = 1 \Rightarrow |\lambda_i^n| = 1$ is sufficiently enough for one to be convinced that power iteration does not converge.

Let $\lambda_k = e^{i\psi_k}$, where $\psi_k \in \mathbb{R}$. Then expanding [eq. \(14\)](#):

$$Q^n x = \frac{1}{e^{i\psi_j \cdot n}} \cdot \sum_{\tau=1}^m e^{i\psi_\tau n} \varphi_\tau v_\tau \quad (15)$$

When $n \rightarrow \infty$ if $\lambda_j = 1$ then we have:

$$Q^n x = \varphi_j v_j + \sum_{\tau \neq j} e^{i\psi_\tau n} \varphi_\tau v_\tau \quad (16)$$

Since no eigenvalue dominates other eigenvalues in the orthogonal case, usually power iteration fails.

4.2. Orthogonal Matrices and Inverse Iteration

Let $Q \in \mathbb{C}^{m \times m}$ be an orthogonal matrix. Using results from [Section 3.2](#), we are interested in applying inverse iteration to Q .

We know that the eigenvalues of Q are on the unit circle, so if μ is close to an eigenvalue λ_j , $\hat{\lambda}_j$ will be huge (dominant), which makes power iteration converge to the eigenvector associated to $\hat{\lambda}_j$, which is the eigenvector associated to λ_j . The fact that the eigenvalues are on the unit circle also contributes to the convergence of the method.

So we conclude that inverse iteration works well for orthogonal matrices, if μ is close to an eigenvalue of Q .

4.3. The 2×2 Case (b)

We will calculate the eigenvalues of:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad (17)$$

With $a, b, c, d \in \mathbb{R}$. The characteristic polynomial gives us:

$$\begin{aligned} \det(A - \lambda I) &= 0 \Leftrightarrow \det \begin{pmatrix} a - \lambda & b \\ c & d - \lambda \end{pmatrix} = 0 \\ \Leftrightarrow (a - \lambda)(d - \lambda) - bc &= 0 \Leftrightarrow \lambda^2 + \lambda(-a - d) + (ad - bc) = 0 \\ \Leftrightarrow \lambda &= (a + d) \pm \frac{\sqrt{(a + d)^2 - 4(ad - bc)}}{2} \end{aligned} \quad (18)$$

So the eigenvalues are:

$$\begin{aligned} \lambda_1 &= \frac{a + d + \sqrt{(a + d)^2 - 4(ad - bc)}}{2} \\ \lambda_2 &= \frac{a + d - \sqrt{(a + d)^2 - 4(ad - bc)}}{2} \end{aligned} \quad (19)$$


4.4. Random Orthogonal Matrices (c)

This code generates orthogonal matrices of order 4×4 generated by the QR factorization of random matrices, and reduces them to Hessenberg form. The eigenvalues of the bottom-right 2×2 block are analytically calculated using [Section 4.3](#).

```

1  def generate_orthogonal_matrix_qr(n=4, seed=None):
2
3      """
4      Generates a random orthogonal matrix using QR decomposition.
5      Args:
6          n (int): Size of the matrix (n x n).
7          seed (int | None): Random seed for reproducibility.
8      Returns:

```

 Python

```

9         np.ndarray: An n x n orthogonal matrix.
10     Raises:
11         None
12     """
13
14     if seed is not None:
15         np.random.seed(seed)
16     A = np.random.randn(n, n)
17     Q, _ = np.linalg.qr(A)
18     return Q
19
20 def analytical_eigenvalues_2x2(a, b, c, d):
21
22     """
23     Calculates the eigenvalues of a 2x2 matrix analytically.
24     Args:
25         a (float): Element at position (0,0).
26         b (float): Element at position (0,1).
27         c (float): Element at position (1,0).
28         d (float): Element at position (1,1).
29     Returns:
30         Tuple[float, float]: The two eigenvalues of the matrix.
31     Raises:
32     """
33
34     trace = a + d
35     det = a * d - b * c
36     discriminant = trace**2 - 4 * det
37
38     #complex if discriminant negative
39     discriminant_root = np.sqrt(discriminant) if discriminant >= 0 else
40     np.sqrt(complex(discriminant))
41
42     lambda1 = (trace + discriminant_root) / 2
43     lambda2 = (trace - discriminant_root) / 2
44
45     return lambda1, lambda2
46
47 def analyze_orthogonal_and_hessenberg(n=4, n_matrices=30):
48
49     """
50     Analyzes orthogonal matrices and their Hessenberg forms.
51     Args:
52         n (int): Size of the matrices (n x n).
53         n_matrices (int): Number of orthogonal matrices to generate and analyze.
54     Returns:

```

```

54         None
55     Raises:
56         None
57     """
58
59     for i in range(n_matrices):
60         print(f"\n--- Orthogonal Matrix Q number {i+1} ---")
61         Q = generate_orthogonal_matrix_qr(n=n)
62         print("Matrix Q:")
63         print(np.array_str(Q, precision=4, suppress_small=True))
64
65         householder_list, H, Q_accum = to_hessenberg(Q)
66
67         print("\nHessenberg Form H (of Q):")
68         print(np.array_str(H, precision=4, suppress_small=True))
69
70         block = Q[2:4, 2:4]
71         a, b, c, d = block[0,0], block[0,1], block[1,0], block[1,1]
72         analytical_eigenvalues = analytical_eigenvalues_2x2(a, b, c, d)
73
74         print("\nBlock Q[3:4,3:4] (indices 2 and 3, 2x2):")
75
76         print(np.array_str(block, precision=4, suppress_small=True))
77
78         print("\nEigenvalues of the 2x2 block (analytically calculated):")
79         for idx, val in enumerate(analytical_eigenvalues):
80             print(f"   $\lambda_{\text{idx}+1} = \{\text{val}\}$  (size =  $\{\text{abs}(\text{val})\}$ )")
81
82         print("-" * 40)
83
84 analyze_orthogonal_and_hessenberg()

```

We ran this code for 30 matrices, the output was:

```

1  --- Orthogonal Matrix Q number 1 ---
2  Matrix Q:
3  [[-0.5629  0.6801  0.4635 -0.0764]
4   [-0.6703 -0.6884  0.2231  0.1645]
5   [-0.1497  0.2337 -0.3793  0.8827]
6   [ 0.4598 -0.0949  0.7691  0.4336]]
7
8  Hessenberg Form H (of Q):
9  [[-0.5629 -0.6779 -0.4534 -0.1342]
10   [ 0.8265 -0.4617 -0.3088 -0.0914]
11   [ 0.      -0.5721  0.7865  0.2329]
12   [-0.      0.      0.2839 -0.9588]]
13

```

```

14 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
15 [[-0.3793  0.8827]
16 [ 0.7691  0.4336]]
17
18 Eigenvalues of the 2x2 block (analytically calculated):
19  $\lambda_1 = 0.9458819605346136$  (size = 0.9459)
20  $\lambda_2 = -0.8915812804514585$  (size = 0.8916)
21 -----
22
23 --- Orthogonal Matrix Q number 2 ---
24 Matrix Q:
25 [[-0.4016  0.3605  0.8354  0.1045]
26 [-0.2846 -0.3518  0.1255 -0.8829]
27 [-0.3325 -0.8166  0.136  0.4519]
28 [ 0.8045 -0.282  0.5176 -0.0734]]
29
30 Hessenberg Form H (of Q):
31 [[-0.4016 -0.3235 -0.1952  0.8343]
32 [ 0.9158 -0.1418 -0.0856  0.3658]
33 [ 0.      -0.9355  0.0805 -0.3439]
34 [ 0.      0.      -0.9737 -0.2278]]
35
36 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
37 [[ 0.136  0.4519]
38 [ 0.5176 -0.0734]]
39
40 Eigenvalues of the 2x2 block (analytically calculated):
41  $\lambda_1 = 0.5261528266779573$  (size = 0.5262)
42  $\lambda_2 = -0.4635182526918817$  (size = 0.4635)
43 -----
44
45 --- Orthogonal Matrix Q number 3 ---
46 Matrix Q:
47 [[-0.0452 -0.9852 -0.1571  0.0523]
48 [ 0.4259  0.0412 -0.0809  0.9002]
49 [ 0.1843  0.1346 -0.9569 -0.1793]
50 [-0.8846  0.0982 -0.2303  0.3934]]
51
52 Hessenberg Form H (of Q):
53 [[-0.0452  0.4954 -0.8604 -0.1112]
54 [-0.999  -0.0224  0.0389  0.005 ]
55 [ 0.      0.8684  0.4918  0.0636]
56 [-0.      0.      0.1282 -0.9917]]
57
58 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
59 [[-0.9569 -0.1793]

```

```

60  [-0.2303  0.3934]]
61
62  Eigenvalues of the 2x2 block (analytically calculated):
63   $\lambda_1 = 0.42331102317930497$  (size = 0.4233)
64   $\lambda_2 = -0.9868680130188092$  (size = 0.9869)
65  -----
66
67  --- Orthogonal Matrix Q number 4 ---
68  Matrix Q:
69  [[-0.2568  0.8228 -0.2287  0.4525]
70   [-0.2848 -0.018  0.9011  0.3265]
71   [-0.6054 -0.5408 -0.3667  0.4544]
72   [ 0.6975 -0.1738 -0.0346  0.6943]]
73
74  Hessenberg Form H (of Q):
75  [[-0.2568  0.2274  0.1895  0.92  ]
76   [ 0.9665  0.0604  0.0503  0.2444]
77   [-0.      -0.9719  0.0475  0.2305]
78   [ 0.       0.      -0.9794  0.2017]]
79
80  Block Q[3:4,3:4] (indices 2 and 3, 2x2):
81  [[-0.3667  0.4544]
82   [-0.0346  0.6943]]
83
84  Eigenvalues of the 2x2 block (analytically calculated):
85   $\lambda_1 = 0.67929095950588$  (size = 0.6793)
86   $\lambda_2 = -0.3516952691053273$  (size = 0.3517)
87  -----
88
89  --- Orthogonal Matrix Q number 5 ---
90  Matrix Q:
91  [[-0.5025 -0.7649 -0.0345  0.4015]
92   [ 0.3718  0.0654  0.6626  0.6469]
93   [ 0.6819 -0.6372  0.0297 -0.358  ]
94   [ 0.3798  0.0679 -0.7476  0.5406]]
95
96  Hessenberg Form H (of Q):
97  [[-0.5025  0.1798 -0.7903  0.3011]
98   [-0.8646 -0.1045  0.4593 -0.175  ]
99   [-0.      -0.9781 -0.1943  0.074  ]
100  [-0.       0.       0.356  0.9345]]
101
102  Block Q[3:4,3:4] (indices 2 and 3, 2x2):
103  [[ 0.0297 -0.358  ]
104   [-0.7476  0.5406]]
105

```

```

106 Eigenvalues of the 2x2 block (analytically calculated):
107    $\lambda_1 = 0.8621172090206812$  (size = 0.8621)
108    $\lambda_2 = -0.2918043055687003$  (size = 0.2918)
109   -----
110
111 --- Orthogonal Matrix Q number 6 ---
112 Matrix Q:
113 [[-0.9092  0.1745  0.2312 -0.2992]
114 [ 0.1618 -0.3496 -0.2497 -0.8884]
115 [-0.1454  0.4629 -0.8736  0.0369]
116 [-0.3551 -0.7956 -0.3479  0.3462]]
117
118 Hessenberg Form H (of Q):
119 [[-0.9092 -0.2422 -0.3011 -0.1552]
120 [-0.4164  0.5288  0.6573  0.3389]
121 [ 0.      0.8134 -0.517  -0.2665]
122 [-0.      0.      0.4582 -0.8888]]
123
124 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
125 [[-0.8736  0.0369]
126 [-0.3479  0.3462]]
127
128 Eigenvalues of the 2x2 block (analytically calculated):
129    $\lambda_1 = 0.3356278613353456$  (size = 0.3356)
130    $\lambda_2 = -0.8630034423078552$  (size = 0.8630)
131   -----
132
133 --- Orthogonal Matrix Q number 7 ---
134 Matrix Q:
135 [[-0.4296  0.6844 -0.4085  0.4245]
136 [ 0.2184 -0.4244 -0.028  0.8783]
137 [ 0.17    -0.3028 -0.9122 -0.2177]
138 [ 0.8596  0.5097 -0.0167  0.032 ]]
139
140 Hessenberg Form H (of Q):
141 [[-0.4296 -0.4928  0.7535  0.0703]
142 [-0.903   0.2344 -0.3584 -0.0334]
143 [-0.     -0.838  -0.5433 -0.0507]
144 [ 0.      0.      0.0928 -0.9957]]
145
146 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
147 [[-0.9122 -0.2177]
148 [-0.0167  0.032 ]]
149
150 Eigenvalues of the 2x2 block (analytically calculated):
151    $\lambda_1 = 0.0358294023777706$  (size = 0.0358)

```

```

152    $\lambda_2 = -0.9159948591320213$  (size = 0.9160)
153 -----
154
155 --- Orthogonal Matrix Q number 8 ---
156 Matrix Q:
157 [[-0.3402 -0.0284 -0.009   0.9399]
158  [-0.7148 -0.6207  0.1662 -0.2759]
159  [-0.5681  0.5973 -0.5323 -0.1927]
160  [ 0.2249 -0.5071 -0.83    0.0581]]
161
162 Hessenberg Form H (of Q):
163 [[-0.3402  0.2518  0.8476  0.3201]
164  [ 0.9403  0.0911  0.3067  0.1158]
165  [ 0.      0.9635 -0.2505 -0.0946]
166  [-0.      -0.      0.3533 -0.9355]]
167
168 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
169 [[-0.5323 -0.1927]
170  [-0.83    0.0581]]
171
172 Eigenvalues of the 2x2 block (analytically calculated):
173    $\lambda_1 = 0.25997158900622225$  (size = 0.2600)
174    $\lambda_2 = -0.7341829044352151$  (size = 0.7342)
175 -----
176
177 --- Orthogonal Matrix Q number 9 ---
178 Matrix Q:
179 [[-0.2692  0.5878 -0.5428  0.5361]
180  [-0.8282  0.0254  0.5486  0.1116]
181  [-0.292   0.3638 -0.2875 -0.8365]
182  [-0.3954 -0.7222 -0.5673  0.0189]]
183
184 Hessenberg Form H (of Q):
185 [[-0.2692 -0.561  -0.1619  0.7659]
186  [ 0.9631 -0.1568 -0.0453  0.2141]
187  [-0.      -0.8128  0.1205 -0.5699]
188  [ 0.      -0.      -0.9784 -0.2069]]
189
190 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
191 [[-0.2875 -0.8365]
192  [-0.5673  0.0189]]
193
194 Eigenvalues of the 2x2 block (analytically calculated):
195    $\lambda_1 = 0.5713848430035342$  (size = 0.5714)
196    $\lambda_2 = -0.8399942790872519$  (size = 0.8400)
197 -----

```

```

198
199 --- Orthogonal Matrix Q number 10 ---
200 Matrix Q:
201 [[-0.4241  0.1349 -0.1773  0.8778]
202  [-0.332   -0.8664 -0.3594 -0.0999]
203  [ 0.8422 -0.2846 -0.2058  0.409 ]
204  [-0.0237 -0.3876  0.8928  0.2284]]
205
206 Hessenberg Form H (of Q):
207 [[-0.4241 -0.2373  0.8677  0.1046]
208  [ 0.9056 -0.1111  0.4063  0.049 ]
209  [-0.      0.9651  0.2602  0.0313]
210  [ 0.      0.      0.1196 -0.9928]]
211
212 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
213 [[-0.2058  0.409 ]
214  [ 0.8928  0.2284]]
215
216 Eigenvalues of the 2x2 block (analytically calculated):
217   $\lambda_1 = 0.6533894082311485$  (size = 0.6534)
218   $\lambda_2 = -0.6307995911677337$  (size = 0.6308)
219 -----
220
221 --- Orthogonal Matrix Q number 11 ---
222 Matrix Q:
223 [[-0.1675 -0.9786 -0.0461 -0.1099]
224  [ 0.7871 -0.1742  0.5819  0.1072]
225  [ 0.3816 -0.1052 -0.6644  0.6339]
226  [-0.4547 -0.0293  0.4667  0.758 ]]
227
228 Hessenberg Form H (of Q):
229 [[-0.1675  0.7485 -0.1241 -0.6295]
230  [-0.9859 -0.1271  0.0211  0.1069]
231  [ 0.      -0.6508 -0.1468 -0.7449]
232  [ 0.      -0.      -0.9811  0.1934]]
233
234 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
235 [[-0.6644  0.6339]
236  [ 0.4667  0.758 ]]
237
238 Eigenvalues of the 2x2 block (analytically calculated):
239   $\lambda_1 = 0.9421402114923986$  (size = 0.9421)
240   $\lambda_2 = -0.8485799008471894$  (size = 0.8486)
241 -----
242
243 --- Orthogonal Matrix Q number 12 ---

```



```

244 Matrix Q:
245 [[-0.3541  0.5585 -0.0616  0.7476]
246  [-0.5108 -0.6958 -0.4435  0.2414]
247  [ 0.1705  0.3507 -0.885   -0.2542]
248  [ 0.7646 -0.2844 -0.1274  0.5641]]
249
250 Hessenberg Form H (of Q):
251 [[-0.3541  0.295   0.7613  0.4561]
252  [ 0.9352  0.1117  0.2883  0.1727]
253  [-0.      0.949   -0.2706 -0.1621]
254  [-0.      -0.      0.5139 -0.8579]]
255
256 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
257 [[-0.885   -0.2542]
258  [-0.1274  0.5641]]
259
260 Eigenvalues of the 2x2 block (analytically calculated):
261   $\lambda_1 = 0.5861402323977607$  (size = 0.5861)
262   $\lambda_2 = -0.9070552039589896$  (size = 0.9071)
263  -----
264
265 --- Orthogonal Matrix Q number 13 ---
266 Matrix Q:
267 [[-0.7015  0.4915  0.3042 -0.417 ]
268  [ 0.4623 -0.1626  0.1522 -0.8583]
269  [ 0.2479  0.6778 -0.6824 -0.1159]
270  [-0.4825 -0.5221 -0.647  -0.2757]]
271
272 Hessenberg Form H (of Q):
273 [[-0.7015 -0.7069 -0.0829 -0.0373]
274  [-0.7127  0.6958  0.0816  0.0367]
275  [ 0.      0.1276 -0.9045 -0.4069]
276  [ 0.      -0.      0.4103 -0.912 ]]
277
278 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
279 [[-0.6824 -0.1159]
280  [-0.647  -0.2757]]
281
282 Eigenvalues of the 2x2 block (analytically calculated):
283   $\lambda_1 = -0.1379150700589457$  (size = 0.1379)
284   $\lambda_2 = -0.820164584090632$  (size = 0.8202)
285  -----
286
287 --- Orthogonal Matrix Q number 14 ---
288 Matrix Q:
289 [[-0.2164 -0.5735  0.7758 -0.1498]

```

```

290  [-0.3581 -0.6722 -0.6278 -0.1609]
291  [-0.1535  0.3054  0.0015 -0.9398]
292  [-0.8952  0.3551  0.0633  0.2617]]
293
294  Hessenberg Form H (of Q):
295  [[-0.2164  0.2256 -0.3541  0.8814]
296  [ 0.9763  0.05   -0.0785  0.1954]
297  [-0.      -0.9729 -0.0862  0.2144]
298  [-0.      0.     -0.9279 -0.3728]]
299
300  Block Q[3:4,3:4] (indices 2 and 3, 2x2):
301  [[ 0.0015 -0.9398]
302  [ 0.0633  0.2617]]
303
304  Eigenvalues of the 2x2 block (analytically calculated):
305   $\lambda_1 = (0.1316169167334142+0.20628124638940884j)$  (size = 0.2447)
306   $\lambda_2 = (0.1316169167334142-0.20628124638940884j)$  (size = 0.2447)
307  -----
308
309  --- Orthogonal Matrix Q number 15 ---
310  Matrix Q:
311  [[-0.467   0.1387  0.7316  0.4769]
312  [ 0.0375 -0.9757  0.0774  0.2017]
313  [-0.7384 -0.0272 -0.627   0.2467]
314  [ 0.485   0.1677 -0.2562  0.8191]]
315
316  Hessenberg Form H (of Q):
317  [[-0.467   0.3435 -0.8022 -0.1428]
318  [-0.8843 -0.1814  0.4236  0.0754]
319  [-0.      0.9215  0.3824  0.068 ]
320  [-0.      -0.     0.1752 -0.9845]]
321
322  Block Q[3:4,3:4] (indices 2 and 3, 2x2):
323  [[-0.627   0.2467]
324  [-0.2562  0.8191]]
325
326  Eigenvalues of the 2x2 block (analytically calculated):
327   $\lambda_1 = 0.7740235941481243$  (size = 0.7740)
328   $\lambda_2 = -0.5818796203145907$  (size = 0.5819)
329  -----
330
331  --- Orthogonal Matrix Q number 16 ---
332  Matrix Q:
333  [[-0.3849  0.0435 -0.7406  0.549 ]
334  [ 0.2525 -0.952  -0.07   0.158 ]
335  [ 0.7807  0.2969  0.0191  0.5495]

```

```

336  [-0.4225 -0.06    0.668   0.6096]]
337
338  Hessenberg Form H (of Q):
339  [[-0.3849  0.866   0.1952  0.2527]
340  [-0.923   -0.3611 -0.0814 -0.1054]
341  [ 0.       0.346  -0.5736 -0.7425]
342  [ 0.       0.      -0.7914  0.6114]]
343
344  Block Q[3:4,3:4] (indices 2 and 3, 2x2):
345  [[0.0191  0.5495]
346  [0.668   0.6096]]
347
348  Eigenvalues of the 2x2 block (analytically calculated):
349   $\lambda_1 = 0.9883202000131572$  (size = 0.9883)
350   $\lambda_2 = -0.3596282457787697$  (size = 0.3596)
351  -----
352
353  --- Orthogonal Matrix Q number 17 ---
354  Matrix Q:
355  [[-0.1066  0.7212  0.0749 -0.6804]
356  [-0.6764 -0.5121  0.3485 -0.3985]
357  [-0.5738  0.1159 -0.8012  0.1245]
358  [-0.4493  0.4519  0.4807  0.6024]]
359
360  Hessenberg Form H (of Q):
361  [[-0.1066 -0.2265 -0.8608 -0.4431]
362  [ 0.9943 -0.0243 -0.0923 -0.0475]
363  [ 0.      -0.9737  0.2025  0.1043]
364  [ 0.       0.      0.4577 -0.8891]]
365
366  Block Q[3:4,3:4] (indices 2 and 3, 2x2):
367  [[-0.8012  0.1245]
368  [ 0.4807  0.6024]]
369
370  Eigenvalues of the 2x2 block (analytically calculated):
371   $\lambda_1 = 0.6437814804055579$  (size = 0.6438)
372   $\lambda_2 = -0.842579679233141$  (size = 0.8426)
373  -----
374
375  --- Orthogonal Matrix Q number 18 ---
376  Matrix Q:
377  [[-0.8255  0.1636  0.5347  0.0763]
378  [-0.5177 -0.2699 -0.6467 -0.4908]
379  [ 0.0194  0.9433 -0.2236 -0.2446]
380  [-0.2238  0.103  -0.4958  0.8327]]
381

```

```

382 Hessenberg Form H (of Q):
383 [[-0.8255 -0.1619 -0.4367 -0.3188]
384 [ 0.5644 -0.2368 -0.6388 -0.4663]
385 [ 0.      0.958  -0.2317 -0.1691]
386 [-0.      0.     -0.5896  0.8077]]
387
388 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
389 [[-0.2236 -0.2446]
390 [-0.4958  0.8327]]
391
392 Eigenvalues of the 2x2 block (analytically calculated):
393  $\lambda_1 = 0.9372008193289816$  (size = 0.9372)
394  $\lambda_2 = -0.32809878403400955$  (size = 0.3281)
395 -----
396
397 --- Orthogonal Matrix Q number 19 ---
398 Matrix Q:
399 [[-0.7885  0.0127  0.4922 -0.3686]
400 [ 0.5982 -0.2155  0.6311 -0.4444]
401 [ 0.0328  0.1282 -0.5621 -0.8164]
402 [-0.1391 -0.968  -0.2085 -0.0141]]
403
404 Hessenberg Form H (of Q):
405 [[-0.7885 -0.122  -0.4769  0.3687]
406 [-0.615   0.1564  0.6114 -0.4727]
407 [ 0.      0.9801 -0.1569  0.1213]
408 [ 0.      0.     -0.6116 -0.7911]]
409
410 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
411 [[-0.5621 -0.8164]
412 [-0.2085 -0.0141]]
413
414 Eigenvalues of the 2x2 block (analytically calculated):
415  $\lambda_1 = 0.2071704757269035$  (size = 0.2072)
416  $\lambda_2 = -0.7833862909149965$  (size = 0.7834)
417 -----
418
419 --- Orthogonal Matrix Q number 20 ---
420 Matrix Q:
421 [[-0.3031  0.0429  0.9481  0.0861]
422 [ 0.1571 -0.9342  0.1195 -0.2972]
423 [-0.8533 -0.0239 -0.2292 -0.4678]
424 [-0.3942 -0.3534 -0.1853  0.8279]]
425
426 Hessenberg Form H (of Q):
427 [[-0.3031  0.8775 -0.2647  0.261 ]

```

```

428 [-0.953 -0.2791 0.0842 -0.083 ]
429 [ 0.      0.3901 0.6557 -0.6464]
430 [ 0.      -0.      -0.702 -0.7121]]
431
432 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
433 [[-0.2292 -0.4678]
434 [-0.1853 0.8279]]
435
436 Eigenvalues of the 2x2 block (analytically calculated):
437  $\lambda_1 = 0.9043475415351031$  (size = 0.9043)
438  $\lambda_2 = -0.30565758703551016$  (size = 0.3057)
439 -----
440
441 --- Orthogonal Matrix Q number 21 ---
442 Matrix Q:
443 [[-0.8149 0.3649 -0.4406 -0.0928]
444 [ 0.0977 -0.6065 -0.7405 0.2726]
445 [ 0.0387 0.372 0.0415 0.9265]
446 [-0.57 -0.6005 0.5058 0.2423]]
447
448 Hessenberg Form H (of Q):
449 [[-0.8149 -0.1234 0.2317 0.5167]
450 [-0.5796 0.1735 -0.3257 -0.7266]
451 [ 0.      -0.9771 -0.0871 -0.1943]
452 [-0.      0.      0.9125 -0.4091]]
453
454 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
455 [[0.0415 0.9265]
456 [0.5058 0.2423]]
457
458 Eigenvalues of the 2x2 block (analytically calculated):
459  $\lambda_1 = 0.8337542266557703$  (size = 0.8338)
460  $\lambda_2 = -0.5499977130761522$  (size = 0.5500)
461 -----
462
463 --- Orthogonal Matrix Q number 22 ---
464 Matrix Q:
465 [[-0.3663 0.4189 -0.6842 -0.4714]
466 [ 0.2919 -0.672 -0.6656 0.142 ]
467 [ 0.5024 0.6076 -0.2758 0.5498]
468 [-0.7268 -0.061 -0.1131 0.6748]]
469
470 Hessenberg Form H (of Q):
471 [[-0.3663 -0.1301 0.2621 0.8833]
472 [-0.9305 0.0512 -0.1032 -0.3477]
473 [-0.      -0.9902 -0.0398 -0.134 ]

```

```

474 [ 0.      -0.      0.9587 -0.2845]]
475
476 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
477 [[-0.2758  0.5498]
478 [-0.1131  0.6748]]
479
480 Eigenvalues of the 2x2 block (analytically calculated):
481  $\lambda_1 = 0.6040601009013424$  (size = 0.6041)
482  $\lambda_2 = -0.20508048336653284$  (size = 0.2051)
483 -----
484
485 --- Orthogonal Matrix Q number 23 ---
486 Matrix Q:
487 [[-0.4467 -0.7825 -0.2287  0.3687]
488 [ 0.4222  0.182  -0.0157  0.8879]
489 [ 0.6922 -0.3543 -0.5694 -0.2666]
490 [-0.3783  0.4786 -0.7895  0.0678]]
491
492 Hessenberg Form H (of Q):
493 [[-0.4467  0.702  -0.0795 -0.5489]
494 [-0.8947 -0.3505  0.0397  0.274 ]
495 [ 0.      -0.62  -0.1125 -0.7765]
496 [ 0.      0.     -0.9897  0.1433]]
497
498 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
499 [[-0.5694 -0.2666]
500 [-0.7895  0.0678]]
501
502 Eigenvalues of the 2x2 block (analytically calculated):
503  $\lambda_1 = 0.3077269347022792$  (size = 0.3077)
504  $\lambda_2 = -0.8093108181365211$  (size = 0.8093)
505 -----
506
507 --- Orthogonal Matrix Q number 24 ---
508 Matrix Q:
509 [[-0.0413 -0.9591  0.2707  0.0721]
510 [ 0.08    -0.274  -0.9584  0.0005]
511 [-0.6777 -0.0389 -0.0459 -0.7328]
512 [-0.7298  0.0604 -0.0778  0.6766]]
513
514 Hessenberg Form H (of Q):
515 [[-0.0413  0.3131  0.5108 -0.7996]
516 [-0.9991 -0.0129 -0.0211  0.0331]
517 [-0.      0.9496 -0.1687  0.2641]
518 [ 0.      0.      0.8427  0.5384]]
519

```

```

520 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
521 [[-0.0459 -0.7328]
522  [-0.0778  0.6766]]
523
524 Eigenvalues of the 2x2 block (analytically calculated):
525    $\lambda_1 = 0.7483535886136121$  (size = 0.7484)
526    $\lambda_2 = -0.1176716985715956$  (size = 0.1177)
527 -----
528
529 --- Orthogonal Matrix Q number 25 ---
530 Matrix Q:
531 [[-0.7483  0.5153 -0.0763  0.4107]
532  [-0.4762 -0.7375 -0.4779 -0.0313]
533  [-0.1713  0.3719 -0.3481 -0.8433]
534  [ 0.4288  0.2287 -0.8029  0.3451]]
535
536 Hessenberg Form H (of Q):
537 [[-0.7483 -0.0847  0.4863  0.4431]
538  [ 0.6633 -0.0956  0.5486  0.4998]
539  [ 0.      0.9918  0.0944  0.086 ]
540  [ 0.      0.      0.6735 -0.7392]]
541
542 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
543 [[-0.3481 -0.8433]
544  [-0.8029  0.3451]]
545
546 Eigenvalues of the 2x2 block (analytically calculated):
547    $\lambda_1 = 0.8914410770554546$  (size = 0.8914)
548    $\lambda_2 = -0.8943450015206609$  (size = 0.8943)
549 -----
550
551 --- Orthogonal Matrix Q number 26 ---
552 Matrix Q:
553 [[-0.9036  0.127 -0.0161  0.4088]
554  [-0.2097 -0.5366 -0.7493 -0.3265]
555  [ 0.3449  0.3256 -0.6074  0.6373]
556  [ 0.1433 -0.7681  0.2634  0.5658]]
557
558 Hessenberg Form H (of Q):
559 [[-0.9036  0.0616  0.2624  0.3329]
560  [ 0.4284  0.13   0.5536  0.7022]
561  [ 0.      0.9896 -0.0891 -0.113 ]
562  [-0.     -0.     0.7853 -0.6191]]
563
564 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
565 [[-0.6074  0.6373]

```

```

566 [ 0.2634  0.5658]]
567
568 Eigenvalues of the 2x2 block (analytically calculated):
569  $\lambda_1 = 0.6947481590186378$  (size = 0.6947)
570  $\lambda_2 = -0.7362843808778513$  (size = 0.7363)
571 -----
572
573 --- Orthogonal Matrix Q number 27 ---
574 Matrix Q:
575 [[-0.3783  0.2175  0.7347 -0.5194]
576 [-0.486  -0.5305 -0.4607 -0.5198]
577 [-0.7609  0.4199 -0.1943  0.455 ]
578 [-0.2046 -0.7036  0.4585  0.5029]]
579
580 Hessenberg Form H (of Q):
581 [[-0.3783 -0.6032 -0.0497 -0.7004]
582 [ 0.9257 -0.2465 -0.0203 -0.2862]
583 [ 0.      0.7585 -0.0461 -0.65  ]
584 [ 0.     -0.     -0.9975  0.0708]]
585
586 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
587 [[-0.1943  0.455 ]
588 [ 0.4585  0.5029]]
589
590 Eigenvalues of the 2x2 block (analytically calculated):
591  $\lambda_1 = 0.7288996874702811$  (size = 0.7289)
592  $\lambda_2 = -0.42026844598624513$  (size = 0.4203)
593 -----
594
595 --- Orthogonal Matrix Q number 28 ---
596 Matrix Q:
597 [[-0.6177 -0.2663 -0.3252  0.6646]
598 [ 0.7345 -0.3401  0.0741  0.5826]
599 [ 0.0015 -0.8476 -0.2571 -0.4641]
600 [ 0.2811  0.308  -0.907  -0.0592]]
601
602 Hessenberg Form H (of Q):
603 [[-0.6177  0.0118  0.5166  0.5928]
604 [-0.7864 -0.0093 -0.4058 -0.4656]
605 [ 0.     -0.9999  0.0099  0.0113]
606 [ 0.     -0.      0.7539 -0.657 ]]
607
608 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
609 [[-0.2571 -0.4641]
610 [-0.907  -0.0592]]
611

```



```

612 Eigenvalues of the 2x2 block (analytically calculated):
613    $\lambda_1 = 0.4981407629675436$  (size = 0.4981)
614    $\lambda_2 = -0.8144343078024388$  (size = 0.8144)
615 -----
616
617 --- Orthogonal Matrix Q number 29 ---
618 Matrix Q:
619 [[-0.889  -0.0288  0.3962 -0.2278]
620  [ 0.1961 -0.4549 -0.0899 -0.864 ]
621  [-0.4017 -0.36   -0.8217  0.1838]
622  [ 0.0995 -0.814   0.3997  0.4095]]
623
624 Hessenberg Form H (of Q):
625 [[-0.889   0.4094  0.14   -0.1501]
626  [-0.458   -0.7947 -0.2718  0.2914]
627  [ 0.      -0.4483  0.6097 -0.6537]
628  [ 0.      -0.      -0.7313 -0.6821]]
629
630 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
631 [[-0.8217  0.1838]
632  [ 0.3997  0.4095]]
633
634 Eigenvalues of the 2x2 block (analytically calculated):
635    $\lambda_1 = 0.46657705244665193$  (size = 0.4666)
636    $\lambda_2 = -0.8787442629640516$  (size = 0.8787)
637 -----
638
639 --- Orthogonal Matrix Q number 30 ---
640 Matrix Q:
641 [[-0.2493  0.2801 -0.5896 -0.7154]
642  [-0.5128 -0.2897  0.6545 -0.4742]
643  [-0.2303 -0.8469 -0.4616  0.1292]
644  [-0.7886  0.3471 -0.1044  0.4967]]
645
646 Hessenberg Form H (of Q):
647 [[-0.2493  0.5744 -0.1969 -0.7544]
648  [ 0.9684  0.1479 -0.0507 -0.1942]
649  [ 0.      -0.8051 -0.1498 -0.5739]
650  [ 0.      -0.      0.9676 -0.2526]]
651
652 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
653 [[-0.4616  0.1292]
654  [-0.1044  0.4967]]
655
656 Eigenvalues of the 2x2 block (analytically calculated):
657    $\lambda_1 = 0.4824356418279282$  (size = 0.4824)

```

```

658     λ_2 = -0.4473584670796439 (size = 0.4474)
659     -----

```

So we observe that in the 2×2 blocks analyzed:

1. Orthogonality is not always preserved
2. The eigenvalues are usually real, with alternating sign and size around 1.

4.5. Shift With an Eigenvalue (d)

Now we use an eigenvalue of the 2×2 block as a shift:

```

1  def pretty(arr: np.ndarray, prec: int = 3) -> str:
2
3      """
4      Compact string for a 1-D NumPy array.
5
6      Args:
7          arr (np.ndarray): Input array to be formatted.
8          prec (int): Precision for the string representation.
9      Returns:
10         str: Formatted string representation of the array.
11      Raises:
12         None
13      """
14
15     return np.array_str(arr, precision=prec, suppress_small=True)
16
17 def qr_iteration_with_fixed_shift(
18     H: np.ndarray,
19     mu: complex,
20     *,
21     max_iter: int = 100,
22     tol: float = 1e-10,
23     debug: bool = False,
24 ):
25     """
26     Fixed-shift QR iteration that optionally shows the sub-diagonal before the
27     first step and after the final step.
28
29     Args:
30         H (np.ndarray): initial matrix in Hessenberg form.
31         mu (complex): fixed shift to be used.
32         max_iter (int): maximum number of iterations.
33         tol (float): tolerance for convergence.
34         debug (bool): if True, print detailed information about each iteration.
35
36     Returns:

```

```

36         Hk (np.ndarray): matrix after iterations.
37         converged (bool): whether it converged to almost upper triangular form.
38         iterations (int): number of iterations performed.
39     Raises:
40         None
41     """
42
43     Hk = H.astype(np.complex128, copy=True)
44     n = Hk.shape[0]
45
46     if debug:
47         init_sub = np.diag(Hk, k=-1)
48         print(" before: subdiag=" + pretty(init_sub) +
49               f", ||·||₂={np.linalg.norm(init_sub):.3e}")
50
51     for k in range(max_iter):
52         Q, R = np.linalg.qr(Hk - mu * np.eye(n))
53         Hk = R @ Q + mu * np.eye(n)
54
55         sub = np.diag(Hk, k=-1)
56         if debug:
57             print(
58                 f" iter {k:02d}: subdiag=" + pretty(sub) +
59                 f", ||·||₂={np.linalg.norm(sub):.3e}"
60             )
61
62         if np.all(np.abs(sub) < tol):
63             break #tests convergence
64
65     if debug: #final sub-diagonal
66         final_sub = np.diag(Hk, k=-1)
67         print(" after : subdiag=" + pretty(final_sub) +
68               f", ||·||₂={np.linalg.norm(final_sub):.3e}")
69
70     converged = np.all(np.abs(np.diag(Hk, k=-1)) < tol)
71     return Hk, converged, min(k + 1, max_iter)
72
73
74 def run_qr_iteration_with_shifts_and_debug(
75     *,
76     n: int = 4,
77     n_matrices: int = 30,
78     max_iter: int = 50,
79     debug: bool = False,
80 ):
81     """

```

```

82     Runs the QR iteration with fixed shifts on randomly generated orthogonal
      matrices,
83     printing a summary for each matrix. Detailed logging appears only when
      debug = True.
84
85     Args:
86         n (int): Size of the matrices (n x n).
87         n_matrices (int): Number of orthogonal matrices to generate and
      analyze.
88         max_iter (int): Maximum number of iterations for the QR iteration.
89         debug (bool): If True, print detailed information about each iteration.
90
91     Returns:
92         None
93     Raises:
94         None
95     """
96
97     for idx in range(1, n_matrices + 1):
98         print(f"\n└─ Matrix {idx:02d}/{n_matrices}  (size {n}x{n})")
99
100        Q = generate_orthogonal_matrix_qr(n)
101        _, H, _ = to_hessenberg(Q)
102
103        a, b, c, d = H[-2:, -2:].ravel()
104        ev1, ev2 = analytical_eigenvalues_2x2(a, b, c, d)
105        mu = ev1 if abs(ev1 - H[-1, -1]) < abs(ev2 - H[-1, -1]) else ev2
106        print(f"|   fixed shift μ = {mu:.6g}  (|μ|={abs(mu):.4f})")
107
108        Hk, ok, iters = qr_iteration_with_fixed_shift(
109            H, mu, max_iter=max_iter, tol=1e-10, debug=debug
110        )
111
112        print(f"|   iterations      = {iters}/{max_iter}")
113        print("|   sub-diag magnitudes after last step:")
114        print("|   ", pretty(np.abs(np.diag(Hk, k=-1))))
115        print(f"└─ converged?      = {'yes' if ok else 'no'}")
116
117
118
119     run_qr_iteration_with_shifts_and_debug(n=4, n_matrices=30, max_iter=100,
      debug=True)

```

An expected output is:

```

1   └─ Matrix 01/30  (size 4x4)
2   |   fixed shift μ = -0.936534  (|μ|=0.9365)

```

```

3    before: subdiag=[ 0.959+0.j -0.993+0.j -0.393+0.j], ||·||2=1.436e+00
4    iter 00: subdiag=[0.731+0.j 0.334+0.j 0.235+0.j], ||·||2=8.375e-01
5    iter 01: subdiag=[ 0.723+0.j -0.045+0.j -0.231+0.j], ||·||2=7.601e-01
6    iter 02: subdiag=[0.723+0.j 0.006+0.j 0.231+0.j], ||·||2=7.586e-01
7    iter 03: subdiag=[ 0.723+0.j -0.001+0.j -0.231+0.j], ||·||2=7.586e-01
8    iter 04: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
9    iter 05: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
10   iter 06: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
11   iter 07: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
12   iter 08: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
13   iter 09: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
14   iter 10: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
15   iter 11: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
16   iter 12: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
17   iter 13: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
18   iter 14: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
19   iter 15: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
20   iter 16: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
21   iter 17: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
22   iter 18: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
23   iter 19: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
24   iter 20: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
25   iter 21: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
26   iter 22: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
27   iter 23: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
28   iter 24: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
29   iter 25: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
30   iter 26: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
31   iter 27: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
32   iter 28: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
33   iter 29: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
34   iter 30: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
35   iter 31: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
36   iter 32: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
37   iter 33: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
38   iter 34: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
39   iter 35: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
40   iter 36: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
41   iter 37: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
42   iter 38: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
43   iter 39: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
44   iter 40: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
45   iter 41: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
46   iter 42: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01
47   iter 43: subdiag=[ 0.723+0.j -0.    +0.j -0.231+0.j], ||·||2=7.586e-01
48   iter 44: subdiag=[0.723+0.j 0.    +0.j 0.231+0.j], ||·||2=7.586e-01

```

[illegible]

```

95   iter 91: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j], ||·||2=7.586e-01
96   iter 92: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j], ||·||2=7.586e-01
97   iter 93: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j], ||·||2=7.586e-01
98   iter 94: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j], ||·||2=7.586e-01
99   iter 95: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j], ||·||2=7.586e-01
100  iter 96: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j], ||·||2=7.586e-01
101  iter 97: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j], ||·||2=7.586e-01
102  iter 98: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j], ||·||2=7.586e-01
103  iter 99: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j], ||·||2=7.586e-01
104  after : subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j], ||·||2=7.586e-01
105  | iterations      = 100/100
106  | sub-diag magnitudes after last step:
107  | [0.723 0.   0.231]
108  └─ converged?     = no
109
110  └─ Matrix 02/30 (size 4x4)
111  | fixed shift  $\mu$  = 0.985706 ( $|\mu|=0.9857$ )
112  before: subdiag=[ 0.942+0.j -0.63 +0.j -0.464+0.j], ||·||2=1.224e+00
113  iter 00: subdiag=[ 0.982+0.j -0.877+0.j -0.004+0.j], ||·||2=1.316e+00
114  iter 01: subdiag=[ 0.993+0.j -0.997+0.j -0.   +0.j], ||·||2=1.407e+00
115  iter 02: subdiag=[ 0.85 +0.j -0.977+0.j -0.   +0.j], ||·||2=1.295e+00
116  iter 03: subdiag=[ 0.581+0.j -0.942+0.j -0.   +0.j], ||·||2=1.107e+00
117  iter 04: subdiag=[ 0.346+0.j -0.925+0.j -0.   +0.j], ||·||2=9.879e-01
118  iter 05: subdiag=[ 0.195+0.j -0.92 +0.j -0.   +0.j], ||·||2=9.402e-01
119  iter 06: subdiag=[ 0.108+0.j -0.918+0.j -0.   +0.j], ||·||2=9.244e-01
120  iter 07: subdiag=[ 0.059+0.j -0.918+0.j -0.   +0.j], ||·||2=9.194e-01
121  iter 08: subdiag=[ 0.033+0.j -0.917+0.j -0.   +0.j], ||·||2=9.179e-01
122  iter 09: subdiag=[ 0.018+0.j -0.917+0.j -0.   +0.j], ||·||2=9.175e-01
123  iter 10: subdiag=[ 0.01 +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
124  iter 11: subdiag=[ 0.005+0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
125  iter 12: subdiag=[ 0.003+0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
126  iter 13: subdiag=[ 0.002+0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
127  iter 14: subdiag=[ 0.001+0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
128  iter 15: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
129  iter 16: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
130  iter 17: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
131  iter 18: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
132  iter 19: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
133  iter 20: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
134  iter 21: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
135  iter 22: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
136  iter 23: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
137  iter 24: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
138  iter 25: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
139  iter 26: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
140  iter 27: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01

```

[illegible]


```

187   iter 74: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
188   iter 75: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
189   iter 76: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
190   iter 77: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
191   iter 78: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
192   iter 79: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
193   iter 80: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
194   iter 81: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
195   iter 82: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
196   iter 83: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
197   iter 84: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
198   iter 85: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
199   iter 86: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
200   iter 87: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
201   iter 88: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
202   iter 89: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
203   iter 90: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
204   iter 91: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
205   iter 92: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
206   iter 93: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
207   iter 94: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
208   iter 95: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
209   iter 96: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
210   iter 97: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
211   iter 98: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
212   iter 99: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
213   after : subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j], ||·||2=9.173e-01
214   | iterations      = 100/100
215   | sub-diag magnitudes after last step:
216   | [0.   0.917 0.   ]
217   | └ converged?    = no
218
219   └─ Matrix 03/30 (size 4x4)
220   | fixed shift  $\mu = 0.999872$  ( $|\mu|=0.9999$ )
221   before: subdiag=[ 0.889+0.j -0.786+0.j 0.008+0.j], ||·||2=1.186e+00
222   iter 00: subdiag=[ 0.692+0.j -0.475+0.j 0.   +0.j], ||·||2=8.397e-01
223   iter 01: subdiag=[ 0.182+0.j -0.444+0.j 0.   +0.j], ||·||2=4.803e-01
224   iter 02: subdiag=[ 0.042+0.j -0.443+0.j 0.   +0.j], ||·||2=4.448e-01
225   iter 03: subdiag=[ 0.009+0.j -0.443+0.j 0.   +0.j], ||·||2=4.428e-01
226   iter 04: subdiag=[ 0.002+0.j -0.443+0.j 0.   +0.j], ||·||2=4.427e-01
227   iter 05: subdiag=[ 0.   +0.j -0.443+0.j 0.   +0.j], ||·||2=4.427e-01
228   iter 06: subdiag=[ 0.   +0.j -0.443+0.j 0.   +0.j], ||·||2=4.427e-01
229   iter 07: subdiag=[ 0.   +0.j -0.443+0.j 0.   +0.j], ||·||2=4.427e-01
230   iter 08: subdiag=[ 0.   +0.j -0.443+0.j 0.   +0.j], ||·||2=4.427e-01
231   iter 09: subdiag=[ 0.   +0.j -0.443+0.j 0.   +0.j], ||·||2=4.427e-01
232   iter 10: subdiag=[ 0.   +0.j -0.443+0.j 0.   +0.j], ||·||2=4.427e-01

```

[illegible]

```

279   iter 57: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
280   iter 58: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
281   iter 59: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
282   iter 60: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
283   iter 61: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
284   iter 62: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
285   iter 63: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
286   iter 64: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
287   iter 65: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
288   iter 66: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
289   iter 67: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
290   iter 68: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
291   iter 69: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
292   iter 70: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
293   iter 71: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
294   iter 72: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
295   iter 73: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
296   iter 74: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
297   iter 75: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
298   iter 76: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
299   iter 77: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
300   iter 78: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
301   iter 79: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
302   iter 80: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
303   iter 81: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
304   iter 82: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
305   iter 83: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
306   iter 84: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
307   iter 85: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
308   iter 86: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
309   iter 87: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
310   iter 88: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
311   iter 89: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
312   iter 90: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
313   iter 91: subdiag=[0.   +0.j 0.443+0.j 0.   +0.j], ||·||2=4.427e-01
314   iter 92: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
315   iter 93: subdiag=[0.   +0.j 0.443+0.j 0.   +0.j], ||·||2=4.427e-01
316   iter 94: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
317   iter 95: subdiag=[0.   +0.j 0.443+0.j 0.   +0.j], ||·||2=4.427e-01
318   iter 96: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
319   iter 97: subdiag=[0.   +0.j 0.443+0.j 0.   +0.j], ||·||2=4.427e-01
320   iter 98: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j], ||·||2=4.427e-01
321   iter 99: subdiag=[0.   +0.j 0.443+0.j 0.   +0.j], ||·||2=4.427e-01
322   after : subdiag=[0.   +0.j 0.443+0.j 0.   +0.j], ||·||2=4.427e-01
323 | iterations      = 100/100
324 | sub-diag magnitudes after last step:

```

```

325 | [0.    0.443 0.   ]
326 └─ converged?      = no
327
328 └─ Matrix 04/30  (size 4x4)
329 | fixed shift  $\mu$  = -0.623567 ( $|\mu|=0.6236$ )
330   before: subdiag=[-0.823+0.j  0.998+0.j -0.726+0.j],  $\|\cdot\|_2=1.483e+00$ 
331   iter 00: subdiag=[-1.    +0.j -0.866+0.j  0.426+0.j],  $\|\cdot\|_2=1.390e+00$ 
332   iter 01: subdiag=[-0.973+0.j  0.433+0.j -0.346+0.j],  $\|\cdot\|_2=1.120e+00$ 
333   iter 02: subdiag=[-0.964+0.j -0.161+0.j  0.334+0.j],  $\|\cdot\|_2=1.033e+00$ 
334   iter 03: subdiag=[-0.963+0.j  0.057+0.j -0.333+0.j],  $\|\cdot\|_2=1.020e+00$ 
335   iter 04: subdiag=[-0.962+0.j -0.02  +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
336   iter 05: subdiag=[-0.962+0.j  0.007+0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
337   iter 06: subdiag=[-0.962+0.j -0.002+0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
338   iter 07: subdiag=[-0.962+0.j  0.001+0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
339   iter 08: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
340   iter 09: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
341   iter 10: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
342   iter 11: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
343   iter 12: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
344   iter 13: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
345   iter 14: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
346   iter 15: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
347   iter 16: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
348   iter 17: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
349   iter 18: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
350   iter 19: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
351   iter 20: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
352   iter 21: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
353   iter 22: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
354   iter 23: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
355   iter 24: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
356   iter 25: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
357   iter 26: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
358   iter 27: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
359   iter 28: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
360   iter 29: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
361   iter 30: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
362   iter 31: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
363   iter 32: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
364   iter 33: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
365   iter 34: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
366   iter 35: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
367   iter 36: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
368   iter 37: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
369   iter 38: subdiag=[-0.962+0.j -0.    +0.j  0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 
370   iter 39: subdiag=[-0.962+0.j  0.    +0.j -0.332+0.j],  $\|\cdot\|_2=1.018e+00$ 

```

[illegible]

```

417   iter 86: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j], ||·||2=1.018e+00
418   iter 87: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j], ||·||2=1.018e+00
419   iter 88: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j], ||·||2=1.018e+00
420   iter 89: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j], ||·||2=1.018e+00
421   iter 90: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j], ||·||2=1.018e+00
422   iter 91: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j], ||·||2=1.018e+00
423   iter 92: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j], ||·||2=1.018e+00
424   iter 93: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j], ||·||2=1.018e+00
425   iter 94: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j], ||·||2=1.018e+00
426   iter 95: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j], ||·||2=1.018e+00
427   iter 96: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j], ||·||2=1.018e+00
428   iter 97: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j], ||·||2=1.018e+00
429   iter 98: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j], ||·||2=1.018e+00
430   iter 99: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j], ||·||2=1.018e+00
431   after : subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j], ||·||2=1.018e+00
432   | iterations      = 100/100
433   | sub-diag magnitudes after last step:
434   | [0.962 0.      0.332]
435   | └ converged?    = no
436
437   └ Matrix 05/30 (size 4x4)
438   | fixed shift  $\mu$  = -0.649367-0.477769j (| $\mu$ |=0.8062)
439   before: subdiag=[-0.78 +0.j -0.76 +0.j  0.617+0.j], ||·||2=1.252e+00
440   iter 00: subdiag=[-0.705-0.j  0.441-0.j  0.209+0.j], ||·||2=8.579e-01
441   iter 01: subdiag=[-0.627+0.j -0.316+0.j  0.04 +0.j], ||·||2=7.032e-01
442   iter 02: subdiag=[-0.53 +0.j  0.232-0.j  0.008+0.j], ||·||2=5.789e-01
443   iter 03: subdiag=[-0.432+0.j -0.173+0.j  0.001+0.j], ||·||2=4.649e-01
444   iter 04: subdiag=[-0.342+0.j  0.13 -0.j  0.   +0.j], ||·||2=3.660e-01
445   iter 05: subdiag=[-0.267+0.j -0.098+0.j  0.   +0.j], ||·||2=2.844e-01
446   iter 06: subdiag=[-0.206+0.j  0.075-0.j  0.   -0.j], ||·||2=2.191e-01
447   iter 07: subdiag=[-0.158+0.j -0.057+0.j  0.   +0.j], ||·||2=1.679e-01
448   iter 08: subdiag=[-0.121+0.j  0.044-0.j  0.   -0.j], ||·||2=1.283e-01
449   iter 09: subdiag=[-0.092+0.j -0.033+0.j  0.   -0.j], ||·||2=9.788e-02
450   iter 10: subdiag=[-0.07 +0.j  0.026-0.j  0.   -0.j], ||·||2=7.459e-02
451   iter 11: subdiag=[-0.053+0.j -0.02 +0.j  0.   -0.j], ||·||2=5.681e-02
452   iter 12: subdiag=[-0.041+0.j  0.015-0.j  0.   -0.j], ||·||2=4.325e-02
453   iter 13: subdiag=[-0.031+0.j -0.012+0.j  0.   -0.j], ||·||2=3.292e-02
454   iter 14: subdiag=[-0.023+0.j  0.009-0.j  0.   -0.j], ||·||2=2.506e-02
455   iter 15: subdiag=[-0.018+0.j -0.007+0.j  0.   -0.j], ||·||2=1.907e-02
456   iter 16: subdiag=[-0.014+0.j  0.005-0.j  0.   -0.j], ||·||2=1.451e-02
457   iter 17: subdiag=[-0.01 +0.j -0.004+0.j  0.   -0.j], ||·||2=1.105e-02
458   iter 18: subdiag=[-0.008+0.j  0.003-0.j  0.   -0.j], ||·||2=8.407e-03
459   iter 19: subdiag=[-0.006+0.j -0.002+0.j  0.   -0.j], ||·||2=6.398e-03
460   iter 20: subdiag=[-0.005+0.j  0.002-0.j  0.   -0.j], ||·||2=4.870e-03
461   iter 21: subdiag=[-0.003+0.j -0.001+0.j  0.   -0.j], ||·||2=3.706e-03
462   iter 22: subdiag=[-0.003+0.j  0.001-0.j  0.   -0.j], ||·||2=2.821e-03

```

463	iter 23: subdiag=[-0.002+0.j -0.001+0.j 0. -0.j], $\ \cdot\ _2=2.147\text{e-}03$
464	iter 24: subdiag=[-0.002+0.j 0.001-0.j 0. -0.j], $\ \cdot\ _2=1.634\text{e-}03$
465	iter 25: subdiag=[-0.001+0.j -0. +0.j 0. -0.j], $\ \cdot\ _2=1.244\text{e-}03$
466	iter 26: subdiag=[-0.001+0.j 0. -0.j 0. -0.j], $\ \cdot\ _2=9.466\text{e-}04$
467	iter 27: subdiag=[-0.001+0.j -0. +0.j 0. -0.j], $\ \cdot\ _2=7.205\text{e-}04$
468	iter 28: subdiag=[-0.001+0.j 0. -0.j 0. -0.j], $\ \cdot\ _2=5.484\text{e-}04$
469	iter 29: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=4.174\text{e-}04$
470	iter 30: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=3.177\text{e-}04$
471	iter 31: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=2.419\text{e-}04$
472	iter 32: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=1.841\text{e-}04$
473	iter 33: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=1.401\text{e-}04$
474	iter 34: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=1.067\text{e-}04$
475	iter 35: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=8.120\text{e-}05$
476	iter 36: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=6.182\text{e-}05$
477	iter 37: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=4.706\text{e-}05$
478	iter 38: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=3.582\text{e-}05$
479	iter 39: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=2.727\text{e-}05$
480	iter 40: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=2.076\text{e-}05$
481	iter 41: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=1.581\text{e-}05$
482	iter 42: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=1.203\text{e-}05$
483	iter 43: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=9.162\text{e-}06$
484	iter 44: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=6.975\text{e-}06$
485	iter 45: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=5.311\text{e-}06$
486	iter 46: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=4.044\text{e-}06$
487	iter 47: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=3.079\text{e-}06$
488	iter 48: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=2.344\text{e-}06$
489	iter 49: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=1.785\text{e-}06$
490	iter 50: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=1.359\text{e-}06$
491	iter 51: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=1.035\text{e-}06$
492	iter 52: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=7.880\text{e-}07$
493	iter 53: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=6.000\text{e-}07$
494	iter 54: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=4.569\text{e-}07$
495	iter 55: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=3.479\text{e-}07$
496	iter 56: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=2.650\text{e-}07$
497	iter 57: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=2.018\text{e-}07$
498	iter 58: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=1.537\text{e-}07$
499	iter 59: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=1.170\text{e-}07$
500	iter 60: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=8.913\text{e-}08$
501	iter 61: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=6.788\text{e-}08$
502	iter 62: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=5.170\text{e-}08$
503	iter 63: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=3.937\text{e-}08$
504	iter 64: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=2.999\text{e-}08$
505	iter 65: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=2.284\text{e-}08$
506	iter 66: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=1.740\text{e-}08$
507	iter 67: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=1.325\text{e-}08$
508	iter 68: subdiag=[-0.+0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=1.009\text{e-}08$

```

509   iter 69: subdiag=[-0.+0.j -0.+0.j  0.-0.j], ||·||2=7.688e-09
510   iter 70: subdiag=[-0.+0.j  0.-0.j  0.-0.j], ||·||2=5.857e-09
511   iter 71: subdiag=[-0.+0.j -0.+0.j  0.-0.j], ||·||2=4.461e-09
512   iter 72: subdiag=[-0.+0.j  0.-0.j  0.-0.j], ||·||2=3.398e-09
513   iter 73: subdiag=[-0.+0.j -0.+0.j  0.-0.j], ||·||2=2.589e-09
514   iter 74: subdiag=[-0.+0.j  0.-0.j  0.-0.j], ||·||2=1.972e-09
515   iter 75: subdiag=[-0.+0.j -0.+0.j  0.-0.j], ||·||2=1.503e-09
516   iter 76: subdiag=[-0.+0.j  0.-0.j  0.-0.j], ||·||2=1.145e-09
517   iter 77: subdiag=[-0.+0.j -0.+0.j  0.-0.j], ||·||2=8.721e-10
518   iter 78: subdiag=[-0.+0.j  0.-0.j  0.-0.j], ||·||2=6.644e-10
519   iter 79: subdiag=[-0.+0.j -0.+0.j  0.-0.j], ||·||2=5.062e-10
520   iter 80: subdiag=[-0.+0.j  0.-0.j  0.-0.j], ||·||2=3.857e-10
521   iter 81: subdiag=[-0.+0.j -0.+0.j  0.-0.j], ||·||2=2.939e-10
522   iter 82: subdiag=[-0.+0.j  0.-0.j  0.-0.j], ||·||2=2.239e-10
523   iter 83: subdiag=[-0.+0.j -0.+0.j  0.-0.j], ||·||2=1.706e-10
524   iter 84: subdiag=[-0.+0.j  0.-0.j  0.-0.j], ||·||2=1.300e-10
525   iter 85: subdiag=[-0.+0.j -0.+0.j  0.-0.j], ||·||2=9.907e-11
526   after : subdiag=[-0.+0.j -0.+0.j  0.-0.j], ||·||2=9.907e-11
527   | iterations      = 86/100
528   | sub-diag magnitudes after last step:
529   | [0. 0. 0.]
530   └─ converged?     = yes
531
532   └─ Matrix 06/30 (size 4x4)
533   | fixed shift  $\mu$  = -0.771151 ( $|\mu|=0.7712$ )
534   before: subdiag=[-0.695+0.j  0.921+0.j  0.395+0.j], ||·||2=1.220e+00
535   iter 00: subdiag=[-0.414+0.j  0.919+0.j  0.099+0.j], ||·||2=1.012e+00
536   iter 01: subdiag=[-0.236+0.j  0.912+0.j  0.023+0.j], ||·||2=9.419e-01
537   iter 02: subdiag=[-0.131+0.j  0.909+0.j  0.005+0.j], ||·||2=9.186e-01
538   iter 03: subdiag=[-0.072+0.j  0.908+0.j  0.001+0.j], ||·||2=9.112e-01
539   iter 04: subdiag=[-0.04 +0.j  0.908+0.j  0. +0.j], ||·||2=9.090e-01
540   iter 05: subdiag=[-0.022+0.j  0.908+0.j  0. +0.j], ||·||2=9.083e-01
541   iter 06: subdiag=[-0.012+0.j  0.908+0.j  0. +0.j], ||·||2=9.081e-01
542   iter 07: subdiag=[-0.007+0.j  0.908+0.j  0. +0.j], ||·||2=9.080e-01
543   iter 08: subdiag=[-0.004+0.j  0.908+0.j  0. +0.j], ||·||2=9.080e-01
544   iter 09: subdiag=[-0.002+0.j  0.908+0.j  0. +0.j], ||·||2=9.080e-01
545   iter 10: subdiag=[-0.001+0.j  0.908+0.j  0. +0.j], ||·||2=9.080e-01
546   iter 11: subdiag=[-0.001+0.j  0.908+0.j  0. +0.j], ||·||2=9.080e-01
547   iter 12: subdiag=[-0. +0.j  0.908+0.j  0. +0.j], ||·||2=9.080e-01
548   iter 13: subdiag=[-0. +0.j  0.908+0.j  0. +0.j], ||·||2=9.080e-01
549   iter 14: subdiag=[-0. +0.j  0.908+0.j  0. +0.j], ||·||2=9.080e-01
550   iter 15: subdiag=[-0. +0.j  0.908+0.j  0. +0.j], ||·||2=9.080e-01
551   iter 16: subdiag=[-0. +0.j  0.908+0.j  0. +0.j], ||·||2=9.080e-01
552   iter 17: subdiag=[-0. +0.j  0.908+0.j  0. +0.j], ||·||2=9.080e-01
553   iter 18: subdiag=[-0. +0.j  0.908+0.j  0. +0.j], ||·||2=9.080e-01
554   iter 19: subdiag=[-0. +0.j  0.908+0.j  0. +0.j], ||·||2=9.080e-01

```


[illegible]

```

601   iter 66: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
602   iter 67: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
603   iter 68: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
604   iter 69: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
605   iter 70: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
606   iter 71: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
607   iter 72: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
608   iter 73: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
609   iter 74: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
610   iter 75: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
611   iter 76: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
612   iter 77: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
613   iter 78: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
614   iter 79: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
615   iter 80: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
616   iter 81: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
617   iter 82: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
618   iter 83: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
619   iter 84: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
620   iter 85: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
621   iter 86: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
622   iter 87: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
623   iter 88: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
624   iter 89: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
625   iter 90: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
626   iter 91: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
627   iter 92: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
628   iter 93: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
629   iter 94: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
630   iter 95: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
631   iter 96: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
632   iter 97: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
633   iter 98: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
634   iter 99: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
635   after : subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j], ||·||2=9.080e-01
636   | iterations      = 100/100
637   | sub-diag magnitudes after last step:
638   | [0.   0.908 0.   ]
639   | └ converged?    = no
640
641   └ Matrix 07/30 (size 4x4)
642   | fixed shift μ = -0.884666 (|μ|=0.8847)
643   before: subdiag=[ 1.   +0.j -0.815+0.j  0.838+0.j], ||·||2=1.538e+00
644   iter 00: subdiag=[ 0.626+0.j  0.831+0.j -0.362+0.j], ||·||2=1.102e+00
645   iter 01: subdiag=[ 0.56 +0.j -0.193+0.j  0.317+0.j], ||·||2=6.720e-01
646   iter 02: subdiag=[ 0.558+0.j  0.035+0.j -0.316+0.j], ||·||2=6.419e-01

```

[illegible]

[illegible]

```

739   iter 95: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j], ||·||2=6.409e-01
740   iter 96: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j], ||·||2=6.409e-01
741   iter 97: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j], ||·||2=6.409e-01
742   iter 98: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j], ||·||2=6.409e-01
743   iter 99: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j], ||·||2=6.409e-01
744   after : subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j], ||·||2=6.409e-01
745   | iterations      = 100/100
746   | sub-diag magnitudes after last step:
747   | [0.558 0.   0.316]
748   └ converged?      = no
749
750   ┌ Matrix 08/30 (size 4x4)
751   | fixed shift  $\mu$  = -0.662036-0.664809j (| $\mu$ |=0.9382)
752   before: subdiag=[ 0.992+0.j -0.474+0.j  0.71 +0.j], ||·||2=1.309e+00
753   iter 00: subdiag=[0.782+0.j 0.413-0.j 0.08 -0.j], ||·||2=8.875e-01
754   iter 01: subdiag=[ 0.462-0.j  0.541-0.j -0.005+0.j], ||·||2=7.119e-01
755   iter 02: subdiag=[0.25 +0.j 0.713-0.j 0.   -0.j], ||·||2=7.559e-01
756   iter 03: subdiag=[ 0.136+0.j  0.876-0.j -0.   +0.j], ||·||2=8.861e-01
757   iter 04: subdiag=[0.078-0.j 0.95 -0.j 0.   -0.j], ||·||2=9.531e-01
758   iter 05: subdiag=[ 0.048+0.j  0.89 -0.j -0.   +0.j], ||·||2=8.914e-01
759   iter 06: subdiag=[0.032+0.j 0.732-0.j 0.   -0.j], ||·||2=7.331e-01
760   iter 07: subdiag=[ 0.022+0.j  0.551-0.j -0.   +0.j], ||·||2=5.514e-01
761   iter 08: subdiag=[0.015+0.j 0.393-0.j 0.   -0.j], ||·||2=3.938e-01
762   iter 09: subdiag=[-0.011-0.j -0.274+0.j -0.   +0.j], ||·||2=2.738e-01
763   iter 10: subdiag=[0.008+0.j 0.188-0.j 0.   -0.j], ||·||2=1.880e-01
764   iter 11: subdiag=[-0.006-0.j -0.128+0.j -0.   +0.j], ||·||2=1.282e-01
765   iter 12: subdiag=[0.004+0.j 0.087-0.j 0.   -0.j], ||·||2=8.725e-02
766   iter 13: subdiag=[-0.003-0.j -0.059+0.j -0.   +0.j], ||·||2=5.928e-02
767   iter 14: subdiag=[0.002+0.j 0.04 -0.j 0.   -0.j], ||·||2=4.025e-02
768   iter 15: subdiag=[-0.002-0.j -0.027+0.j -0.   +0.j], ||·||2=2.732e-02
769   iter 16: subdiag=[0.001+0.j 0.019-0.j 0.   -0.j], ||·||2=1.855e-02
770   iter 17: subdiag=[-0.001-0.j -0.013+0.j -0.   +0.j], ||·||2=1.259e-02
771   iter 18: subdiag=[0.001+0.j 0.009-0.j 0.   -0.j], ||·||2=8.545e-03
772   iter 19: subdiag=[-0.   -0.j -0.006+0.j -0.   +0.j], ||·||2=5.800e-03
773   iter 20: subdiag=[0.   +0.j 0.004-0.j 0.   -0.j], ||·||2=3.937e-03
774   iter 21: subdiag=[-0.   -0.j -0.003+0.j -0.   +0.j], ||·||2=2.673e-03
775   iter 22: subdiag=[0.   +0.j 0.002-0.j 0.   -0.j], ||·||2=1.814e-03
776   iter 23: subdiag=[-0.   -0.j -0.001+0.j -0.   +0.j], ||·||2=1.232e-03
777   iter 24: subdiag=[0.   +0.j 0.001-0.j 0.   -0.j], ||·||2=8.364e-04
778   iter 25: subdiag=[-0.   -0.j -0.001+0.j -0.   +0.j], ||·||2=5.679e-04
779   iter 26: subdiag=[0.+0.j 0.-0.j 0.+0.j], ||·||2=3.857e-04
780   iter 27: subdiag=[-0.-0.j -0.+0.j -0.+0.j], ||·||2=2.619e-04
781   iter 28: subdiag=[0.+0.j 0.-0.j 0.+0.j], ||·||2=1.779e-04
782   iter 29: subdiag=[-0.-0.j -0.+0.j -0.-0.j], ||·||2=1.209e-04
783   iter 30: subdiag=[0.+0.j 0.-0.j 0.+0.j], ||·||2=8.213e-05
784   iter 31: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=5.581e-05

```

```

785   iter 32: subdiag=[0.+0.j 0.-0.j 0.+0.j], ||·||2=3.794e-05
786   iter 33: subdiag=[-0.-0.j -0.+0.j -0.-0.j], ||·||2=2.579e-05
787   iter 34: subdiag=[0.+0.j 0.+0.j 0.+0.j], ||·||2=1.754e-05
788   iter 35: subdiag=[-0.-0.j -0.-0.j -0.+0.j], ||·||2=1.193e-05
789   iter 36: subdiag=[0.+0.j 0.+0.j 0.-0.j], ||·||2=8.118e-06
790   iter 37: subdiag=[-0.-0.j -0.-0.j -0.+0.j], ||·||2=5.526e-06
791   iter 38: subdiag=[0.+0.j 0.+0.j 0.-0.j], ||·||2=3.763e-06
792   iter 39: subdiag=[-0.-0.j -0.-0.j -0.+0.j], ||·||2=2.564e-06
793   iter 40: subdiag=[0.+0.j 0.+0.j 0.+0.j], ||·||2=1.748e-06
794   iter 41: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=1.192e-06
795   iter 42: subdiag=[0.+0.j 0.+0.j 0.+0.j], ||·||2=8.135e-07
796   iter 43: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=5.556e-07
797   iter 44: subdiag=[0.+0.j 0.+0.j 0.+0.j], ||·||2=3.797e-07
798   iter 45: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=2.598e-07
799   iter 46: subdiag=[0.+0.j 0.+0.j 0.+0.j], ||·||2=1.779e-07
800   iter 47: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=1.219e-07
801   iter 48: subdiag=[0.+0.j 0.+0.j 0.+0.j], ||·||2=8.368e-08
802   iter 49: subdiag=[-0.-0.j -0.-0.j -0.+0.j], ||·||2=5.750e-08
803   iter 50: subdiag=[0.+0.j 0.+0.j 0.-0.j], ||·||2=3.956e-08
804   iter 51: subdiag=[-0.-0.j -0.-0.j -0.+0.j], ||·||2=2.726e-08
805   iter 52: subdiag=[0.+0.j 0.+0.j 0.-0.j], ||·||2=1.881e-08
806   iter 53: subdiag=[-0.-0.j -0.-0.j -0.+0.j], ||·||2=1.300e-08
807   iter 54: subdiag=[0.+0.j 0.+0.j 0.+0.j], ||·||2=9.004e-09
808   iter 55: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=6.246e-09
809   iter 56: subdiag=[0.+0.j 0.+0.j 0.+0.j], ||·||2=4.341e-09
810   iter 57: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=3.023e-09
811   iter 58: subdiag=[0.+0.j 0.+0.j 0.+0.j], ||·||2=2.110e-09
812   iter 59: subdiag=[-0.-0.j -0.-0.j -0.+0.j], ||·||2=1.475e-09
813   iter 60: subdiag=[0.+0.j 0.+0.j 0.+0.j], ||·||2=1.034e-09
814   iter 61: subdiag=[-0.-0.j -0.+0.j -0.-0.j], ||·||2=7.257e-10
815   iter 62: subdiag=[0.+0.j 0.-0.j 0.+0.j], ||·||2=5.106e-10
816   iter 63: subdiag=[-0.-0.j -0.+0.j -0.-0.j], ||·||2=3.599e-10
817   iter 64: subdiag=[0.+0.j 0.-0.j 0.+0.j], ||·||2=2.542e-10
818   iter 65: subdiag=[-0.-0.j -0.+0.j -0.-0.j], ||·||2=1.799e-10
819   iter 66: subdiag=[0.+0.j 0.-0.j 0.+0.j], ||·||2=1.276e-10
820   iter 67: subdiag=[-0.-0.j -0.+0.j -0.+0.j], ||·||2=9.060e-11
821   after : subdiag=[-0.-0.j -0.+0.j -0.+0.j], ||·||2=9.060e-11
822 | iterations      = 68/100
823 | sub-diag magnitudes after last step:
824 | [0. 0. 0.]
825 └ converged?     = yes
826
827 └ Matrix 09/30 (size 4x4)
828 | fixed shift μ = 0.995354 (|μ|=0.9954)
829   before: subdiag=[ 0.805+0.j 0.682+0.j -0.243+0.j], ||·||2=1.083e+00
830   iter 00: subdiag=[ 0.618+0.j -0.241+0.j 0.043+0.j], ||·||2=6.650e-01

```

[illegible]

[illegible]


```

923   iter 93: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j], ||·||2=6.158e-01
924   iter 94: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j], ||·||2=6.158e-01
925   iter 95: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j], ||·||2=6.158e-01
926   iter 96: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j], ||·||2=6.158e-01
927   iter 97: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j], ||·||2=6.158e-01
928   iter 98: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j], ||·||2=6.158e-01
929   iter 99: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j], ||·||2=6.158e-01
930   after : subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j], ||·||2=6.158e-01
931   | iterations      = 100/100
932   | sub-diag magnitudes after last step:
933   | [0.614 0.      0.043]
934   | └ converged?    = no
935
936   └ Matrix 10/30 (size 4x4)
937   | fixed shift μ = -0.926518 (|μ|=0.9265)
938   before: subdiag=[ 0.914+0.j  0.658+0.j -0.888+0.j], ||·||2=1.434e+00
939   iter 00: subdiag=[ 0.572+0.j -0.901+0.j  0.244+0.j], ||·||2=1.095e+00
940   iter 01: subdiag=[ 0.491+0.j  0.144+0.j -0.205+0.j], ||·||2=5.513e-01
941   iter 02: subdiag=[ 0.49 +0.j -0.016+0.j  0.204+0.j], ||·||2=5.311e-01
942   iter 03: subdiag=[ 0.49 +0.j  0.002+0.j -0.204+0.j], ||·||2=5.308e-01
943   iter 04: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
944   iter 05: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
945   iter 06: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
946   iter 07: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
947   iter 08: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
948   iter 09: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
949   iter 10: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
950   iter 11: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
951   iter 12: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
952   iter 13: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
953   iter 14: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
954   iter 15: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
955   iter 16: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
956   iter 17: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
957   iter 18: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
958   iter 19: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
959   iter 20: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
960   iter 21: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
961   iter 22: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
962   iter 23: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
963   iter 24: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
964   iter 25: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
965   iter 26: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
966   iter 27: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
967   iter 28: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
968   iter 29: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01

```

[illegible]

```

1015   iter 76: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
1016   iter 77: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
1017   iter 78: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
1018   iter 79: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
1019   iter 80: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
1020   iter 81: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
1021   iter 82: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
1022   iter 83: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
1023   iter 84: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
1024   iter 85: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
1025   iter 86: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
1026   iter 87: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
1027   iter 88: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
1028   iter 89: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
1029   iter 90: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
1030   iter 91: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
1031   iter 92: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
1032   iter 93: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
1033   iter 94: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
1034   iter 95: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
1035   iter 96: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
1036   iter 97: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
1037   iter 98: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j], ||·||2=5.308e-01
1038   iter 99: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
1039   after : subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j], ||·||2=5.308e-01
1040 |   iterations      = 100/100
1041 |   sub-diag magnitudes after last step:
1042 |   [0.49  0.   0.204]
1043 |   converged?      = no
1044
1045 |   Matrix 11/30 (size 4x4)
1046 |   fixed shift  $\mu$  = -0.553226-0.269054j ( $|\mu|=0.6152$ )
1047   before: subdiag=[-0.998+0.j  0.926+0.j  0.596+0.j], ||·||2=1.486e+00
1048   iter 00: subdiag=[-0.938+0.j -0.68 +0.j  0.382+0.j], ||·||2=1.219e+00
1049   iter 01: subdiag=[-0.81 +0.j  0.462-0.j  0.206-0.j], ||·||2=9.552e-01
1050   iter 02: subdiag=[-0.661+0.j -0.318+0.j  0.103+0.j], ||·||2=7.404e-01
1051   iter 03: subdiag=[-0.517-0.j  0.221-0.j  0.051+0.j], ||·||2=5.644e-01
1052   iter 04: subdiag=[-0.394-0.j -0.156+0.j  0.025+0.j], ||·||2=4.240e-01
1053   iter 05: subdiag=[-0.295-0.j  0.11 -0.j  0.012+0.j], ||·||2=3.154e-01
1054   iter 06: subdiag=[-0.22 -0.j -0.078+0.j  0.006-0.j], ||·||2=2.332e-01
1055   iter 07: subdiag=[-0.163+0.j  0.055-0.j  0.003+0.j], ||·||2=1.718e-01
1056   iter 08: subdiag=[-0.12 -0.j -0.039+0.j  0.001+0.j], ||·||2=1.263e-01
1057   iter 09: subdiag=[-0.089-0.j  0.028-0.j  0.001-0.j], ||·||2=9.283e-02
1058   iter 10: subdiag=[-0.065-0.j -0.02 +0.j  0.   -0.j], ||·||2=6.818e-02
1059   iter 11: subdiag=[-0.048+0.j  0.014-0.j  0.   -0.j], ||·||2=5.006e-02
1060   iter 12: subdiag=[-0.035+0.j -0.01 +0.j  0.   -0.j], ||·||2=3.676e-02

```

1061	iter 13: subdiag=[-0.026+0.j 0.007-0.j 0. +0.j], $\ \cdot\ _2=2.699\text{e-}02$
1062	iter 14: subdiag=[-0.019+0.j -0.005+0.j 0. +0.j], $\ \cdot\ _2=1.982\text{e-}02$
1063	iter 15: subdiag=[-0.014+0.j 0.004-0.j 0. +0.j], $\ \cdot\ _2=1.456\text{e-}02$
1064	iter 16: subdiag=[-0.01 +0.j -0.003+0.j 0. +0.j], $\ \cdot\ _2=1.069\text{e-}02$
1065	iter 17: subdiag=[-0.008+0.j 0.002-0.j 0. +0.j], $\ \cdot\ _2=7.855\text{e-}03$
1066	iter 18: subdiag=[-0.006+0.j -0.001+0.j 0. -0.j], $\ \cdot\ _2=5.771\text{e-}03$
1067	iter 19: subdiag=[-0.004+0.j 0.001-0.j 0. -0.j], $\ \cdot\ _2=4.240\text{e-}03$
1068	iter 20: subdiag=[-0.003-0.j -0.001-0.j 0. -0.j], $\ \cdot\ _2=3.116\text{e-}03$
1069	iter 21: subdiag=[-0.002-0.j 0. +0.j 0. -0.j], $\ \cdot\ _2=2.290\text{e-}03$
1070	iter 22: subdiag=[-0.002-0.j -0. -0.j 0. +0.j], $\ \cdot\ _2=1.683\text{e-}03$
1071	iter 23: subdiag=[-0.001-0.j 0. -0.j 0. +0.j], $\ \cdot\ _2=1.237\text{e-}03$
1072	iter 24: subdiag=[-0.001-0.j -0. +0.j 0. -0.j], $\ \cdot\ _2=9.094\text{e-}04$
1073	iter 25: subdiag=[-0.001-0.j 0. +0.j 0. -0.j], $\ \cdot\ _2=6.685\text{e-}04$
1074	iter 26: subdiag=[-0.-0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=4.915\text{e-}04$
1075	iter 27: subdiag=[-0.-0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=3.614\text{e-}04$
1076	iter 28: subdiag=[-0.+0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=2.657\text{e-}04$
1077	iter 29: subdiag=[-0.-0.j 0.-0.j 0.+0.j], $\ \cdot\ _2=1.954\text{e-}04$
1078	iter 30: subdiag=[-0.+0.j -0.+0.j 0.+0.j], $\ \cdot\ _2=1.437\text{e-}04$
1079	iter 31: subdiag=[-0.-0.j 0.-0.j 0.+0.j], $\ \cdot\ _2=1.057\text{e-}04$
1080	iter 32: subdiag=[-0.-0.j -0.+0.j 0.+0.j], $\ \cdot\ _2=7.772\text{e-}05$
1081	iter 33: subdiag=[-0.-0.j 0.-0.j 0.+0.j], $\ \cdot\ _2=5.716\text{e-}05$
1082	iter 34: subdiag=[-0.-0.j -0.+0.j 0.+0.j], $\ \cdot\ _2=4.204\text{e-}05$
1083	iter 35: subdiag=[-0.-0.j 0.-0.j 0.+0.j], $\ \cdot\ _2=3.093\text{e-}05$
1084	iter 36: subdiag=[-0.-0.j -0.+0.j 0.+0.j], $\ \cdot\ _2=2.275\text{e-}05$
1085	iter 37: subdiag=[-0.-0.j 0.-0.j 0.+0.j], $\ \cdot\ _2=1.673\text{e-}05$
1086	iter 38: subdiag=[-0.-0.j -0.+0.j 0.+0.j], $\ \cdot\ _2=1.231\text{e-}05$
1087	iter 39: subdiag=[-0.-0.j 0.+0.j 0.+0.j], $\ \cdot\ _2=9.056\text{e-}06$
1088	iter 40: subdiag=[-0.-0.j -0.-0.j 0.+0.j], $\ \cdot\ _2=6.662\text{e-}06$
1089	iter 41: subdiag=[-0.-0.j 0.+0.j 0.+0.j], $\ \cdot\ _2=4.902\text{e-}06$
1090	iter 42: subdiag=[-0.-0.j -0.+0.j 0.+0.j], $\ \cdot\ _2=3.606\text{e-}06$
1091	iter 43: subdiag=[-0.-0.j 0.-0.j 0.+0.j], $\ \cdot\ _2=2.653\text{e-}06$
1092	iter 44: subdiag=[-0.-0.j -0.+0.j 0.+0.j], $\ \cdot\ _2=1.952\text{e-}06$
1093	iter 45: subdiag=[-0.-0.j 0.-0.j 0.+0.j], $\ \cdot\ _2=1.436\text{e-}06$
1094	iter 46: subdiag=[-0.-0.j -0.+0.j 0.+0.j], $\ \cdot\ _2=1.057\text{e-}06$
1095	iter 47: subdiag=[-0.-0.j 0.-0.j 0.+0.j], $\ \cdot\ _2=7.776\text{e-}07$
1096	iter 48: subdiag=[-0.-0.j -0.+0.j 0.+0.j], $\ \cdot\ _2=5.722\text{e-}07$
1097	iter 49: subdiag=[-0.-0.j 0.-0.j 0.+0.j], $\ \cdot\ _2=4.210\text{e-}07$
1098	iter 50: subdiag=[-0.-0.j -0.+0.j 0.+0.j], $\ \cdot\ _2=3.098\text{e-}07$
1099	iter 51: subdiag=[-0.-0.j 0.-0.j 0.+0.j], $\ \cdot\ _2=2.280\text{e-}07$
1100	iter 52: subdiag=[-0.-0.j -0.+0.j 0.+0.j], $\ \cdot\ _2=1.677\text{e-}07$
1101	iter 53: subdiag=[-0.-0.j 0.-0.j 0.+0.j], $\ \cdot\ _2=1.234\text{e-}07$
1102	iter 54: subdiag=[-0.-0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=9.083\text{e-}08$
1103	iter 55: subdiag=[-0.-0.j 0.-0.j 0.+0.j], $\ \cdot\ _2=6.684\text{e-}08$
1104	iter 56: subdiag=[-0.-0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=4.919\text{e-}08$
1105	iter 57: subdiag=[-0.-0.j 0.-0.j 0.-0.j], $\ \cdot\ _2=3.620\text{e-}08$
1106	iter 58: subdiag=[-0.-0.j -0.+0.j 0.-0.j], $\ \cdot\ _2=2.664\text{e-}08$

```

1107   iter 59: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ||·||2=1.960e-08
1108   iter 60: subdiag=[-0.-0.j -0.-0.j  0.-0.j],  ||·||2=1.443e-08
1109   iter 61: subdiag=[-0.-0.j  0.+0.j  0.-0.j],  ||·||2=1.062e-08
1110   iter 62: subdiag=[-0.-0.j -0.-0.j  0.-0.j],  ||·||2=7.812e-09
1111   iter 63: subdiag=[-0.-0.j  0.+0.j  0.-0.j],  ||·||2=5.749e-09
1112   iter 64: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ||·||2=4.231e-09
1113   iter 65: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ||·||2=3.114e-09
1114   iter 66: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ||·||2=2.292e-09
1115   iter 67: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ||·||2=1.686e-09
1116   iter 68: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ||·||2=1.241e-09
1117   iter 69: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ||·||2=9.134e-10
1118   iter 70: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ||·||2=6.722e-10
1119   iter 71: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ||·||2=4.947e-10
1120   iter 72: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ||·||2=3.641e-10
1121   iter 73: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ||·||2=2.680e-10
1122   iter 74: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ||·||2=1.972e-10
1123   iter 75: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ||·||2=1.451e-10
1124   iter 76: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ||·||2=1.068e-10
1125   iter 77: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ||·||2=7.861e-11
1126   after : subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ||·||2=7.861e-11
1127 | iterations      = 78/100
1128 | sub-diag magnitudes after last step:
1129 | [0. 0. 0.]
1130 └ converged?      = yes
1131
1132 └ Matrix 12/30  (size 4x4)
1133 | fixed shift μ = 0.918038 (|μ|=0.9180)
1134   before: subdiag=[-0.634+0.j  0.891+0.j  0.631+0.j],  ||·||2=1.263e+00
1135   iter 00: subdiag=[-0.868+0.j -0.701+0.j -0.147+0.j],  ||·||2=1.126e+00
1136   iter 01: subdiag=[-0.823+0.j  0.075+0.j  0.133+0.j],  ||·||2=8.371e-01
1137   iter 02: subdiag=[-0.823+0.j -0.007+0.j -0.133+0.j],  ||·||2=8.333e-01
1138   iter 03: subdiag=[-0.823+0.j  0.001+0.j  0.133+0.j],  ||·||2=8.333e-01
1139   iter 04: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ||·||2=8.333e-01
1140   iter 05: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ||·||2=8.333e-01
1141   iter 06: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ||·||2=8.333e-01
1142   iter 07: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ||·||2=8.333e-01
1143   iter 08: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ||·||2=8.333e-01
1144   iter 09: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ||·||2=8.333e-01
1145   iter 10: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ||·||2=8.333e-01
1146   iter 11: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ||·||2=8.333e-01
1147   iter 12: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ||·||2=8.333e-01
1148   iter 13: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ||·||2=8.333e-01
1149   iter 14: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ||·||2=8.333e-01
1150   iter 15: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ||·||2=8.333e-01
1151   iter 16: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ||·||2=8.333e-01
1152   iter 17: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ||·||2=8.333e-01

```

[illegible]

```

1199   iter 64: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1200   iter 65: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1201   iter 66: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1202   iter 67: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1203   iter 68: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1204   iter 69: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1205   iter 70: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1206   iter 71: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1207   iter 72: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1208   iter 73: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1209   iter 74: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1210   iter 75: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1211   iter 76: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1212   iter 77: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1213   iter 78: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1214   iter 79: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1215   iter 80: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1216   iter 81: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1217   iter 82: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1218   iter 83: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1219   iter 84: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1220   iter 85: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1221   iter 86: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1222   iter 87: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1223   iter 88: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1224   iter 89: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1225   iter 90: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1226   iter 91: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1227   iter 92: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1228   iter 93: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1229   iter 94: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1230   iter 95: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1231   iter 96: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1232   iter 97: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1233   iter 98: subdiag=[-0.823+0.j  -0.   +0.j  -0.133+0.j],  ||·||2=8.333e-01
1234   iter 99: subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1235   after : subdiag=[-0.823+0.j   0.   +0.j   0.133+0.j],  ||·||2=8.333e-01
1236 |   iterations      = 100/100
1237 |   sub-diag magnitudes after last step:
1238 |   [0.823 0.      0.133]
1239 └─ converged?      = no
1240
1241 └─ Matrix 13/30   (size 4x4)
1242 |   fixed shift μ = -0.0259786-0.624268j (|μ|=0.6248)
1243   before: subdiag=[ 0.996+0.j -0.921+0.j  0.999+0.j],  ||·||2=1.685e+00
1244   iter 00: subdiag=[ 0.78 +0.j -0.999+0.j  0.781+0.j],  ||·||2=1.488e+00

```

```

1245 iter 01: subdiag=[-0.6 +0.j -0.718+0.j -0.6 -0.j], ||·||2=1.111e+00
1246 iter 02: subdiag=[0.56 +0.j 0.319-0.j 0.559+0.j], ||·||2=8.526e-01
1247 iter 03: subdiag=[-0.553+0.j -0.124-0.j -0.551+0.j], ||·||2=7.910e-01
1248 iter 04: subdiag=[0.552+0.j 0.047-0.j 0.549+0.j], ||·||2=7.802e-01
1249 iter 05: subdiag=[-0.551-0.j 0.018-0.j -0.548+0.j], ||·||2=7.772e-01
1250 iter 06: subdiag=[0.549+0.j 0.007-0.j 0.547+0.j], ||·||2=7.750e-01
1251 iter 07: subdiag=[-0.548-0.j 0.003+0.j -0.545+0.j], ||·||2=7.726e-01
1252 iter 08: subdiag=[0.546+0.j 0.001+0.j 0.543-0.j], ||·||2=7.698e-01
1253 iter 09: subdiag=[-0.543-0.j 0. +0.j -0.541+0.j], ||·||2=7.666e-01
1254 iter 10: subdiag=[0.54 +0.j 0. +0.j 0.539-0.j], ||·||2=7.631e-01
1255 iter 11: subdiag=[-0.537-0.j 0. +0.j -0.536+0.j], ||·||2=7.591e-01
1256 iter 12: subdiag=[0.534-0.j 0. +0.j 0.534+0.j], ||·||2=7.548e-01
1257 iter 13: subdiag=[-0.53 +0.j 0. +0.j -0.531-0.j], ||·||2=7.502e-01
1258 iter 14: subdiag=[0.526-0.j 0. +0.j 0.528+0.j], ||·||2=7.452e-01
1259 iter 15: subdiag=[-0.521+0.j 0. +0.j -0.525+0.j], ||·||2=7.399e-01
1260 iter 16: subdiag=[0.517-0.j 0. +0.j 0.522+0.j], ||·||2=7.343e-01
1261 iter 17: subdiag=[-0.512-0.j 0. +0.j -0.518-0.j], ||·||2=7.283e-01
1262 iter 18: subdiag=[0.506+0.j 0. +0.j 0.515+0.j], ||·||2=7.221e-01
1263 iter 19: subdiag=[-0.501-0.j 0. +0.j -0.511-0.j], ||·||2=7.157e-01
1264 iter 20: subdiag=[0.495+0.j 0. +0.j 0.507+0.j], ||·||2=7.089e-01
1265 iter 21: subdiag=[-0.489-0.j 0. +0.j -0.503+0.j], ||·||2=7.020e-01
1266 iter 22: subdiag=[0.483+0.j 0. +0.j 0.499-0.j], ||·||2=6.948e-01
1267 iter 23: subdiag=[-0.477-0.j 0. +0.j -0.495+0.j], ||·||2=6.874e-01
1268 iter 24: subdiag=[0.47 +0.j 0. +0.j 0.491-0.j], ||·||2=6.798e-01
1269 iter 25: subdiag=[-0.464-0.j 0. +0.j -0.486+0.j], ||·||2=6.720e-01
1270 iter 26: subdiag=[0.457+0.j 0. +0.j 0.482-0.j], ||·||2=6.641e-01
1271 iter 27: subdiag=[-0.45 -0.j 0. +0.j -0.477+0.j], ||·||2=6.560e-01
1272 iter 28: subdiag=[0.443+0.j 0. +0.j 0.472+0.j], ||·||2=6.478e-01
1273 iter 29: subdiag=[-0.436-0.j 0. +0.j -0.468+0.j], ||·||2=6.395e-01
1274 iter 30: subdiag=[0.429+0.j 0. +0.j 0.463+0.j], ||·||2=6.311e-01
1275 iter 31: subdiag=[-0.422-0.j 0. +0.j -0.458+0.j], ||·||2=6.226e-01
1276 iter 32: subdiag=[0.415+0.j 0. +0.j 0.453-0.j], ||·||2=6.140e-01
1277 iter 33: subdiag=[-0.408-0.j 0. +0.j -0.448+0.j], ||·||2=6.054e-01
1278 iter 34: subdiag=[0.4 +0.j 0. +0.j 0.443+0.j], ||·||2=5.967e-01
1279 iter 35: subdiag=[-0.393-0.j 0. +0.j -0.437+0.j], ||·||2=5.880e-01
1280 iter 36: subdiag=[0.386+0.j 0. +0.j 0.432-0.j], ||·||2=5.792e-01
1281 iter 37: subdiag=[-0.378-0.j 0. +0.j -0.427+0.j], ||·||2=5.704e-01
1282 iter 38: subdiag=[0.371+0.j 0. +0.j 0.422+0.j], ||·||2=5.616e-01
1283 iter 39: subdiag=[-0.364-0.j 0. +0.j -0.416+0.j], ||·||2=5.529e-01
1284 iter 40: subdiag=[0.357+0.j 0. +0.j 0.411+0.j], ||·||2=5.441e-01
1285 iter 41: subdiag=[-0.349-0.j 0. +0.j -0.406+0.j], ||·||2=5.354e-01
1286 iter 42: subdiag=[0.342+0.j 0. +0.j 0.4 -0.j], ||·||2=5.266e-01
1287 iter 43: subdiag=[-0.335-0.j 0. +0.j -0.395+0.j], ||·||2=5.180e-01
1288 iter 44: subdiag=[0.328+0.j 0. +0.j 0.389-0.j], ||·||2=5.093e-01
1289 iter 45: subdiag=[-0.321-0.j 0. +0.j -0.384+0.j], ||·||2=5.008e-01
1290 iter 46: subdiag=[0.314+0.j 0. +0.j 0.379-0.j], ||·||2=4.922e-01

```


1291	iter 47: subdiag=[-0.308-0.j 0. +0.j -0.373+0.j], $\ \cdot\ _2=4.838e-01$
1292	iter 48: subdiag=[0.301+0.j 0. +0.j 0.368-0.j], $\ \cdot\ _2=4.754e-01$
1293	iter 49: subdiag=[-0.294-0.j 0. +0.j -0.363+0.j], $\ \cdot\ _2=4.671e-01$
1294	iter 50: subdiag=[0.288+0.j 0. +0.j 0.357-0.j], $\ \cdot\ _2=4.588e-01$
1295	iter 51: subdiag=[-0.281-0.j 0. +0.j -0.352+0.j], $\ \cdot\ _2=4.507e-01$
1296	iter 52: subdiag=[0.275+0.j 0. +0.j 0.347-0.j], $\ \cdot\ _2=4.426e-01$
1297	iter 53: subdiag=[-0.269-0.j 0. +0.j -0.342+0.j], $\ \cdot\ _2=4.346e-01$
1298	iter 54: subdiag=[0.263+0.j 0. +0.j 0.336+0.j], $\ \cdot\ _2=4.267e-01$
1299	iter 55: subdiag=[-0.257-0.j 0. +0.j -0.331+0.j], $\ \cdot\ _2=4.189e-01$
1300	iter 56: subdiag=[0.251+0.j 0. +0.j 0.326+0.j], $\ \cdot\ _2=4.112e-01$
1301	iter 57: subdiag=[-0.245-0.j 0. +0.j -0.321-0.j], $\ \cdot\ _2=4.036e-01$
1302	iter 58: subdiag=[0.239+0.j 0. +0.j 0.316+0.j], $\ \cdot\ _2=3.961e-01$
1303	iter 59: subdiag=[-0.233-0.j 0. +0.j -0.311-0.j], $\ \cdot\ _2=3.887e-01$
1304	iter 60: subdiag=[0.228+0.j 0. +0.j 0.306+0.j], $\ \cdot\ _2=3.814e-01$
1305	iter 61: subdiag=[-0.222-0.j 0. +0.j -0.301-0.j], $\ \cdot\ _2=3.742e-01$
1306	iter 62: subdiag=[0.217+0.j 0. +0.j 0.296+0.j], $\ \cdot\ _2=3.671e-01$
1307	iter 63: subdiag=[-0.212-0.j 0. +0.j -0.291-0.j], $\ \cdot\ _2=3.601e-01$
1308	iter 64: subdiag=[0.207+0.j 0. +0.j 0.286+0.j], $\ \cdot\ _2=3.532e-01$
1309	iter 65: subdiag=[-0.202-0.j 0. +0.j -0.282-0.j], $\ \cdot\ _2=3.464e-01$
1310	iter 66: subdiag=[0.197+0.j 0. +0.j 0.277+0.j], $\ \cdot\ _2=3.398e-01$
1311	iter 67: subdiag=[-0.192-0.j 0. +0.j -0.272-0.j], $\ \cdot\ _2=3.332e-01$
1312	iter 68: subdiag=[0.187+0.j 0. +0.j 0.268+0.j], $\ \cdot\ _2=3.267e-01$
1313	iter 69: subdiag=[-0.183-0.j 0. +0.j -0.263+0.j], $\ \cdot\ _2=3.204e-01$
1314	iter 70: subdiag=[0.178+0.j 0. +0.j 0.259+0.j], $\ \cdot\ _2=3.141e-01$
1315	iter 71: subdiag=[-0.174-0.j 0. +0.j -0.254-0.j], $\ \cdot\ _2=3.080e-01$
1316	iter 72: subdiag=[0.169+0.j 0. +0.j 0.25 +0.j], $\ \cdot\ _2=3.020e-01$
1317	iter 73: subdiag=[-0.165-0.j 0. +0.j -0.246-0.j], $\ \cdot\ _2=2.960e-01$
1318	iter 74: subdiag=[0.161+0.j 0. +0.j 0.241+0.j], $\ \cdot\ _2=2.902e-01$
1319	iter 75: subdiag=[-0.157-0.j 0. +0.j -0.237-0.j], $\ \cdot\ _2=2.845e-01$
1320	iter 76: subdiag=[0.153+0.j 0. +0.j 0.233+0.j], $\ \cdot\ _2=2.789e-01$
1321	iter 77: subdiag=[-0.149-0.j 0. +0.j -0.229+0.j], $\ \cdot\ _2=2.733e-01$
1322	iter 78: subdiag=[0.146+0.j 0. +0.j 0.225-0.j], $\ \cdot\ _2=2.679e-01$
1323	iter 79: subdiag=[-0.142-0.j 0. +0.j -0.221+0.j], $\ \cdot\ _2=2.626e-01$
1324	iter 80: subdiag=[0.138+0.j 0. +0.j 0.217+0.j], $\ \cdot\ _2=2.574e-01$
1325	iter 81: subdiag=[-0.135-0.j 0. +0.j -0.213+0.j], $\ \cdot\ _2=2.522e-01$
1326	iter 82: subdiag=[0.131+0.j 0. +0.j 0.209+0.j], $\ \cdot\ _2=2.472e-01$
1327	iter 83: subdiag=[-0.128-0.j 0. +0.j -0.206-0.j], $\ \cdot\ _2=2.423e-01$
1328	iter 84: subdiag=[0.125+0.j 0. +0.j 0.202+0.j], $\ \cdot\ _2=2.374e-01$
1329	iter 85: subdiag=[-0.122-0.j 0. +0.j -0.198-0.j], $\ \cdot\ _2=2.327e-01$
1330	iter 86: subdiag=[0.119+0.j 0. +0.j 0.195+0.j], $\ \cdot\ _2=2.280e-01$
1331	iter 87: subdiag=[-0.116-0.j 0. +0.j -0.191-0.j], $\ \cdot\ _2=2.234e-01$
1332	iter 88: subdiag=[0.113+0.j 0. +0.j 0.188+0.j], $\ \cdot\ _2=2.189e-01$
1333	iter 89: subdiag=[-0.11 -0.j 0. +0.j -0.184+0.j], $\ \cdot\ _2=2.145e-01$
1334	iter 90: subdiag=[0.107+0.j 0. +0.j 0.181-0.j], $\ \cdot\ _2=2.102e-01$
1335	iter 91: subdiag=[-0.104-0.j 0. +0.j -0.178+0.j], $\ \cdot\ _2=2.060e-01$
1336	iter 92: subdiag=[0.101+0.j 0. +0.j 0.174-0.j], $\ \cdot\ _2=2.018e-01$

```

1337   iter 93: subdiag=[-0.099-0.j  0.   +0.j -0.171+0.j], ||·||2=1.978e-01
1338   iter 94: subdiag=[0.096+0.j 0.   +0.j 0.168+0.j], ||·||2=1.938e-01
1339   iter 95: subdiag=[-0.094-0.j 0.   +0.j -0.165-0.j], ||·||2=1.899e-01
1340   iter 96: subdiag=[0.091+0.j 0.   +0.j 0.162+0.j], ||·||2=1.860e-01
1341   iter 97: subdiag=[-0.089-0.j 0.   +0.j -0.159-0.j], ||·||2=1.823e-01
1342   iter 98: subdiag=[0.087+0.j 0.   +0.j 0.156+0.j], ||·||2=1.786e-01
1343   iter 99: subdiag=[-0.085-0.j 0.   +0.j -0.153+0.j], ||·||2=1.750e-01
1344   after : subdiag=[-0.085-0.j 0.   +0.j -0.153+0.j], ||·||2=1.750e-01
1345   | iterations      = 100/100
1346   | sub-diag magnitudes after last step:
1347   | [0.085 0.   0.153]
1348   └ converged?      = no
1349
1350   └ Matrix 14/30 (size 4x4)
1351   | fixed shift  $\mu = 0.842538$  ( $|\mu|=0.8425$ )
1352   before: subdiag=[-0.944+0.j 0.964+0.j 0.696+0.j], ||·||2=1.518e+00
1353   iter 00: subdiag=[-0.662+0.j -0.644+0.j -0.413+0.j], ||·||2=1.012e+00
1354   iter 01: subdiag=[-0.628+0.j 0.166+0.j 0.387+0.j], ||·||2=7.558e-01
1355   iter 02: subdiag=[-0.626+0.j -0.038+0.j -0.386+0.j], ||·||2=7.360e-01
1356   iter 03: subdiag=[-0.626+0.j 0.009+0.j 0.386+0.j], ||·||2=7.349e-01
1357   iter 04: subdiag=[-0.626+0.j -0.002+0.j -0.386+0.j], ||·||2=7.349e-01
1358   iter 05: subdiag=[-0.626+0.j 0.   +0.j 0.386+0.j], ||·||2=7.349e-01
1359   iter 06: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j], ||·||2=7.349e-01
1360   iter 07: subdiag=[-0.626+0.j 0.   +0.j 0.386+0.j], ||·||2=7.349e-01
1361   iter 08: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j], ||·||2=7.349e-01
1362   iter 09: subdiag=[-0.626+0.j 0.   +0.j 0.386+0.j], ||·||2=7.349e-01
1363   iter 10: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j], ||·||2=7.349e-01
1364   iter 11: subdiag=[-0.626+0.j 0.   +0.j 0.386+0.j], ||·||2=7.349e-01
1365   iter 12: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j], ||·||2=7.349e-01
1366   iter 13: subdiag=[-0.626+0.j 0.   +0.j 0.386+0.j], ||·||2=7.349e-01
1367   iter 14: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j], ||·||2=7.349e-01
1368   iter 15: subdiag=[-0.626+0.j 0.   +0.j 0.386+0.j], ||·||2=7.349e-01
1369   iter 16: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j], ||·||2=7.349e-01
1370   iter 17: subdiag=[-0.626+0.j 0.   +0.j 0.386+0.j], ||·||2=7.349e-01
1371   iter 18: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j], ||·||2=7.349e-01
1372   iter 19: subdiag=[-0.626+0.j 0.   +0.j 0.386+0.j], ||·||2=7.349e-01
1373   iter 20: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j], ||·||2=7.349e-01
1374   iter 21: subdiag=[-0.626+0.j 0.   +0.j 0.386+0.j], ||·||2=7.349e-01
1375   iter 22: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j], ||·||2=7.349e-01
1376   iter 23: subdiag=[-0.626+0.j 0.   +0.j 0.386+0.j], ||·||2=7.349e-01
1377   iter 24: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j], ||·||2=7.349e-01
1378   iter 25: subdiag=[-0.626+0.j 0.   +0.j 0.386+0.j], ||·||2=7.349e-01
1379   iter 26: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j], ||·||2=7.349e-01
1380   iter 27: subdiag=[-0.626+0.j 0.   +0.j 0.386+0.j], ||·||2=7.349e-01
1381   iter 28: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j], ||·||2=7.349e-01
1382   iter 29: subdiag=[-0.626+0.j 0.   +0.j 0.386+0.j], ||·||2=7.349e-01

```

[illegible]

```

1429   iter 76: subdiag=[-0.626+0.j  -0.   +0.j  -0.386+0.j], ||·||2=7.349e-01
1430   iter 77: subdiag=[-0.626+0.j   0.   +0.j   0.386+0.j], ||·||2=7.349e-01
1431   iter 78: subdiag=[-0.626+0.j  -0.   +0.j  -0.386+0.j], ||·||2=7.349e-01
1432   iter 79: subdiag=[-0.626+0.j   0.   +0.j   0.386+0.j], ||·||2=7.349e-01
1433   iter 80: subdiag=[-0.626+0.j  -0.   +0.j  -0.386+0.j], ||·||2=7.349e-01
1434   iter 81: subdiag=[-0.626+0.j   0.   +0.j   0.386+0.j], ||·||2=7.349e-01
1435   iter 82: subdiag=[-0.626+0.j  -0.   +0.j  -0.386+0.j], ||·||2=7.349e-01
1436   iter 83: subdiag=[-0.626+0.j   0.   +0.j   0.386+0.j], ||·||2=7.349e-01
1437   iter 84: subdiag=[-0.626+0.j  -0.   +0.j  -0.386+0.j], ||·||2=7.349e-01
1438   iter 85: subdiag=[-0.626+0.j   0.   +0.j   0.386+0.j], ||·||2=7.349e-01
1439   iter 86: subdiag=[-0.626+0.j  -0.   +0.j  -0.386+0.j], ||·||2=7.349e-01
1440   iter 87: subdiag=[-0.626+0.j   0.   +0.j   0.386+0.j], ||·||2=7.349e-01
1441   iter 88: subdiag=[-0.626+0.j  -0.   +0.j  -0.386+0.j], ||·||2=7.349e-01
1442   iter 89: subdiag=[-0.626+0.j   0.   +0.j   0.386+0.j], ||·||2=7.349e-01
1443   iter 90: subdiag=[-0.626+0.j  -0.   +0.j  -0.386+0.j], ||·||2=7.349e-01
1444   iter 91: subdiag=[-0.626+0.j   0.   +0.j   0.386+0.j], ||·||2=7.349e-01
1445   iter 92: subdiag=[-0.626+0.j  -0.   +0.j  -0.386+0.j], ||·||2=7.349e-01
1446   iter 93: subdiag=[-0.626+0.j   0.   +0.j   0.386+0.j], ||·||2=7.349e-01
1447   iter 94: subdiag=[-0.626+0.j  -0.   +0.j  -0.386+0.j], ||·||2=7.349e-01
1448   iter 95: subdiag=[-0.626+0.j   0.   +0.j   0.386+0.j], ||·||2=7.349e-01
1449   iter 96: subdiag=[-0.626+0.j  -0.   +0.j  -0.386+0.j], ||·||2=7.349e-01
1450   iter 97: subdiag=[-0.626+0.j   0.   +0.j   0.386+0.j], ||·||2=7.349e-01
1451   iter 98: subdiag=[-0.626+0.j  -0.   +0.j  -0.386+0.j], ||·||2=7.349e-01
1452   iter 99: subdiag=[-0.626+0.j   0.   +0.j   0.386+0.j], ||·||2=7.349e-01
1453   after : subdiag=[-0.626+0.j   0.   +0.j   0.386+0.j], ||·||2=7.349e-01
1454 | iterations      = 100/100
1455 | sub-diag magnitudes after last step:
1456 | [0.626 0.      0.386]
1457 └ converged?      = no
1458
1459 └ Matrix 15/30 (size 4x4)
1460 | fixed shift μ = 0.941615 (|μ|=0.9416)
1461   before: subdiag=[-0.745+0.j  0.93 +0.j -0.49 +0.j], ||·||2=1.288e+00
1462   iter 00: subdiag=[-0.99 +0.j  0.927+0.j -0.031+0.j], ||·||2=1.356e+00
1463   iter 01: subdiag=[-0.767+0.j  0.739+0.j -0.002+0.j], ||·||2=1.065e+00
1464   iter 02: subdiag=[-0.338+0.j  0.69 +0.j -0.   +0.j], ||·||2=7.680e-01
1465   iter 03: subdiag=[-0.127+0.j  0.682+0.j -0.   +0.j], ||·||2=6.941e-01
1466   iter 04: subdiag=[-0.047+0.j  0.681+0.j -0.   +0.j], ||·||2=6.829e-01
1467   iter 05: subdiag=[-0.017+0.j  0.681+0.j -0.   +0.j], ||·||2=6.813e-01
1468   iter 06: subdiag=[-0.006+0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1469   iter 07: subdiag=[-0.002+0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1470   iter 08: subdiag=[-0.001+0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1471   iter 09: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1472   iter 10: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1473   iter 11: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1474   iter 12: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01

```

[illegible]

```

1521   iter 59: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1522   iter 60: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1523   iter 61: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1524   iter 62: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1525   iter 63: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1526   iter 64: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1527   iter 65: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1528   iter 66: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1529   iter 67: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1530   iter 68: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1531   iter 69: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1532   iter 70: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1533   iter 71: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1534   iter 72: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1535   iter 73: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1536   iter 74: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1537   iter 75: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1538   iter 76: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1539   iter 77: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1540   iter 78: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1541   iter 79: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1542   iter 80: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1543   iter 81: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1544   iter 82: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1545   iter 83: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1546   iter 84: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1547   iter 85: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1548   iter 86: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1549   iter 87: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1550   iter 88: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1551   iter 89: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1552   iter 90: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1553   iter 91: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1554   iter 92: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1555   iter 93: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1556   iter 94: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1557   iter 95: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1558   iter 96: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1559   iter 97: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1560   iter 98: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1561   iter 99: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1562   after : subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j], ||·||2=6.811e-01
1563 | iterations      = 100/100
1564 | sub-diag magnitudes after last step:
1565 | [0.   0.681 0.   ]
1566 | └ converged?     = no

```

```

1567
1568 └─ Matrix 16/30 (size 4x4)
1569 │ fixed shift  $\mu = 0.533835$  ( $|\mu|=0.5338$ )
1570 before: subdiag=[ 0.92 +0.j  0.998+0.j -0.89 +0.j],  $\|\cdot\|_2=1.624e+00$ 
1571 iter 00: subdiag=[ 0.987+0.j -0.962+0.j  0.616+0.j],  $\|\cdot\|_2=1.510e+00$ 
1572 iter 01: subdiag=[ 0.908+0.j  0.654+0.j -0.485+0.j],  $\|\cdot\|_2=1.219e+00$ 
1573 iter 02: subdiag=[ 0.877+0.j -0.318+0.j  0.453+0.j],  $\|\cdot\|_2=1.037e+00$ 
1574 iter 03: subdiag=[ 0.87 +0.j  0.139+0.j -0.447+0.j],  $\|\cdot\|_2=9.880e-01$ 
1575 iter 04: subdiag=[ 0.869+0.j -0.06 +0.j  0.446+0.j],  $\|\cdot\|_2=9.783e-01$ 
1576 iter 05: subdiag=[ 0.869+0.j  0.025+0.j -0.445+0.j],  $\|\cdot\|_2=9.766e-01$ 
1577 iter 06: subdiag=[ 0.869+0.j -0.011+0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1578 iter 07: subdiag=[ 0.869+0.j  0.005+0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1579 iter 08: subdiag=[ 0.869+0.j -0.002+0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1580 iter 09: subdiag=[ 0.869+0.j  0.001+0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1581 iter 10: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1582 iter 11: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1583 iter 12: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1584 iter 13: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1585 iter 14: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1586 iter 15: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1587 iter 16: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1588 iter 17: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1589 iter 18: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1590 iter 19: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1591 iter 20: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1592 iter 21: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1593 iter 22: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1594 iter 23: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1595 iter 24: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1596 iter 25: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1597 iter 26: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1598 iter 27: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1599 iter 28: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1600 iter 29: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1601 iter 30: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1602 iter 31: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1603 iter 32: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1604 iter 33: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1605 iter 34: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1606 iter 35: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1607 iter 36: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1608 iter 37: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1609 iter 38: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1610 iter 39: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1611 iter 40: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 
1612 iter 41: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  $\|\cdot\|_2=9.762e-01$ 

```

[illegible]


```

1659   iter 88: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j], ||·||2=9.762e-01
1660   iter 89: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j], ||·||2=9.762e-01
1661   iter 90: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j], ||·||2=9.762e-01
1662   iter 91: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j], ||·||2=9.762e-01
1663   iter 92: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j], ||·||2=9.762e-01
1664   iter 93: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j], ||·||2=9.762e-01
1665   iter 94: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j], ||·||2=9.762e-01
1666   iter 95: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j], ||·||2=9.762e-01
1667   iter 96: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j], ||·||2=9.762e-01
1668   iter 97: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j], ||·||2=9.762e-01
1669   iter 98: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j], ||·||2=9.762e-01
1670   iter 99: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j], ||·||2=9.762e-01
1671   after : subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j], ||·||2=9.762e-01
1672 | iterations      = 100/100
1673 | sub-diag magnitudes after last step:
1674 | [0.869 0.      0.445]
1675 └ converged?      = no
1676
1677 ┌ Matrix 17/30 (size 4x4)
1678 | fixed shift  $\mu$  = 0.0598199-0.0955454j ( $|\mu|=0.1127$ )
1679   before: subdiag=[0.866+0.j 1.   +0.j 0.993+0.j], ||·||2=1.654e+00
1680   iter 00: subdiag=[-0.888+0.j -0.998-0.j 0.979+0.j], ||·||2=1.657e+00
1681   iter 01: subdiag=[0.904-0.j 0.995+0.j 0.951+0.j], ||·||2=1.647e+00
1682   iter 02: subdiag=[-0.913+0.j -0.992-0.j 0.912+0.j], ||·||2=1.627e+00
1683   iter 03: subdiag=[0.913-0.j 0.987+0.j 0.862+0.j], ||·||2=1.597e+00
1684   iter 04: subdiag=[-0.906+0.j -0.98 -0.j 0.808+0.j], ||·||2=1.560e+00
1685   iter 05: subdiag=[0.893-0.j 0.968+0.j 0.751+0.j], ||·||2=1.516e+00
1686   iter 06: subdiag=[-0.876+0.j -0.95 +0.j 0.696-0.j], ||·||2=1.468e+00
1687   iter 07: subdiag=[0.857-0.j 0.924-0.j 0.643-0.j], ||·||2=1.415e+00
1688   iter 08: subdiag=[-0.838+0.j -0.89 +0.j 0.595-0.j], ||·||2=1.359e+00
1689   iter 09: subdiag=[0.819-0.j 0.847+0.j 0.552-0.j], ||·||2=1.301e+00
1690   iter 10: subdiag=[ 0.801-0.j -0.798-0.j 0.513-0.j], ||·||2=1.241e+00
1691   iter 11: subdiag=[0.786-0.j 0.743+0.j 0.478-0.j], ||·||2=1.182e+00
1692   iter 12: subdiag=[ 0.772-0.j -0.686+0.j 0.447-0.j], ||·||2=1.125e+00
1693   iter 13: subdiag=[0.759-0.j 0.628+0.j 0.419-0.j], ||·||2=1.071e+00
1694   iter 14: subdiag=[ 0.749-0.j -0.571+0.j 0.394-0.j], ||·||2=1.020e+00
1695   iter 15: subdiag=[0.739-0.j 0.516-0.j 0.371-0.j], ||·||2=9.747e-01
1696   iter 16: subdiag=[ 0.731-0.j -0.464+0.j 0.35 -0.j], ||·||2=9.337e-01
1697   iter 17: subdiag=[0.724-0.j 0.416-0.j 0.33 -0.j], ||·||2=8.975e-01
1698   iter 18: subdiag=[ 0.717-0.j -0.371+0.j 0.312-0.j], ||·||2=8.656e-01
1699   iter 19: subdiag=[0.711-0.j 0.331-0.j 0.295-0.j], ||·||2=8.378e-01
1700   iter 20: subdiag=[ 0.705-0.j -0.294+0.j 0.279-0.j], ||·||2=8.136e-01
1701   iter 21: subdiag=[0.7 -0.j 0.261-0.j 0.264-0.j], ||·||2=7.925e-01
1702   iter 22: subdiag=[ 0.695-0.j -0.232+0.j 0.249-0.j], ||·||2=7.741e-01
1703   iter 23: subdiag=[0.69 -0.j 0.206-0.j 0.236-0.j], ||·||2=7.580e-01
1704   iter 24: subdiag=[ 0.686-0.j -0.182+0.j 0.223-0.j], ||·||2=7.437e-01

```

1705	iter 25: subdiag=[0.681-0.j 0.162-0.j 0.211-0.j], $\ \cdot\ _2=7.311e-01$
1706	iter 26: subdiag=[0.677-0.j -0.143+0.j 0.2 -0.j], $\ \cdot\ _2=7.197e-01$
1707	iter 27: subdiag=[0.672-0.j 0.127-0.j 0.189-0.j], $\ \cdot\ _2=7.095e-01$
1708	iter 28: subdiag=[0.668-0.j -0.112+0.j 0.178-0.j], $\ \cdot\ _2=7.001e-01$
1709	iter 29: subdiag=[0.663-0.j 0.099-0.j 0.168-0.j], $\ \cdot\ _2=6.915e-01$
1710	iter 30: subdiag=[0.659-0.j -0.088+0.j 0.159-0.j], $\ \cdot\ _2=6.835e-01$
1711	iter 31: subdiag=[0.655-0.j 0.078-0.j 0.15 -0.j], $\ \cdot\ _2=6.761e-01$
1712	iter 32: subdiag=[0.65 -0.j -0.069+0.j 0.142-0.j], $\ \cdot\ _2=6.690e-01$
1713	iter 33: subdiag=[0.646-0.j 0.061-0.j 0.134-0.j], $\ \cdot\ _2=6.624e-01$
1714	iter 34: subdiag=[0.641-0.j -0.054+0.j 0.127-0.j], $\ \cdot\ _2=6.560e-01$
1715	iter 35: subdiag=[0.637-0.j 0.048-0.j 0.12 -0.j], $\ \cdot\ _2=6.499e-01$
1716	iter 36: subdiag=[0.633-0.j -0.043+0.j 0.113-0.j], $\ \cdot\ _2=6.440e-01$
1717	iter 37: subdiag=[-0.628+0.j 0.038-0.j 0.107-0.j], $\ \cdot\ _2=6.382e-01$
1718	iter 38: subdiag=[0.624-0.j -0.033+0.j 0.101-0.j], $\ \cdot\ _2=6.327e-01$
1719	iter 39: subdiag=[-0.619+0.j 0.03 -0.j 0.095-0.j], $\ \cdot\ _2=6.272e-01$
1720	iter 40: subdiag=[0.615-0.j -0.026+0.j 0.09 -0.j], $\ \cdot\ _2=6.219e-01$
1721	iter 41: subdiag=[-0.61 +0.j 0.023-0.j 0.084-0.j], $\ \cdot\ _2=6.166e-01$
1722	iter 42: subdiag=[0.606-0.j -0.021+0.j 0.08 -0.j], $\ \cdot\ _2=6.115e-01$
1723	iter 43: subdiag=[-0.601-0.j 0.018-0.j 0.075-0.j], $\ \cdot\ _2=6.064e-01$
1724	iter 44: subdiag=[0.597+0.j -0.016+0.j 0.071-0.j], $\ \cdot\ _2=6.014e-01$
1725	iter 45: subdiag=[-0.592-0.j 0.014-0.j 0.067-0.j], $\ \cdot\ _2=5.964e-01$
1726	iter 46: subdiag=[0.588+0.j -0.013+0.j 0.063-0.j], $\ \cdot\ _2=5.915e-01$
1727	iter 47: subdiag=[-0.584-0.j 0.011-0.j 0.06 -0.j], $\ \cdot\ _2=5.866e-01$
1728	iter 48: subdiag=[0.579+0.j -0.01 +0.j 0.056-0.j], $\ \cdot\ _2=5.818e-01$
1729	iter 49: subdiag=[-0.575-0.j 0.009-0.j 0.053-0.j], $\ \cdot\ _2=5.770e-01$
1730	iter 50: subdiag=[0.57 +0.j -0.008+0.j 0.05 -0.j], $\ \cdot\ _2=5.723e-01$
1731	iter 51: subdiag=[-0.566+0.j 0.007-0.j 0.047-0.j], $\ \cdot\ _2=5.675e-01$
1732	iter 52: subdiag=[0.561+0.j -0.006+0.j 0.044-0.j], $\ \cdot\ _2=5.628e-01$
1733	iter 53: subdiag=[-0.557+0.j 0.005-0.j 0.042-0.j], $\ \cdot\ _2=5.581e-01$
1734	iter 54: subdiag=[0.552-0.j -0.005+0.j 0.04 -0.j], $\ \cdot\ _2=5.535e-01$
1735	iter 55: subdiag=[-0.548+0.j 0.004-0.j 0.037-0.j], $\ \cdot\ _2=5.488e-01$
1736	iter 56: subdiag=[0.543-0.j -0.004+0.j 0.035-0.j], $\ \cdot\ _2=5.442e-01$
1737	iter 57: subdiag=[-0.539+0.j 0.003-0.j 0.033-0.j], $\ \cdot\ _2=5.396e-01$
1738	iter 58: subdiag=[0.534+0.j -0.003+0.j 0.031-0.j], $\ \cdot\ _2=5.351e-01$
1739	iter 59: subdiag=[-0.53 -0.j 0.003-0.j 0.029-0.j], $\ \cdot\ _2=5.305e-01$
1740	iter 60: subdiag=[0.525+0.j -0.002+0.j 0.028-0.j], $\ \cdot\ _2=5.260e-01$
1741	iter 61: subdiag=[-0.521-0.j 0.002-0.j 0.026-0.j], $\ \cdot\ _2=5.215e-01$
1742	iter 62: subdiag=[0.516+0.j -0.002+0.j 0.025-0.j], $\ \cdot\ _2=5.170e-01$
1743	iter 63: subdiag=[-0.512-0.j 0.002-0.j 0.023-0.j], $\ \cdot\ _2=5.125e-01$
1744	iter 64: subdiag=[0.508+0.j -0.001+0.j 0.022-0.j], $\ \cdot\ _2=5.080e-01$
1745	iter 65: subdiag=[-0.503+0.j 0.001-0.j 0.021-0.j], $\ \cdot\ _2=5.036e-01$
1746	iter 66: subdiag=[0.499-0.j -0.001+0.j 0.02 -0.j], $\ \cdot\ _2=4.992e-01$
1747	iter 67: subdiag=[-0.494+0.j 0.001-0.j 0.018-0.j], $\ \cdot\ _2=4.948e-01$
1748	iter 68: subdiag=[0.49 -0.j -0.001+0.j 0.017-0.j], $\ \cdot\ _2=4.904e-01$
1749	iter 69: subdiag=[-0.486+0.j 0.001-0.j 0.016-0.j], $\ \cdot\ _2=4.860e-01$
1750	iter 70: subdiag=[0.481-0.j -0.001+0.j 0.015-0.j], $\ \cdot\ _2=4.817e-01$

```

1751   iter 71: subdiag=[-0.477+0.j  0.001-0.j  0.015-0.j], ||·||2=4.774e-01
1752   iter 72: subdiag=[ 0.473-0.j -0.001+0.j  0.014-0.j], ||·||2=4.731e-01
1753   iter 73: subdiag=[-0.469+0.j  0.   -0.j  0.013-0.j], ||·||2=4.688e-01
1754   iter 74: subdiag=[ 0.464-0.j -0.   +0.j  0.012-0.j], ||·||2=4.645e-01
1755   iter 75: subdiag=[-0.46 +0.j  0.   -0.j  0.012-0.j], ||·||2=4.603e-01
1756   iter 76: subdiag=[ 0.456-0.j -0.   +0.j  0.011-0.j], ||·||2=4.561e-01
1757   iter 77: subdiag=[-0.452-0.j  0.   -0.j  0.01 -0.j], ||·||2=4.519e-01
1758   iter 78: subdiag=[ 0.448+0.j -0.   +0.j  0.01 -0.j], ||·||2=4.477e-01
1759   iter 79: subdiag=[-0.443-0.j  0.   -0.j  0.009-0.j], ||·||2=4.436e-01
1760   iter 80: subdiag=[ 0.439+0.j -0.   +0.j  0.009-0.j], ||·||2=4.394e-01
1761   iter 81: subdiag=[-0.435-0.j  0.   -0.j  0.008-0.j], ||·||2=4.353e-01
1762   iter 82: subdiag=[ 0.431+0.j -0.   +0.j  0.008-0.j], ||·||2=4.313e-01
1763   iter 83: subdiag=[-0.427-0.j  0.   -0.j  0.007-0.j], ||·||2=4.272e-01
1764   iter 84: subdiag=[ 0.423+0.j -0.   +0.j  0.007-0.j], ||·||2=4.232e-01
1765   iter 85: subdiag=[-0.419-0.j  0.   -0.j  0.006-0.j], ||·||2=4.192e-01
1766   iter 86: subdiag=[ 0.415+0.j -0.   +0.j  0.006-0.j], ||·||2=4.152e-01
1767   iter 87: subdiag=[-0.411-0.j  0.   -0.j  0.006-0.j], ||·||2=4.112e-01
1768   iter 88: subdiag=[ 0.407+0.j -0.   +0.j  0.005-0.j], ||·||2=4.073e-01
1769   iter 89: subdiag=[-0.403-0.j  0.   -0.j  0.005-0.j], ||·||2=4.034e-01
1770   iter 90: subdiag=[ 0.399+0.j -0.   +0.j  0.005-0.j], ||·||2=3.995e-01
1771   iter 91: subdiag=[-0.396-0.j  0.   -0.j  0.005-0.j], ||·||2=3.957e-01
1772   iter 92: subdiag=[ 0.392+0.j -0.   +0.j  0.004-0.j], ||·||2=3.918e-01
1773   iter 93: subdiag=[-0.388-0.j  0.   -0.j  0.004-0.j], ||·||2=3.880e-01
1774   iter 94: subdiag=[ 0.384+0.j -0.   +0.j  0.004-0.j], ||·||2=3.842e-01
1775   iter 95: subdiag=[-0.38 -0.j  0.   -0.j  0.004-0.j], ||·||2=3.805e-01
1776   iter 96: subdiag=[ 0.377+0.j -0.   +0.j  0.003-0.j], ||·||2=3.768e-01
1777   iter 97: subdiag=[-0.373+0.j  0.   -0.j  0.003-0.j], ||·||2=3.731e-01
1778   iter 98: subdiag=[ 0.369+0.j -0.   +0.j  0.003-0.j], ||·||2=3.694e-01
1779   iter 99: subdiag=[-0.366-0.j  0.   -0.j  0.003-0.j], ||·||2=3.658e-01
1780   after : subdiag=[-0.366-0.j  0.   -0.j  0.003-0.j], ||·||2=3.658e-01
1781 | iterations      = 100/100
1782 | sub-diag magnitudes after last step:
1783 | [0.366 0.   0.003]
1784 └─ converged?     = no
1785
1786 └─ Matrix 18/30 (size 4x4)
1787 | fixed shift μ = 0.0412469-0.965376j (|μ|=0.9663)
1788   before: subdiag=[-0.986+0.j -0.358+0.j -0.999+0.j], ||·||2=1.449e+00
1789   iter 00: subdiag=[-0.281-0.j  0.975-0.j -0.203-0.j], ||·||2=1.035e+00
1790   iter 01: subdiag=[-0.184-0.j  0.285-0.j  0.078+0.j], ||·||2=3.481e-01
1791   iter 02: subdiag=[ 0.183+0.j -0.037+0.j -0.043-0.j], ||·||2=1.915e-01
1792   iter 03: subdiag=[-0.184-0.j  0.005-0.j  0.023+0.j], ||·||2=1.853e-01
1793   iter 04: subdiag=[ 0.185+0.j -0.001+0.j -0.012-0.j], ||·||2=1.850e-01
1794   iter 05: subdiag=[ 0.185+0.j -0.   +0.j  0.007+0.j], ||·||2=1.854e-01
1795   iter 06: subdiag=[ 0.186+0.j -0.   +0.j -0.004-0.j], ||·||2=1.860e-01
1796   iter 07: subdiag=[ 0.187+0.j -0.   +0.j  0.002+0.j], ||·||2=1.866e-01

```

1797	iter 08: subdiag=[0.187+0.j -0. +0.j -0.001-0.j], $\ \cdot\ _2=1.872e-01$
1798	iter 09: subdiag=[0.188+0.j -0. +0.j 0.001+0.j], $\ \cdot\ _2=1.878e-01$
1799	iter 10: subdiag=[0.188+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.884e-01$
1800	iter 11: subdiag=[0.189+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.889e-01$
1801	iter 12: subdiag=[0.189+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.894e-01$
1802	iter 13: subdiag=[0.19+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.898e-01$
1803	iter 14: subdiag=[0.19+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.902e-01$
1804	iter 15: subdiag=[0.191+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.906e-01$
1805	iter 16: subdiag=[0.191+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.909e-01$
1806	iter 17: subdiag=[0.191+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.912e-01$
1807	iter 18: subdiag=[0.191+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.915e-01$
1808	iter 19: subdiag=[0.192+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.917e-01$
1809	iter 20: subdiag=[0.192+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.919e-01$
1810	iter 21: subdiag=[0.192+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.920e-01$
1811	iter 22: subdiag=[0.192+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.921e-01$
1812	iter 23: subdiag=[0.192+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.922e-01$
1813	iter 24: subdiag=[0.192+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.922e-01$
1814	iter 25: subdiag=[0.192+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.922e-01$
1815	iter 26: subdiag=[0.192+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.921e-01$
1816	iter 27: subdiag=[0.192+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.920e-01$
1817	iter 28: subdiag=[0.192+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.919e-01$
1818	iter 29: subdiag=[0.192+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.918e-01$
1819	iter 30: subdiag=[0.192+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.915e-01$
1820	iter 31: subdiag=[0.191+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.913e-01$
1821	iter 32: subdiag=[0.191+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.910e-01$
1822	iter 33: subdiag=[0.191+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.907e-01$
1823	iter 34: subdiag=[0.19+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.903e-01$
1824	iter 35: subdiag=[0.19+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.900e-01$
1825	iter 36: subdiag=[0.19+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.895e-01$
1826	iter 37: subdiag=[0.189+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.891e-01$
1827	iter 38: subdiag=[0.189+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.886e-01$
1828	iter 39: subdiag=[0.188+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.880e-01$
1829	iter 40: subdiag=[0.187+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.875e-01$
1830	iter 41: subdiag=[0.187+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.869e-01$
1831	iter 42: subdiag=[0.186+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.862e-01$
1832	iter 43: subdiag=[0.186+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.855e-01$
1833	iter 44: subdiag=[0.185+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.848e-01$
1834	iter 45: subdiag=[0.184+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.841e-01$
1835	iter 46: subdiag=[0.183+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.833e-01$
1836	iter 47: subdiag=[0.183+0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.825e-01$
1837	iter 48: subdiag=[0.182+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.817e-01$
1838	iter 49: subdiag=[-0.181-0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.809e-01$
1839	iter 50: subdiag=[0.18+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.800e-01$
1840	iter 51: subdiag=[-0.179-0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.791e-01$
1841	iter 52: subdiag=[0.178+0.j -0. +0.j -0. -0.j], $\ \cdot\ _2=1.781e-01$
1842	iter 53: subdiag=[-0.177-0.j -0. +0.j 0. +0.j], $\ \cdot\ _2=1.772e-01$

1843	iter 54: subdiag=[0.176+0.j -0. +0.j -0. -0.j], · ₂ =1.762e-01
1844	iter 55: subdiag=[-0.175-0.j -0. +0.j 0. +0.j], · ₂ =1.752e-01
1845	iter 56: subdiag=[0.174+0.j -0. +0.j -0. -0.j], · ₂ =1.741e-01
1846	iter 57: subdiag=[-0.173-0.j -0. +0.j 0. +0.j], · ₂ =1.731e-01
1847	iter 58: subdiag=[0.172+0.j -0. +0.j -0. -0.j], · ₂ =1.720e-01
1848	iter 59: subdiag=[-0.171-0.j -0. +0.j 0. +0.j], · ₂ =1.709e-01
1849	iter 60: subdiag=[0.17+0.j -0. +0.j -0. -0.j], · ₂ =1.698e-01
1850	iter 61: subdiag=[-0.169-0.j -0. +0.j 0. +0.j], · ₂ =1.687e-01
1851	iter 62: subdiag=[0.167+0.j -0. +0.j -0. -0.j], · ₂ =1.675e-01
1852	iter 63: subdiag=[-0.166-0.j -0. +0.j 0. +0.j], · ₂ =1.663e-01
1853	iter 64: subdiag=[0.165+0.j -0. +0.j -0. -0.j], · ₂ =1.651e-01
1854	iter 65: subdiag=[-0.164-0.j -0. +0.j 0. +0.j], · ₂ =1.639e-01
1855	iter 66: subdiag=[0.163+0.j -0. +0.j -0. -0.j], · ₂ =1.627e-01
1856	iter 67: subdiag=[-0.161-0.j -0. +0.j 0. +0.j], · ₂ =1.615e-01
1857	iter 68: subdiag=[0.16+0.j -0. +0.j -0. -0.j], · ₂ =1.602e-01
1858	iter 69: subdiag=[-0.159-0.j -0. +0.j 0. +0.j], · ₂ =1.590e-01
1859	iter 70: subdiag=[0.158+0.j -0. +0.j -0. -0.j], · ₂ =1.577e-01
1860	iter 71: subdiag=[-0.156-0.j -0. +0.j 0. +0.j], · ₂ =1.564e-01
1861	iter 72: subdiag=[0.155+0.j -0. +0.j -0. -0.j], · ₂ =1.551e-01
1862	iter 73: subdiag=[-0.154-0.j -0. +0.j 0. +0.j], · ₂ =1.538e-01
1863	iter 74: subdiag=[0.152+0.j -0. +0.j -0. -0.j], · ₂ =1.525e-01
1864	iter 75: subdiag=[-0.151-0.j -0. +0.j 0. +0.j], · ₂ =1.512e-01
1865	iter 76: subdiag=[0.15-0.j -0. +0.j -0. -0.j], · ₂ =1.498e-01
1866	iter 77: subdiag=[-0.148-0.j -0. +0.j 0. +0.j], · ₂ =1.485e-01
1867	iter 78: subdiag=[0.147+0.j -0. +0.j -0. -0.j], · ₂ =1.472e-01
1868	iter 79: subdiag=[-0.146+0.j -0. +0.j 0. +0.j], · ₂ =1.458e-01
1869	iter 80: subdiag=[0.144+0.j -0. +0.j -0. -0.j], · ₂ =1.445e-01
1870	iter 81: subdiag=[-0.143-0.j -0. +0.j 0. +0.j], · ₂ =1.431e-01
1871	iter 82: subdiag=[0.142+0.j -0. +0.j -0. -0.j], · ₂ =1.418e-01
1872	iter 83: subdiag=[-0.14-0.j -0. +0.j 0. +0.j], · ₂ =1.404e-01
1873	iter 84: subdiag=[0.139+0.j -0. +0.j -0. -0.j], · ₂ =1.390e-01
1874	iter 85: subdiag=[-0.138-0.j -0. +0.j 0. +0.j], · ₂ =1.377e-01
1875	iter 86: subdiag=[0.136+0.j -0. +0.j -0. -0.j], · ₂ =1.363e-01
1876	iter 87: subdiag=[-0.135+0.j -0. +0.j 0. +0.j], · ₂ =1.349e-01
1877	iter 88: subdiag=[0.134+0.j -0. +0.j -0. -0.j], · ₂ =1.336e-01
1878	iter 89: subdiag=[-0.132+0.j -0. +0.j 0. +0.j], · ₂ =1.322e-01
1879	iter 90: subdiag=[0.131+0.j -0. +0.j -0. -0.j], · ₂ =1.309e-01
1880	iter 91: subdiag=[-0.129-0.j -0. +0.j 0. +0.j], · ₂ =1.295e-01
1881	iter 92: subdiag=[0.128+0.j -0. +0.j -0. -0.j], · ₂ =1.281e-01
1882	iter 93: subdiag=[-0.127+0.j -0. +0.j 0. +0.j], · ₂ =1.268e-01
1883	iter 94: subdiag=[0.125+0.j -0. +0.j -0. -0.j], · ₂ =1.254e-01
1884	iter 95: subdiag=[-0.124+0.j -0. +0.j 0. +0.j], · ₂ =1.241e-01
1885	iter 96: subdiag=[0.123+0.j -0. +0.j -0. -0.j], · ₂ =1.227e-01
1886	iter 97: subdiag=[-0.121+0.j -0. +0.j 0. +0.j], · ₂ =1.214e-01
1887	iter 98: subdiag=[0.12-0.j -0. +0.j -0. -0.j], · ₂ =1.201e-01
1888	iter 99: subdiag=[-0.119-0.j -0. +0.j 0. +0.j], · ₂ =1.187e-01

```

1889   after : subdiag=[-0.119-0.j  -0.   +0.j  0.   +0.j], ||·||2=1.187e-01
1890 |   iterations      = 100/100
1891 |   sub-diag magnitudes after last step:
1892 |   [0.119 0.   0.   ]
1893 └─ converged?      = no
1894
1895 └─ Matrix 19/30   (size 4x4)
1896 |   fixed shift μ = -0.993087 (|μ|=0.9931)
1897   before: subdiag=[-0.745+0.j  0.986+0.j  0.139+0.j], ||·||2=1.244e+00
1898   iter 00: subdiag=[-0.592+0.j  0.997+0.j  0.001+0.j], ||·||2=1.159e+00
1899   iter 01: subdiag=[-0.445+0.j  1.   +0.j  0.   +0.j], ||·||2=1.094e+00
1900   iter 02: subdiag=[-0.324+0.j  1.   +0.j  0.   +0.j], ||·||2=1.051e+00
1901   iter 03: subdiag=[-0.231+0.j  1.   +0.j  0.   +0.j], ||·||2=1.026e+00
1902   iter 04: subdiag=[-0.163+0.j  1.   +0.j  0.   +0.j], ||·||2=1.013e+00
1903   iter 05: subdiag=[-0.114+0.j  1.   +0.j  0.   +0.j], ||·||2=1.006e+00
1904   iter 06: subdiag=[-0.08+0.j  1.   +0.j  0.   +0.j], ||·||2=1.003e+00
1905   iter 07: subdiag=[-0.056+0.j  1.   +0.j  0.   +0.j], ||·||2=1.001e+00
1906   iter 08: subdiag=[-0.039+0.j  1.   +0.j  0.   +0.j], ||·||2=1.000e+00
1907   iter 09: subdiag=[-0.027+0.j  1.   +0.j  0.   +0.j], ||·||2=1.000e+00
1908   iter 10: subdiag=[-0.019+0.j  1.   +0.j  0.   +0.j], ||·||2=9.999e-01
1909   iter 11: subdiag=[-0.013+0.j  1.   +0.j  0.   +0.j], ||·||2=9.998e-01
1910   iter 12: subdiag=[-0.009+0.j  1.   +0.j  0.   +0.j], ||·||2=9.997e-01
1911   iter 13: subdiag=[-0.006+0.j  1.   +0.j  0.   +0.j], ||·||2=9.997e-01
1912   iter 14: subdiag=[-0.005+0.j  1.   +0.j  0.   +0.j], ||·||2=9.997e-01
1913   iter 15: subdiag=[-0.003+0.j  1.   +0.j  0.   +0.j], ||·||2=9.997e-01
1914   iter 16: subdiag=[-0.002+0.j  1.   +0.j  0.   +0.j], ||·||2=9.997e-01
1915   iter 17: subdiag=[-0.002+0.j  1.   +0.j  0.   +0.j], ||·||2=9.997e-01
1916   iter 18: subdiag=[-0.001+0.j  1.   +0.j  0.   +0.j], ||·||2=9.997e-01
1917   iter 19: subdiag=[-0.001+0.j  1.   +0.j  0.   +0.j], ||·||2=9.997e-01
1918   iter 20: subdiag=[-0.001+0.j  1.   +0.j  0.   +0.j], ||·||2=9.997e-01
1919   iter 21: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1920   iter 22: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1921   iter 23: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1922   iter 24: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1923   iter 25: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1924   iter 26: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1925   iter 27: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1926   iter 28: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1927   iter 29: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1928   iter 30: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1929   iter 31: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1930   iter 32: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1931   iter 33: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1932   iter 34: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1933   iter 35: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01
1934   iter 36: subdiag=[-0.+0.j  1.+0.j  0.+0.j], ||·||2=9.997e-01

```

[illegible]

```

1981   iter 83: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1982   iter 84: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1983   iter 85: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1984   iter 86: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1985   iter 87: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1986   iter 88: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1987   iter 89: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1988   iter 90: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1989   iter 91: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1990   iter 92: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1991   iter 93: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1992   iter 94: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1993   iter 95: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1994   iter 96: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1995   iter 97: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1996   iter 98: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1997   iter 99: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1998   after : subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ||·||2=9.997e-01
1999 | iterations      = 100/100
2000 | sub-diag magnitudes after last step:
2001 | [0. 1. 0.]
2002 └ converged?     = no
2003
2004 └ Matrix 20/30   (size 4x4)
2005 | fixed shift μ = 0.943718 (|μ|=0.9437)
2006   before: subdiag=[-0.429+0.j -0.972+0.j  0.419+0.j],  ||·||2=1.142e+00
2007   iter 00: subdiag=[-0.982+0.j -0.64 +0.j  0.047+0.j],  ||·||2=1.173e+00
2008   iter 01: subdiag=[-0.51 +0.j -0.44 +0.j  0.006+0.j],  ||·||2=6.740e-01
2009   iter 02: subdiag=[-0.12 +0.j -0.426+0.j  0.001+0.j],  ||·||2=4.427e-01
2010   iter 03: subdiag=[-0.027+0.j -0.425+0.j  0.   +0.j],  ||·||2=4.262e-01
2011   iter 04: subdiag=[-0.006+0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2012   iter 05: subdiag=[-0.001+0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2013   iter 06: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2014   iter 07: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2015   iter 08: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2016   iter 09: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2017   iter 10: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2018   iter 11: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2019   iter 12: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2020   iter 13: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2021   iter 14: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2022   iter 15: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2023   iter 16: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2024   iter 17: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2025   iter 18: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01
2026   iter 19: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ||·||2=4.253e-01

```


[illegible]

```

2073   iter 66: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2074   iter 67: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2075   iter 68: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2076   iter 69: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2077   iter 70: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2078   iter 71: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2079   iter 72: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2080   iter 73: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2081   iter 74: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2082   iter 75: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2083   iter 76: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2084   iter 77: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2085   iter 78: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2086   iter 79: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2087   iter 80: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2088   iter 81: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2089   iter 82: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2090   iter 83: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2091   iter 84: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2092   iter 85: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2093   iter 86: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2094   iter 87: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2095   iter 88: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2096   iter 89: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2097   iter 90: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2098   iter 91: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2099   iter 92: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2100   iter 93: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2101   iter 94: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2102   iter 95: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2103   iter 96: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2104   iter 97: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2105   iter 98: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2106   iter 99: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2107   after : subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j], ||·||2=4.253e-01
2108 |   iterations      = 100/100
2109 |   sub-diag magnitudes after last step:
2110 |   [0.   0.425 0.   ]
2111 |   converged?      = no
2112
2113 |   Matrix 21/30   (size 4x4)
2114 |   fixed shift μ = -0.645528 (|μ|=0.6455)
2115   before: subdiag=[-0.998+0.j  0.999+0.j  0.737+0.j], ||·||2=1.593e+00
2116   iter 00: subdiag=[-0.902+0.j  0.984+0.j  0.314+0.j], ||·||2=1.371e+00
2117   iter 01: subdiag=[-0.666+0.j  0.955+0.j  0.114+0.j], ||·||2=1.169e+00
2118   iter 02: subdiag=[-0.431+0.j  0.936+0.j  0.041+0.j], ||·||2=1.032e+00

```

[illegible]

[illegible]

```

2211 iter 95: subdiag=[-0.  +0.j  0.924+0.j  0.  +0.j], ||·||2=9.238e-01
2212 iter 96: subdiag=[-0.  +0.j  0.924+0.j  0.  +0.j], ||·||2=9.238e-01
2213 iter 97: subdiag=[-0.  +0.j  0.924+0.j  0.  +0.j], ||·||2=9.238e-01
2214 iter 98: subdiag=[-0.  +0.j  0.924+0.j  0.  +0.j], ||·||2=9.238e-01
2215 iter 99: subdiag=[-0.  +0.j  0.924+0.j  0.  +0.j], ||·||2=9.238e-01
2216 after : subdiag=[-0.  +0.j  0.924+0.j  0.  +0.j], ||·||2=9.238e-01
2217 | iterations      = 100/100
2218 | sub-diag magnitudes after last step:
2219 | [0.    0.924 0.    ]
2220 └ converged?      = no
2221
2222 └ Matrix 22/30 (size 4x4)
2223 | fixed shift μ = 0.833114 (|μ|=0.8331)
2224 before: subdiag=[-0.476+0.j  0.996+0.j  0.602+0.j], ||·||2=1.258e+00
2225 iter 00: subdiag=[-0.329+0.j -0.495+0.j -0.426+0.j], ||·||2=7.312e-01
2226 iter 01: subdiag=[-0.318+0.j  0.123+0.j  0.413+0.j], ||·||2=5.355e-01
2227 iter 02: subdiag=[-0.318+0.j -0.029+0.j -0.412+0.j], ||·||2=5.211e-01
2228 iter 03: subdiag=[-0.318+0.j  0.007+0.j  0.412+0.j], ||·||2=5.203e-01
2229 iter 04: subdiag=[-0.318+0.j -0.002+0.j -0.412+0.j], ||·||2=5.202e-01
2230 iter 05: subdiag=[-0.318+0.j  0.  +0.j  0.412+0.j], ||·||2=5.202e-01
2231 iter 06: subdiag=[-0.318+0.j -0.  +0.j -0.412+0.j], ||·||2=5.202e-01
2232 iter 07: subdiag=[-0.318+0.j  0.  +0.j  0.412+0.j], ||·||2=5.202e-01
2233 iter 08: subdiag=[-0.318+0.j -0.  +0.j -0.412+0.j], ||·||2=5.202e-01
2234 iter 09: subdiag=[-0.318+0.j  0.  +0.j  0.412+0.j], ||·||2=5.202e-01
2235 iter 10: subdiag=[-0.318+0.j -0.  +0.j -0.412+0.j], ||·||2=5.202e-01
2236 iter 11: subdiag=[-0.318+0.j  0.  +0.j  0.412+0.j], ||·||2=5.202e-01
2237 iter 12: subdiag=[-0.318+0.j -0.  +0.j -0.412+0.j], ||·||2=5.202e-01
2238 iter 13: subdiag=[-0.318+0.j  0.  +0.j  0.412+0.j], ||·||2=5.202e-01
2239 iter 14: subdiag=[-0.318+0.j -0.  +0.j -0.412+0.j], ||·||2=5.202e-01
2240 iter 15: subdiag=[-0.318+0.j  0.  +0.j  0.412+0.j], ||·||2=5.202e-01
2241 iter 16: subdiag=[-0.318+0.j -0.  +0.j -0.412+0.j], ||·||2=5.202e-01
2242 iter 17: subdiag=[-0.318+0.j  0.  +0.j  0.412+0.j], ||·||2=5.202e-01
2243 iter 18: subdiag=[-0.318+0.j -0.  +0.j -0.412+0.j], ||·||2=5.202e-01
2244 iter 19: subdiag=[-0.318+0.j  0.  +0.j  0.412+0.j], ||·||2=5.202e-01
2245 iter 20: subdiag=[-0.318+0.j -0.  +0.j -0.412+0.j], ||·||2=5.202e-01
2246 iter 21: subdiag=[-0.318+0.j  0.  +0.j  0.412+0.j], ||·||2=5.202e-01
2247 iter 22: subdiag=[-0.318+0.j -0.  +0.j -0.412+0.j], ||·||2=5.202e-01
2248 iter 23: subdiag=[-0.318+0.j  0.  +0.j  0.412+0.j], ||·||2=5.202e-01
2249 iter 24: subdiag=[-0.318+0.j -0.  +0.j -0.412+0.j], ||·||2=5.202e-01
2250 iter 25: subdiag=[-0.318+0.j  0.  +0.j  0.412+0.j], ||·||2=5.202e-01
2251 iter 26: subdiag=[-0.318+0.j -0.  +0.j -0.412+0.j], ||·||2=5.202e-01
2252 iter 27: subdiag=[-0.318+0.j  0.  +0.j  0.412+0.j], ||·||2=5.202e-01
2253 iter 28: subdiag=[-0.318+0.j -0.  +0.j -0.412+0.j], ||·||2=5.202e-01
2254 iter 29: subdiag=[-0.318+0.j  0.  +0.j  0.412+0.j], ||·||2=5.202e-01
2255 iter 30: subdiag=[-0.318+0.j -0.  +0.j -0.412+0.j], ||·||2=5.202e-01
2256 iter 31: subdiag=[-0.318+0.j  0.  +0.j  0.412+0.j], ||·||2=5.202e-01

```

[illegible]

```

2303   iter 78: subdiag=[-0.318+0.j  -0.   +0.j  -0.412+0.j],  ||·||2=5.202e-01
2304   iter 79: subdiag=[-0.318+0.j   0.   +0.j   0.412+0.j],  ||·||2=5.202e-01
2305   iter 80: subdiag=[-0.318+0.j  -0.   +0.j  -0.412+0.j],  ||·||2=5.202e-01
2306   iter 81: subdiag=[-0.318+0.j   0.   +0.j   0.412+0.j],  ||·||2=5.202e-01
2307   iter 82: subdiag=[-0.318+0.j  -0.   +0.j  -0.412+0.j],  ||·||2=5.202e-01
2308   iter 83: subdiag=[-0.318+0.j   0.   +0.j   0.412+0.j],  ||·||2=5.202e-01
2309   iter 84: subdiag=[-0.318+0.j  -0.   +0.j  -0.412+0.j],  ||·||2=5.202e-01
2310   iter 85: subdiag=[-0.318+0.j   0.   +0.j   0.412+0.j],  ||·||2=5.202e-01
2311   iter 86: subdiag=[-0.318+0.j  -0.   +0.j  -0.412+0.j],  ||·||2=5.202e-01
2312   iter 87: subdiag=[-0.318+0.j   0.   +0.j   0.412+0.j],  ||·||2=5.202e-01
2313   iter 88: subdiag=[-0.318+0.j  -0.   +0.j  -0.412+0.j],  ||·||2=5.202e-01
2314   iter 89: subdiag=[-0.318+0.j   0.   +0.j   0.412+0.j],  ||·||2=5.202e-01
2315   iter 90: subdiag=[-0.318+0.j  -0.   +0.j  -0.412+0.j],  ||·||2=5.202e-01
2316   iter 91: subdiag=[-0.318+0.j   0.   +0.j   0.412+0.j],  ||·||2=5.202e-01
2317   iter 92: subdiag=[-0.318+0.j  -0.   +0.j  -0.412+0.j],  ||·||2=5.202e-01
2318   iter 93: subdiag=[-0.318+0.j   0.   +0.j   0.412+0.j],  ||·||2=5.202e-01
2319   iter 94: subdiag=[-0.318+0.j  -0.   +0.j  -0.412+0.j],  ||·||2=5.202e-01
2320   iter 95: subdiag=[-0.318+0.j   0.   +0.j   0.412+0.j],  ||·||2=5.202e-01
2321   iter 96: subdiag=[-0.318+0.j  -0.   +0.j  -0.412+0.j],  ||·||2=5.202e-01
2322   iter 97: subdiag=[-0.318+0.j   0.   +0.j   0.412+0.j],  ||·||2=5.202e-01
2323   iter 98: subdiag=[-0.318+0.j  -0.   +0.j  -0.412+0.j],  ||·||2=5.202e-01
2324   iter 99: subdiag=[-0.318+0.j   0.   +0.j   0.412+0.j],  ||·||2=5.202e-01
2325   after : subdiag=[-0.318+0.j   0.   +0.j   0.412+0.j],  ||·||2=5.202e-01
2326 |   iterations      = 100/100
2327 |   sub-diag magnitudes after last step:
2328 |   [0.318 0.      0.412]
2329 |   └─ converged?    = no
2330
2331 └─ Matrix 23/30   (size 4x4)
2332 |   fixed shift μ = 0.475111-0.570494j (|μ|=0.7424)
2333   before: subdiag=[0.907+0.j 0.834+0.j 0.79 +0.j],  ||·||2=1.464e+00
2334   iter 00: subdiag=[ 0.727-0.j -0.911-0.j  0.447+0.j],  ||·||2=1.248e+00
2335   iter 01: subdiag=[ 0.484-0.j -0.857-0.j -0.193-0.j],  ||·||2=1.003e+00
2336   iter 02: subdiag=[-0.345+0.j  0.585+0.j  0.093+0.j],  ||·||2=6.857e-01
2337   iter 03: subdiag=[ 0.253-0.j -0.345-0.j -0.048+0.j],  ||·||2=4.310e-01
2338   iter 04: subdiag=[-0.185+0.j  0.195+0.j  0.026-0.j],  ||·||2=2.705e-01
2339   iter 05: subdiag=[ 0.135-0.j -0.109-0.j -0.014-0.j],  ||·||2=1.741e-01
2340   iter 06: subdiag=[-0.097+0.j  0.061+0.j  0.007+0.j],  ||·||2=1.152e-01
2341   iter 07: subdiag=[ 0.07 -0.j -0.034-0.j -0.004-0.j],  ||·||2=7.812e-02
2342   iter 08: subdiag=[-0.05 +0.j  0.019+0.j  0.002-0.j],  ||·||2=5.393e-02
2343   iter 09: subdiag=[ 0.036-0.j -0.011-0.j -0.001+0.j],  ||·||2=3.772e-02
2344   iter 10: subdiag=[-0.026+0.j  0.006+0.j  0.001+0.j],  ||·||2=2.661e-02
2345   iter 11: subdiag=[ 0.019-0.j -0.003-0.j -0.   -0.j],  ||·||2=1.888e-02
2346   iter 12: subdiag=[-0.013+0.j  0.002+0.j  0.   +0.j],  ||·||2=1.345e-02
2347   iter 13: subdiag=[ 0.01 -0.j -0.001-0.j -0.   -0.j],  ||·||2=9.598e-03
2348   iter 14: subdiag=[-0.007+0.j  0.001+0.j  0.   +0.j],  ||·||2=6.862e-03

```

2349	iter 15: subdiag=[0.005-0.j -0. -0.j -0. -0.j], $\ \cdot\ _2=4.910e-03$
2350	iter 16: subdiag=[-0.004+0.j 0. +0.j 0. +0.j], $\ \cdot\ _2=3.515e-03$
2351	iter 17: subdiag=[0.003-0.j -0. -0.j -0. -0.j], $\ \cdot\ _2=2.517e-03$
2352	iter 18: subdiag=[-0.002+0.j 0. +0.j 0. +0.j], $\ \cdot\ _2=1.803e-03$
2353	iter 19: subdiag=[0.001-0.j -0. -0.j -0. -0.j], $\ \cdot\ _2=1.292e-03$
2354	iter 20: subdiag=[-0.001+0.j 0. +0.j 0. +0.j], $\ \cdot\ _2=9.255e-04$
2355	iter 21: subdiag=[0.001-0.j -0. -0.j -0. -0.j], $\ \cdot\ _2=6.631e-04$
2356	iter 22: subdiag=[-0.+0.j 0.+0.j 0.+0.j], $\ \cdot\ _2=4.752e-04$
2357	iter 23: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=3.405e-04$
2358	iter 24: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=2.440e-04$
2359	iter 25: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=1.748e-04$
2360	iter 26: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=1.253e-04$
2361	iter 27: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=8.975e-05$
2362	iter 28: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=6.431e-05$
2363	iter 29: subdiag=[0.-0.j -0.-0.j -0.-0.j], $\ \cdot\ _2=4.608e-05$
2364	iter 30: subdiag=[-0.+0.j 0.+0.j 0.+0.j], $\ \cdot\ _2=3.302e-05$
2365	iter 31: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=2.366e-05$
2366	iter 32: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=1.695e-05$
2367	iter 33: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=1.215e-05$
2368	iter 34: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=8.705e-06$
2369	iter 35: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=6.238e-06$
2370	iter 36: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=4.470e-06$
2371	iter 37: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=3.203e-06$
2372	iter 38: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=2.295e-06$
2373	iter 39: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=1.645e-06$
2374	iter 40: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=1.178e-06$
2375	iter 41: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=8.444e-07$
2376	iter 42: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=6.050e-07$
2377	iter 43: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=4.335e-07$
2378	iter 44: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=3.107e-07$
2379	iter 45: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=2.226e-07$
2380	iter 46: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=1.595e-07$
2381	iter 47: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=1.143e-07$
2382	iter 48: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=8.190e-08$
2383	iter 49: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=5.868e-08$
2384	iter 50: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=4.205e-08$
2385	iter 51: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=3.013e-08$
2386	iter 52: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=2.159e-08$
2387	iter 53: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=1.547e-08$
2388	iter 54: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=1.109e-08$
2389	iter 55: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=7.944e-09$
2390	iter 56: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=5.692e-09$
2391	iter 57: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=4.079e-09$
2392	iter 58: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=2.923e-09$
2393	iter 59: subdiag=[0.-0.j -0.-0.j -0.+0.j], $\ \cdot\ _2=2.094e-09$
2394	iter 60: subdiag=[-0.+0.j 0.+0.j 0.-0.j], $\ \cdot\ _2=1.501e-09$


```

2395   iter 61: subdiag=[ 0.-0.j -0.-0.j -0.+0.j], ||·||2=1.075e-09
2396   iter 62: subdiag=[-0.+0.j 0.+0.j 0.-0.j], ||·||2=7.705e-10
2397   iter 63: subdiag=[ 0.-0.j -0.-0.j -0.+0.j], ||·||2=5.521e-10
2398   iter 64: subdiag=[-0.+0.j 0.+0.j 0.-0.j], ||·||2=3.956e-10
2399   iter 65: subdiag=[ 0.-0.j -0.-0.j -0.+0.j], ||·||2=2.835e-10
2400   iter 66: subdiag=[-0.+0.j 0.+0.j 0.-0.j], ||·||2=2.031e-10
2401   iter 67: subdiag=[ 0.-0.j -0.-0.j -0.+0.j], ||·||2=1.455e-10
2402   iter 68: subdiag=[-0.+0.j 0.+0.j 0.-0.j], ||·||2=1.043e-10
2403   iter 69: subdiag=[ 0.-0.j -0.-0.j -0.+0.j], ||·||2=7.473e-11
2404   after : subdiag=[ 0.-0.j -0.-0.j -0.+0.j], ||·||2=7.473e-11
2405 | iterations      = 70/100
2406 | sub-diag magnitudes after last step:
2407 | [0. 0. 0.]
2408 └ converged?      = yes
2409
2410 └ Matrix 24/30 (size 4x4)
2411 | fixed shift μ = 0.921576-0.227859j (|μ|=0.9493)
2412   before: subdiag=[-0.916+0.j 0.433+0.j -0.245+0.j], ||·||2=1.042e+00
2413   iter 00: subdiag=[-0.888+0.j -0.146-0.j -0.052-0.j], ||·||2=9.016e-01
2414   iter 01: subdiag=[-0.756-0.j 0.066+0.j -0.006-0.j], ||·||2=7.589e-01
2415   iter 02: subdiag=[-0.595+0.j -0.031-0.j -0.001+0.j], ||·||2=5.955e-01
2416   iter 03: subdiag=[-0.446-0.j 0.015+0.j -0.   +0.j], ||·||2=4.459e-01
2417   iter 04: subdiag=[-0.325-0.j -0.007-0.j -0.   +0.j], ||·||2=3.251e-01
2418   iter 05: subdiag=[-0.234-0.j 0.003+0.j -0.   -0.j], ||·||2=2.336e-01
2419   iter 06: subdiag=[-0.167-0.j -0.002-0.j -0.   -0.j], ||·||2=1.666e-01
2420   iter 07: subdiag=[-0.118-0.j 0.001+0.j -0.   -0.j], ||·||2=1.184e-01
2421   iter 08: subdiag=[-0.084-0.j -0.   -0.j -0.   -0.j], ||·||2=8.395e-02
2422   iter 09: subdiag=[-0.059-0.j 0.   +0.j -0.   -0.j], ||·||2=5.947e-02
2423   iter 10: subdiag=[-0.042-0.j -0.   -0.j -0.   -0.j], ||·||2=4.211e-02
2424   iter 11: subdiag=[-0.03-0.j 0.   +0.j -0.   +0.j], ||·||2=2.981e-02
2425   iter 12: subdiag=[-0.021-0.j -0.   -0.j -0.   -0.j], ||·||2=2.110e-02
2426   iter 13: subdiag=[-0.015+0.j 0.   +0.j -0.   +0.j], ||·||2=1.493e-02
2427   iter 14: subdiag=[-0.011-0.j -0.   -0.j -0.   +0.j], ||·||2=1.057e-02
2428   iter 15: subdiag=[-0.007-0.j 0.   +0.j -0.   -0.j], ||·||2=7.480e-03
2429   iter 16: subdiag=[-0.005-0.j -0.   -0.j -0.   -0.j], ||·||2=5.294e-03
2430   iter 17: subdiag=[-0.004-0.j 0.   +0.j -0.   -0.j], ||·||2=3.747e-03
2431   iter 18: subdiag=[-0.003-0.j -0.   -0.j -0.   -0.j], ||·||2=2.652e-03
2432   iter 19: subdiag=[-0.002-0.j 0.   +0.j -0.   -0.j], ||·||2=1.877e-03
2433   iter 20: subdiag=[-0.001-0.j -0.   -0.j -0.   -0.j], ||·||2=1.328e-03
2434   iter 21: subdiag=[-0.001+0.j 0.   +0.j -0.   -0.j], ||·||2=9.400e-04
2435   iter 22: subdiag=[-0.001+0.j -0.   -0.j -0.   -0.j], ||·||2=6.652e-04
2436   iter 23: subdiag=[-0.+0.j 0.+0.j -0.+0.j], ||·||2=4.708e-04
2437   iter 24: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=3.332e-04
2438   iter 25: subdiag=[-0.+0.j 0.+0.j -0.-0.j], ||·||2=2.358e-04
2439   iter 26: subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=1.669e-04
2440   iter 27: subdiag=[-0.+0.j 0.+0.j -0.-0.j], ||·||2=1.181e-04

```

```

2441   iter 28: subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=8.359e-05
2442   iter 29: subdiag=[-0.+0.j  0.+0.j -0.-0.j], ||·||2=5.916e-05
2443   iter 30: subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=4.187e-05
2444   iter 31: subdiag=[-0.+0.j  0.+0.j -0.-0.j], ||·||2=2.963e-05
2445   iter 32: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=2.097e-05
2446   iter 33: subdiag=[-0.-0.j  0.+0.j -0.-0.j], ||·||2=1.484e-05
2447   iter 34: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=1.050e-05
2448   iter 35: subdiag=[-0.-0.j  0.+0.j -0.-0.j], ||·||2=7.434e-06
2449   iter 36: subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=5.261e-06
2450   iter 37: subdiag=[-0.+0.j  0.+0.j -0.-0.j], ||·||2=3.724e-06
2451   iter 38: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=2.635e-06
2452   iter 39: subdiag=[-0.-0.j  0.+0.j -0.-0.j], ||·||2=1.865e-06
2453   iter 40: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=1.320e-06
2454   iter 41: subdiag=[-0.+0.j  0.+0.j -0.-0.j], ||·||2=9.342e-07
2455   iter 42: subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=6.611e-07
2456   iter 43: subdiag=[-0.+0.j  0.+0.j -0.-0.j], ||·||2=4.679e-07
2457   iter 44: subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=3.311e-07
2458   iter 45: subdiag=[-0.-0.j  0.+0.j -0.-0.j], ||·||2=2.344e-07
2459   iter 46: subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=1.659e-07
2460   iter 47: subdiag=[-0.+0.j  0.+0.j -0.-0.j], ||·||2=1.174e-07
2461   iter 48: subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=8.308e-08
2462   iter 49: subdiag=[-0.+0.j  0.+0.j -0.-0.j], ||·||2=5.880e-08
2463   iter 50: subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=4.161e-08
2464   iter 51: subdiag=[-0.+0.j  0.+0.j -0.-0.j], ||·||2=2.945e-08
2465   iter 52: subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=2.084e-08
2466   iter 53: subdiag=[-0.+0.j  0.+0.j -0.-0.j], ||·||2=1.475e-08
2467   iter 54: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=1.044e-08
2468   iter 55: subdiag=[-0.-0.j  0.+0.j -0.-0.j], ||·||2=7.388e-09
2469   iter 56: subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=5.229e-09
2470   iter 57: subdiag=[-0.-0.j  0.+0.j -0.-0.j], ||·||2=3.701e-09
2471   iter 58: subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=2.619e-09
2472   iter 59: subdiag=[-0.-0.j  0.+0.j -0.-0.j], ||·||2=1.854e-09
2473   iter 60: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=1.312e-09
2474   iter 61: subdiag=[-0.-0.j  0.+0.j -0.-0.j], ||·||2=9.284e-10
2475   iter 62: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=6.570e-10
2476   iter 63: subdiag=[-0.+0.j  0.+0.j -0.-0.j], ||·||2=4.650e-10
2477   iter 64: subdiag=[-0.-0.j -0.-0.j -0.-0.j], ||·||2=3.291e-10
2478   iter 65: subdiag=[-0.+0.j  0.+0.j -0.-0.j], ||·||2=2.329e-10
2479   iter 66: subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=1.648e-10
2480   iter 67: subdiag=[-0.+0.j  0.+0.j -0.-0.j], ||·||2=1.167e-10
2481   iter 68: subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=8.256e-11
2482   after : subdiag=[-0.+0.j -0.-0.j -0.-0.j], ||·||2=8.256e-11
2483 | iterations      = 69/100
2484 | sub-diag magnitudes after last step:
2485 | [0. 0. 0.]
2486 └ converged?     = yes

```

```

2487
2488 └─ Matrix 25/30 (size 4x4)
2489 | fixed shift  $\mu = -0.65739-0.393338j$  ( $|\mu|=0.7661$ )
2490 before: subdiag=[ 0.282+0.j 0.81 +0.j -0.56 +0.j],  $\|\cdot\|_2=1.024e+00$ 
2491 iter 00: subdiag=[-0.423+0.j 0.699+0.j 0.406+0.j],  $\|\cdot\|_2=9.128e-01$ 
2492 iter 01: subdiag=[ 0.525-0.j 0.476+0.j -0.313+0.j],  $\|\cdot\|_2=7.751e-01$ 
2493 iter 02: subdiag=[ 0.389-0.j -0.366-0.j 0.245+0.j],  $\|\cdot\|_2=5.879e-01$ 
2494 iter 03: subdiag=[-0.207-0.j 0.318+0.j -0.189+0.j],  $\|\cdot\|_2=4.242e-01$ 
2495 iter 04: subdiag=[ 0.1 -0.j -0.287-0.j 0.143+0.j],  $\|\cdot\|_2=3.359e-01$ 
2496 iter 05: subdiag=[-0.047+0.j 0.259+0.j -0.108-0.j],  $\|\cdot\|_2=2.846e-01$ 
2497 iter 06: subdiag=[ 0.022+0.j -0.233-0.j 0.081+0.j],  $\|\cdot\|_2=2.481e-01$ 
2498 iter 07: subdiag=[-0.011-0.j 0.21 +0.j -0.061-0.j],  $\|\cdot\|_2=2.187e-01$ 
2499 iter 08: subdiag=[ 0.005-0.j -0.188+0.j 0.045+0.j],  $\|\cdot\|_2=1.937e-01$ 
2500 iter 09: subdiag=[-0.002-0.j 0.169+0.j -0.034-0.j],  $\|\cdot\|_2=1.721e-01$ 
2501 iter 10: subdiag=[ 0.001+0.j -0.151-0.j 0.025+0.j],  $\|\cdot\|_2=1.531e-01$ 
2502 iter 11: subdiag=[-0.001-0.j 0.135-0.j -0.019+0.j],  $\|\cdot\|_2=1.363e-01$ 
2503 iter 12: subdiag=[ 0. +0.j -0.121-0.j 0.014+0.j],  $\|\cdot\|_2=1.215e-01$ 
2504 iter 13: subdiag=[-0. +0.j 0.108+0.j -0.011+0.j],  $\|\cdot\|_2=1.083e-01$ 
2505 iter 14: subdiag=[ 0. +0.j -0.096+0.j 0.008+0.j],  $\|\cdot\|_2=9.653e-02$ 
2506 iter 15: subdiag=[-0. -0.j 0.086+0.j -0.006-0.j],  $\|\cdot\|_2=8.606e-02$ 
2507 iter 16: subdiag=[ 0. +0.j -0.077-0.j 0.004+0.j],  $\|\cdot\|_2=7.673e-02$ 
2508 iter 17: subdiag=[-0. -0.j 0.068+0.j -0.003-0.j],  $\|\cdot\|_2=6.842e-02$ 
2509 iter 18: subdiag=[ 0. +0.j -0.061-0.j 0.003+0.j],  $\|\cdot\|_2=6.100e-02$ 
2510 iter 19: subdiag=[-0. +0.j 0.054+0.j -0.002+0.j],  $\|\cdot\|_2=5.438e-02$ 
2511 iter 20: subdiag=[ 0. -0.j -0.048-0.j 0.001+0.j],  $\|\cdot\|_2=4.848e-02$ 
2512 iter 21: subdiag=[-0. -0.j 0.043-0.j -0.001+0.j],  $\|\cdot\|_2=4.322e-02$ 
2513 iter 22: subdiag=[ 0. +0.j -0.039-0.j 0.001+0.j],  $\|\cdot\|_2=3.853e-02$ 
2514 iter 23: subdiag=[-0. -0.j 0.034+0.j -0.001+0.j],  $\|\cdot\|_2=3.434e-02$ 
2515 iter 24: subdiag=[ 0. +0.j -0.031-0.j 0. +0.j],  $\|\cdot\|_2=3.061e-02$ 
2516 iter 25: subdiag=[-0. -0.j 0.027+0.j -0. +0.j],  $\|\cdot\|_2=2.729e-02$ 
2517 iter 26: subdiag=[ 0. +0.j -0.024-0.j 0. +0.j],  $\|\cdot\|_2=2.432e-02$ 
2518 iter 27: subdiag=[-0. -0.j 0.022+0.j -0. +0.j],  $\|\cdot\|_2=2.168e-02$ 
2519 iter 28: subdiag=[ 0. -0.j -0.019-0.j 0. -0.j],  $\|\cdot\|_2=1.932e-02$ 
2520 iter 29: subdiag=[-0. +0.j 0.017-0.j -0. +0.j],  $\|\cdot\|_2=1.722e-02$ 
2521 iter 30: subdiag=[ 0. -0.j -0.015-0.j 0. -0.j],  $\|\cdot\|_2=1.535e-02$ 
2522 iter 31: subdiag=[-0. +0.j 0.014+0.j -0. +0.j],  $\|\cdot\|_2=1.368e-02$ 
2523 iter 32: subdiag=[ 0. +0.j -0.012-0.j 0. +0.j],  $\|\cdot\|_2=1.220e-02$ 
2524 iter 33: subdiag=[-0. -0.j 0.011+0.j -0. +0.j],  $\|\cdot\|_2=1.087e-02$ 
2525 iter 34: subdiag=[ 0. +0.j -0.01-0.j 0. +0.j],  $\|\cdot\|_2=9.688e-03$ 
2526 iter 35: subdiag=[-0. -0.j 0.009+0.j -0. -0.j],  $\|\cdot\|_2=8.635e-03$ 
2527 iter 36: subdiag=[ 0. +0.j -0.008-0.j 0. +0.j],  $\|\cdot\|_2=7.696e-03$ 
2528 iter 37: subdiag=[-0. -0.j 0.007+0.j -0. -0.j],  $\|\cdot\|_2=6.860e-03$ 
2529 iter 38: subdiag=[ 0. +0.j -0.006-0.j 0. +0.j],  $\|\cdot\|_2=6.114e-03$ 
2530 iter 39: subdiag=[-0. -0.j 0.005+0.j -0. -0.j],  $\|\cdot\|_2=5.449e-03$ 
2531 iter 40: subdiag=[ 0. +0.j -0.005-0.j 0. +0.j],  $\|\cdot\|_2=4.857e-03$ 
2532 iter 41: subdiag=[-0. -0.j 0.004+0.j -0. +0.j],  $\|\cdot\|_2=4.329e-03$ 

```

2533	iter 42: subdiag=[0. +0.j -0.004+0.j 0. -0.j], · ₂ =3.858e-03
2534	iter 43: subdiag=[-0. -0.j 0.003-0.j -0. +0.j], · ₂ =3.439e-03
2535	iter 44: subdiag=[0. +0.j -0.003+0.j 0. -0.j], · ₂ =3.065e-03
2536	iter 45: subdiag=[-0. -0.j 0.003+0.j -0. +0.j], · ₂ =2.731e-03
2537	iter 46: subdiag=[0. +0.j -0.002-0.j 0. -0.j], · ₂ =2.434e-03
2538	iter 47: subdiag=[-0. -0.j 0.002+0.j -0. +0.j], · ₂ =2.170e-03
2539	iter 48: subdiag=[0. +0.j -0.002-0.j 0. -0.j], · ₂ =1.934e-03
2540	iter 49: subdiag=[-0. -0.j 0.002+0.j -0. +0.j], · ₂ =1.724e-03
2541	iter 50: subdiag=[0. +0.j -0.002-0.j 0. -0.j], · ₂ =1.536e-03
2542	iter 51: subdiag=[-0. -0.j 0.001+0.j -0. +0.j], · ₂ =1.369e-03
2543	iter 52: subdiag=[0. +0.j -0.001-0.j 0. -0.j], · ₂ =1.220e-03
2544	iter 53: subdiag=[-0. -0.j 0.001-0.j -0. +0.j], · ₂ =1.088e-03
2545	iter 54: subdiag=[0. +0.j -0.001-0.j 0. -0.j], · ₂ =9.694e-04
2546	iter 55: subdiag=[-0. -0.j 0.001+0.j -0. +0.j], · ₂ =8.640e-04
2547	iter 56: subdiag=[0. +0.j -0.001-0.j 0. -0.j], · ₂ =7.701e-04
2548	iter 57: subdiag=[-0. -0.j 0.001+0.j -0. +0.j], · ₂ =6.863e-04
2549	iter 58: subdiag=[0. +0.j -0.001-0.j 0. -0.j], · ₂ =6.117e-04
2550	iter 59: subdiag=[-0. -0.j 0.001+0.j -0. +0.j], · ₂ =5.452e-04
2551	iter 60: subdiag=[0.+0.j -0.-0.j 0.+0.j], · ₂ =4.859e-04
2552	iter 61: subdiag=[-0.-0.j 0.+0.j -0.-0.j], · ₂ =4.331e-04
2553	iter 62: subdiag=[0.+0.j -0.-0.j 0.+0.j], · ₂ =3.860e-04
2554	iter 63: subdiag=[-0.-0.j 0.+0.j -0.-0.j], · ₂ =3.440e-04
2555	iter 64: subdiag=[0.+0.j -0.-0.j 0.+0.j], · ₂ =3.066e-04
2556	iter 65: subdiag=[-0.-0.j 0.+0.j -0.-0.j], · ₂ =2.733e-04
2557	iter 66: subdiag=[0.+0.j -0.-0.j 0.+0.j], · ₂ =2.436e-04
2558	iter 67: subdiag=[-0.-0.j 0.+0.j -0.-0.j], · ₂ =2.171e-04
2559	iter 68: subdiag=[0.+0.j -0.-0.j 0.+0.j], · ₂ =1.935e-04
2560	iter 69: subdiag=[-0.-0.j 0.+0.j -0.-0.j], · ₂ =1.725e-04
2561	iter 70: subdiag=[0.+0.j -0.-0.j 0.+0.j], · ₂ =1.537e-04
2562	iter 71: subdiag=[-0.-0.j 0.+0.j -0.-0.j], · ₂ =1.370e-04
2563	iter 72: subdiag=[0.+0.j -0.-0.j 0.+0.j], · ₂ =1.221e-04
2564	iter 73: subdiag=[-0.-0.j 0.+0.j -0.-0.j], · ₂ =1.088e-04
2565	iter 74: subdiag=[0.+0.j -0.-0.j 0.+0.j], · ₂ =9.699e-05
2566	iter 75: subdiag=[-0.-0.j 0.+0.j -0.-0.j], · ₂ =8.644e-05
2567	iter 76: subdiag=[0.+0.j -0.-0.j 0.+0.j], · ₂ =7.704e-05
2568	iter 77: subdiag=[-0.-0.j 0.+0.j -0.-0.j], · ₂ =6.867e-05
2569	iter 78: subdiag=[0.+0.j -0.-0.j 0.+0.j], · ₂ =6.120e-05
2570	iter 79: subdiag=[-0.-0.j 0.+0.j -0.-0.j], · ₂ =5.455e-05
2571	iter 80: subdiag=[0.+0.j -0.-0.j 0.+0.j], · ₂ =4.862e-05
2572	iter 81: subdiag=[-0.-0.j 0.+0.j -0.+0.j], · ₂ =4.333e-05
2573	iter 82: subdiag=[0.+0.j -0.-0.j 0.-0.j], · ₂ =3.862e-05
2574	iter 83: subdiag=[-0.-0.j 0.+0.j -0.+0.j], · ₂ =3.442e-05
2575	iter 84: subdiag=[0.+0.j -0.-0.j 0.-0.j], · ₂ =3.068e-05
2576	iter 85: subdiag=[-0.-0.j 0.+0.j -0.+0.j], · ₂ =2.734e-05
2577	iter 86: subdiag=[0.+0.j -0.-0.j 0.+0.j], · ₂ =2.437e-05
2578	iter 87: subdiag=[-0.-0.j 0.+0.j -0.-0.j], · ₂ =2.172e-05

```

2579   iter 88: subdiag=[ 0.+0.j -0.-0.j  0.+0.j],  ||·||2=1.936e-05
2580   iter 89: subdiag=[-0.-0.j  0.+0.j -0.+0.j],  ||·||2=1.725e-05
2581   iter 90: subdiag=[ 0.+0.j -0.-0.j  0.-0.j],  ||·||2=1.538e-05
2582   iter 91: subdiag=[-0.-0.j  0.+0.j -0.+0.j],  ||·||2=1.371e-05
2583   iter 92: subdiag=[ 0.+0.j -0.-0.j  0.-0.j],  ||·||2=1.222e-05
2584   iter 93: subdiag=[-0.-0.j  0.+0.j -0.+0.j],  ||·||2=1.089e-05
2585   iter 94: subdiag=[ 0.+0.j -0.-0.j  0.-0.j],  ||·||2=9.704e-06
2586   iter 95: subdiag=[-0.-0.j  0.-0.j -0.+0.j],  ||·||2=8.649e-06
2587   iter 96: subdiag=[ 0.+0.j -0.+0.j  0.-0.j],  ||·||2=7.708e-06
2588   iter 97: subdiag=[-0.-0.j  0.-0.j -0.+0.j],  ||·||2=6.870e-06
2589   iter 98: subdiag=[ 0.+0.j -0.+0.j  0.-0.j],  ||·||2=6.123e-06
2590   iter 99: subdiag=[-0.-0.j  0.-0.j -0.+0.j],  ||·||2=5.458e-06
2591   after : subdiag=[-0.-0.j  0.-0.j -0.+0.j],  ||·||2=5.458e-06
2592 | iterations      = 100/100
2593 | sub-diag magnitudes after last step:
2594 | [0. 0. 0.]
2595 └ converged?      = no
2596
2597 └ Matrix 26/30  (size 4x4)
2598 | fixed shift μ = 0.755595 (|μ|=0.7556)
2599   before: subdiag=[-0.994+0.j -0.988+0.j -0.76 +0.j],  ||·||2=1.594e+00
2600   iter 00: subdiag=[-0.882+0.j -0.998+0.j -0.19 +0.j],  ||·||2=1.345e+00
2601   iter 01: subdiag=[-0.701+0.j -0.997+0.j -0.039+0.j],  ||·||2=1.219e+00
2602   iter 02: subdiag=[-0.503+0.j -0.988+0.j -0.008+0.j],  ||·||2=1.109e+00
2603   iter 03: subdiag=[-0.338+0.j -0.982+0.j -0.002+0.j],  ||·||2=1.039e+00
2604   iter 04: subdiag=[-0.221+0.j -0.98 +0.j -0.   +0.j],  ||·||2=1.004e+00
2605   iter 05: subdiag=[-0.142+0.j -0.978+0.j -0.   +0.j],  ||·||2=9.887e-01
2606   iter 06: subdiag=[-0.091+0.j -0.978+0.j -0.   +0.j],  ||·||2=9.821e-01
2607   iter 07: subdiag=[-0.058+0.j -0.978+0.j -0.   +0.j],  ||·||2=9.794e-01
2608   iter 08: subdiag=[-0.037+0.j -0.978+0.j -0.   +0.j],  ||·||2=9.783e-01
2609   iter 09: subdiag=[-0.024+0.j -0.978+0.j -0.   +0.j],  ||·||2=9.779e-01
2610   iter 10: subdiag=[-0.015+0.j -0.978+0.j -0.   +0.j],  ||·||2=9.777e-01
2611   iter 11: subdiag=[-0.01 +0.j -0.978+0.j -0.   +0.j],  ||·||2=9.776e-01
2612   iter 12: subdiag=[-0.006+0.j -0.978+0.j -0.   +0.j],  ||·||2=9.776e-01
2613   iter 13: subdiag=[-0.004+0.j -0.978+0.j -0.   +0.j],  ||·||2=9.776e-01
2614   iter 14: subdiag=[-0.002+0.j -0.978+0.j -0.   +0.j],  ||·||2=9.776e-01
2615   iter 15: subdiag=[-0.002+0.j -0.978+0.j -0.   +0.j],  ||·||2=9.775e-01
2616   iter 16: subdiag=[-0.001+0.j -0.978+0.j -0.   +0.j],  ||·||2=9.775e-01
2617   iter 17: subdiag=[-0.001+0.j -0.978+0.j -0.   +0.j],  ||·||2=9.775e-01
2618   iter 18: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ||·||2=9.775e-01
2619   iter 19: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ||·||2=9.775e-01
2620   iter 20: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ||·||2=9.775e-01
2621   iter 21: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ||·||2=9.775e-01
2622   iter 22: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ||·||2=9.775e-01
2623   iter 23: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ||·||2=9.775e-01
2624   iter 24: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ||·||2=9.775e-01

```

[illegible]

```

2671   iter 71: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2672   iter 72: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2673   iter 73: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2674   iter 74: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2675   iter 75: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2676   iter 76: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2677   iter 77: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2678   iter 78: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2679   iter 79: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2680   iter 80: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2681   iter 81: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2682   iter 82: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2683   iter 83: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2684   iter 84: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2685   iter 85: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2686   iter 86: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2687   iter 87: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2688   iter 88: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2689   iter 89: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2690   iter 90: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2691   iter 91: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2692   iter 92: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2693   iter 93: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2694   iter 94: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2695   iter 95: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2696   iter 96: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2697   iter 97: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2698   iter 98: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2699   iter 99: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2700   after : subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j], ||·||2=9.775e-01
2701 | iterations      = 100/100
2702 | sub-diag magnitudes after last step:
2703 | [0.   0.978 0.   ]
2704 | └─ converged?    = no
2705
2706 └─ Matrix 27/30 (size 4x4)
2707 | fixed shift μ = -0.995116 (|μ|=0.9951)
2708   before: subdiag=[-0.999+0.j  0.99 +0.j -0.085+0.j], ||·||2=1.409e+00
2709   iter 00: subdiag=[-0.804+0.j  0.902+0.j -0.   +0.j], ||·||2=1.208e+00
2710   iter 01: subdiag=[-0.466+0.j  0.855+0.j -0.   +0.j], ||·||2=9.735e-01
2711   iter 02: subdiag=[-0.234+0.j  0.841+0.j -0.   +0.j], ||·||2=8.733e-01
2712   iter 03: subdiag=[-0.113+0.j  0.838+0.j -0.   +0.j], ||·||2=8.458e-01
2713   iter 04: subdiag=[-0.054+0.j  0.838+0.j -0.   +0.j], ||·||2=8.392e-01
2714   iter 05: subdiag=[-0.026+0.j  0.837+0.j -0.   +0.j], ||·||2=8.377e-01
2715   iter 06: subdiag=[-0.012+0.j  0.837+0.j -0.   +0.j], ||·||2=8.374e-01
2716   iter 07: subdiag=[-0.006+0.j  0.837+0.j -0.   +0.j], ||·||2=8.373e-01

```

[illegible]

[illegible]

```

2809   after : subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j], ||·||2=8.373e-01
2810 | iterations      = 100/100
2811 | sub-diag magnitudes after last step:
2812 | [0.   0.837 0.   ]
2813 └─ converged?      = no
2814
2815 └─ Matrix 28/30 (size 4x4)
2816 | fixed shift  $\mu$  = -0.976473 ( $|\mu|=0.9765$ )
2817   before: subdiag=[-0.799+0.j  0.936+0.j  0.149+0.j], ||·||2=1.239e+00
2818   iter 00: subdiag=[-0.683+0.j -0.093+0.j -0.118+0.j], ||·||2=6.991e-01
2819   iter 01: subdiag=[-0.682+0.j  0.006+0.j  0.118+0.j], ||·||2=6.923e-01
2820   iter 02: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2821   iter 03: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2822   iter 04: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2823   iter 05: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2824   iter 06: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2825   iter 07: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2826   iter 08: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2827   iter 09: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2828   iter 10: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2829   iter 11: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2830   iter 12: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2831   iter 13: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2832   iter 14: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2833   iter 15: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2834   iter 16: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2835   iter 17: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2836   iter 18: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2837   iter 19: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2838   iter 20: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2839   iter 21: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2840   iter 22: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2841   iter 23: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2842   iter 24: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2843   iter 25: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2844   iter 26: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2845   iter 27: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2846   iter 28: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2847   iter 29: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2848   iter 30: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2849   iter 31: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2850   iter 32: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2851   iter 33: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2852   iter 34: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2853   iter 35: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2854   iter 36: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01

```

[illegible]

```

2901   iter 83: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2902   iter 84: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2903   iter 85: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2904   iter 86: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2905   iter 87: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2906   iter 88: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2907   iter 89: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2908   iter 90: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2909   iter 91: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2910   iter 92: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2911   iter 93: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2912   iter 94: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2913   iter 95: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2914   iter 96: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2915   iter 97: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2916   iter 98: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j], ||·||2=6.922e-01
2917   iter 99: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2918   after : subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j], ||·||2=6.922e-01
2919 |   iterations      = 100/100
2920 |   sub-diag magnitudes after last step:
2921 |   [0.682 0.   0.118]
2922 └─ converged?      = no
2923
2924 └─ Matrix 29/30   (size 4x4)
2925 |   fixed shift μ = 0.790166 (|μ|=0.7902)
2926   before: subdiag=[ 0.811+0.j -0.867+0.j  0.948+0.j], ||·||2=1.519e+00
2927   iter 00: subdiag=[ 0.726+0.j -0.795+0.j  0.199+0.j], ||·||2=1.095e+00
2928   iter 01: subdiag=[ 0.664+0.j -0.832+0.j  0.027+0.j], ||·||2=1.064e+00
2929   iter 02: subdiag=[ 0.596+0.j -0.861+0.j  0.004+0.j], ||·||2=1.047e+00
2930   iter 03: subdiag=[ 0.525+0.j -0.883+0.j  0.   +0.j], ||·||2=1.028e+00
2931   iter 04: subdiag=[ 0.456+0.j -0.899+0.j  0.   +0.j], ||·||2=1.008e+00
2932   iter 05: subdiag=[ 0.391+0.j -0.909+0.j  0.   +0.j], ||·||2=9.899e-01
2933   iter 06: subdiag=[ 0.332+0.j -0.917+0.j  0.   +0.j], ||·||2=9.751e-01
2934   iter 07: subdiag=[ 0.28 +0.j -0.922+0.j  0.   +0.j], ||·||2=9.635e-01
2935   iter 08: subdiag=[ 0.234+0.j -0.925+0.j  0.   +0.j], ||·||2=9.547e-01
2936   iter 09: subdiag=[ 0.196+0.j -0.928+0.j  0.   +0.j], ||·||2=9.483e-01
2937   iter 10: subdiag=[ 0.163+0.j -0.93 +0.j  0.   +0.j], ||·||2=9.437e-01
2938   iter 11: subdiag=[ 0.135+0.j -0.931+0.j  0.   +0.j], ||·||2=9.405e-01
2939   iter 12: subdiag=[ 0.112+0.j -0.931+0.j  0.   +0.j], ||·||2=9.382e-01
2940   iter 13: subdiag=[ 0.093+0.j -0.932+0.j  0.   +0.j], ||·||2=9.366e-01
2941   iter 14: subdiag=[ 0.077+0.j -0.932+0.j  0.   +0.j], ||·||2=9.355e-01
2942   iter 15: subdiag=[ 0.064+0.j -0.933+0.j  0.   +0.j], ||·||2=9.348e-01
2943   iter 16: subdiag=[ 0.053+0.j -0.933+0.j  0.   +0.j], ||·||2=9.342e-01
2944   iter 17: subdiag=[ 0.044+0.j -0.933+0.j  0.   +0.j], ||·||2=9.339e-01
2945   iter 18: subdiag=[ 0.036+0.j -0.933+0.j  0.   +0.j], ||·||2=9.336e-01
2946   iter 19: subdiag=[ 0.03 +0.j -0.933+0.j  0.   +0.j], ||·||2=9.335e-01

```

[illegible]

```

2993   iter 66: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
2994   iter 67: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
2995   iter 68: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
2996   iter 69: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
2997   iter 70: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
2998   iter 71: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
2999   iter 72: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3000   iter 73: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3001   iter 74: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3002   iter 75: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3003   iter 76: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3004   iter 77: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3005   iter 78: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3006   iter 79: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3007   iter 80: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3008   iter 81: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3009   iter 82: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3010   iter 83: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3011   iter 84: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3012   iter 85: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3013   iter 86: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3014   iter 87: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3015   iter 88: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3016   iter 89: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3017   iter 90: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3018   iter 91: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3019   iter 92: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3020   iter 93: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3021   iter 94: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3022   iter 95: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3023   iter 96: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3024   iter 97: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3025   iter 98: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3026   iter 99: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3027   after : subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j], ||·||2=9.331e-01
3028 | iterations      = 100/100
3029 | sub-diag magnitudes after last step:
3030 | [0.   0.933 0.   ]
3031 └ converged?      = no
3032
3033 └ Matrix 30/30 (size 4x4)
3034 | fixed shift μ = -0.0383725-0.203948j (|μ|=0.2075)
3035   before: subdiag=[-0.744+0.j -0.999+0.j -0.997+0.j], ||·||2=1.596e+00
3036   iter 00: subdiag=[-0.752-0.j -0.99 +0.j -0.977+0.j], ||·||2=1.581e+00
3037   iter 01: subdiag=[0.757+0.j 0.957+0.j 0.928-0.j], ||·||2=1.533e+00
3038   iter 02: subdiag=[-0.758-0.j -0.894+0.j -0.87 +0.j], ||·||2=1.460e+00

```

3039	iter 03: subdiag=[0.76 +0.j 0.803-0.j 0.814-0.j], $\ \cdot\ _2=1.372e+00$
3040	iter 04: subdiag=[-0.761-0.j -0.694-0.j -0.766+0.j], $\ \cdot\ _2=1.284e+00$
3041	iter 05: subdiag=[0.761+0.j 0.582-0.j 0.729-0.j], $\ \cdot\ _2=1.204e+00$
3042	iter 06: subdiag=[-0.758-0.j -0.477+0.j -0.699+0.j], $\ \cdot\ _2=1.136e+00$
3043	iter 07: subdiag=[-0.751-0.j -0.385-0.j 0.674-0.j], $\ \cdot\ _2=1.080e+00$
3044	iter 08: subdiag=[-0.738-0.j -0.309+0.j -0.653+0.j], $\ \cdot\ _2=1.033e+00$
3045	iter 09: subdiag=[-0.719-0.j -0.247+0.j 0.635-0.j], $\ \cdot\ _2=9.907e-01$
3046	iter 10: subdiag=[0.694+0.j -0.197-0.j -0.618+0.j], $\ \cdot\ _2=9.504e-01$
3047	iter 11: subdiag=[-0.665-0.j -0.158-0.j 0.602-0.j], $\ \cdot\ _2=9.105e-01$
3048	iter 12: subdiag=[0.631+0.j -0.126-0.j -0.586+0.j], $\ \cdot\ _2=8.704e-01$
3049	iter 13: subdiag=[-0.594-0.j -0.102-0.j 0.571-0.j], $\ \cdot\ _2=8.301e-01$
3050	iter 14: subdiag=[0.555+0.j -0.082-0.j -0.556+0.j], $\ \cdot\ _2=7.899e-01$
3051	iter 15: subdiag=[-0.516+0.j -0.066-0.j 0.541+0.j], $\ \cdot\ _2=7.503e-01$
3052	iter 16: subdiag=[0.476+0.j -0.054-0.j -0.526+0.j], $\ \cdot\ _2=7.118e-01$
3053	iter 17: subdiag=[-0.438+0.j -0.044-0.j 0.511+0.j], $\ \cdot\ _2=6.747e-01$
3054	iter 18: subdiag=[0.401-0.j -0.036-0.j -0.496-0.j], $\ \cdot\ _2=6.393e-01$
3055	iter 19: subdiag=[-0.366-0.j -0.029+0.j 0.482+0.j], $\ \cdot\ _2=6.059e-01$
3056	iter 20: subdiag=[0.333+0.j -0.024+0.j -0.467-0.j], $\ \cdot\ _2=5.745e-01$
3057	iter 21: subdiag=[-0.303+0.j -0.019+0.j 0.453+0.j], $\ \cdot\ _2=5.451e-01$
3058	iter 22: subdiag=[0.275+0.j -0.016+0.j -0.439+0.j], $\ \cdot\ _2=5.176e-01$
3059	iter 23: subdiag=[-0.248+0.j -0.013+0.j 0.425-0.j], $\ \cdot\ _2=4.921e-01$
3060	iter 24: subdiag=[0.224-0.j -0.011+0.j -0.411+0.j], $\ \cdot\ _2=4.682e-01$
3061	iter 25: subdiag=[-0.203+0.j -0.009-0.j 0.397-0.j], $\ \cdot\ _2=4.460e-01$
3062	iter 26: subdiag=[0.183+0.j -0.007-0.j -0.384+0.j], $\ \cdot\ _2=4.253e-01$
3063	iter 27: subdiag=[-0.165-0.j -0.006-0.j 0.371-0.j], $\ \cdot\ _2=4.060e-01$
3064	iter 28: subdiag=[0.148+0.j -0.005-0.j -0.358+0.j], $\ \cdot\ _2=3.878e-01$
3065	iter 29: subdiag=[-0.134+0.j -0.004-0.j 0.346-0.j], $\ \cdot\ _2=3.708e-01$
3066	iter 30: subdiag=[0.12 -0.j -0.003-0.j -0.334+0.j], $\ \cdot\ _2=3.548e-01$
3067	iter 31: subdiag=[-0.108-0.j -0.003-0.j 0.322-0.j], $\ \cdot\ _2=3.397e-01$
3068	iter 32: subdiag=[0.097+0.j -0.002-0.j -0.311+0.j], $\ \cdot\ _2=3.255e-01$
3069	iter 33: subdiag=[-0.088+0.j -0.002-0.j 0.299-0.j], $\ \cdot\ _2=3.120e-01$
3070	iter 34: subdiag=[0.079+0.j -0.002-0.j -0.289+0.j], $\ \cdot\ _2=2.992e-01$
3071	iter 35: subdiag=[-0.071-0.j -0.001-0.j 0.278-0.j], $\ \cdot\ _2=2.870e-01$
3072	iter 36: subdiag=[0.064-0.j -0.001+0.j -0.268+0.j], $\ \cdot\ _2=2.754e-01$
3073	iter 37: subdiag=[-0.057+0.j -0.001+0.j 0.258-0.j], $\ \cdot\ _2=2.644e-01$
3074	iter 38: subdiag=[0.051-0.j -0.001+0.j -0.249+0.j], $\ \cdot\ _2=2.538e-01$
3075	iter 39: subdiag=[-0.046+0.j -0.001+0.j 0.239-0.j], $\ \cdot\ _2=2.437e-01$
3076	iter 40: subdiag=[0.042-0.j -0. +0.j -0.23 +0.j], $\ \cdot\ _2=2.341e-01$
3077	iter 41: subdiag=[-0.037+0.j -0. -0.j 0.222-0.j], $\ \cdot\ _2=2.249e-01$
3078	iter 42: subdiag=[0.034+0.j -0. -0.j -0.213+0.j], $\ \cdot\ _2=2.160e-01$
3079	iter 43: subdiag=[-0.03 +0.j -0. -0.j 0.205-0.j], $\ \cdot\ _2=2.075e-01$
3080	iter 44: subdiag=[0.027-0.j -0. -0.j -0.198+0.j], $\ \cdot\ _2=1.994e-01$
3081	iter 45: subdiag=[-0.024+0.j -0. -0.j 0.19 -0.j], $\ \cdot\ _2=1.916e-01$
3082	iter 46: subdiag=[0.022-0.j -0. -0.j -0.183+0.j], $\ \cdot\ _2=1.841e-01$
3083	iter 47: subdiag=[-0.02 +0.j -0. -0.j 0.176-0.j], $\ \cdot\ _2=1.769e-01$
3084	iter 48: subdiag=[0.018-0.j -0. -0.j -0.169+0.j], $\ \cdot\ _2=1.700e-01$

3085	iter 49: subdiag=[-0.016+0.j -0. -0.j 0.163-0.j], $\ \cdot\ _2=1.634e-01$
3086	iter 50: subdiag=[0.014-0.j -0. -0.j -0.156+0.j], $\ \cdot\ _2=1.570e-01$
3087	iter 51: subdiag=[-0.013+0.j -0. -0.j 0.15 -0.j], $\ \cdot\ _2=1.509e-01$
3088	iter 52: subdiag=[0.012-0.j -0. -0.j -0.145+0.j], $\ \cdot\ _2=1.450e-01$
3089	iter 53: subdiag=[-0.01 +0.j -0. -0.j 0.139+0.j], $\ \cdot\ _2=1.393e-01$
3090	iter 54: subdiag=[0.009-0.j -0. +0.j -0.134-0.j], $\ \cdot\ _2=1.339e-01$
3091	iter 55: subdiag=[-0.008-0.j -0. +0.j 0.128+0.j], $\ \cdot\ _2=1.286e-01$
3092	iter 56: subdiag=[0.008-0.j -0. +0.j -0.123-0.j], $\ \cdot\ _2=1.236e-01$
3093	iter 57: subdiag=[-0.007+0.j -0. +0.j 0.119+0.j], $\ \cdot\ _2=1.188e-01$
3094	iter 58: subdiag=[0.006-0.j -0. +0.j -0.114-0.j], $\ \cdot\ _2=1.141e-01$
3095	iter 59: subdiag=[-0.005+0.j -0. +0.j 0.11 +0.j], $\ \cdot\ _2=1.097e-01$
3096	iter 60: subdiag=[0.005-0.j -0. +0.j -0.105-0.j], $\ \cdot\ _2=1.054e-01$
3097	iter 61: subdiag=[-0.004+0.j -0. +0.j 0.101+0.j], $\ \cdot\ _2=1.013e-01$
3098	iter 62: subdiag=[0.004-0.j -0. -0.j -0.097-0.j], $\ \cdot\ _2=9.730e-02$
3099	iter 63: subdiag=[-0.004+0.j -0. -0.j 0.093+0.j], $\ \cdot\ _2=9.349e-02$
3100	iter 64: subdiag=[0.003-0.j -0. -0.j -0.09 -0.j], $\ \cdot\ _2=8.983e-02$
3101	iter 65: subdiag=[-0.003+0.j -0. -0.j 0.086+0.j], $\ \cdot\ _2=8.631e-02$
3102	iter 66: subdiag=[0.003-0.j -0. -0.j -0.083-0.j], $\ \cdot\ _2=8.292e-02$
3103	iter 67: subdiag=[-0.002+0.j -0. -0.j 0.08 +0.j], $\ \cdot\ _2=7.967e-02$
3104	iter 68: subdiag=[0.002-0.j -0. -0.j -0.077-0.j], $\ \cdot\ _2=7.655e-02$
3105	iter 69: subdiag=[-0.002+0.j -0. -0.j 0.074+0.j], $\ \cdot\ _2=7.355e-02$
3106	iter 70: subdiag=[0.002-0.j -0. -0.j -0.071-0.j], $\ \cdot\ _2=7.066e-02$
3107	iter 71: subdiag=[-0.002+0.j -0. -0.j 0.068+0.j], $\ \cdot\ _2=6.789e-02$
3108	iter 72: subdiag=[0.001-0.j -0. -0.j -0.065-0.j], $\ \cdot\ _2=6.522e-02$
3109	iter 73: subdiag=[-0.001+0.j -0. -0.j 0.063+0.j], $\ \cdot\ _2=6.266e-02$
3110	iter 74: subdiag=[0.001-0.j -0. -0.j -0.06 -0.j], $\ \cdot\ _2=6.020e-02$
3111	iter 75: subdiag=[-0.001+0.j -0. -0.j 0.058+0.j], $\ \cdot\ _2=5.783e-02$
3112	iter 76: subdiag=[0.001-0.j -0. -0.j -0.056-0.j], $\ \cdot\ _2=5.556e-02$
3113	iter 77: subdiag=[-0.001+0.j -0. -0.j 0.053+0.j], $\ \cdot\ _2=5.338e-02$
3114	iter 78: subdiag=[0.001-0.j -0. -0.j -0.051-0.j], $\ \cdot\ _2=5.128e-02$
3115	iter 79: subdiag=[-0.001+0.j -0. -0.j 0.049+0.j], $\ \cdot\ _2=4.926e-02$
3116	iter 80: subdiag=[0.001-0.j -0. -0.j -0.047-0.j], $\ \cdot\ _2=4.733e-02$
3117	iter 81: subdiag=[-0.001+0.j -0. -0.j 0.045+0.j], $\ \cdot\ _2=4.547e-02$
3118	iter 82: subdiag=[0. -0.j -0. -0.j -0.044-0.j], $\ \cdot\ _2=4.368e-02$
3119	iter 83: subdiag=[-0. +0.j -0. -0.j 0.042+0.j], $\ \cdot\ _2=4.196e-02$
3120	iter 84: subdiag=[0. -0.j -0. -0.j -0.04-0.j], $\ \cdot\ _2=4.031e-02$
3121	iter 85: subdiag=[-0. +0.j -0. -0.j 0.039+0.j], $\ \cdot\ _2=3.872e-02$
3122	iter 86: subdiag=[0. -0.j -0. -0.j -0.037-0.j], $\ \cdot\ _2=3.720e-02$
3123	iter 87: subdiag=[-0. +0.j -0. -0.j 0.036+0.j], $\ \cdot\ _2=3.574e-02$
3124	iter 88: subdiag=[0. -0.j -0. -0.j -0.034-0.j], $\ \cdot\ _2=3.433e-02$
3125	iter 89: subdiag=[-0. +0.j -0. -0.j 0.033+0.j], $\ \cdot\ _2=3.298e-02$
3126	iter 90: subdiag=[0. -0.j -0. -0.j -0.032-0.j], $\ \cdot\ _2=3.168e-02$
3127	iter 91: subdiag=[-0. +0.j -0. -0.j 0.03+0.j], $\ \cdot\ _2=3.043e-02$
3128	iter 92: subdiag=[0. -0.j -0. -0.j -0.029-0.j], $\ \cdot\ _2=2.924e-02$
3129	iter 93: subdiag=[-0. +0.j -0. -0.j 0.028+0.j], $\ \cdot\ _2=2.809e-02$
3130	iter 94: subdiag=[0. -0.j -0. -0.j -0.027-0.j], $\ \cdot\ _2=2.698e-02$


```

3131 iter 95: subdiag=[-0.  +0.j -0.  -0.j  0.026+0.j], ||·||2=2.592e-02
3132 iter 96: subdiag=[ 0.  -0.j -0.  -0.j -0.025-0.j], ||·||2=2.490e-02
3133 iter 97: subdiag=[-0.  +0.j -0.  -0.j  0.024+0.j], ||·||2=2.392e-02
3134 iter 98: subdiag=[ 0.  -0.j -0.  -0.j -0.023-0.j], ||·||2=2.298e-02
3135 iter 99: subdiag=[-0.  +0.j -0.  -0.j  0.022+0.j], ||·||2=2.207e-02
3136 after : subdiag=[-0.  +0.j -0.  -0.j  0.022+0.j], ||·||2=2.207e-02
3137 | iterations      = 100/100
3138 | sub-diag magnitudes after last step:
3139 | [0.    0.    0.022]
3140 └ converged?      = no

```

From this monstrosity we conclude that using one constant shift μ (taken from the trailing 2×2 block) attacks only the last sub-diagonal element. That entry shrinks quickly, but the higher sub-diagonals are influenced only through round-off-sized couplings, so they plateau at some non-zero value $\approx 10^{-2}$. Consequently the matrix rarely satisfies the stringent “ $|H_{i,i-1}| < 10^{-10}$ ” test, and the script reports converged = no even after 100 iterations.

Using the Wilkinson shift would definitely force convergence, but since we are using one $\mu = \lambda_1$ of the trailing 2×2 block, we can expect convergence only for the last sub-diagonal element.

Bibliography

[1] L. N. Trefethen and D. Bau, *Numerical Linear Algebra*. 1997.