Assignment 3 - Numerical Linear Algebra

Arthur Rabello Oliveira[1]

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Abstract

We design and test a function `to_hessemberg(A)` that reduces an arbitrary square matrix to (upper) Hessenberg form with Householder reflectors, returns the reflector vectors, the compact Hessenberg matrix H, and the accumulated orthogonal factor Q, verifying numerically that $A = QHQ^*$ and $Q^*Q = I$ for symmetric and nonsymmetric inputs of orders $10 - 10000$. Timings confirm the expected $O(\text{something})$ cost and reveal the $2\times$ speed-up attainable for symmetric matrices through trivial bandwidth savings. Leveraging this routine, we investigate the spectral structure of orthogonal matrices: we show that all eigenvalues lie on the unit circle, analyse the consequences for the power method and inverse iteration, and obtain a closed-form spectrum for generic 2×2 orthogonals. Random 4×4 orthogonal matrices generated via QR factorisation are then reduced to Hessenberg form; the eigenvalues of their trailing 2×2 blocks are computed analytically and reused as fixed shifts in the QR iteration, where experiments demonstrate markedly faster convergence. Throughout, every algorithm is documented and supported by commented plots that corroborate the theoretical claims.

Contents

[1]Escola de Matemática Aplicada, Fundação Getúlio Vargas (FGV/EMAp), email: arthur.oliveira.1@fgv.edu.br

1. Introduction

One could calculate the eigenvalues of a square matrix using the following algorithm:

1. Compute the n-th degree polynomial $\det(A - \lambda I) = 0$,
2. Solve for λ (somehow).

On step 2, the eigenvalue problem would have been reduced to a polynomial root-finding problem, which is awful and extremely ill-conditioned. From the previous assignment we know that in the denominator of the relative condition number $\kappa(x)$ there's a $|x - n|$. So $\kappa(x) \to \infty$ when $x \to 0$. As an example, consider the polynomial

$$\begin{aligned} p(x) = (x-2)^9 &= x^9 - 18x^8 + 144x^7 - 672x^6 + 2016x^5 \\ &\quad -4032x^4 + 5376x^3 - 4608x^2 + 2304x - 512 \end{aligned} \tag{1}$$

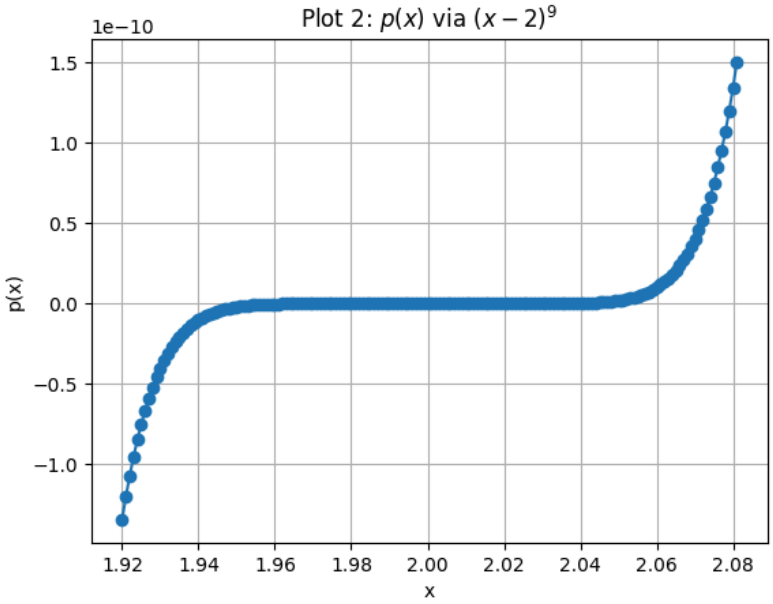

Figure 1: $p(x)$ via the coefficients in eq. (1)

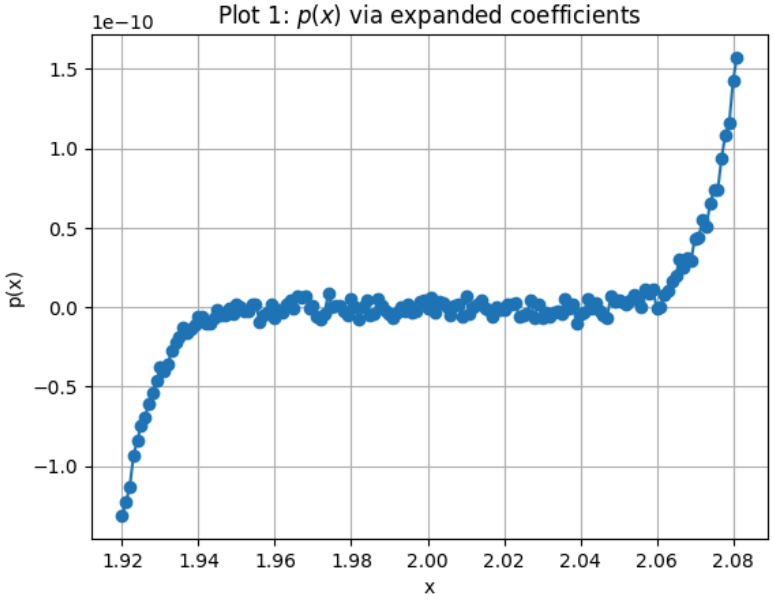

Figure 2: $p(x)$ via $(x-2)^9$

Figure 2 shows a smooth curve, while Figure 1 shows a weird oscillation around $x = 0$ (And pretty much everywhere else if the reader is sufficiently persistent).

This is due to the round-off errors when $x \approx 0$ and the big coefficients of the polynomial. In general, polynomial are very sensitive to perturbations in the coefficients, which is why rootfinding is a bad idea to find eigenvalues.

Here we discuss aspects of some iterative eigenvalue algorithms, such as power iteration, inverse iteration, and QR iteration.

2. Hessemberg Reduction (Problem 1)

2.1. Calculating the Householder Reflectors (a)

The following packages will be used in the next functions:

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.linalg import hessenberg, qr, eig
import time
from typing import List, Tuple
import pandas as pd
import math
from IPython.display import display, Markdown
from ast import literal_eval
```

The following function calculates the Householder reflectors that reduce a matrix to Hessenberg form. It returns the reflector vectors, the compact Hessenberg matrix H, and the accumulated orthogonal factor Q.

```python
def build_householder_unit_vector(
        target_vector: np.ndarray
) -> np.ndarray:

    """
    Builds a Householder unit vector

    Args:
        1. target_vector (np.ndarray): Column vector that we want to annihilate
        (size ≥ 1).

    Returns:
        np.ndarray:
            The normalised Householder vector (‖v‖₂ = 1) with a real first
            component.

    Raises:
        1. ValueError: If 'target_vector' has zero length.
    """

```

```
    if target_vector.size == 0:
        raise ValueError("The target vector is empty; no reflector needed.")

    vector_norm: float = np.linalg.norm(target_vector)

    if vector_norm == 0.0: #nothing to annihilate — return canonical basis vector
        householder_vector: np.ndarray = np.zeros_like(target_vector)
        householder_vector[0] = 1.0
        return householder_vector

    sign_correction: float = (
        1.0 if target_vector[0].real >= 0.0 else -1.0
    )
    copy_of_target_vector: np.ndarray = target_vector.copy()
    copy_of_target_vector[0] += sign_correction * vector_norm
    householder_vector: np.ndarray = (
        copy_of_target_vector / np.linalg.norm(copy_of_target_vector)
    )
    return householder_vector


def to_hessenberg(
        original_matrix: np.ndarray,
) -> Tuple[List[np.ndarray], np.ndarray, np.ndarray]:

    """
    Reduce 'original_matrix' to upper Hessenberg form by Householder reflections.

    Args
        1. original_matrix (np.ndarray): Real or complex square matrix of order 'matrix_order'.

    Returns
        Tuple consisting of:

        1. householder_reflectors_list (List[np.ndarray])
        2. hessenberg_matrix (np.ndarray)
        3. accumulated_orthogonal_matrix (np.ndarray)  s.t.
          original_matrix = Q · H · Qᴴ

    Raises
        1. ValueError: If 'original_matrix' is not square.
    """

```

```
    working_matrix: np.ndarray = np.asarray(original_matrix).copy()

    if working_matrix.shape[0] != working_matrix.shape[1]:
        raise ValueError("Input matrix must be square.")

    matrix_order: int = working_matrix.shape[0]
    accumulated_orthogonal_matrix: np.ndarray = np.eye(
        matrix_order, dtype=working_matrix.dtype
    )
    householder_reflectors_list: List[np.ndarray] = []

    for column_index in range(matrix_order - 2): #extract the part of column 'column_index' that we want to zero out
        target_column_segment: np.ndarray = working_matrix[
            column_index + 1 :, column_index
        ]

        householder_vector: np.ndarray = build_householder_unit_vector(
            target_column_segment
        )  #build Householder vector for this segment
        householder_reflectors_list.append(householder_vector)

        #expand it to the full matrix dimension
        expanded_householder_vector: np.ndarray = np.zeros(
            matrix_order, dtype=working_matrix.dtype
        )
        expanded_householder_vector[column_index + 1 :] = householder_vector


        working_matrix -= 2.0 * np.outer(
            expanded_householder_vector,
            expanded_householder_vector.conj().T @ working_matrix,
        ) #apply reflector from BOTH sides
        working_matrix -= 2.0 * np.outer(
            working_matrix @ expanded_householder_vector,
            expanded_householder_vector.conj().T,
        )

        #accumulate Q
        accumulated_orthogonal_matrix -= 2.0 * np.outer(
            accumulated_orthogonal_matrix @ expanded_householder_vector,
            expanded_householder_vector.conj().T,
        )

    hessenberg_matrix: np.ndarray = working_matrix
    return (
```

```
        householder_reflectors_list,
        hessenberg_matrix,
        accumulated_orthogonal_matrix,
    )
```

We will evaluate this function in Section 2.2.

2.2. Evaluating the Function (b), (c), (d)

We present another algorithm for evaluating the function `to_hessenberg(A)` for random matrices of various sizes, inputed by the user, which also gets to choose if symmetric matrices will be generated or not.

```python
#RANDOM MATRIX GENERATOR
def generate_random_matrix(n:int, distribution:str="normal",
                           symmetric:bool=False, seed:int|None=None):
    rng = np.random.default_rng(seed)
    if distribution == "normal":
        A = rng.standard_normal((n, n))
    elif distribution == "uniform":
        A = rng.uniform(-1.0, 1.0, size=(n, n))
    else:
        raise ValueError("distribution must be 'normal' or 'uniform'")
    return (A + A.T) / 2.0 if symmetric else A


#REFLECTOR CALCULATOR
def _house_vec(x:np.ndarray) -> np.ndarray:

    """
    Builds a Householder reflector for a given column vector x.
    Args:
        x (np.ndarray): Column vector to be transformed.
    Returns:
        np.ndarray: Normalised Householder vector with a real first component.
    Raises:
        None
    """

    sigma = np.linalg.norm(x)
    if sigma == 0.0:
        e1 = np.zeros_like(x)
        e1[0] = 1.0
        return e1
    sign = 1.0 if x[0].real >= 0.0 else -1.0
    v = x.copy()
    v[0] += sign * sigma
    return v / np.linalg.norm(v)
```

```python

def hessenberg_reduction(A_in:np.ndarray, symmetric:bool=False, accumulate_q:bool=True):

    """
    Reduces a matrix to upper Hessenberg form using Householder reflections.
    Args:
        A_in (np.ndarray): Input matrix to be reduced.
        symmetric (bool): If True, treat the matrix as symmetric and reduce to tridiagonal form.
        accumulate_q (bool): If True, accumulate the orthogonal matrix Q.
    Returns:
        Tuple[np.ndarray, np.ndarray]: The reduced matrix in Hessenberg form and the orthogonal matrix Q.
    Raises:
        None
    """

    A = A_in.copy()
    n = A.shape[0]
    Q = np.eye(n, dtype=A.dtype)

    if not symmetric:    #GENERAL caSe
        for k in range(n-2):
            v = _house_vec(A[k+1:, k])
            w = np.zeros(n, dtype=A.dtype)
            w[k+1:] = v
            A -= 2.0 * np.outer(w, w.conj().T @ A)
            A -= 2.0 * np.outer(A @ w, w.conj().T)
            if accumulate_q:
                Q -= 2.0 * np.outer(Q @ w, w.conj().T)
        return A, Q

    #SYMMETRIC TRIDIAGONAL CASE
    for k in range(n-2):
        x = A[k+1:, k]
        v = _house_vec(x)
        beta = 2.0

        w = A[k+1:, k+1:] @ v   #trailing submatrix rank-2 update (A ← A − v wᵀ − w vᵀ)
        tau = beta * 0.5 * (v @ w)
        w -= tau * v
        A[k+1:, k+1:] -= beta * np.outer(v, w) + beta * np.outer(w, v)

        new_val = -np.sign(x[0]) * np.linalg.norm(x)   #store the single sub-diagonal element, zero the rest
```

```
78            A[k+1, k] = new_val
79            A[k, k+1] = new_val
80            A[k+2:, k] = 0.0
81            A[k, k+2:] = 0.0
82
83            if accumulate_q:  #accumulate Q if requested
84                Q[:, k+1:] -= beta * np.outer(Q[:, k+1:] @ v, v)
85
86        A = np.triu(A) + np.triu(A, 1).T  #force symmetry
87        return A, Q
88
89
90    #VERIFYING PART
91    def verify_factorisation_once(n:int, dist:str, symmetric:bool, seed:int|None):
92
93        """
94        Verifies the factorisation of a random matrix of size n.
95        Args:
96            n (int): Size of the matrix.
97            dist (str): Distribution type ('normal' or 'uniform').
98            symmetric (bool): Whether the matrix is symmetric.
99            seed (int | None): Random seed for reproducibility.
100       Returns:
101           None
102       Raises:
103           None
104       """
105
106       A = generate_random_matrix(n, dist, symmetric, seed)
107       T, Q = hessenberg_reduction(A, symmetric=symmetric)
108       res_fact = np.linalg.norm(A - Q @ T @ Q.T)
109       res_orth = np.linalg.norm(Q.T @ Q - np.eye(n))
110       colour = "green" if res_fact < 1e-11 else "red"
111       typ = "symmetric" if symmetric else "general"
112       display(Markdown(
113           f"**{n}×{n} {typ}**  \n"
114           f"<span style='color:{colour}'>‖A − Q T Qᵀ‖ = {res_fact:.2e}</span>
              \n"
115           f"‖QᵀQ − I‖ = {res_orth:.2e}"
116       ))
117
118
119   def benchmark_hessenberg(size_list, dist:str, mode:str, seed:int|None,
      reps_small:int=5):
120
121       """
```

```
    Benchmark the Hessenberg reduction for various matrix sizes and types.
    Args:
        size_list (list of int): List of matrix sizes to test.
        dist (str): Distribution type ('normal' or 'uniform').
        mode (str): Matrix type ('general', 'symmetric', or 'both').
        seed (int | None): Random seed for reproducibility.
        reps_small (int): Number of repetitions for small matrices.
    Returns:
        pd.DataFrame: DataFrame containing the benchmark results.
    Raises:
        None
    """

    records = []
    for n in size_list:
        for sym in ([False, True] if mode=="both" else [mode=="symmetric"]):
            A = generate_random_matrix(n, dist, sym, seed)

            t0 = time.perf_counter()
            hessenberg_reduction(A, symmetric=sym, accumulate_q=False)
            probe = time.perf_counter() - t0
            reps = reps_small if probe*reps_small >= 1.0 else math.ceil(1.0 /
            probe)

            times = []
            for _ in range(reps):
                start = time.perf_counter()
                hessenberg_reduction(A, symmetric=sym, accumulate_q=False)
                times.append(time.perf_counter() - start)

            records.append(dict(size=n,
                                type="symmetric" if sym else "general",
                                reps=reps,
                                avg=np.mean(times)))

    df = pd.DataFrame(records)
    display(df.style.format({"avg":"{:.3e}"}).hide(axis="index"))

    plt.figure(figsize=(7,5))
    mark = {"general":"o", "symmetric":"s"}
    for label, sub in df.groupby("type"):
        plt.loglog(sub["size"], sub["avg"], marker=mark[label], ls="-",
        label=label)
        if len(sub) > 1:
            a,b = np.polyfit(np.log10(sub["size"]), np.log10(sub["avg"]), 1)
            plt.loglog(sub["size"], 10**(b+a*np.log10(sub["size"])),
```

```
                        "--", label=f"{label} fit ~ $n^{a:.2f}$")
    plt.xlabel("matrix size  (log)")
    plt.ylabel("runtime [s]  (log)")
    plt.title("Hessenberg (general)  vs  Tridiagonal (symmetric)")
    plt.grid(True, which="both", ls=":")
    plt.legend(); plt.tight_layout(); plt.show()
    return df


#===INTERACTIVE PART=============================================================
try:
    raw = input("\nMatrix sizes (Python list) (e.g): [64,128,256,512,1024]: ")
    sizes = literal_eval(raw) if raw.strip() else [64,128,256,512,1024]
except Exception:
    print("Bad list -> using default.")
    sizes = [64,128,256,512,1024]

dist = input("Distribution ('normal'/'uniform')  [normal]: ").strip().lower()
or "normal"
mode_txt = input("Matrix type g=general, s=symmetric, b=both  [g]:
").strip().lower() or "g"
mode = "symmetric" if mode_txt=="s" else "both" if mode_txt=="b" else "general"
seed_txt = input("Random seed (None/int) [None]: ").strip()
seed_val = None if seed_txt.lower() in {"", "none"} else int(seed_txt)

# accuracy on *all* requested sizes
for n in sizes:
    for sym in ([False, True] if mode=="both" else [mode=="symmetric"]):
        verify_factorisation_once(n, dist, sym, seed_val)


benchmark_hessenberg(sizes, dist, mode, seed_val)
```

The reader should be aware that my poor Dell Inspiron 5590 has crashed precisely 5 times while i was writing this (i might have tried with matrices of order $10^6 \times 10^6$). Unfortunately the runtime was around 4 minutes for a matrix $A \approx 10^3 \times 10^3$.

An expected output is:

```
64×64 general
‖A − Q T Qᵀ‖ = 7.51e-14
‖QᵀQ − I‖ = 7.07e-15

64×64 symmetric
‖A − Q T Qᵀ‖ = 4.83e-14
‖QᵀQ − I‖ = 7.39e-15

128×128 general
```

```
||A - Q T Q^T|| = 1.84e-13
||Q^TQ - I|| = 1.26e-14

128×128 symmetric
||A - Q T Q^T|| = 1.14e-13
||Q^TQ - I|| = 1.25e-14

256×256 general
||A - Q T Q^T|| = 4.70e-13
||Q^TQ - I|| = 2.28e-14

256×256 symmetric
||A - Q T Q^T|| = 2.78e-13
||Q^TQ - I|| = 2.25e-14

512×512 general
||A - Q T Q^T|| = 1.16e-12
||Q^TQ - I|| = 4.10e-14

512×512 symmetric
||A - Q T Q^T|| = 7.10e-13
||Q^TQ - I|| = 4.09e-14

1024×1024 general
||A - Q T Q^T|| = 3.05e-12
||Q^TQ - I|| = 7.57e-14

1024×1024 symmetric
||A - Q T Q^T|| = 1.84e-12
||Q^TQ - I|| = 7.64e-14
```

As n grows, we observe that the residuals also grow, but still in machine precision. The difference between the symmetric and nonsymmetric cases are more pronounced in larger matrices.

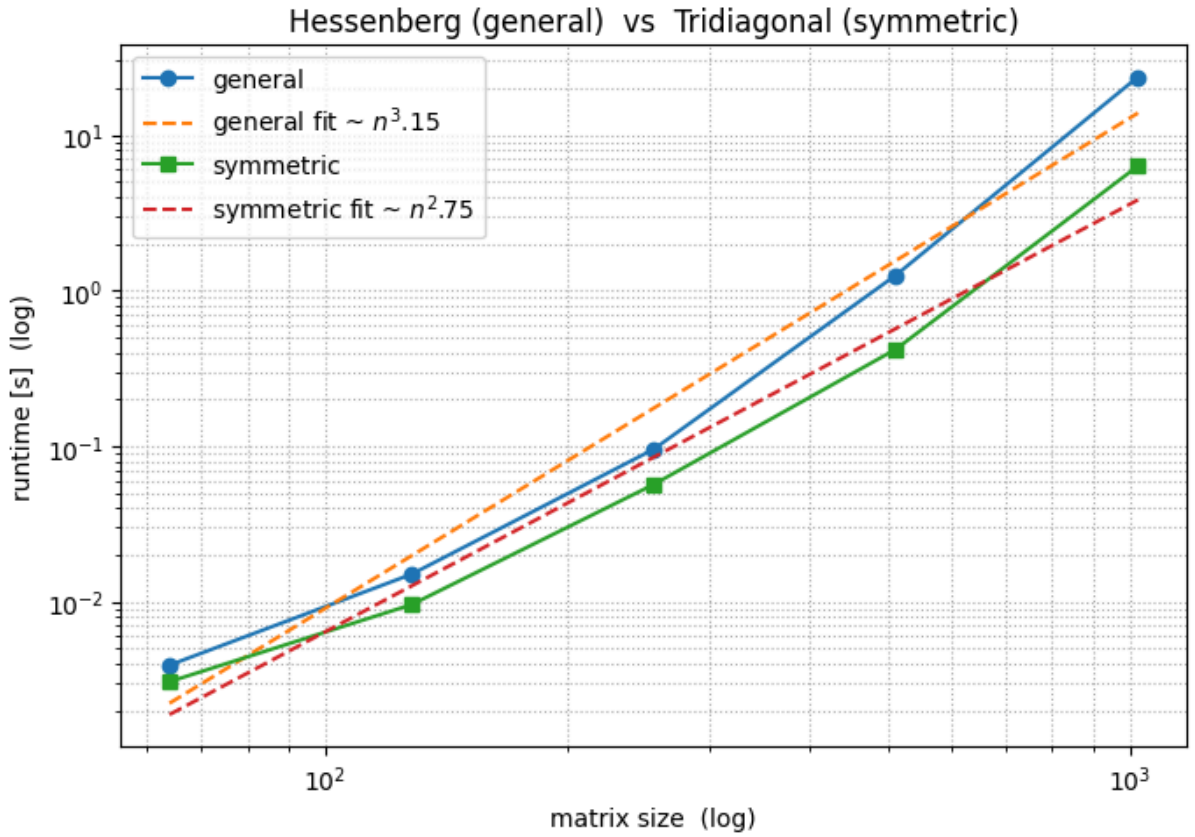

Figure 3: Runtime of the Hessenberg reduction for ordinary and symmetric matrices

2.2.1. Complexity (c)

Figure 3 shows the expected $O(n^3)$ complexity for the general case and $O(n^2)$ for the symmetric case. The latter is better discussed in Section 2.2.2.

To understand why the complexity is $O(n^3)$ in the general case, we can look at the algorithm. The outer loop runs $n-2$ times, and inside it, we have two matrix-vector products and two outer products, which are all $O(n^2)$. Thus, the total complexity is $O(n^3)$.

2.2.2. The Symmetric Case (d)

On the symmetric case we know that reflectors will be applied in only one side of the matrix, since $v^T A = Av^T$. That is precisely what the function `generate_random_matrix` does. Which cuts complexity from the expected $O(n^3)$ seen in the previous section to a $O(n^2)$[2]. [1]

3. Eigenvalues and Iterative Methods

3.1. Power iteration

The power iteration consists on computing large powers of the sequence:

$$\frac{x}{\|x\|}, \frac{Ax}{\|Ax\|}, \frac{A^2x}{\|A^2x\|}, ..., A \in \mathbb{C}^{m\times m} \tag{2}$$

To see why this sequence converges(under good assumptions), let A be diagonalizable. And write:

[2]See page 194 of Trefethen & Bau's Numerical Linear Algebra book

$$x = \sum_{i=1}^{m} \varphi_i v_i \tag{3}$$

In a basis of eigenvectors v_i with respective eigenvalues λ_i. Then for $x \in \mathbb{C}^m$ we have:

$$Ax = \sum_{i=1}^{m} \lambda_i \varphi_i v_i \tag{4}$$

Or even better:

$$A^n x = \sum_{i=1}^{m} \lambda_i^n \varphi_i v_i \tag{5}$$

Let v_j be the eigenvector associated to the biggest eigenvalue λ_j, then we have:

$$A^n x = \frac{1}{\lambda_j^n} \cdot \sum_{i=1}^{m} \lambda_i^n \varphi_i v_i = \frac{\lambda_1^n}{\lambda_j^n} \varphi_1 v_1 + ... + \varphi_j v_j + ... + \frac{\lambda_m^n}{\lambda_j^n} \varphi_m v_m \tag{6}$$

When $n \to \infty$ all of the smaller $\frac{\lambda_k}{\lambda_j}$ will approach 0, so we have:

$$\lim_{n \to \infty} A^n x = \varphi_j v_j \tag{7}$$

So the denominator on the original expression becomes

$$\|A^n x\| = \|\varphi_j v_j\| = |\varphi_j| \|v_j\| \tag{8}$$

And the limit is:

$$\lim_{n \to \infty} \frac{A^n x}{\|A^n x\|} = \frac{\varphi_j v_j}{|\varphi_j| \|v_j\|} \tag{9}$$

Since $\frac{\varphi_j}{|\varphi_j|} = \pm 1$, the sequence converges to the eigenvector v_j associated to the eigenvalue λ_j.

3.2. Inverse Iteration

Consider $\mu \in \mathbb{R} \setminus \Lambda$, where Λ is the set of eigenvalues of A. The eigenvalues $\hat{\lambda}$ of $(A - \mu I)^{-1}$ are:

$$\begin{aligned} \det\left(A - \mu I - \hat{\lambda} I\right) = 0 \Leftrightarrow \det\left(A - \left(\mu + \hat{\lambda}\right) I\right) = 0 \\ \Leftrightarrow \hat{\lambda}_j = \frac{1}{\lambda_j - \mu} \end{aligned} \tag{10}$$

Where λ_j are the eigenvalues of A. So if μ is close to an eigenvalue, then $\hat{\lambda}$ will be large. Power iteration seems interesting here, so the sequence:

$$\frac{x}{\|x\|}, \frac{(A - \mu I)^{-1} x}{\|(A - \mu I)^{-1} x\|}, \frac{(A - \mu I)^{-2} x}{\|(A - \mu I)^{-2} x\|}, ... \tag{11}$$

Converges to the eigenvector associated to the eigenvalue $\hat{\lambda}$.

4. Orthogonal Matrices (Problem 2) (a)

Here we will discuss how orthogonal matrices behave when we appluy the iterations discussed in Section 3.1, and Section 3.2.

So let $Q \in \mathbb{C}^{m \times m}$ be an orthogonal matrix. We are interested in its eigenvalues λ. We know that:

$$\begin{aligned} Qx = \lambda x &\Leftrightarrow x^T Qx = \lambda x^T x \\ &\Leftrightarrow Q\langle x, x\rangle = \lambda\langle x, x\rangle \end{aligned} \tag{12}$$

Since Q preserves inner product, we have:

$$\begin{aligned} Q\langle x, x\rangle = \lambda\langle x, x\rangle &\Leftrightarrow \langle x, x\rangle = \lambda\langle x, x\rangle \\ &\Leftrightarrow |\lambda| = 1 \end{aligned} \tag{13}$$

So λ lies in the unit circle, i.e $\lambda = e^{i\varphi}, \varphi \in \mathbb{R}$. We now discuss how this affects efficiency of some iterative methods

4.1. Orthogonal Matrices and the Power Iteration

The power method is better discussed in Section 3.1. Here we will write straight forward the result:

$$Q^n x = \frac{1}{\lambda_j^n} \cdot \sum_{i=1}^{m} \lambda_i^n \varphi_i v_i \tag{14}$$

Where λ_i are the eigenvalues of $Q \in \mathbb{C}^{m\times m}$, φ_i are the coefficients of the expansion of x in the basis of eigenvectors v_i. Since we have that $|\lambda_i| = 1$, we have:

The fact that $|\lambda_i| = 1 \Rightarrow |\lambda_i^n| = 1$ is sufficiently enough for one to be convinced that power iteration does not converge.

Let $\lambda_k = e^{i\psi_k}$, where $\psi_k \in \mathbb{R}$. Then expanding eq. (14):

$$Q^n x = \frac{1}{e^{i\psi_j \cdot n}} \cdot \sum_{\tau=1}^{m} e^{i\psi_\tau n} \varphi_\tau v_\tau \tag{15}$$

When $n \to \infty$ if $\lambda_j = 1$ then we have:

$$Q^n x = \varphi_j v_j + \sum_{\tau \neq j} e^{i\psi_\tau n} \varphi_\tau v_\tau \tag{16}$$

Since no eigenvalue dominates other eigenvalues in the orthogonal case, usually power iteration fails.

4.2. Orthogonal Matrices and Inverse Iteration

We know that if $A \in \mathbb{C}^{m\times m}, \det(A) \neq 0$ has $\Lambda \subset \mathbb{R} := \{\lambda_j\}$ as eigenvalues, then the eigenvalues of A^{-1} are λ_j^{-1}. Similarly the eigenvalues of $A + \varphi I$ are $\lambda_j + \varphi$. So the eigenvalues of $(A - \mu I)^{-1}, \mu \in \mathbb{R} \setminus \Lambda$ are:

$$\hat{\lambda_j} = \frac{1}{\lambda_j - \mu} \tag{17}$$

So let $Q \in \mathbb{C}^{m\times m}$ be an orthogonal matrix. We are interested in applying invers iteration to Q.

We know that the eigenvalues of Q are on the unit circle, so if μ is close to an eigenvalue λ_j, $\hat{\lambda}_j$ will be huge (dominant), which makes power iteration converge to the eigenvector associated to $\hat{\lambda}_j$, which is the eigenvector associated to λ_j. The fact that the eigenvalues are on the unit circle also contributes to the convergence of the method.

So we concude that inverse iteration works well for orthogonal matrices, *if μ is close to an eigenvalue of Q*.

4.3. The 2×2 Case (b)

We will calculate the eigenvalues of:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{18}$$

With $a, b, c, d \in \mathbb{R}$. The characteristic polynomial gives us:

$$\begin{aligned} &\det(A - \lambda I) = 0 \Leftrightarrow \det\begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix} = 0 \\ \Leftrightarrow\ &(a-\lambda)(d-\lambda) - bc = 0 \Leftrightarrow \lambda^2 + \lambda(-a-d) + (ad-bc) = 0 \\ \Leftrightarrow\ &\lambda = (a+d) \pm \frac{\sqrt{(a+d)^2 - 4(ad-bc)}}{2} \end{aligned} \tag{19}$$

So the eigenvalues are:

$$\begin{aligned} \lambda_1 &= \frac{a+d+\sqrt{(a+d)^2 - 4(ad-bc)}}{2} \\ \lambda_2 &= \frac{a+d-\sqrt{(a+d)^2 - 4(ad-bc)}}{2} \end{aligned} \tag{20}$$

4.4. Random Orthogonal Matrices (c)

This code generates orthogonal matrices of order 4×4 generated by the QR factorization of random matrices, and reduces the to Hessenberg form. The eigenvalues of the bottom-right 2×2 block are analytically calculated using Section 4.3.

Python

```python
def generate_orthogonal_matrix_qr(n=4, seed=None):

    """
    Generates a random orthogonal matrix using QR decomposition.
    Args:
        n (int): Size of the matrix (n x n).
        seed (int | None): Random seed for reproducibility.
    Returns:
        np.ndarray: An n x n orthogonal matrix.
    Raises:
        None
    """

    if seed is not None:
        np.random.seed(seed)
    A = np.random.randn(n, n)
    Q, _ = np.linalg.qr(A)
    return Q

def analytical_eigenvalues_2x2(a, b, c, d):

    """
    Calculates the eigenvalues of a 2x2 matrix analytically.
    Args:
        a (float): Element at position (0,0).
```

```
        b (float): Element at position (0,1).
        c (float): Element at position (1,0).
        d (float): Element at position (1,1).
    Returns:
        Tuple[float, float]: The two eigenvalues of the matrix.
    Raises:
    """

    trace = a + d
    det = a * d - b * c
    discriminant = trace**2 - 4 * det

    #complex if discriminant negative
    discriminant_root = np.sqrt(discriminant) if discriminant >= 0 else
    np.sqrt(complex(discriminant))

    lambda1 = (trace + discriminant_root) / 2
    lambda2 = (trace - discriminant_root) / 2

    return lambda1, lambda2

def analyze_orthogonal_and_hessenberg(n=4, n_matrices=30):

    """
    Analyzes orthogonal matrices and their Hessenberg forms.
    Args:
        n (int): Size of the matrices (n x n).
        n_matrices (int): Number of orthogonal matrices to generate and analyze.
    Returns:
        None
    Raises:
        None
    """

    for i in range(n_matrices):
        print(f"\n--- Orthogonal Matrix Q number {i+1} ---")
        Q = generate_orthogonal_matrix_qr(n=n)
        print("Matrix Q:")
        print(np.array_str(Q, precision=4, suppress_small=True))

        householder_list, H, Q_accum = to_hessenberg(Q)

        print("\nHessenberg Form H (of Q):")
        print(np.array_str(H, precision=4, suppress_small=True))

        block = Q[2:4, 2:4]
```

```
        a, b, c, d = block[0,0], block[0,1], block[1,0], block[1,1]
        analytical_eigenvalues = analytical_eigenvalues_2x2(a, b, c, d)

        print("\nBlock Q[3:4,3:4] (indices 2 and 3, 2x2):")

        print(np.array_str(block, precision=4, suppress_small=True))

        print("\nEigenvalues of the 2x2 block (analytically calculated):")
        for idx, val in enumerate(analytical_eigenvalues):
            print(f"  λ_{idx+1} = {val} (size = {abs(val):.4f})")

        print("-" * 40)

analyze_orthogonal_and_hessenberg()
```

We ran this code for 30 matrices, the output was:

```
--- Orthogonal Matrix Q number 1 ---
Matrix Q:
[[-0.5629  0.6801  0.4635 -0.0764]
 [-0.6703 -0.6884  0.2231  0.1645]
 [-0.1497  0.2337 -0.3793  0.8827]
 [ 0.4598 -0.0949  0.7691  0.4336]]

Hessenberg Form H (of Q):
[[-0.5629 -0.6779 -0.4534 -0.1342]
 [ 0.8265 -0.4617 -0.3088 -0.0914]
 [ 0.     -0.5721  0.7865  0.2329]
 [-0.      0.      0.2839 -0.9588]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.3793  0.8827]
 [ 0.7691  0.4336]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.9458819605346136 (size = 0.9459)
  λ_2 = -0.8915812804514585 (size = 0.8916)
----------------------------------------

--- Orthogonal Matrix Q number 2 ---
Matrix Q:
[[-0.4016  0.3605  0.8354  0.1045]
 [-0.2846 -0.3518  0.1255 -0.8829]
 [-0.3325 -0.8166  0.136   0.4519]
 [ 0.8045 -0.282   0.5176 -0.0734]]

Hessenberg Form H (of Q):
```

```
[[-0.4016 -0.3235 -0.1952  0.8343]
 [ 0.9158 -0.1418 -0.0856  0.3658]
 [ 0.     -0.9355  0.0805 -0.3439]
 [ 0.      0.     -0.9737 -0.2278]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[ 0.136   0.4519]
 [ 0.5176 -0.0734]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.5261528266779573 (size = 0.5262)
  λ_2 = -0.4635182526918817 (size = 0.4635)
----------------------------------------

--- Orthogonal Matrix Q number 3 ---
Matrix Q:
[[-0.0452 -0.9852 -0.1571  0.0523]
 [ 0.4259  0.0412 -0.0809  0.9002]
 [ 0.1843  0.1346 -0.9569 -0.1793]
 [-0.8846  0.0982 -0.2303  0.3934]]

Hessenberg Form H (of Q):
[[-0.0452  0.4954 -0.8604 -0.1112]
 [-0.999  -0.0224  0.0389  0.005 ]
 [ 0.      0.8684  0.4918  0.0636]
 [-0.      0.      0.1282 -0.9917]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.9569 -0.1793]
 [-0.2303  0.3934]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.42331102317930497 (size = 0.4233)
  λ_2 = -0.9868680130188092 (size = 0.9869)
----------------------------------------

--- Orthogonal Matrix Q number 4 ---
Matrix Q:
[[-0.2568  0.8228 -0.2287  0.4525]
 [-0.2848 -0.018   0.9011  0.3265]
 [-0.6054 -0.5408 -0.3667  0.4544]
 [ 0.6975 -0.1738 -0.0346  0.6943]]

Hessenberg Form H (of Q):
[[-0.2568  0.2274  0.1895  0.92  ]
 [ 0.9665  0.0604  0.0503  0.2444]
```

```
 [-0.     -0.9719  0.0475  0.2305]
 [ 0.      0.     -0.9794  0.2017]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.3667  0.4544]
 [-0.0346  0.6943]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.67929095950588 (size = 0.6793)
  λ_2 = -0.3516952691053273 (size = 0.3517)
----------------------------------------

--- Orthogonal Matrix Q number 5 ---
Matrix Q:
[[-0.5025 -0.7649 -0.0345  0.4015]
 [ 0.3718  0.0654  0.6626  0.6469]
 [ 0.6819 -0.6372  0.0297 -0.358 ]
 [ 0.3798  0.0679 -0.7476  0.5406]]

Hessenberg Form H (of Q):
[[-0.5025  0.1798 -0.7903  0.3011]
 [-0.8646 -0.1045  0.4593 -0.175 ]
 [-0.     -0.9781 -0.1943  0.074 ]
 [-0.      0.      0.356   0.9345]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[ 0.0297 -0.358 ]
 [-0.7476  0.5406]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.8621172090206812 (size = 0.8621)
  λ_2 = -0.2918043055687003 (size = 0.2918)
----------------------------------------

--- Orthogonal Matrix Q number 6 ---
Matrix Q:
[[-0.9092  0.1745  0.2312 -0.2992]
 [ 0.1618 -0.3496 -0.2497 -0.8884]
 [-0.1454  0.4629 -0.8736  0.0369]
 [-0.3551 -0.7956 -0.3479  0.3462]]

Hessenberg Form H (of Q):
[[-0.9092 -0.2422 -0.3011 -0.1552]
 [-0.4164  0.5288  0.6573  0.3389]
 [ 0.      0.8134 -0.517  -0.2665]
 [-0.      0.      0.4582 -0.8888]]
```

```
123
124 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
125 [[-0.8736  0.0369]
126  [-0.3479  0.3462]]
127
128 Eigenvalues of the 2x2 block (analytically calculated):
129   λ_1 = 0.3356278613353456 (size = 0.3356)
130   λ_2 = -0.8630034423078552 (size = 0.8630)
131 ----------------------------------------
132
133 --- Orthogonal Matrix Q number 7 ---
134 Matrix Q:
135 [[-0.4296  0.6844 -0.4085  0.4245]
136  [ 0.2184 -0.4244 -0.028   0.8783]
137  [ 0.17   -0.3028 -0.9122 -0.2177]
138  [ 0.8596  0.5097 -0.0167  0.032 ]]
139
140 Hessenberg Form H (of Q):
141 [[-0.4296 -0.4928  0.7535  0.0703]
142  [-0.903   0.2344 -0.3584 -0.0334]
143  [-0.     -0.838  -0.5433 -0.0507]
144  [ 0.      0.      0.0928 -0.9957]]
145
146 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
147 [[-0.9122 -0.2177]
148  [-0.0167  0.032 ]]
149
150 Eigenvalues of the 2x2 block (analytically calculated):
151   λ_1 = 0.0358294023777706 (size = 0.0358)
152   λ_2 = -0.9159948591320213 (size = 0.9160)
153 ----------------------------------------
154
155 --- Orthogonal Matrix Q number 8 ---
156 Matrix Q:
157 [[-0.3402 -0.0284 -0.009   0.9399]
158  [-0.7148 -0.6207  0.1662 -0.2759]
159  [-0.5681  0.5973 -0.5323 -0.1927]
160  [ 0.2249 -0.5071 -0.83    0.0581]]
161
162 Hessenberg Form H (of Q):
163 [[-0.3402  0.2518  0.8476  0.3201]
164  [ 0.9403  0.0911  0.3067  0.1158]
165  [ 0.      0.9635 -0.2505 -0.0946]
166  [-0.     -0.      0.3533 -0.9355]]
167
168 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
```

```
[[-0.5323 -0.1927]
 [-0.83    0.0581]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.25997158900622225 (size = 0.2600)
  λ_2 = -0.7341829044352151 (size = 0.7342)
----------------------------------------

--- Orthogonal Matrix Q number 9 ---
Matrix Q:
[[-0.2692  0.5878 -0.5428  0.5361]
 [-0.8282  0.0254  0.5486  0.1116]
 [-0.292   0.3638 -0.2875 -0.8365]
 [-0.3954 -0.7222 -0.5673  0.0189]]

Hessenberg Form H (of Q):
[[-0.2692 -0.561  -0.1619  0.7659]
 [ 0.9631 -0.1568 -0.0453  0.2141]
 [-0.     -0.8128  0.1205 -0.5699]
 [ 0.     -0.     -0.9784 -0.2069]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.2875 -0.8365]
 [-0.5673  0.0189]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.5713848430035342 (size = 0.5714)
  λ_2 = -0.8399942790872519 (size = 0.8400)
----------------------------------------

--- Orthogonal Matrix Q number 10 ---
Matrix Q:
[[-0.4241  0.1349 -0.1773  0.8778]
 [-0.332  -0.8664 -0.3594 -0.0999]
 [ 0.8422 -0.2846 -0.2058  0.409 ]
 [-0.0237 -0.3876  0.8928  0.2284]]

Hessenberg Form H (of Q):
[[-0.4241 -0.2373  0.8677  0.1046]
 [ 0.9056 -0.1111  0.4063  0.049 ]
 [-0.      0.9651  0.2602  0.0313]
 [ 0.      0.      0.1196 -0.9928]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.2058  0.409 ]
 [ 0.8928  0.2284]]
```

```
215
216 Eigenvalues of the 2x2 block (analytically calculated):
217   λ_1 = 0.6533894082311485 (size = 0.6534)
218   λ_2 = -0.6307995911677337 (size = 0.6308)
219 ----------------------------------------
220
221 --- Orthogonal Matrix Q number 11 ---
222 Matrix Q:
223 [[-0.1675 -0.9786 -0.0461 -0.1099]
224  [ 0.7871 -0.1742  0.5819  0.1072]
225  [ 0.3816 -0.1052 -0.6644  0.6339]
226  [-0.4547 -0.0293  0.4667  0.758 ]]
227
228 Hessenberg Form H (of Q):
229 [[-0.1675  0.7485 -0.1241 -0.6295]
230  [-0.9859 -0.1271  0.0211  0.1069]
231  [ 0.     -0.6508 -0.1468 -0.7449]
232  [ 0.     -0.     -0.9811  0.1934]]
233
234 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
235 [[-0.6644  0.6339]
236  [ 0.4667  0.758 ]]
237
238 Eigenvalues of the 2x2 block (analytically calculated):
239   λ_1 = 0.9421402114923986 (size = 0.9421)
240   λ_2 = -0.8485799008471894 (size = 0.8486)
241 ----------------------------------------
242
243 --- Orthogonal Matrix Q number 12 ---
244 Matrix Q:
245 [[-0.3541  0.5585 -0.0616  0.7476]
246  [-0.5108 -0.6958 -0.4435  0.2414]
247  [ 0.1705  0.3507 -0.885  -0.2542]
248  [ 0.7646 -0.2844 -0.1274  0.5641]]
249
250 Hessenberg Form H (of Q):
251 [[-0.3541  0.295   0.7613  0.4561]
252  [ 0.9352  0.1117  0.2883  0.1727]
253  [-0.      0.949  -0.2706 -0.1621]
254  [-0.     -0.      0.5139 -0.8579]]
255
256 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
257 [[-0.885  -0.2542]
258  [-0.1274  0.5641]]
259
260 Eigenvalues of the 2x2 block (analytically calculated):
```

```
  λ_1 = 0.5861402323977607 (size = 0.5861)
  λ_2 = -0.9070552039589896 (size = 0.9071)
----------------------------------------

--- Orthogonal Matrix Q number 13 ---
Matrix Q:
[[-0.7015  0.4915  0.3042 -0.417 ]
 [ 0.4623 -0.1626  0.1522 -0.8583]
 [ 0.2479  0.6778 -0.6824 -0.1159]
 [-0.4825 -0.5221 -0.647  -0.2757]]

Hessenberg Form H (of Q):
[[-0.7015 -0.7069 -0.0829 -0.0373]
 [-0.7127  0.6958  0.0816  0.0367]
 [ 0.      0.1276 -0.9045 -0.4069]
 [ 0.     -0.      0.4103 -0.912 ]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.6824 -0.1159]
 [-0.647  -0.2757]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = -0.1379150700589457 (size = 0.1379)
  λ_2 = -0.820164584090632 (size = 0.8202)
----------------------------------------

--- Orthogonal Matrix Q number 14 ---
Matrix Q:
[[-0.2164 -0.5735  0.7758 -0.1498]
 [-0.3581 -0.6722 -0.6278 -0.1609]
 [-0.1535  0.3054  0.0015 -0.9398]
 [-0.8952  0.3551  0.0633  0.2617]]

Hessenberg Form H (of Q):
[[-0.2164  0.2256 -0.3541  0.8814]
 [ 0.9763  0.05   -0.0785  0.1954]
 [-0.     -0.9729 -0.0862  0.2144]
 [-0.      0.     -0.9279 -0.3728]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[ 0.0015 -0.9398]
 [ 0.0633  0.2617]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = (0.1316169167334142+0.20628124638940884j) (size = 0.2447)
  λ_2 = (0.1316169167334142-0.20628124638940884j) (size = 0.2447)
```

```
----------------------------------------

--- Orthogonal Matrix Q number 15 ---
Matrix Q:
[[-0.467   0.1387  0.7316  0.4769]
 [ 0.0375 -0.9757  0.0774  0.2017]
 [-0.7384 -0.0272 -0.627   0.2467]
 [ 0.485   0.1677 -0.2562  0.8191]]

Hessenberg Form H (of Q):
[[-0.467   0.3435 -0.8022 -0.1428]
 [-0.8843 -0.1814  0.4236  0.0754]
 [-0.      0.9215  0.3824  0.068 ]
 [-0.     -0.      0.1752 -0.9845]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.627   0.2467]
 [-0.2562  0.8191]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.7740235941481243 (size = 0.7740)
  λ_2 = -0.5818796203145907 (size = 0.5819)
----------------------------------------

--- Orthogonal Matrix Q number 16 ---
Matrix Q:
[[-0.3849  0.0435 -0.7406  0.549 ]
 [ 0.2525 -0.952  -0.07    0.158 ]
 [ 0.7807  0.2969  0.0191  0.5495]
 [-0.4225 -0.06    0.668   0.6096]]

Hessenberg Form H (of Q):
[[-0.3849  0.866   0.1952  0.2527]
 [-0.923  -0.3611 -0.0814 -0.1054]
 [ 0.      0.346  -0.5736 -0.7425]
 [ 0.      0.     -0.7914  0.6114]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[0.0191 0.5495]
 [0.668  0.6096]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.9883202000131572 (size = 0.9883)
  λ_2 = -0.3596282457787697 (size = 0.3596)
----------------------------------------

```

```
--- Orthogonal Matrix Q number 17 ---
Matrix Q:
[[-0.1066  0.7212  0.0749 -0.6804]
 [-0.6764 -0.5121  0.3485 -0.3985]
 [-0.5738  0.1159 -0.8012  0.1245]
 [-0.4493  0.4519  0.4807  0.6024]]

Hessenberg Form H (of Q):
[[-0.1066 -0.2265 -0.8608 -0.4431]
 [ 0.9943 -0.0243 -0.0923 -0.0475]
 [ 0.     -0.9737  0.2025  0.1043]
 [ 0.      0.      0.4577 -0.8891]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.8012  0.1245]
 [ 0.4807  0.6024]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.6437814804055579 (size = 0.6438)
  λ_2 = -0.842579679233141 (size = 0.8426)
----------------------------------------

--- Orthogonal Matrix Q number 18 ---
Matrix Q:
[[-0.8255  0.1636  0.5347  0.0763]
 [-0.5177 -0.2699 -0.6467 -0.4908]
 [ 0.0194  0.9433 -0.2236 -0.2446]
 [-0.2238  0.103  -0.4958  0.8327]]

Hessenberg Form H (of Q):
[[-0.8255 -0.1619 -0.4367 -0.3188]
 [ 0.5644 -0.2368 -0.6388 -0.4663]
 [ 0.      0.958  -0.2317 -0.1691]
 [-0.      0.     -0.5896  0.8077]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.2236 -0.2446]
 [-0.4958  0.8327]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.9372008193289816 (size = 0.9372)
  λ_2 = -0.32809878403400955 (size = 0.3281)
----------------------------------------

--- Orthogonal Matrix Q number 19 ---
Matrix Q:
```

```
[[-0.7885  0.0127  0.4922 -0.3686]
 [ 0.5982 -0.2155  0.6311 -0.4444]
 [ 0.0328  0.1282 -0.5621 -0.8164]
 [-0.1391 -0.968  -0.2085 -0.0141]]

Hessenberg Form H (of Q):
[[-0.7885 -0.122  -0.4769  0.3687]
 [-0.615   0.1564  0.6114 -0.4727]
 [ 0.      0.9801 -0.1569  0.1213]
 [ 0.      0.     -0.6116 -0.7911]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.5621 -0.8164]
 [-0.2085 -0.0141]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.2071704757269035 (size = 0.2072)
  λ_2 = -0.7833862909149965 (size = 0.7834)
----------------------------------------

--- Orthogonal Matrix Q number 20 ---
Matrix Q:
[[-0.3031  0.0429  0.9481  0.0861]
 [ 0.1571 -0.9342  0.1195 -0.2972]
 [-0.8533 -0.0239 -0.2292 -0.4678]
 [-0.3942 -0.3534 -0.1853  0.8279]]

Hessenberg Form H (of Q):
[[-0.3031  0.8775 -0.2647  0.261 ]
 [-0.953  -0.2791  0.0842 -0.083 ]
 [ 0.      0.3901  0.6557 -0.6464]
 [ 0.     -0.     -0.702  -0.7121]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.2292 -0.4678]
 [-0.1853  0.8279]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.9043475415351031 (size = 0.9043)
  λ_2 = -0.30565758703551016 (size = 0.3057)
----------------------------------------

--- Orthogonal Matrix Q number 21 ---
Matrix Q:
[[-0.8149  0.3649 -0.4406 -0.0928]
 [ 0.0977 -0.6065 -0.7405  0.2726]
```

```
 [ 0.0387  0.372   0.0415  0.9265]
 [-0.57   -0.6005  0.5058  0.2423]]

Hessenberg Form H (of Q):
[[-0.8149 -0.1234  0.2317  0.5167]
 [-0.5796  0.1735 -0.3257 -0.7266]
 [ 0.     -0.9771 -0.0871 -0.1943]
 [-0.      0.      0.9125 -0.4091]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[0.0415 0.9265]
 [0.5058 0.2423]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.8337542266557703 (size = 0.8338)
  λ_2 = -0.5499977130761522 (size = 0.5500)
----------------------------------------

--- Orthogonal Matrix Q number 22 ---
Matrix Q:
[[-0.3663  0.4189 -0.6842 -0.4714]
 [ 0.2919 -0.672  -0.6656  0.142 ]
 [ 0.5024  0.6076 -0.2758  0.5498]
 [-0.7268 -0.061  -0.1131  0.6748]]

Hessenberg Form H (of Q):
[[-0.3663 -0.1301  0.2621  0.8833]
 [-0.9305  0.0512 -0.1032 -0.3477]
 [-0.     -0.9902 -0.0398 -0.134 ]
 [ 0.     -0.      0.9587 -0.2845]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.2758  0.5498]
 [-0.1131  0.6748]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.6040601009013424 (size = 0.6041)
  λ_2 = -0.20508048336653284 (size = 0.2051)
----------------------------------------

--- Orthogonal Matrix Q number 23 ---
Matrix Q:
[[-0.4467 -0.7825 -0.2287  0.3687]
 [ 0.4222  0.182  -0.0157  0.8879]
 [ 0.6922 -0.3543 -0.5694 -0.2666]
 [-0.3783  0.4786 -0.7895  0.0678]]
```

```
491
492 Hessenberg Form H (of Q):
493 [[-0.4467  0.702  -0.0795 -0.5489]
494  [-0.8947 -0.3505  0.0397  0.274 ]
495  [ 0.     -0.62   -0.1125 -0.7765]
496  [ 0.      0.     -0.9897  0.1433]]
497
498 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
499 [[-0.5694 -0.2666]
500  [-0.7895  0.0678]]
501
502 Eigenvalues of the 2x2 block (analytically calculated):
503   λ_1 = 0.3077269347022792 (size = 0.3077)
504   λ_2 = -0.8093108181365211 (size = 0.8093)
505 ----------------------------------------
506
507 --- Orthogonal Matrix Q number 24 ---
508 Matrix Q:
509 [[-0.0413 -0.9591  0.2707  0.0721]
510  [ 0.08   -0.274  -0.9584  0.0005]
511  [-0.6777 -0.0389 -0.0459 -0.7328]
512  [-0.7298  0.0604 -0.0778  0.6766]]
513
514 Hessenberg Form H (of Q):
515 [[-0.0413  0.3131  0.5108 -0.7996]
516  [-0.9991 -0.0129 -0.0211  0.0331]
517  [-0.      0.9496 -0.1687  0.2641]
518  [ 0.      0.      0.8427  0.5384]]
519
520 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
521 [[-0.0459 -0.7328]
522  [-0.0778  0.6766]]
523
524 Eigenvalues of the 2x2 block (analytically calculated):
525   λ_1 = 0.7483535886136121 (size = 0.7484)
526   λ_2 = -0.1176716985715956 (size = 0.1177)
527 ----------------------------------------
528
529 --- Orthogonal Matrix Q number 25 ---
530 Matrix Q:
531 [[-0.7483  0.5153 -0.0763  0.4107]
532  [-0.4762 -0.7375 -0.4779 -0.0313]
533  [-0.1713  0.3719 -0.3481 -0.8433]
534  [ 0.4288  0.2287 -0.8029  0.3451]]
535
536 Hessenberg Form H (of Q):
```

```
[[-0.7483 -0.0847  0.4863  0.4431]
 [ 0.6633 -0.0956  0.5486  0.4998]
 [ 0.      0.9918  0.0944  0.086 ]
 [ 0.      0.      0.6735 -0.7392]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.3481 -0.8433]
 [-0.8029  0.3451]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.8914410770554546 (size = 0.8914)
  λ_2 = -0.8943450015206609 (size = 0.8943)
----------------------------------------

--- Orthogonal Matrix Q number 26 ---
Matrix Q:
[[-0.9036  0.127  -0.0161  0.4088]
 [-0.2097 -0.5366 -0.7493 -0.3265]
 [ 0.3449  0.3256 -0.6074  0.6373]
 [ 0.1433 -0.7681  0.2634  0.5658]]

Hessenberg Form H (of Q):
[[-0.9036  0.0616  0.2624  0.3329]
 [ 0.4284  0.13    0.5536  0.7022]
 [ 0.      0.9896 -0.0891 -0.113 ]
 [-0.     -0.      0.7853 -0.6191]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.6074  0.6373]
 [ 0.2634  0.5658]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.6947481590186378 (size = 0.6947)
  λ_2 = -0.7362843808778513 (size = 0.7363)
----------------------------------------

--- Orthogonal Matrix Q number 27 ---
Matrix Q:
[[-0.3783  0.2175  0.7347 -0.5194]
 [-0.486  -0.5305 -0.4607 -0.5198]
 [-0.7609  0.4199 -0.1943  0.455 ]
 [-0.2046 -0.7036  0.4585  0.5029]]

Hessenberg Form H (of Q):
[[-0.3783 -0.6032 -0.0497 -0.7004]
 [ 0.9257 -0.2465 -0.0203 -0.2862]
```

```
 [ 0.      0.7585 -0.0461 -0.65  ]
 [ 0.     -0.     -0.9975  0.0708]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.1943  0.455 ]
 [ 0.4585  0.5029]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.7288996874702811 (size = 0.7289)
  λ_2 = -0.42026844598624513 (size = 0.4203)
----------------------------------------

--- Orthogonal Matrix Q number 28 ---
Matrix Q:
[[-0.6177 -0.2663 -0.3252  0.6646]
 [ 0.7345 -0.3401  0.0741  0.5826]
 [ 0.0015 -0.8476 -0.2571 -0.4641]
 [ 0.2811  0.308  -0.907  -0.0592]]

Hessenberg Form H (of Q):
[[-0.6177  0.0118  0.5166  0.5928]
 [-0.7864 -0.0093 -0.4058 -0.4656]
 [ 0.     -0.9999  0.0099  0.0113]
 [ 0.     -0.      0.7539 -0.657 ]]

Block Q[3:4,3:4] (indices 2 and 3, 2x2):
[[-0.2571 -0.4641]
 [-0.907  -0.0592]]

Eigenvalues of the 2x2 block (analytically calculated):
  λ_1 = 0.4981407629675436 (size = 0.4981)
  λ_2 = -0.8144343078024388 (size = 0.8144)
----------------------------------------

--- Orthogonal Matrix Q number 29 ---
Matrix Q:
[[-0.889  -0.0288  0.3962 -0.2278]
 [ 0.1961 -0.4549 -0.0899 -0.864 ]
 [-0.4017 -0.36   -0.8217  0.1838]
 [ 0.0995 -0.814   0.3997  0.4095]]

Hessenberg Form H (of Q):
[[-0.889   0.4094  0.14   -0.1501]
 [-0.458  -0.7947 -0.2718  0.2914]
 [ 0.     -0.4483  0.6097 -0.6537]
 [ 0.     -0.     -0.7313 -0.6821]]
```

```
629
630 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
631 [[-0.8217  0.1838]
632  [ 0.3997  0.4095]]
633
634 Eigenvalues of the 2x2 block (analytically calculated):
635   λ_1 = 0.46657705244665193 (size = 0.4666)
636   λ_2 = -0.8787442629640516 (size = 0.8787)
637 ----------------------------------------
638
639 --- Orthogonal Matrix Q number 30 ---
640 Matrix Q:
641 [[-0.2493  0.2801 -0.5896 -0.7154]
642  [-0.5128 -0.2897  0.6545 -0.4742]
643  [-0.2303 -0.8469 -0.4616  0.1292]
644  [-0.7886  0.3471 -0.1044  0.4967]]
645
646 Hessenberg Form H (of Q):
647 [[-0.2493  0.5744 -0.1969 -0.7544]
648  [ 0.9684  0.1479 -0.0507 -0.1942]
649  [ 0.     -0.8051 -0.1498 -0.5739]
650  [ 0.     -0.      0.9676 -0.2526]]
651
652 Block Q[3:4,3:4] (indices 2 and 3, 2x2):
653 [[-0.4616  0.1292]
654  [-0.1044  0.4967]]
655
656 Eigenvalues of the 2x2 block (analytically calculated):
657   λ_1 = 0.4824356418279282 (size = 0.4824)
658   λ_2 = -0.4473584670796439 (size = 0.4474)
659 ----------------------------------------
```

So we observe that in the 2×2 blocks analyzed:

1. Orthogonality is not always preserved
2. The eigenvalues are usually real, with alternating sign and size around 1.

4.5. Shift With an Eigenvalue (d)

Now we use an eigenvalue of the 2×2 block as a shift:

```python
def pretty(arr: np.ndarray, prec: int = 3) -> str:
    """
    Compact string for a 1-D NumPy array.

    Args:
        arr (np.ndarray): Input array to be formatted.
```

```
        prec (int): Precision for the string representation.
    Returns:
        str: Formatted string representation of the array.
    Raises:
        None
    """

    return np.array_str(arr, precision=prec, suppress_small=True)

def qr_iteration_with_fixed_shift(
    H: np.ndarray,
    mu: complex,
    *,
    max_iter: int = 100,
    tol: float = 1e-10,
    debug: bool = False,
):
    """
    Fixed-shift QR iteration that optionally shows the sub-diagonal before the
    first step and after the final step.

    Args:
        H (np.ndarray): initial matrix in Hessenberg form.
        mu (complex): fixed shift to be used.
        max_iter (int): maximum number of iterations.
        tol (float): tolerance for convergence.
        debug (bool): if True, print detailed information about each iteration.

    Returns:
        Hk (np.ndarray): matrix after iterations.
        converged (bool): whether it converged to almost upper triangular form.
        iterations (int): number of iterations performed.
    Raises:
        None
    """

    Hk = H.astype(np.complex128, copy=True)
    n = Hk.shape[0]

    if debug:
        init_sub = np.diag(Hk, k=-1)
        print("  before: subdiag=" + pretty(init_sub) +
              f",  ‖·‖₂={np.linalg.norm(init_sub):.3e}")

    for k in range(max_iter):
        Q, R = np.linalg.qr(Hk - mu * np.eye(n))
```

```
        Hk = R @ Q + mu * np.eye(n)

        sub = np.diag(Hk, k=-1)
        if debug:
            print(
                f"  iter {k:02d}: subdiag=" + pretty(sub) +
                f",  ‖·‖₂={np.linalg.norm(sub):.3e}"
            )

        if np.all(np.abs(sub) < tol):
            break  #tests convergence

    if debug: #final sub-diagonal
        final_sub = np.diag(Hk, k=-1)
        print("  after : subdiag=" + pretty(final_sub) +
              f",  ‖·‖₂={np.linalg.norm(final_sub):.3e}")

    converged = np.all(np.abs(np.diag(Hk, k=-1)) < tol)
    return Hk, converged, min(k + 1, max_iter)


def run_qr_iteration_with_shifts_and_debug(
    *,
    n: int = 4,
    n_matrices: int = 30,
    max_iter: int = 50,
    debug: bool = False,
):
    """
    Runs the QR iteration with fixed shifts on randomly generated orthogonal matrices,
    printing a summary for each matrix. Detailed logging appears only when debug = True.

    Args:
        n (int): Size of the matrices (n x n).
        n_matrices (int): Number of orthogonal matrices to generate and analyze.
        max_iter (int): Maximum number of iterations for the QR iteration.
        debug (bool): If True, print detailed information about each iteration.

    Returns:
        None
    Raises:
        None
    """
```

```
96
97      for idx in range(1, n_matrices + 1):
98          print(f"\n┌─ Matrix {idx:02d}/{n_matrices}  (size {n}x{n})")
99
100         Q = generate_orthogonal_matrix_qr(n)
101         _, H, _ = to_hessenberg(Q)
102
103         a, b, c, d = H[-2:, -2:].ravel()
104         ev1, ev2 = analytical_eigenvalues_2x2(a, b, c, d)
105         mu = ev1 if abs(ev1 - H[-1, -1]) < abs(ev2 - H[-1, -1]) else ev2
106         print(f"│  fixed shift μ = {mu:.6g} (|μ|={abs(mu):.4f})")
107
108         Hk, ok, iters = qr_iteration_with_fixed_shift(
109             H, mu, max_iter=max_iter, tol=1e-10, debug=debug
110         )
111
112         print(f"│  iterations    = {iters}/{max_iter}")
113         print("│  sub-diag magnitudes after last step:")
114         print("│ ", pretty(np.abs(np.diag(Hk, k=-1))))
115         print(f"└─ converged?    = {'yes' if ok else 'no'}")
116
117
118
119 run_qr_iteration_with_shifts_and_debug(n=4, n_matrices=30, max_iter=100,
    debug=True)
```

An expected output is:

```
1     ┌─ Matrix 01/30  (size 4x4)
2     │  fixed shift μ = -0.936534 (|μ|=0.9365)
3       before: subdiag=[ 0.959+0.j -0.993+0.j -0.393+0.j],  ‖·‖₂=1.436e+00
4       iter 00: subdiag=[0.731+0.j 0.334+0.j 0.235+0.j],  ‖·‖₂=8.375e-01
5       iter 01: subdiag=[ 0.723+0.j -0.045+0.j -0.231+0.j],  ‖·‖₂=7.601e-01
6       iter 02: subdiag=[0.723+0.j 0.006+0.j 0.231+0.j],  ‖·‖₂=7.586e-01
7       iter 03: subdiag=[ 0.723+0.j -0.001+0.j -0.231+0.j],  ‖·‖₂=7.586e-01
8       iter 04: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
9       iter 05: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
10      iter 06: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
11      iter 07: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
12      iter 08: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
13      iter 09: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
14      iter 10: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
15      iter 11: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
16      iter 12: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
17      iter 13: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
18      iter 14: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
19      iter 15: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
```

```
    iter 16: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 17: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 18: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 19: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 20: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 21: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 22: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 23: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 24: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 25: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 26: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 27: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 28: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 29: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 30: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 31: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 32: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 33: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 34: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 35: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 36: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 37: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 38: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 39: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 40: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 41: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 42: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 43: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 44: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 45: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 46: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 47: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 48: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 49: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 50: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 51: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 52: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 53: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 54: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 55: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 56: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 57: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 58: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 59: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
    iter 60: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
    iter 61: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
```

```
  iter 62: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 63: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 64: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 65: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 66: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 67: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 68: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 69: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 70: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 71: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 72: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 73: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 74: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 75: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 76: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 77: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 78: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 79: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 80: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 81: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 82: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 83: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 84: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 85: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 86: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 87: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 88: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 89: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 90: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 91: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 92: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 93: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 94: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 95: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 96: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 97: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  iter 98: subdiag=[0.723+0.j 0.   +0.j 0.231+0.j],  ‖·‖₂=7.586e-01
  iter 99: subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  after : subdiag=[ 0.723+0.j -0.   +0.j -0.231+0.j],  ‖·‖₂=7.586e-01
  │  iterations    = 100/100
  │  sub-diag magnitudes after last step:
  │  [0.723 0.    0.231]
  └─ converged?    = no

  ┌─ Matrix 02/30  (size 4x4)
  │  fixed shift μ = 0.985706 (|μ|=0.9857)
```

```
    before: subdiag=[ 0.942+0.j -0.63 +0.j -0.464+0.j],  ‖·‖₂=1.224e+00
    iter 00: subdiag=[ 0.982+0.j -0.877+0.j -0.004+0.j],  ‖·‖₂=1.316e+00
    iter 01: subdiag=[ 0.993+0.j -0.997+0.j -0.   +0.j],  ‖·‖₂=1.407e+00
    iter 02: subdiag=[ 0.85 +0.j -0.977+0.j -0.   +0.j],  ‖·‖₂=1.295e+00
    iter 03: subdiag=[ 0.581+0.j -0.942+0.j -0.   +0.j],  ‖·‖₂=1.107e+00
    iter 04: subdiag=[ 0.346+0.j -0.925+0.j -0.   +0.j],  ‖·‖₂=9.879e-01
    iter 05: subdiag=[ 0.195+0.j -0.92 +0.j -0.   +0.j],  ‖·‖₂=9.402e-01
    iter 06: subdiag=[ 0.108+0.j -0.918+0.j -0.   +0.j],  ‖·‖₂=9.244e-01
    iter 07: subdiag=[ 0.059+0.j -0.918+0.j -0.   +0.j],  ‖·‖₂=9.194e-01
    iter 08: subdiag=[ 0.033+0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.179e-01
    iter 09: subdiag=[ 0.018+0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.175e-01
    iter 10: subdiag=[ 0.01 +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 11: subdiag=[ 0.005+0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 12: subdiag=[ 0.003+0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 13: subdiag=[ 0.002+0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 14: subdiag=[ 0.001+0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 15: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 16: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 17: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 18: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 19: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 20: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 21: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 22: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 23: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 24: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 25: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 26: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 27: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 28: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 29: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 30: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 31: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 32: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 33: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 34: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 35: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 36: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 37: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 38: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 39: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 40: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 41: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 42: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 43: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
    iter 44: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
```

```
iter 45: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 46: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 47: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 48: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 49: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 50: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 51: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 52: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 53: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 54: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 55: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 56: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 57: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 58: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 59: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 60: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 61: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 62: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 63: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 64: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 65: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 66: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 67: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 68: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 69: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 70: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 71: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 72: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 73: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 74: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 75: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 76: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 77: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 78: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 79: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 80: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 81: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 82: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 83: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 84: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 85: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 86: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 87: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 88: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 89: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
iter 90: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
```

```
  iter 91: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
  iter 92: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
  iter 93: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
  iter 94: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
  iter 95: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
  iter 96: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
  iter 97: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
  iter 98: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
  iter 99: subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
  after : subdiag=[ 0.   +0.j -0.917+0.j -0.   +0.j],  ‖·‖₂=9.173e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.    0.917 0.   ]
└─ converged?    = no

┌─ Matrix 03/30  (size 4x4)
│  fixed shift μ = 0.999872 (|μ|=0.9999)
  before: subdiag=[ 0.889+0.j -0.786+0.j  0.008+0.j],  ‖·‖₂=1.186e+00
  iter 00: subdiag=[ 0.692+0.j -0.475+0.j  0.   +0.j],  ‖·‖₂=8.397e-01
  iter 01: subdiag=[ 0.182+0.j -0.444+0.j  0.   +0.j],  ‖·‖₂=4.803e-01
  iter 02: subdiag=[ 0.042+0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.448e-01
  iter 03: subdiag=[ 0.009+0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.428e-01
  iter 04: subdiag=[ 0.002+0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 05: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 06: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 07: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 08: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 09: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 10: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 11: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 12: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 13: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 14: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 15: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 16: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 17: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 18: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 19: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 20: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 21: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 22: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 23: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 24: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 25: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 26: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 27: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
```

```
iter 28: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 29: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 30: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 31: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 32: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 33: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 34: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 35: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 36: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 37: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 38: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 39: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 40: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 41: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 42: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 43: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 44: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 45: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 46: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 47: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 48: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 49: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 50: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 51: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 52: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 53: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 54: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 55: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 56: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 57: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 58: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 59: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 60: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 61: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 62: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 63: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 64: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 65: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 66: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 67: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 68: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 69: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 70: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 71: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 72: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
iter 73: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
```

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  iter 74: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 75: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 76: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 77: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 78: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 79: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 80: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 81: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 82: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 83: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 84: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 85: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 86: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 87: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 88: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 89: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 90: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 91: subdiag=[0.   +0.j 0.443+0.j 0.   +0.j],  ‖·‖₂=4.427e-01
  iter 92: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 93: subdiag=[0.   +0.j 0.443+0.j 0.   +0.j],  ‖·‖₂=4.427e-01
  iter 94: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 95: subdiag=[0.   +0.j 0.443+0.j 0.   +0.j],  ‖·‖₂=4.427e-01
  iter 96: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 97: subdiag=[0.   +0.j 0.443+0.j 0.   +0.j],  ‖·‖₂=4.427e-01
  iter 98: subdiag=[ 0.   +0.j -0.443+0.j  0.   +0.j],  ‖·‖₂=4.427e-01
  iter 99: subdiag=[0.   +0.j 0.443+0.j 0.   +0.j],  ‖·‖₂=4.427e-01
  after : subdiag=[0.   +0.j 0.443+0.j 0.   +0.j],  ‖·‖₂=4.427e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.    0.443 0.   ]
└─ converged?    = no

┌─ Matrix 04/30  (size 4x4)
│  fixed shift μ = -0.623567 (|μ|=0.6236)
  before: subdiag=[-0.823+0.j  0.998+0.j -0.726+0.j],  ‖·‖₂=1.483e+00
  iter 00: subdiag=[-1.   +0.j -0.866+0.j  0.426+0.j],  ‖·‖₂=1.390e+00
  iter 01: subdiag=[-0.973+0.j  0.433+0.j -0.346+0.j],  ‖·‖₂=1.120e+00
  iter 02: subdiag=[-0.964+0.j -0.161+0.j  0.334+0.j],  ‖·‖₂=1.033e+00
  iter 03: subdiag=[-0.963+0.j  0.057+0.j -0.333+0.j],  ‖·‖₂=1.020e+00
  iter 04: subdiag=[-0.962+0.j -0.02 +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 05: subdiag=[-0.962+0.j  0.007+0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 06: subdiag=[-0.962+0.j -0.002+0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 07: subdiag=[-0.962+0.j  0.001+0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 08: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 09: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 10: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
```

```
    iter 11: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 12: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 13: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 14: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 15: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 16: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 17: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 18: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 19: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 20: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 21: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 22: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 23: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 24: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 25: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 26: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 27: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 28: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 29: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 30: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 31: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 32: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 33: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 34: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 35: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 36: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 37: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 38: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 39: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 40: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 41: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 42: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 43: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 44: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 45: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 46: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 47: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 48: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 49: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 50: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 51: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 52: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 53: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 54: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
    iter 55: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
    iter 56: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
```

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  iter 57: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 58: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 59: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 60: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 61: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 62: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 63: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 64: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 65: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 66: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 67: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 68: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 69: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 70: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 71: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 72: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 73: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 74: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 75: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 76: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 77: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 78: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 79: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 80: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 81: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 82: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 83: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 84: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 85: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 86: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 87: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 88: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 89: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 90: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 91: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 92: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 93: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 94: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 95: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 96: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 97: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  iter 98: subdiag=[-0.962+0.j -0.   +0.j  0.332+0.j],  ‖·‖₂=1.018e+00
  iter 99: subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
  after : subdiag=[-0.962+0.j  0.   +0.j -0.332+0.j],  ‖·‖₂=1.018e+00
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
```

```
│  [0.962 0.    0.332]
└─ converged?    = no

┌─ Matrix 05/30  (size 4x4)
│  fixed shift μ = -0.649367-0.477769j (|μ|=0.8062)
  before: subdiag=[-0.78 +0.j -0.76 +0.j  0.617+0.j],  ‖·‖₂=1.252e+00
  iter 00: subdiag=[-0.705-0.j  0.441-0.j  0.209+0.j],  ‖·‖₂=8.579e-01
  iter 01: subdiag=[-0.627+0.j -0.316+0.j  0.04 +0.j],  ‖·‖₂=7.032e-01
  iter 02: subdiag=[-0.53 +0.j  0.232-0.j  0.008+0.j],  ‖·‖₂=5.789e-01
  iter 03: subdiag=[-0.432+0.j -0.173+0.j  0.001+0.j],  ‖·‖₂=4.649e-01
  iter 04: subdiag=[-0.342+0.j  0.13 -0.j  0.   +0.j],  ‖·‖₂=3.660e-01
  iter 05: subdiag=[-0.267+0.j -0.098+0.j  0.   +0.j],  ‖·‖₂=2.844e-01
  iter 06: subdiag=[-0.206+0.j  0.075-0.j  0.   -0.j],  ‖·‖₂=2.191e-01
  iter 07: subdiag=[-0.158+0.j -0.057+0.j  0.   +0.j],  ‖·‖₂=1.679e-01
  iter 08: subdiag=[-0.121+0.j  0.044-0.j  0.   -0.j],  ‖·‖₂=1.283e-01
  iter 09: subdiag=[-0.092+0.j -0.033+0.j  0.   -0.j],  ‖·‖₂=9.788e-02
  iter 10: subdiag=[-0.07 +0.j  0.026-0.j  0.   -0.j],  ‖·‖₂=7.459e-02
  iter 11: subdiag=[-0.053+0.j -0.02 +0.j  0.   -0.j],  ‖·‖₂=5.681e-02
  iter 12: subdiag=[-0.041+0.j  0.015-0.j  0.   -0.j],  ‖·‖₂=4.325e-02
  iter 13: subdiag=[-0.031+0.j -0.012+0.j  0.   -0.j],  ‖·‖₂=3.292e-02
  iter 14: subdiag=[-0.023+0.j  0.009-0.j  0.   -0.j],  ‖·‖₂=2.506e-02
  iter 15: subdiag=[-0.018+0.j -0.007+0.j  0.   -0.j],  ‖·‖₂=1.907e-02
  iter 16: subdiag=[-0.014+0.j  0.005-0.j  0.   -0.j],  ‖·‖₂=1.451e-02
  iter 17: subdiag=[-0.01 +0.j -0.004+0.j  0.   -0.j],  ‖·‖₂=1.105e-02
  iter 18: subdiag=[-0.008+0.j  0.003-0.j  0.   -0.j],  ‖·‖₂=8.407e-03
  iter 19: subdiag=[-0.006+0.j -0.002+0.j  0.   -0.j],  ‖·‖₂=6.398e-03
  iter 20: subdiag=[-0.005+0.j  0.002-0.j  0.   -0.j],  ‖·‖₂=4.870e-03
  iter 21: subdiag=[-0.003+0.j -0.001+0.j  0.   -0.j],  ‖·‖₂=3.706e-03
  iter 22: subdiag=[-0.003+0.j  0.001-0.j  0.   -0.j],  ‖·‖₂=2.821e-03
  iter 23: subdiag=[-0.002+0.j -0.001+0.j  0.   -0.j],  ‖·‖₂=2.147e-03
  iter 24: subdiag=[-0.002+0.j  0.001-0.j  0.   -0.j],  ‖·‖₂=1.634e-03
  iter 25: subdiag=[-0.001+0.j -0.   +0.j  0.   -0.j],  ‖·‖₂=1.244e-03
  iter 26: subdiag=[-0.001+0.j  0.   -0.j  0.   -0.j],  ‖·‖₂=9.466e-04
  iter 27: subdiag=[-0.001+0.j -0.   +0.j  0.   -0.j],  ‖·‖₂=7.205e-04
  iter 28: subdiag=[-0.001+0.j  0.   -0.j  0.   -0.j],  ‖·‖₂=5.484e-04
  iter 29: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=4.174e-04
  iter 30: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=3.177e-04
  iter 31: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=2.419e-04
  iter 32: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=1.841e-04
  iter 33: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=1.401e-04
  iter 34: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=1.067e-04
  iter 35: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=8.120e-05
  iter 36: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=6.182e-05
  iter 37: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=4.706e-05
  iter 38: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=3.582e-05
  iter 39: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=2.727e-05
```

```
    iter 40: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=2.076e-05
    iter 41: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=1.581e-05
    iter 42: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=1.203e-05
    iter 43: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=9.162e-06
    iter 44: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=6.975e-06
    iter 45: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=5.311e-06
    iter 46: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=4.044e-06
    iter 47: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=3.079e-06
    iter 48: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=2.344e-06
    iter 49: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=1.785e-06
    iter 50: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=1.359e-06
    iter 51: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=1.035e-06
    iter 52: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=7.880e-07
    iter 53: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=6.000e-07
    iter 54: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=4.569e-07
    iter 55: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=3.479e-07
    iter 56: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=2.650e-07
    iter 57: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=2.018e-07
    iter 58: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=1.537e-07
    iter 59: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=1.170e-07
    iter 60: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=8.913e-08
    iter 61: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=6.788e-08
    iter 62: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=5.170e-08
    iter 63: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=3.937e-08
    iter 64: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=2.999e-08
    iter 65: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=2.284e-08
    iter 66: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=1.740e-08
    iter 67: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=1.325e-08
    iter 68: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=1.009e-08
    iter 69: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=7.688e-09
    iter 70: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=5.857e-09
    iter 71: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=4.461e-09
    iter 72: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=3.398e-09
    iter 73: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=2.589e-09
    iter 74: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=1.972e-09
    iter 75: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=1.503e-09
    iter 76: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=1.145e-09
    iter 77: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=8.721e-10
    iter 78: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=6.644e-10
    iter 79: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=5.062e-10
    iter 80: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=3.857e-10
    iter 81: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=2.939e-10
    iter 82: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=2.239e-10
    iter 83: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=1.706e-10
    iter 84: subdiag=[-0.+0.j  0.-0.j  0.-0.j],  ‖·‖₂=1.300e-10
    iter 85: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=9.907e-11
```

```
  after : subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=9.907e-11
│  iterations    = 86/100
│  sub-diag magnitudes after last step:
│  [0. 0. 0.]
└─ converged?    = yes

┌─ Matrix 06/30  (size 4x4)
│  fixed shift μ = -0.771151 (|μ|=0.7712)
  before: subdiag=[-0.695+0.j  0.921+0.j  0.395+0.j],  ‖·‖₂=1.220e+00
  iter 00: subdiag=[-0.414+0.j  0.919+0.j  0.099+0.j],  ‖·‖₂=1.012e+00
  iter 01: subdiag=[-0.236+0.j  0.912+0.j  0.023+0.j],  ‖·‖₂=9.419e-01
  iter 02: subdiag=[-0.131+0.j  0.909+0.j  0.005+0.j],  ‖·‖₂=9.186e-01
  iter 03: subdiag=[-0.072+0.j  0.908+0.j  0.001+0.j],  ‖·‖₂=9.112e-01
  iter 04: subdiag=[-0.04 +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.090e-01
  iter 05: subdiag=[-0.022+0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.083e-01
  iter 06: subdiag=[-0.012+0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.081e-01
  iter 07: subdiag=[-0.007+0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 08: subdiag=[-0.004+0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 09: subdiag=[-0.002+0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 10: subdiag=[-0.001+0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 11: subdiag=[-0.001+0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 12: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 13: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 14: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 15: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 16: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 17: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 18: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 19: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 20: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 21: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 22: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 23: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 24: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 25: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 26: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 27: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 28: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 29: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 30: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 31: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 32: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 33: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 34: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 35: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 36: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
```

```
iter 37: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 38: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 39: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 40: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 41: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 42: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 43: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 44: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 45: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 46: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 47: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 48: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 49: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 50: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 51: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 52: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 53: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 54: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 55: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 56: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 57: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 58: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 59: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 60: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 61: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 62: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 63: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 64: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 65: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 66: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 67: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 68: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 69: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 70: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 71: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 72: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 73: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 74: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 75: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 76: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 77: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 78: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 79: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 80: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 81: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
iter 82: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
```

```
  iter 83: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 84: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 85: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 86: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 87: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 88: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 89: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 90: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 91: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 92: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 93: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 94: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 95: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 96: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 97: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 98: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  iter 99: subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
  after : subdiag=[-0.   +0.j  0.908+0.j  0.   +0.j],  ‖·‖₂=9.080e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.    0.908 0.   ]
└─ converged?    = no

┌─ Matrix 07/30  (size 4x4)
│  fixed shift μ = -0.884666 (|μ|=0.8847)
  before: subdiag=[ 1.   +0.j -0.815+0.j  0.838+0.j],  ‖·‖₂=1.538e+00
  iter 00: subdiag=[ 0.626+0.j  0.831+0.j -0.362+0.j],  ‖·‖₂=1.102e+00
  iter 01: subdiag=[ 0.56 +0.j -0.193+0.j  0.317+0.j],  ‖·‖₂=6.720e-01
  iter 02: subdiag=[ 0.558+0.j  0.035+0.j -0.316+0.j],  ‖·‖₂=6.419e-01
  iter 03: subdiag=[ 0.558+0.j -0.006+0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 04: subdiag=[ 0.558+0.j  0.001+0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 05: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 06: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 07: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 08: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 09: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 10: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 11: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 12: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 13: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 14: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 15: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 16: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 17: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 18: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 19: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
```

```
    iter 20: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 21: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 22: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 23: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 24: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 25: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 26: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 27: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 28: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 29: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 30: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 31: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 32: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 33: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 34: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 35: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 36: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 37: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 38: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 39: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 40: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 41: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 42: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 43: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 44: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 45: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 46: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 47: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 48: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 49: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 50: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 51: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 52: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 53: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 54: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 55: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 56: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 57: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 58: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 59: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 60: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 61: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 62: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 63: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
    iter 64: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
    iter 65: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
```

```
  iter 66: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 67: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 68: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 69: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 70: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 71: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 72: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 73: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 74: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 75: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 76: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 77: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 78: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 79: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 80: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 81: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 82: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 83: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 84: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 85: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 86: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 87: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 88: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 89: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 90: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 91: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 92: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 93: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 94: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 95: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 96: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 97: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  iter 98: subdiag=[ 0.558+0.j  0.   +0.j -0.316+0.j],  ‖·‖₂=6.409e-01
  iter 99: subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
  after : subdiag=[ 0.558+0.j -0.   +0.j  0.316+0.j],  ‖·‖₂=6.409e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.558 0.    0.316]
└─ converged?    = no

┌─ Matrix 08/30  (size 4x4)
│  fixed shift μ = -0.662036-0.664809j (|μ|=0.9382)
  before: subdiag=[ 0.992+0.j -0.474+0.j  0.71 +0.j],  ‖·‖₂=1.309e+00
  iter 00: subdiag=[0.782+0.j 0.413-0.j 0.08 -0.j],  ‖·‖₂=8.875e-01
  iter 01: subdiag=[ 0.462-0.j  0.541-0.j -0.005+0.j],  ‖·‖₂=7.119e-01
  iter 02: subdiag=[0.25 +0.j 0.713-0.j 0.   -0.j],  ‖·‖₂=7.559e-01
```

```
    iter 03: subdiag=[ 0.136+0.j  0.876-0.j -0.   +0.j],  ‖·‖₂=8.861e-01
    iter 04: subdiag=[0.078-0.j 0.95 -0.j 0.   -0.j],  ‖·‖₂=9.531e-01
    iter 05: subdiag=[ 0.048+0.j  0.89 -0.j -0.   +0.j],  ‖·‖₂=8.914e-01
    iter 06: subdiag=[0.032+0.j 0.732-0.j 0.   -0.j],  ‖·‖₂=7.331e-01
    iter 07: subdiag=[ 0.022+0.j  0.551-0.j -0.   +0.j],  ‖·‖₂=5.514e-01
    iter 08: subdiag=[0.015+0.j 0.393-0.j 0.   -0.j],  ‖·‖₂=3.938e-01
    iter 09: subdiag=[-0.011-0.j -0.274+0.j -0.   +0.j],  ‖·‖₂=2.738e-01
    iter 10: subdiag=[0.008+0.j 0.188-0.j 0.   -0.j],  ‖·‖₂=1.880e-01
    iter 11: subdiag=[-0.006-0.j -0.128+0.j -0.   +0.j],  ‖·‖₂=1.282e-01
    iter 12: subdiag=[0.004+0.j 0.087-0.j 0.   -0.j],  ‖·‖₂=8.725e-02
    iter 13: subdiag=[-0.003-0.j -0.059+0.j -0.   +0.j],  ‖·‖₂=5.928e-02
    iter 14: subdiag=[0.002+0.j 0.04 -0.j 0.   -0.j],  ‖·‖₂=4.025e-02
    iter 15: subdiag=[-0.002-0.j -0.027+0.j -0.   +0.j],  ‖·‖₂=2.732e-02
    iter 16: subdiag=[0.001+0.j 0.019-0.j 0.   -0.j],  ‖·‖₂=1.855e-02
    iter 17: subdiag=[-0.001-0.j -0.013+0.j -0.   +0.j],  ‖·‖₂=1.259e-02
    iter 18: subdiag=[0.001+0.j 0.009-0.j 0.   -0.j],  ‖·‖₂=8.545e-03
    iter 19: subdiag=[-0.   -0.j -0.006+0.j -0.   +0.j],  ‖·‖₂=5.800e-03
    iter 20: subdiag=[0.   +0.j 0.004-0.j 0.   -0.j],  ‖·‖₂=3.937e-03
    iter 21: subdiag=[-0.   -0.j -0.003+0.j -0.   +0.j],  ‖·‖₂=2.673e-03
    iter 22: subdiag=[0.   +0.j 0.002-0.j 0.   -0.j],  ‖·‖₂=1.814e-03
    iter 23: subdiag=[-0.   -0.j -0.001+0.j -0.   +0.j],  ‖·‖₂=1.232e-03
    iter 24: subdiag=[0.   +0.j 0.001-0.j 0.   -0.j],  ‖·‖₂=8.364e-04
    iter 25: subdiag=[-0.   -0.j -0.001+0.j -0.   +0.j],  ‖·‖₂=5.679e-04
    iter 26: subdiag=[0.+0.j 0.-0.j 0.+0.j],  ‖·‖₂=3.857e-04
    iter 27: subdiag=[-0.-0.j -0.+0.j -0.+0.j],  ‖·‖₂=2.619e-04
    iter 28: subdiag=[0.+0.j 0.-0.j 0.+0.j],  ‖·‖₂=1.779e-04
    iter 29: subdiag=[-0.-0.j -0.+0.j -0.-0.j],  ‖·‖₂=1.209e-04
    iter 30: subdiag=[0.+0.j 0.-0.j 0.+0.j],  ‖·‖₂=8.213e-05
    iter 31: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=5.581e-05
    iter 32: subdiag=[0.+0.j 0.-0.j 0.+0.j],  ‖·‖₂=3.794e-05
    iter 33: subdiag=[-0.-0.j -0.+0.j -0.-0.j],  ‖·‖₂=2.579e-05
    iter 34: subdiag=[0.+0.j 0.+0.j 0.+0.j],  ‖·‖₂=1.754e-05
    iter 35: subdiag=[-0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=1.193e-05
    iter 36: subdiag=[0.+0.j 0.+0.j 0.-0.j],  ‖·‖₂=8.118e-06
    iter 37: subdiag=[-0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=5.526e-06
    iter 38: subdiag=[0.+0.j 0.+0.j 0.-0.j],  ‖·‖₂=3.763e-06
    iter 39: subdiag=[-0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=2.564e-06
    iter 40: subdiag=[0.+0.j 0.+0.j 0.+0.j],  ‖·‖₂=1.748e-06
    iter 41: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=1.192e-06
    iter 42: subdiag=[0.+0.j 0.+0.j 0.+0.j],  ‖·‖₂=8.135e-07
    iter 43: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=5.556e-07
    iter 44: subdiag=[0.+0.j 0.+0.j 0.+0.j],  ‖·‖₂=3.797e-07
    iter 45: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=2.598e-07
    iter 46: subdiag=[0.+0.j 0.+0.j 0.+0.j],  ‖·‖₂=1.779e-07
    iter 47: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=1.219e-07
    iter 48: subdiag=[0.+0.j 0.+0.j 0.+0.j],  ‖·‖₂=8.368e-08
```

```
    iter 49: subdiag=[-0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=5.750e-08
    iter 50: subdiag=[0.+0.j 0.+0.j 0.-0.j],  ‖·‖₂=3.956e-08
    iter 51: subdiag=[-0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=2.726e-08
    iter 52: subdiag=[0.+0.j 0.+0.j 0.-0.j],  ‖·‖₂=1.881e-08
    iter 53: subdiag=[-0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=1.300e-08
    iter 54: subdiag=[0.+0.j 0.+0.j 0.+0.j],  ‖·‖₂=9.004e-09
    iter 55: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=6.246e-09
    iter 56: subdiag=[0.+0.j 0.+0.j 0.+0.j],  ‖·‖₂=4.341e-09
    iter 57: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=3.023e-09
    iter 58: subdiag=[0.+0.j 0.+0.j 0.+0.j],  ‖·‖₂=2.110e-09
    iter 59: subdiag=[-0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=1.475e-09
    iter 60: subdiag=[0.+0.j 0.+0.j 0.+0.j],  ‖·‖₂=1.034e-09
    iter 61: subdiag=[-0.-0.j -0.+0.j -0.-0.j],  ‖·‖₂=7.257e-10
    iter 62: subdiag=[0.+0.j 0.-0.j 0.+0.j],  ‖·‖₂=5.106e-10
    iter 63: subdiag=[-0.-0.j -0.+0.j -0.-0.j],  ‖·‖₂=3.599e-10
    iter 64: subdiag=[0.+0.j 0.-0.j 0.+0.j],  ‖·‖₂=2.542e-10
    iter 65: subdiag=[-0.-0.j -0.+0.j -0.-0.j],  ‖·‖₂=1.799e-10
    iter 66: subdiag=[0.+0.j 0.-0.j 0.+0.j],  ‖·‖₂=1.276e-10
    iter 67: subdiag=[-0.-0.j -0.+0.j -0.+0.j],  ‖·‖₂=9.060e-11
    after : subdiag=[-0.-0.j -0.+0.j -0.+0.j],  ‖·‖₂=9.060e-11
  │  iterations    = 68/100
  │  sub-diag magnitudes after last step:
  │  [0. 0. 0.]
  └─ converged?    = yes

  ┌─ Matrix 09/30  (size 4x4)
  │  fixed shift μ = 0.995354 (|μ|=0.9954)
    before: subdiag=[ 0.805+0.j  0.682+0.j -0.243+0.j],  ‖·‖₂=1.083e+00
    iter 00: subdiag=[ 0.618+0.j -0.241+0.j  0.043+0.j],  ‖·‖₂=6.650e-01
    iter 01: subdiag=[ 0.614+0.j  0.006+0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 02: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 03: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 04: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 05: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 06: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 07: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 08: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 09: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 10: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 11: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 12: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 13: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 14: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 15: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 16: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 17: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
```

```
    iter 18: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 19: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 20: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 21: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 22: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 23: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 24: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 25: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 26: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 27: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 28: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 29: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 30: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 31: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 32: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 33: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 34: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 35: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 36: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 37: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 38: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 39: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 40: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 41: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 42: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 43: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 44: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 45: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 46: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 47: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 48: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 49: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 50: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 51: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 52: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 53: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 54: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 55: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 56: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 57: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 58: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 59: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 60: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 61: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
    iter 62: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
    iter 63: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
```

```
  iter 64: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 65: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 66: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 67: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 68: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 69: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 70: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 71: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 72: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 73: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 74: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 75: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 76: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 77: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 78: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 79: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 80: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 81: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 82: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 83: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 84: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 85: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 86: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 87: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 88: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 89: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 90: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 91: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 92: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 93: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 94: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 95: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 96: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 97: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  iter 98: subdiag=[ 0.614+0.j -0.   +0.j  0.043+0.j],  ‖·‖₂=6.158e-01
  iter 99: subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
  after : subdiag=[ 0.614+0.j  0.   +0.j -0.043+0.j],  ‖·‖₂=6.158e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.614 0.    0.043]
└─ converged?    = no

┌─ Matrix 10/30  (size 4x4)
│  fixed shift μ = -0.926518 (|μ|=0.9265)
  before: subdiag=[ 0.914+0.j  0.658+0.j -0.888+0.j],  ‖·‖₂=1.434e+00
  iter 00: subdiag=[ 0.572+0.j -0.901+0.j  0.244+0.j],  ‖·‖₂=1.095e+00
```

```
    iter 01: subdiag=[ 0.491+0.j  0.144+0.j -0.205+0.j],  ‖·‖₂=5.513e-01
    iter 02: subdiag=[ 0.49 +0.j -0.016+0.j  0.204+0.j],  ‖·‖₂=5.311e-01
    iter 03: subdiag=[ 0.49 +0.j  0.002+0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 04: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 05: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 06: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 07: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 08: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 09: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 10: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 11: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 12: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 13: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 14: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 15: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 16: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 17: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 18: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 19: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 20: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 21: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 22: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 23: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 24: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 25: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 26: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 27: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 28: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 29: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 30: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 31: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 32: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 33: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 34: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 35: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 36: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 37: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 38: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 39: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 40: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 41: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 42: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 43: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 44: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
    iter 45: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
    iter 46: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
```

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986     iter 47: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
987     iter 48: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
988     iter 49: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
989     iter 50: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
990     iter 51: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
991     iter 52: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
992     iter 53: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
993     iter 54: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
994     iter 55: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
995     iter 56: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
996     iter 57: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
997     iter 58: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
998     iter 59: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
999     iter 60: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1000    iter 61: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1001    iter 62: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1002    iter 63: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1003    iter 64: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1004    iter 65: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1005    iter 66: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1006    iter 67: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1007    iter 68: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1008    iter 69: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1009    iter 70: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1010    iter 71: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1011    iter 72: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1012    iter 73: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1013    iter 74: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1014    iter 75: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1015    iter 76: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1016    iter 77: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1017    iter 78: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1018    iter 79: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1019    iter 80: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1020    iter 81: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1021    iter 82: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1022    iter 83: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1023    iter 84: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1024    iter 85: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1025    iter 86: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1026    iter 87: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1027    iter 88: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1028    iter 89: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1029    iter 90: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
1030    iter 91: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
1031    iter 92: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
```

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  iter 93: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
  iter 94: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
  iter 95: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
  iter 96: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
  iter 97: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
  iter 98: subdiag=[ 0.49 +0.j -0.   +0.j  0.204+0.j],  ‖·‖₂=5.308e-01
  iter 99: subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
  after : subdiag=[ 0.49 +0.j  0.   +0.j -0.204+0.j],  ‖·‖₂=5.308e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.49  0.    0.204]
└─ converged?    = no

┌─ Matrix 11/30  (size 4x4)
│  fixed shift μ = -0.553226-0.269054j (|μ|=0.6152)
  before: subdiag=[-0.998+0.j  0.926+0.j  0.596+0.j],  ‖·‖₂=1.486e+00
  iter 00: subdiag=[-0.938+0.j -0.68 +0.j  0.382+0.j],  ‖·‖₂=1.219e+00
  iter 01: subdiag=[-0.81 +0.j  0.462-0.j  0.206-0.j],  ‖·‖₂=9.552e-01
  iter 02: subdiag=[-0.661+0.j -0.318+0.j  0.103+0.j],  ‖·‖₂=7.404e-01
  iter 03: subdiag=[-0.517-0.j  0.221-0.j  0.051+0.j],  ‖·‖₂=5.644e-01
  iter 04: subdiag=[-0.394-0.j -0.156+0.j  0.025+0.j],  ‖·‖₂=4.240e-01
  iter 05: subdiag=[-0.295-0.j  0.11 -0.j  0.012+0.j],  ‖·‖₂=3.154e-01
  iter 06: subdiag=[-0.22 -0.j -0.078+0.j  0.006-0.j],  ‖·‖₂=2.332e-01
  iter 07: subdiag=[-0.163+0.j  0.055-0.j  0.003+0.j],  ‖·‖₂=1.718e-01
  iter 08: subdiag=[-0.12 -0.j -0.039+0.j  0.001+0.j],  ‖·‖₂=1.263e-01
  iter 09: subdiag=[-0.089-0.j  0.028-0.j  0.001-0.j],  ‖·‖₂=9.283e-02
  iter 10: subdiag=[-0.065-0.j -0.02 +0.j  0.   -0.j],  ‖·‖₂=6.818e-02
  iter 11: subdiag=[-0.048+0.j  0.014-0.j  0.   -0.j],  ‖·‖₂=5.006e-02
  iter 12: subdiag=[-0.035+0.j -0.01 +0.j  0.   -0.j],  ‖·‖₂=3.676e-02
  iter 13: subdiag=[-0.026+0.j  0.007-0.j  0.   +0.j],  ‖·‖₂=2.699e-02
  iter 14: subdiag=[-0.019+0.j -0.005+0.j  0.   +0.j],  ‖·‖₂=1.982e-02
  iter 15: subdiag=[-0.014+0.j  0.004-0.j  0.   +0.j],  ‖·‖₂=1.456e-02
  iter 16: subdiag=[-0.01 +0.j -0.003+0.j  0.   +0.j],  ‖·‖₂=1.069e-02
  iter 17: subdiag=[-0.008+0.j  0.002-0.j  0.   +0.j],  ‖·‖₂=7.855e-03
  iter 18: subdiag=[-0.006+0.j -0.001+0.j  0.   -0.j],  ‖·‖₂=5.771e-03
  iter 19: subdiag=[-0.004+0.j  0.001-0.j  0.   -0.j],  ‖·‖₂=4.240e-03
  iter 20: subdiag=[-0.003-0.j -0.001-0.j  0.   -0.j],  ‖·‖₂=3.116e-03
  iter 21: subdiag=[-0.002-0.j  0.   +0.j  0.   -0.j],  ‖·‖₂=2.290e-03
  iter 22: subdiag=[-0.002-0.j -0.   -0.j  0.   +0.j],  ‖·‖₂=1.683e-03
  iter 23: subdiag=[-0.001-0.j  0.   -0.j  0.   +0.j],  ‖·‖₂=1.237e-03
  iter 24: subdiag=[-0.001-0.j -0.   +0.j  0.   -0.j],  ‖·‖₂=9.094e-04
  iter 25: subdiag=[-0.001-0.j  0.   +0.j  0.   -0.j],  ‖·‖₂=6.685e-04
  iter 26: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ‖·‖₂=4.915e-04
  iter 27: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ‖·‖₂=3.614e-04
  iter 28: subdiag=[-0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=2.657e-04
  iter 29: subdiag=[-0.-0.j  0.-0.j  0.+0.j],  ‖·‖₂=1.954e-04
```

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  iter 30: subdiag=[-0.+0.j -0.+0.j  0.+0.j],  ‖·‖₂=1.437e-04
  iter 31: subdiag=[-0.-0.j  0.-0.j  0.+0.j],  ‖·‖₂=1.057e-04
  iter 32: subdiag=[-0.-0.j -0.+0.j  0.+0.j],  ‖·‖₂=7.772e-05
  iter 33: subdiag=[-0.-0.j  0.-0.j  0.+0.j],  ‖·‖₂=5.716e-05
  iter 34: subdiag=[-0.-0.j -0.+0.j  0.+0.j],  ‖·‖₂=4.204e-05
  iter 35: subdiag=[-0.-0.j  0.-0.j  0.+0.j],  ‖·‖₂=3.093e-05
  iter 36: subdiag=[-0.-0.j -0.+0.j  0.+0.j],  ‖·‖₂=2.275e-05
  iter 37: subdiag=[-0.-0.j  0.-0.j  0.+0.j],  ‖·‖₂=1.673e-05
  iter 38: subdiag=[-0.-0.j -0.+0.j  0.+0.j],  ‖·‖₂=1.231e-05
  iter 39: subdiag=[-0.-0.j  0.+0.j  0.+0.j],  ‖·‖₂=9.056e-06
  iter 40: subdiag=[-0.-0.j -0.-0.j  0.+0.j],  ‖·‖₂=6.662e-06
  iter 41: subdiag=[-0.-0.j  0.+0.j  0.+0.j],  ‖·‖₂=4.902e-06
  iter 42: subdiag=[-0.-0.j -0.+0.j  0.+0.j],  ‖·‖₂=3.606e-06
  iter 43: subdiag=[-0.-0.j  0.-0.j  0.+0.j],  ‖·‖₂=2.653e-06
  iter 44: subdiag=[-0.-0.j -0.+0.j  0.+0.j],  ‖·‖₂=1.952e-06
  iter 45: subdiag=[-0.-0.j  0.-0.j  0.+0.j],  ‖·‖₂=1.436e-06
  iter 46: subdiag=[-0.-0.j -0.+0.j  0.+0.j],  ‖·‖₂=1.057e-06
  iter 47: subdiag=[-0.-0.j  0.-0.j  0.+0.j],  ‖·‖₂=7.776e-07
  iter 48: subdiag=[-0.-0.j -0.+0.j  0.+0.j],  ‖·‖₂=5.722e-07
  iter 49: subdiag=[-0.-0.j  0.-0.j  0.+0.j],  ‖·‖₂=4.210e-07
  iter 50: subdiag=[-0.-0.j -0.+0.j  0.+0.j],  ‖·‖₂=3.098e-07
  iter 51: subdiag=[-0.-0.j  0.-0.j  0.+0.j],  ‖·‖₂=2.280e-07
  iter 52: subdiag=[-0.-0.j -0.+0.j  0.+0.j],  ‖·‖₂=1.677e-07
  iter 53: subdiag=[-0.-0.j  0.-0.j  0.+0.j],  ‖·‖₂=1.234e-07
  iter 54: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ‖·‖₂=9.083e-08
  iter 55: subdiag=[-0.-0.j  0.-0.j  0.+0.j],  ‖·‖₂=6.684e-08
  iter 56: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ‖·‖₂=4.919e-08
  iter 57: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ‖·‖₂=3.620e-08
  iter 58: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ‖·‖₂=2.664e-08
  iter 59: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ‖·‖₂=1.960e-08
  iter 60: subdiag=[-0.-0.j -0.-0.j  0.-0.j],  ‖·‖₂=1.443e-08
  iter 61: subdiag=[-0.-0.j  0.+0.j  0.-0.j],  ‖·‖₂=1.062e-08
  iter 62: subdiag=[-0.-0.j -0.-0.j  0.-0.j],  ‖·‖₂=7.812e-09
  iter 63: subdiag=[-0.-0.j  0.+0.j  0.-0.j],  ‖·‖₂=5.749e-09
  iter 64: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ‖·‖₂=4.231e-09
  iter 65: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ‖·‖₂=3.114e-09
  iter 66: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ‖·‖₂=2.292e-09
  iter 67: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ‖·‖₂=1.686e-09
  iter 68: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ‖·‖₂=1.241e-09
  iter 69: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ‖·‖₂=9.134e-10
  iter 70: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ‖·‖₂=6.722e-10
  iter 71: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ‖·‖₂=4.947e-10
  iter 72: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ‖·‖₂=3.641e-10
  iter 73: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ‖·‖₂=2.680e-10
  iter 74: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ‖·‖₂=1.972e-10
  iter 75: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ‖·‖₂=1.451e-10
```

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  iter 76: subdiag=[-0.-0.j -0.+0.j  0.-0.j],  ‖·‖₂=1.068e-10
  iter 77: subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ‖·‖₂=7.861e-11
  after : subdiag=[-0.-0.j  0.-0.j  0.-0.j],  ‖·‖₂=7.861e-11
│  iterations    = 78/100
│  sub-diag magnitudes after last step:
│  [0. 0. 0.]
└─ converged?    = yes

┌─ Matrix 12/30  (size 4x4)
│  fixed shift μ = 0.918038 (|μ|=0.9180)
  before: subdiag=[-0.634+0.j  0.891+0.j  0.631+0.j],  ‖·‖₂=1.263e+00
  iter 00: subdiag=[-0.868+0.j -0.701+0.j -0.147+0.j],  ‖·‖₂=1.126e+00
  iter 01: subdiag=[-0.823+0.j  0.075+0.j  0.133+0.j],  ‖·‖₂=8.371e-01
  iter 02: subdiag=[-0.823+0.j -0.007+0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 03: subdiag=[-0.823+0.j  0.001+0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 04: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 05: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 06: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 07: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 08: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 09: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 10: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 11: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 12: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 13: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 14: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 15: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 16: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 17: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 18: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 19: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 20: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 21: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 22: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 23: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 24: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 25: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 26: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 27: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 28: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 29: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 30: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 31: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 32: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 33: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 34: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
```

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  iter 35: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 36: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 37: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 38: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 39: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 40: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 41: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 42: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 43: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 44: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 45: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 46: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 47: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 48: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 49: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 50: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 51: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 52: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 53: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 54: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 55: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 56: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 57: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 58: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 59: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 60: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 61: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 62: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 63: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 64: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 65: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 66: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 67: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 68: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 69: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 70: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 71: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 72: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 73: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 74: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 75: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 76: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 77: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 78: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 79: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 80: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
```

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  iter 81: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 82: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 83: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 84: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 85: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 86: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 87: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 88: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 89: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 90: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 91: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 92: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 93: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 94: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 95: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 96: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 97: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  iter 98: subdiag=[-0.823+0.j -0.   +0.j -0.133+0.j],  ‖·‖₂=8.333e-01
  iter 99: subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
  after : subdiag=[-0.823+0.j  0.   +0.j  0.133+0.j],  ‖·‖₂=8.333e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.823 0.    0.133]
└─ converged?    = no

┌─ Matrix 13/30  (size 4x4)
│  fixed shift μ = -0.0259786-0.624268j (|μ|=0.6248)
  before: subdiag=[ 0.996+0.j -0.921+0.j  0.999+0.j],  ‖·‖₂=1.685e+00
  iter 00: subdiag=[ 0.78 +0.j -0.999+0.j  0.781+0.j],  ‖·‖₂=1.488e+00
  iter 01: subdiag=[-0.6  +0.j -0.718+0.j -0.6  -0.j],  ‖·‖₂=1.111e+00
  iter 02: subdiag=[0.56 +0.j 0.319-0.j 0.559+0.j],  ‖·‖₂=8.526e-01
  iter 03: subdiag=[-0.553+0.j -0.124-0.j -0.551+0.j],  ‖·‖₂=7.910e-01
  iter 04: subdiag=[0.552+0.j 0.047-0.j 0.549+0.j],  ‖·‖₂=7.802e-01
  iter 05: subdiag=[-0.551-0.j  0.018-0.j -0.548+0.j],  ‖·‖₂=7.772e-01
  iter 06: subdiag=[0.549+0.j 0.007-0.j 0.547+0.j],  ‖·‖₂=7.750e-01
  iter 07: subdiag=[-0.548-0.j  0.003+0.j -0.545+0.j],  ‖·‖₂=7.726e-01
  iter 08: subdiag=[0.546+0.j 0.001+0.j 0.543-0.j],  ‖·‖₂=7.698e-01
  iter 09: subdiag=[-0.543-0.j  0.   +0.j -0.541+0.j],  ‖·‖₂=7.666e-01
  iter 10: subdiag=[0.54 +0.j 0.   +0.j 0.539-0.j],  ‖·‖₂=7.631e-01
  iter 11: subdiag=[-0.537-0.j  0.   +0.j -0.536+0.j],  ‖·‖₂=7.591e-01
  iter 12: subdiag=[0.534-0.j 0.   +0.j 0.534+0.j],  ‖·‖₂=7.548e-01
  iter 13: subdiag=[-0.53 +0.j  0.   +0.j -0.531-0.j],  ‖·‖₂=7.502e-01
  iter 14: subdiag=[0.526-0.j 0.   +0.j 0.528+0.j],  ‖·‖₂=7.452e-01
  iter 15: subdiag=[-0.521+0.j  0.   +0.j -0.525+0.j],  ‖·‖₂=7.399e-01
  iter 16: subdiag=[0.517-0.j 0.   +0.j 0.522+0.j],  ‖·‖₂=7.343e-01
  iter 17: subdiag=[-0.512-0.j  0.   +0.j -0.518-0.j],  ‖·‖₂=7.283e-01
```

```
  iter 18: subdiag=[0.506+0.j 0.   +0.j 0.515+0.j],  ‖·‖₂=7.221e-01
  iter 19: subdiag=[-0.501-0.j  0.   +0.j -0.511-0.j],  ‖·‖₂=7.157e-01
  iter 20: subdiag=[0.495+0.j 0.   +0.j 0.507+0.j],  ‖·‖₂=7.089e-01
  iter 21: subdiag=[-0.489-0.j  0.   +0.j -0.503+0.j],  ‖·‖₂=7.020e-01
  iter 22: subdiag=[0.483+0.j 0.   +0.j 0.499-0.j],  ‖·‖₂=6.948e-01
  iter 23: subdiag=[-0.477-0.j  0.   +0.j -0.495+0.j],  ‖·‖₂=6.874e-01
  iter 24: subdiag=[0.47 +0.j 0.   +0.j 0.491-0.j],  ‖·‖₂=6.798e-01
  iter 25: subdiag=[-0.464-0.j  0.   +0.j -0.486+0.j],  ‖·‖₂=6.720e-01
  iter 26: subdiag=[0.457+0.j 0.   +0.j 0.482-0.j],  ‖·‖₂=6.641e-01
  iter 27: subdiag=[-0.45 -0.j  0.   +0.j -0.477+0.j],  ‖·‖₂=6.560e-01
  iter 28: subdiag=[0.443+0.j 0.   +0.j 0.472+0.j],  ‖·‖₂=6.478e-01
  iter 29: subdiag=[-0.436-0.j  0.   +0.j -0.468+0.j],  ‖·‖₂=6.395e-01
  iter 30: subdiag=[0.429+0.j 0.   +0.j 0.463+0.j],  ‖·‖₂=6.311e-01
  iter 31: subdiag=[-0.422-0.j  0.   +0.j -0.458+0.j],  ‖·‖₂=6.226e-01
  iter 32: subdiag=[0.415+0.j 0.   +0.j 0.453-0.j],  ‖·‖₂=6.140e-01
  iter 33: subdiag=[-0.408-0.j  0.   +0.j -0.448+0.j],  ‖·‖₂=6.054e-01
  iter 34: subdiag=[0.4  +0.j 0.   +0.j 0.443+0.j],  ‖·‖₂=5.967e-01
  iter 35: subdiag=[-0.393-0.j  0.   +0.j -0.437+0.j],  ‖·‖₂=5.880e-01
  iter 36: subdiag=[0.386+0.j 0.   +0.j 0.432-0.j],  ‖·‖₂=5.792e-01
  iter 37: subdiag=[-0.378-0.j  0.   +0.j -0.427+0.j],  ‖·‖₂=5.704e-01
  iter 38: subdiag=[0.371+0.j 0.   +0.j 0.422+0.j],  ‖·‖₂=5.616e-01
  iter 39: subdiag=[-0.364-0.j  0.   +0.j -0.416+0.j],  ‖·‖₂=5.529e-01
  iter 40: subdiag=[0.357+0.j 0.   +0.j 0.411+0.j],  ‖·‖₂=5.441e-01
  iter 41: subdiag=[-0.349-0.j  0.   +0.j -0.406+0.j],  ‖·‖₂=5.354e-01
  iter 42: subdiag=[0.342+0.j 0.   +0.j 0.4  -0.j],  ‖·‖₂=5.266e-01
  iter 43: subdiag=[-0.335-0.j  0.   +0.j -0.395+0.j],  ‖·‖₂=5.180e-01
  iter 44: subdiag=[0.328+0.j 0.   +0.j 0.389-0.j],  ‖·‖₂=5.093e-01
  iter 45: subdiag=[-0.321-0.j  0.   +0.j -0.384+0.j],  ‖·‖₂=5.008e-01
  iter 46: subdiag=[0.314+0.j 0.   +0.j 0.379-0.j],  ‖·‖₂=4.922e-01
  iter 47: subdiag=[-0.308-0.j  0.   +0.j -0.373+0.j],  ‖·‖₂=4.838e-01
  iter 48: subdiag=[0.301+0.j 0.   +0.j 0.368-0.j],  ‖·‖₂=4.754e-01
  iter 49: subdiag=[-0.294-0.j  0.   +0.j -0.363+0.j],  ‖·‖₂=4.671e-01
  iter 50: subdiag=[0.288+0.j 0.   +0.j 0.357-0.j],  ‖·‖₂=4.588e-01
  iter 51: subdiag=[-0.281-0.j  0.   +0.j -0.352+0.j],  ‖·‖₂=4.507e-01
  iter 52: subdiag=[0.275+0.j 0.   +0.j 0.347-0.j],  ‖·‖₂=4.426e-01
  iter 53: subdiag=[-0.269-0.j  0.   +0.j -0.342+0.j],  ‖·‖₂=4.346e-01
  iter 54: subdiag=[0.263+0.j 0.   +0.j 0.336+0.j],  ‖·‖₂=4.267e-01
  iter 55: subdiag=[-0.257-0.j  0.   +0.j -0.331+0.j],  ‖·‖₂=4.189e-01
  iter 56: subdiag=[0.251+0.j 0.   +0.j 0.326+0.j],  ‖·‖₂=4.112e-01
  iter 57: subdiag=[-0.245-0.j  0.   +0.j -0.321-0.j],  ‖·‖₂=4.036e-01
  iter 58: subdiag=[0.239+0.j 0.   +0.j 0.316+0.j],  ‖·‖₂=3.961e-01
  iter 59: subdiag=[-0.233-0.j  0.   +0.j -0.311-0.j],  ‖·‖₂=3.887e-01
  iter 60: subdiag=[0.228+0.j 0.   +0.j 0.306+0.j],  ‖·‖₂=3.814e-01
  iter 61: subdiag=[-0.222-0.j  0.   +0.j -0.301-0.j],  ‖·‖₂=3.742e-01
  iter 62: subdiag=[0.217+0.j 0.   +0.j 0.296+0.j],  ‖·‖₂=3.671e-01
  iter 63: subdiag=[-0.212-0.j  0.   +0.j -0.291-0.j],  ‖·‖₂=3.601e-01
```

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  iter 64: subdiag=[0.207+0.j 0.   +0.j 0.286+0.j],  ‖·‖₂=3.532e-01
  iter 65: subdiag=[-0.202-0.j  0.   +0.j -0.282-0.j],  ‖·‖₂=3.464e-01
  iter 66: subdiag=[0.197+0.j 0.   +0.j 0.277+0.j],  ‖·‖₂=3.398e-01
  iter 67: subdiag=[-0.192-0.j  0.   +0.j -0.272-0.j],  ‖·‖₂=3.332e-01
  iter 68: subdiag=[0.187+0.j 0.   +0.j 0.268+0.j],  ‖·‖₂=3.267e-01
  iter 69: subdiag=[-0.183-0.j  0.   +0.j -0.263+0.j],  ‖·‖₂=3.204e-01
  iter 70: subdiag=[0.178+0.j 0.   +0.j 0.259+0.j],  ‖·‖₂=3.141e-01
  iter 71: subdiag=[-0.174-0.j  0.   +0.j -0.254-0.j],  ‖·‖₂=3.080e-01
  iter 72: subdiag=[0.169+0.j 0.   +0.j 0.25 +0.j],  ‖·‖₂=3.020e-01
  iter 73: subdiag=[-0.165-0.j  0.   +0.j -0.246-0.j],  ‖·‖₂=2.960e-01
  iter 74: subdiag=[0.161+0.j 0.   +0.j 0.241+0.j],  ‖·‖₂=2.902e-01
  iter 75: subdiag=[-0.157-0.j  0.   +0.j -0.237-0.j],  ‖·‖₂=2.845e-01
  iter 76: subdiag=[0.153+0.j 0.   +0.j 0.233+0.j],  ‖·‖₂=2.789e-01
  iter 77: subdiag=[-0.149-0.j  0.   +0.j -0.229+0.j],  ‖·‖₂=2.733e-01
  iter 78: subdiag=[0.146+0.j 0.   +0.j 0.225-0.j],  ‖·‖₂=2.679e-01
  iter 79: subdiag=[-0.142-0.j  0.   +0.j -0.221+0.j],  ‖·‖₂=2.626e-01
  iter 80: subdiag=[0.138+0.j 0.   +0.j 0.217+0.j],  ‖·‖₂=2.574e-01
  iter 81: subdiag=[-0.135-0.j  0.   +0.j -0.213+0.j],  ‖·‖₂=2.522e-01
  iter 82: subdiag=[0.131+0.j 0.   +0.j 0.209+0.j],  ‖·‖₂=2.472e-01
  iter 83: subdiag=[-0.128-0.j  0.   +0.j -0.206-0.j],  ‖·‖₂=2.423e-01
  iter 84: subdiag=[0.125+0.j 0.   +0.j 0.202+0.j],  ‖·‖₂=2.374e-01
  iter 85: subdiag=[-0.122-0.j  0.   +0.j -0.198-0.j],  ‖·‖₂=2.327e-01
  iter 86: subdiag=[0.119+0.j 0.   +0.j 0.195+0.j],  ‖·‖₂=2.280e-01
  iter 87: subdiag=[-0.116-0.j  0.   +0.j -0.191-0.j],  ‖·‖₂=2.234e-01
  iter 88: subdiag=[0.113+0.j 0.   +0.j 0.188+0.j],  ‖·‖₂=2.189e-01
  iter 89: subdiag=[-0.11 -0.j  0.   +0.j -0.184+0.j],  ‖·‖₂=2.145e-01
  iter 90: subdiag=[0.107+0.j 0.   +0.j 0.181-0.j],  ‖·‖₂=2.102e-01
  iter 91: subdiag=[-0.104-0.j  0.   +0.j -0.178+0.j],  ‖·‖₂=2.060e-01
  iter 92: subdiag=[0.101+0.j 0.   +0.j 0.174-0.j],  ‖·‖₂=2.018e-01
  iter 93: subdiag=[-0.099-0.j  0.   +0.j -0.171+0.j],  ‖·‖₂=1.978e-01
  iter 94: subdiag=[0.096+0.j 0.   +0.j 0.168+0.j],  ‖·‖₂=1.938e-01
  iter 95: subdiag=[-0.094-0.j  0.   +0.j -0.165-0.j],  ‖·‖₂=1.899e-01
  iter 96: subdiag=[0.091+0.j 0.   +0.j 0.162+0.j],  ‖·‖₂=1.860e-01
  iter 97: subdiag=[-0.089-0.j  0.   +0.j -0.159-0.j],  ‖·‖₂=1.823e-01
  iter 98: subdiag=[0.087+0.j 0.   +0.j 0.156+0.j],  ‖·‖₂=1.786e-01
  iter 99: subdiag=[-0.085-0.j  0.   +0.j -0.153+0.j],  ‖·‖₂=1.750e-01
  after : subdiag=[-0.085-0.j  0.   +0.j -0.153+0.j],  ‖·‖₂=1.750e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.085 0.    0.153]
└─ converged?    = no

┌─ Matrix 14/30  (size 4x4)
│  fixed shift μ = 0.842538 (|μ|=0.8425)
  before: subdiag=[-0.944+0.j  0.964+0.j  0.696+0.j],  ‖·‖₂=1.518e+00
  iter 00: subdiag=[-0.662+0.j -0.644+0.j -0.413+0.j],  ‖·‖₂=1.012e+00
```

```
  iter 01: subdiag=[-0.628+0.j  0.166+0.j  0.387+0.j],  ‖·‖₂=7.558e-01
  iter 02: subdiag=[-0.626+0.j -0.038+0.j -0.386+0.j],  ‖·‖₂=7.360e-01
  iter 03: subdiag=[-0.626+0.j  0.009+0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 04: subdiag=[-0.626+0.j -0.002+0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 05: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 06: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 07: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 08: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 09: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 10: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 11: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 12: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 13: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 14: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 15: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 16: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 17: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 18: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 19: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 20: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 21: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 22: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 23: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 24: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 25: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 26: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 27: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 28: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 29: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 30: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 31: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 32: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 33: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 34: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 35: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 36: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 37: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 38: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 39: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 40: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 41: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 42: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 43: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 44: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 45: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 46: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
```

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  iter 47: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 48: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 49: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 50: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 51: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 52: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 53: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 54: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 55: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 56: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 57: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 58: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 59: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 60: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 61: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 62: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 63: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 64: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 65: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 66: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 67: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 68: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 69: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 70: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 71: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 72: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 73: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 74: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 75: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 76: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 77: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 78: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 79: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 80: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 81: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 82: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 83: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 84: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 85: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 86: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 87: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 88: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 89: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 90: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 91: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 92: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
```

```
  iter 93: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 94: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 95: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 96: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 97: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  iter 98: subdiag=[-0.626+0.j -0.   +0.j -0.386+0.j],  ‖·‖₂=7.349e-01
  iter 99: subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
  after : subdiag=[-0.626+0.j  0.   +0.j  0.386+0.j],  ‖·‖₂=7.349e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.626 0.    0.386]
└─ converged?    = no

┌─ Matrix 15/30  (size 4x4)
│  fixed shift μ = 0.941615 (|μ|=0.9416)
  before: subdiag=[-0.745+0.j  0.93 +0.j -0.49 +0.j],  ‖·‖₂=1.288e+00
  iter 00: subdiag=[-0.99 +0.j  0.927+0.j -0.031+0.j],  ‖·‖₂=1.356e+00
  iter 01: subdiag=[-0.767+0.j  0.739+0.j -0.002+0.j],  ‖·‖₂=1.065e+00
  iter 02: subdiag=[-0.338+0.j  0.69 +0.j -0.   +0.j],  ‖·‖₂=7.680e-01
  iter 03: subdiag=[-0.127+0.j  0.682+0.j -0.   +0.j],  ‖·‖₂=6.941e-01
  iter 04: subdiag=[-0.047+0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.829e-01
  iter 05: subdiag=[-0.017+0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.813e-01
  iter 06: subdiag=[-0.006+0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 07: subdiag=[-0.002+0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 08: subdiag=[-0.001+0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 09: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 10: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 11: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 12: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 13: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 14: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 15: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 16: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 17: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 18: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 19: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 20: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 21: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 22: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 23: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 24: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 25: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 26: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 27: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 28: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 29: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
```

```
iter 30: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 31: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 32: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 33: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 34: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 35: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 36: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 37: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 38: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 39: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 40: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 41: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 42: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 43: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 44: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 45: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 46: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 47: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 48: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 49: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 50: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 51: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 52: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 53: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 54: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 55: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 56: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 57: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 58: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 59: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 60: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 61: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 62: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 63: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 64: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 65: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 66: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 67: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 68: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 69: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 70: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 71: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 72: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 73: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 74: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
iter 75: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
```

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  iter 76: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 77: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 78: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 79: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 80: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 81: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 82: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 83: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 84: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 85: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 86: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 87: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 88: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 89: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 90: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 91: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 92: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 93: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 94: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 95: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 96: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 97: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 98: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  iter 99: subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
  after : subdiag=[-0.   +0.j  0.681+0.j -0.   +0.j],  ‖·‖₂=6.811e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.    0.681 0.   ]
└─ converged?    = no

┌─ Matrix 16/30  (size 4x4)
│  fixed shift μ = 0.533835 (|μ|=0.5338)
  before: subdiag=[ 0.92 +0.j  0.998+0.j -0.89 +0.j],  ‖·‖₂=1.624e+00
  iter 00: subdiag=[ 0.987+0.j -0.962+0.j  0.616+0.j],  ‖·‖₂=1.510e+00
  iter 01: subdiag=[ 0.908+0.j  0.654+0.j -0.485+0.j],  ‖·‖₂=1.219e+00
  iter 02: subdiag=[ 0.877+0.j -0.318+0.j  0.453+0.j],  ‖·‖₂=1.037e+00
  iter 03: subdiag=[ 0.87 +0.j  0.139+0.j -0.447+0.j],  ‖·‖₂=9.880e-01
  iter 04: subdiag=[ 0.869+0.j -0.06 +0.j  0.446+0.j],  ‖·‖₂=9.783e-01
  iter 05: subdiag=[ 0.869+0.j  0.025+0.j -0.445+0.j],  ‖·‖₂=9.766e-01
  iter 06: subdiag=[ 0.869+0.j -0.011+0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 07: subdiag=[ 0.869+0.j  0.005+0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 08: subdiag=[ 0.869+0.j -0.002+0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 09: subdiag=[ 0.869+0.j  0.001+0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 10: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 11: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 12: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
```

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  iter 13: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 14: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 15: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 16: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 17: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 18: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 19: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 20: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 21: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 22: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 23: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 24: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 25: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 26: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 27: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 28: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 29: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 30: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 31: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 32: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 33: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 34: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 35: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 36: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 37: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 38: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 39: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 40: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 41: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 42: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 43: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 44: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 45: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 46: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 47: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 48: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 49: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 50: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 51: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 52: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 53: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 54: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 55: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 56: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 57: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 58: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
```

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  iter 59: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 60: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 61: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 62: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 63: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 64: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 65: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 66: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 67: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 68: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 69: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 70: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 71: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 72: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 73: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 74: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 75: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 76: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 77: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 78: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 79: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 80: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 81: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 82: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 83: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 84: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 85: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 86: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 87: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 88: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 89: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 90: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 91: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 92: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 93: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 94: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 95: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 96: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 97: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  iter 98: subdiag=[ 0.869+0.j -0.   +0.j  0.445+0.j],  ‖·‖₂=9.762e-01
  iter 99: subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
  after : subdiag=[ 0.869+0.j  0.   +0.j -0.445+0.j],  ‖·‖₂=9.762e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.869 0.    0.445]
└─ converged?    = no
```

```
┌─ Matrix 17/30  (size 4x4)
│  fixed shift μ = 0.0598199-0.0955454j (|μ|=0.1127)
  before: subdiag=[0.866+0.j 1.   +0.j 0.993+0.j],  ‖·‖₂=1.654e+00
  iter 00: subdiag=[-0.888+0.j -0.998-0.j  0.979+0.j],  ‖·‖₂=1.657e+00
  iter 01: subdiag=[0.904-0.j 0.995+0.j 0.951+0.j],  ‖·‖₂=1.647e+00
  iter 02: subdiag=[-0.913+0.j -0.992-0.j  0.912+0.j],  ‖·‖₂=1.627e+00
  iter 03: subdiag=[0.913-0.j 0.987+0.j 0.862+0.j],  ‖·‖₂=1.597e+00
  iter 04: subdiag=[-0.906+0.j -0.98 -0.j  0.808+0.j],  ‖·‖₂=1.560e+00
  iter 05: subdiag=[0.893-0.j 0.968+0.j 0.751+0.j],  ‖·‖₂=1.516e+00
  iter 06: subdiag=[-0.876+0.j -0.95 +0.j  0.696-0.j],  ‖·‖₂=1.468e+00
  iter 07: subdiag=[0.857-0.j 0.924-0.j 0.643-0.j],  ‖·‖₂=1.415e+00
  iter 08: subdiag=[-0.838+0.j -0.89 +0.j  0.595-0.j],  ‖·‖₂=1.359e+00
  iter 09: subdiag=[0.819-0.j 0.847+0.j 0.552-0.j],  ‖·‖₂=1.301e+00
  iter 10: subdiag=[ 0.801-0.j -0.798-0.j  0.513-0.j],  ‖·‖₂=1.241e+00
  iter 11: subdiag=[0.786-0.j 0.743+0.j 0.478-0.j],  ‖·‖₂=1.182e+00
  iter 12: subdiag=[ 0.772-0.j -0.686+0.j  0.447-0.j],  ‖·‖₂=1.125e+00
  iter 13: subdiag=[0.759-0.j 0.628+0.j 0.419-0.j],  ‖·‖₂=1.071e+00
  iter 14: subdiag=[ 0.749-0.j -0.571+0.j  0.394-0.j],  ‖·‖₂=1.020e+00
  iter 15: subdiag=[0.739-0.j 0.516-0.j 0.371-0.j],  ‖·‖₂=9.747e-01
  iter 16: subdiag=[ 0.731-0.j -0.464+0.j  0.35 -0.j],  ‖·‖₂=9.337e-01
  iter 17: subdiag=[0.724-0.j 0.416-0.j 0.33 -0.j],  ‖·‖₂=8.975e-01
  iter 18: subdiag=[ 0.717-0.j -0.371+0.j  0.312-0.j],  ‖·‖₂=8.656e-01
  iter 19: subdiag=[0.711-0.j 0.331-0.j 0.295-0.j],  ‖·‖₂=8.378e-01
  iter 20: subdiag=[ 0.705-0.j -0.294+0.j  0.279-0.j],  ‖·‖₂=8.136e-01
  iter 21: subdiag=[0.7  -0.j 0.261-0.j 0.264-0.j],  ‖·‖₂=7.925e-01
  iter 22: subdiag=[ 0.695-0.j -0.232+0.j  0.249-0.j],  ‖·‖₂=7.741e-01
  iter 23: subdiag=[0.69 -0.j 0.206-0.j 0.236-0.j],  ‖·‖₂=7.580e-01
  iter 24: subdiag=[ 0.686-0.j -0.182+0.j  0.223-0.j],  ‖·‖₂=7.437e-01
  iter 25: subdiag=[0.681-0.j 0.162-0.j 0.211-0.j],  ‖·‖₂=7.311e-01
  iter 26: subdiag=[ 0.677-0.j -0.143+0.j  0.2  -0.j],  ‖·‖₂=7.197e-01
  iter 27: subdiag=[0.672-0.j 0.127-0.j 0.189-0.j],  ‖·‖₂=7.095e-01
  iter 28: subdiag=[ 0.668-0.j -0.112+0.j  0.178-0.j],  ‖·‖₂=7.001e-01
  iter 29: subdiag=[0.663-0.j 0.099-0.j 0.168-0.j],  ‖·‖₂=6.915e-01
  iter 30: subdiag=[ 0.659-0.j -0.088+0.j  0.159-0.j],  ‖·‖₂=6.835e-01
  iter 31: subdiag=[0.655-0.j 0.078-0.j 0.15 -0.j],  ‖·‖₂=6.761e-01
  iter 32: subdiag=[ 0.65 -0.j -0.069+0.j  0.142-0.j],  ‖·‖₂=6.690e-01
  iter 33: subdiag=[0.646-0.j 0.061-0.j 0.134-0.j],  ‖·‖₂=6.624e-01
  iter 34: subdiag=[ 0.641-0.j -0.054+0.j  0.127-0.j],  ‖·‖₂=6.560e-01
  iter 35: subdiag=[0.637-0.j 0.048-0.j 0.12 -0.j],  ‖·‖₂=6.499e-01
  iter 36: subdiag=[ 0.633-0.j -0.043+0.j  0.113-0.j],  ‖·‖₂=6.440e-01
  iter 37: subdiag=[-0.628+0.j  0.038-0.j  0.107-0.j],  ‖·‖₂=6.382e-01
  iter 38: subdiag=[ 0.624-0.j -0.033+0.j  0.101-0.j],  ‖·‖₂=6.327e-01
  iter 39: subdiag=[-0.619+0.j  0.03 -0.j  0.095-0.j],  ‖·‖₂=6.272e-01
  iter 40: subdiag=[ 0.615-0.j -0.026+0.j  0.09 -0.j],  ‖·‖₂=6.219e-01
  iter 41: subdiag=[-0.61 +0.j  0.023-0.j  0.084-0.j],  ‖·‖₂=6.166e-01
```

```
  iter 42: subdiag=[ 0.606-0.j -0.021+0.j  0.08 -0.j],  ‖·‖₂=6.115e-01
  iter 43: subdiag=[-0.601-0.j  0.018-0.j  0.075-0.j],  ‖·‖₂=6.064e-01
  iter 44: subdiag=[ 0.597+0.j -0.016+0.j  0.071-0.j],  ‖·‖₂=6.014e-01
  iter 45: subdiag=[-0.592-0.j  0.014-0.j  0.067-0.j],  ‖·‖₂=5.964e-01
  iter 46: subdiag=[ 0.588+0.j -0.013+0.j  0.063-0.j],  ‖·‖₂=5.915e-01
  iter 47: subdiag=[-0.584-0.j  0.011-0.j  0.06 -0.j],  ‖·‖₂=5.866e-01
  iter 48: subdiag=[ 0.579+0.j -0.01 +0.j  0.056-0.j],  ‖·‖₂=5.818e-01
  iter 49: subdiag=[-0.575-0.j  0.009-0.j  0.053-0.j],  ‖·‖₂=5.770e-01
  iter 50: subdiag=[ 0.57 +0.j -0.008+0.j  0.05 -0.j],  ‖·‖₂=5.723e-01
  iter 51: subdiag=[-0.566+0.j  0.007-0.j  0.047-0.j],  ‖·‖₂=5.675e-01
  iter 52: subdiag=[ 0.561+0.j -0.006+0.j  0.044-0.j],  ‖·‖₂=5.628e-01
  iter 53: subdiag=[-0.557+0.j  0.005-0.j  0.042-0.j],  ‖·‖₂=5.581e-01
  iter 54: subdiag=[ 0.552-0.j -0.005+0.j  0.04 -0.j],  ‖·‖₂=5.535e-01
  iter 55: subdiag=[-0.548+0.j  0.004-0.j  0.037-0.j],  ‖·‖₂=5.488e-01
  iter 56: subdiag=[ 0.543-0.j -0.004+0.j  0.035-0.j],  ‖·‖₂=5.442e-01
  iter 57: subdiag=[-0.539+0.j  0.003-0.j  0.033-0.j],  ‖·‖₂=5.396e-01
  iter 58: subdiag=[ 0.534+0.j -0.003+0.j  0.031-0.j],  ‖·‖₂=5.351e-01
  iter 59: subdiag=[-0.53 -0.j  0.003-0.j  0.029-0.j],  ‖·‖₂=5.305e-01
  iter 60: subdiag=[ 0.525+0.j -0.002+0.j  0.028-0.j],  ‖·‖₂=5.260e-01
  iter 61: subdiag=[-0.521-0.j  0.002-0.j  0.026-0.j],  ‖·‖₂=5.215e-01
  iter 62: subdiag=[ 0.516+0.j -0.002+0.j  0.025-0.j],  ‖·‖₂=5.170e-01
  iter 63: subdiag=[-0.512-0.j  0.002-0.j  0.023-0.j],  ‖·‖₂=5.125e-01
  iter 64: subdiag=[ 0.508+0.j -0.001+0.j  0.022-0.j],  ‖·‖₂=5.080e-01
  iter 65: subdiag=[-0.503+0.j  0.001-0.j  0.021-0.j],  ‖·‖₂=5.036e-01
  iter 66: subdiag=[ 0.499-0.j -0.001+0.j  0.02 -0.j],  ‖·‖₂=4.992e-01
  iter 67: subdiag=[-0.494+0.j  0.001-0.j  0.018-0.j],  ‖·‖₂=4.948e-01
  iter 68: subdiag=[ 0.49 -0.j -0.001+0.j  0.017-0.j],  ‖·‖₂=4.904e-01
  iter 69: subdiag=[-0.486+0.j  0.001-0.j  0.016-0.j],  ‖·‖₂=4.860e-01
  iter 70: subdiag=[ 0.481-0.j -0.001+0.j  0.015-0.j],  ‖·‖₂=4.817e-01
  iter 71: subdiag=[-0.477+0.j  0.001-0.j  0.015-0.j],  ‖·‖₂=4.774e-01
  iter 72: subdiag=[ 0.473-0.j -0.001+0.j  0.014-0.j],  ‖·‖₂=4.731e-01
  iter 73: subdiag=[-0.469+0.j  0.   -0.j  0.013-0.j],  ‖·‖₂=4.688e-01
  iter 74: subdiag=[ 0.464-0.j -0.   +0.j  0.012-0.j],  ‖·‖₂=4.645e-01
  iter 75: subdiag=[-0.46 +0.j  0.   -0.j  0.012-0.j],  ‖·‖₂=4.603e-01
  iter 76: subdiag=[ 0.456-0.j -0.   +0.j  0.011-0.j],  ‖·‖₂=4.561e-01
  iter 77: subdiag=[-0.452-0.j  0.   -0.j  0.01 -0.j],  ‖·‖₂=4.519e-01
  iter 78: subdiag=[ 0.448+0.j -0.   +0.j  0.01 -0.j],  ‖·‖₂=4.477e-01
  iter 79: subdiag=[-0.443-0.j  0.   -0.j  0.009-0.j],  ‖·‖₂=4.436e-01
  iter 80: subdiag=[ 0.439+0.j -0.   +0.j  0.009-0.j],  ‖·‖₂=4.394e-01
  iter 81: subdiag=[-0.435-0.j  0.   -0.j  0.008-0.j],  ‖·‖₂=4.353e-01
  iter 82: subdiag=[ 0.431+0.j -0.   +0.j  0.008-0.j],  ‖·‖₂=4.313e-01
  iter 83: subdiag=[-0.427-0.j  0.   -0.j  0.007-0.j],  ‖·‖₂=4.272e-01
  iter 84: subdiag=[ 0.423+0.j -0.   +0.j  0.007-0.j],  ‖·‖₂=4.232e-01
  iter 85: subdiag=[-0.419-0.j  0.   -0.j  0.006-0.j],  ‖·‖₂=4.192e-01
  iter 86: subdiag=[ 0.415+0.j -0.   +0.j  0.006-0.j],  ‖·‖₂=4.152e-01
  iter 87: subdiag=[-0.411-0.j  0.   -0.j  0.006-0.j],  ‖·‖₂=4.112e-01
```

```
  iter 88: subdiag=[ 0.407+0.j -0.   +0.j  0.005-0.j],  ‖·‖₂=4.073e-01
  iter 89: subdiag=[-0.403-0.j  0.   -0.j  0.005-0.j],  ‖·‖₂=4.034e-01
  iter 90: subdiag=[ 0.399+0.j -0.   +0.j  0.005-0.j],  ‖·‖₂=3.995e-01
  iter 91: subdiag=[-0.396-0.j  0.   -0.j  0.005-0.j],  ‖·‖₂=3.957e-01
  iter 92: subdiag=[ 0.392+0.j -0.   +0.j  0.004-0.j],  ‖·‖₂=3.918e-01
  iter 93: subdiag=[-0.388-0.j  0.   -0.j  0.004-0.j],  ‖·‖₂=3.880e-01
  iter 94: subdiag=[ 0.384+0.j -0.   +0.j  0.004-0.j],  ‖·‖₂=3.842e-01
  iter 95: subdiag=[-0.38 -0.j  0.   -0.j  0.004-0.j],  ‖·‖₂=3.805e-01
  iter 96: subdiag=[ 0.377+0.j -0.   +0.j  0.003-0.j],  ‖·‖₂=3.768e-01
  iter 97: subdiag=[-0.373+0.j  0.   -0.j  0.003-0.j],  ‖·‖₂=3.731e-01
  iter 98: subdiag=[ 0.369+0.j -0.   +0.j  0.003-0.j],  ‖·‖₂=3.694e-01
  iter 99: subdiag=[-0.366-0.j  0.   -0.j  0.003-0.j],  ‖·‖₂=3.658e-01
  after : subdiag=[-0.366-0.j  0.   -0.j  0.003-0.j],  ‖·‖₂=3.658e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.366 0.    0.003]
└─ converged?    = no

┌─ Matrix 18/30  (size 4x4)
│  fixed shift μ = 0.0412469-0.965376j (|μ|=0.9663)
  before: subdiag=[-0.986+0.j -0.358+0.j -0.999+0.j],  ‖·‖₂=1.449e+00
  iter 00: subdiag=[-0.281-0.j  0.975-0.j -0.203-0.j],  ‖·‖₂=1.035e+00
  iter 01: subdiag=[-0.184-0.j  0.285-0.j  0.078+0.j],  ‖·‖₂=3.481e-01
  iter 02: subdiag=[ 0.183+0.j -0.037+0.j -0.043-0.j],  ‖·‖₂=1.915e-01
  iter 03: subdiag=[-0.184-0.j  0.005-0.j  0.023+0.j],  ‖·‖₂=1.853e-01
  iter 04: subdiag=[ 0.185+0.j -0.001+0.j -0.012-0.j],  ‖·‖₂=1.850e-01
  iter 05: subdiag=[ 0.185+0.j -0.   +0.j  0.007+0.j],  ‖·‖₂=1.854e-01
  iter 06: subdiag=[ 0.186+0.j -0.   +0.j -0.004-0.j],  ‖·‖₂=1.860e-01
  iter 07: subdiag=[ 0.187+0.j -0.   +0.j  0.002+0.j],  ‖·‖₂=1.866e-01
  iter 08: subdiag=[ 0.187+0.j -0.   +0.j -0.001-0.j],  ‖·‖₂=1.872e-01
  iter 09: subdiag=[ 0.188+0.j -0.   +0.j  0.001+0.j],  ‖·‖₂=1.878e-01
  iter 10: subdiag=[ 0.188+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.884e-01
  iter 11: subdiag=[ 0.189+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.889e-01
  iter 12: subdiag=[ 0.189+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.894e-01
  iter 13: subdiag=[ 0.19+0.j -0.  +0.j  0.  +0.j],  ‖·‖₂=1.898e-01
  iter 14: subdiag=[ 0.19+0.j -0.  +0.j -0.  -0.j],  ‖·‖₂=1.902e-01
  iter 15: subdiag=[ 0.191+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.906e-01
  iter 16: subdiag=[ 0.191+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.909e-01
  iter 17: subdiag=[ 0.191+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.912e-01
  iter 18: subdiag=[ 0.191+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.915e-01
  iter 19: subdiag=[ 0.192+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.917e-01
  iter 20: subdiag=[ 0.192+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.919e-01
  iter 21: subdiag=[ 0.192+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.920e-01
  iter 22: subdiag=[ 0.192+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.921e-01
  iter 23: subdiag=[ 0.192+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.922e-01
  iter 24: subdiag=[ 0.192+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.922e-01
```

```
  iter 25: subdiag=[ 0.192+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.922e-01
  iter 26: subdiag=[ 0.192+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.921e-01
  iter 27: subdiag=[ 0.192+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.920e-01
  iter 28: subdiag=[ 0.192+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.919e-01
  iter 29: subdiag=[ 0.192+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.918e-01
  iter 30: subdiag=[ 0.192+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.915e-01
  iter 31: subdiag=[ 0.191+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.913e-01
  iter 32: subdiag=[ 0.191+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.910e-01
  iter 33: subdiag=[ 0.191+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.907e-01
  iter 34: subdiag=[ 0.19+0.j -0.  +0.j -0.  -0.j],  ‖·‖₂=1.903e-01
  iter 35: subdiag=[ 0.19+0.j -0.  +0.j  0.  +0.j],  ‖·‖₂=1.900e-01
  iter 36: subdiag=[ 0.19+0.j -0.  +0.j -0.  -0.j],  ‖·‖₂=1.895e-01
  iter 37: subdiag=[ 0.189+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.891e-01
  iter 38: subdiag=[ 0.189+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.886e-01
  iter 39: subdiag=[ 0.188+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.880e-01
  iter 40: subdiag=[ 0.187+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.875e-01
  iter 41: subdiag=[ 0.187+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.869e-01
  iter 42: subdiag=[ 0.186+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.862e-01
  iter 43: subdiag=[ 0.186+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.855e-01
  iter 44: subdiag=[ 0.185+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.848e-01
  iter 45: subdiag=[ 0.184+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.841e-01
  iter 46: subdiag=[ 0.183+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.833e-01
  iter 47: subdiag=[ 0.183+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.825e-01
  iter 48: subdiag=[ 0.182+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.817e-01
  iter 49: subdiag=[-0.181-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.809e-01
  iter 50: subdiag=[ 0.18+0.j -0.  +0.j -0.  -0.j],  ‖·‖₂=1.800e-01
  iter 51: subdiag=[-0.179-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.791e-01
  iter 52: subdiag=[ 0.178+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.781e-01
  iter 53: subdiag=[-0.177-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.772e-01
  iter 54: subdiag=[ 0.176+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.762e-01
  iter 55: subdiag=[-0.175-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.752e-01
  iter 56: subdiag=[ 0.174+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.741e-01
  iter 57: subdiag=[-0.173-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.731e-01
  iter 58: subdiag=[ 0.172+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.720e-01
  iter 59: subdiag=[-0.171-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.709e-01
  iter 60: subdiag=[ 0.17+0.j -0.  +0.j -0.  -0.j],  ‖·‖₂=1.698e-01
  iter 61: subdiag=[-0.169-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.687e-01
  iter 62: subdiag=[ 0.167+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.675e-01
  iter 63: subdiag=[-0.166-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.663e-01
  iter 64: subdiag=[ 0.165+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.651e-01
  iter 65: subdiag=[-0.164-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.639e-01
  iter 66: subdiag=[ 0.163+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.627e-01
  iter 67: subdiag=[-0.161-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.615e-01
  iter 68: subdiag=[ 0.16+0.j -0.  +0.j -0.  -0.j],  ‖·‖₂=1.602e-01
  iter 69: subdiag=[-0.159-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.590e-01
  iter 70: subdiag=[ 0.158+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.577e-01
```

```
  iter 71: subdiag=[-0.156-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.564e-01
  iter 72: subdiag=[ 0.155-0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.551e-01
  iter 73: subdiag=[-0.154-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.538e-01
  iter 74: subdiag=[ 0.152-0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.525e-01
  iter 75: subdiag=[-0.151-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.512e-01
  iter 76: subdiag=[ 0.15-0.j -0.  +0.j -0.  -0.j],  ‖·‖₂=1.498e-01
  iter 77: subdiag=[-0.148-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.485e-01
  iter 78: subdiag=[ 0.147+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.472e-01
  iter 79: subdiag=[-0.146+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.458e-01
  iter 80: subdiag=[ 0.144-0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.445e-01
  iter 81: subdiag=[-0.143-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.431e-01
  iter 82: subdiag=[ 0.142+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.418e-01
  iter 83: subdiag=[-0.14-0.j -0.  +0.j  0.  +0.j],  ‖·‖₂=1.404e-01
  iter 84: subdiag=[ 0.139+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.390e-01
  iter 85: subdiag=[-0.138-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.377e-01
  iter 86: subdiag=[ 0.136-0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.363e-01
  iter 87: subdiag=[-0.135+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.349e-01
  iter 88: subdiag=[ 0.134-0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.336e-01
  iter 89: subdiag=[-0.132+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.322e-01
  iter 90: subdiag=[ 0.131+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.309e-01
  iter 91: subdiag=[-0.129-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.295e-01
  iter 92: subdiag=[ 0.128-0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.281e-01
  iter 93: subdiag=[-0.127+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.268e-01
  iter 94: subdiag=[ 0.125+0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.254e-01
  iter 95: subdiag=[-0.124+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.241e-01
  iter 96: subdiag=[ 0.123-0.j -0.   +0.j -0.   -0.j],  ‖·‖₂=1.227e-01
  iter 97: subdiag=[-0.121+0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.214e-01
  iter 98: subdiag=[ 0.12-0.j -0.  +0.j -0.  -0.j],  ‖·‖₂=1.201e-01
  iter 99: subdiag=[-0.119-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.187e-01
  after : subdiag=[-0.119-0.j -0.   +0.j  0.   +0.j],  ‖·‖₂=1.187e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.119 0.    0.   ]
└─ converged?    = no

┌─ Matrix 19/30  (size 4x4)
│  fixed shift μ = -0.993087 (|μ|=0.9931)
  before: subdiag=[-0.745+0.j  0.986+0.j  0.139+0.j],  ‖·‖₂=1.244e+00
  iter 00: subdiag=[-0.592+0.j  0.997+0.j  0.001+0.j],  ‖·‖₂=1.159e+00
  iter 01: subdiag=[-0.445+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=1.094e+00
  iter 02: subdiag=[-0.324+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=1.051e+00
  iter 03: subdiag=[-0.231+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=1.026e+00
  iter 04: subdiag=[-0.163+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=1.013e+00
  iter 05: subdiag=[-0.114+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=1.006e+00
  iter 06: subdiag=[-0.08+0.j  1.  +0.j  0.  +0.j],  ‖·‖₂=1.003e+00
  iter 07: subdiag=[-0.056+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=1.001e+00
```

```
  iter 08: subdiag=[-0.039+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=1.000e+00
  iter 09: subdiag=[-0.027+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=1.000e+00
  iter 10: subdiag=[-0.019+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=9.999e-01
  iter 11: subdiag=[-0.013+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=9.998e-01
  iter 12: subdiag=[-0.009+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=9.997e-01
  iter 13: subdiag=[-0.006+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=9.997e-01
  iter 14: subdiag=[-0.005+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=9.997e-01
  iter 15: subdiag=[-0.003+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=9.997e-01
  iter 16: subdiag=[-0.002+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=9.997e-01
  iter 17: subdiag=[-0.002+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=9.997e-01
  iter 18: subdiag=[-0.001+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=9.997e-01
  iter 19: subdiag=[-0.001+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=9.997e-01
  iter 20: subdiag=[-0.001+0.j  1.   +0.j  0.   +0.j],  ‖·‖₂=9.997e-01
  iter 21: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 22: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 23: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 24: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 25: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 26: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 27: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 28: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 29: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 30: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 31: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 32: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 33: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 34: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 35: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 36: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 37: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 38: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 39: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 40: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 41: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 42: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 43: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 44: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 45: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 46: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 47: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 48: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 49: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 50: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 51: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 52: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
  iter 53: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
```

```
iter 54: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 55: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 56: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 57: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 58: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 59: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 60: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 61: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 62: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 63: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 64: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 65: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 66: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 67: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 68: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 69: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 70: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 71: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 72: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 73: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 74: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 75: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 76: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 77: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 78: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 79: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 80: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 81: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 82: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 83: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 84: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 85: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 86: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 87: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 88: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 89: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 90: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 91: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 92: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 93: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 94: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 95: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 96: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 97: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 98: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
iter 99: subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
```

```
  after : subdiag=[-0.+0.j  1.+0.j  0.+0.j],  ‖·‖₂=9.997e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0. 1. 0.]
└─ converged?    = no

┌─ Matrix 20/30  (size 4x4)
│  fixed shift μ = 0.943718 (|μ|=0.9437)
  before: subdiag=[-0.429+0.j -0.972+0.j  0.419+0.j],  ‖·‖₂=1.142e+00
  iter 00: subdiag=[-0.982+0.j -0.64 +0.j  0.047+0.j],  ‖·‖₂=1.173e+00
  iter 01: subdiag=[-0.51 +0.j -0.44 +0.j  0.006+0.j],  ‖·‖₂=6.740e-01
  iter 02: subdiag=[-0.12 +0.j -0.426+0.j  0.001+0.j],  ‖·‖₂=4.427e-01
  iter 03: subdiag=[-0.027+0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.262e-01
  iter 04: subdiag=[-0.006+0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 05: subdiag=[-0.001+0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 06: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 07: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 08: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 09: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 10: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 11: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 12: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 13: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 14: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 15: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 16: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 17: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 18: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 19: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 20: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 21: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 22: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 23: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 24: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 25: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 26: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 27: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 28: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 29: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 30: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 31: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 32: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 33: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 34: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 35: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 36: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
```

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2044    iter 37: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2045    iter 38: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2046    iter 39: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2047    iter 40: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2048    iter 41: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2049    iter 42: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2050    iter 43: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2051    iter 44: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2052    iter 45: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2053    iter 46: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2054    iter 47: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2055    iter 48: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2056    iter 49: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2057    iter 50: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2058    iter 51: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2059    iter 52: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2060    iter 53: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2061    iter 54: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2062    iter 55: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2063    iter 56: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2064    iter 57: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2065    iter 58: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2066    iter 59: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2067    iter 60: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2068    iter 61: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2069    iter 62: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2070    iter 63: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2071    iter 64: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2072    iter 65: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2073    iter 66: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2074    iter 67: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2075    iter 68: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2076    iter 69: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2077    iter 70: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2078    iter 71: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2079    iter 72: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2080    iter 73: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2081    iter 74: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2082    iter 75: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2083    iter 76: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2084    iter 77: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2085    iter 78: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2086    iter 79: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2087    iter 80: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2088    iter 81: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
2089    iter 82: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
```

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  iter 83: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 84: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 85: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 86: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 87: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 88: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 89: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 90: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 91: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 92: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 93: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 94: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 95: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 96: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 97: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 98: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  iter 99: subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
  after : subdiag=[-0.   +0.j -0.425+0.j  0.   +0.j],  ‖·‖₂=4.253e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.    0.425 0.   ]
└─ converged?    = no

┌─ Matrix 21/30  (size 4x4)
│  fixed shift μ = -0.645528 (|μ|=0.6455)
  before: subdiag=[-0.998+0.j  0.999+0.j  0.737+0.j],  ‖·‖₂=1.593e+00
  iter 00: subdiag=[-0.902+0.j  0.984+0.j  0.314+0.j],  ‖·‖₂=1.371e+00
  iter 01: subdiag=[-0.666+0.j  0.955+0.j  0.114+0.j],  ‖·‖₂=1.169e+00
  iter 02: subdiag=[-0.431+0.j  0.936+0.j  0.041+0.j],  ‖·‖₂=1.032e+00
  iter 03: subdiag=[-0.262+0.j  0.928+0.j  0.015+0.j],  ‖·‖₂=9.647e-01
  iter 04: subdiag=[-0.155+0.j  0.925+0.j  0.006+0.j],  ‖·‖₂=9.383e-01
  iter 05: subdiag=[-0.091+0.j  0.924+0.j  0.002+0.j],  ‖·‖₂=9.288e-01
  iter 06: subdiag=[-0.053+0.j  0.924+0.j  0.001+0.j],  ‖·‖₂=9.255e-01
  iter 07: subdiag=[-0.031+0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.244e-01
  iter 08: subdiag=[-0.018+0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.240e-01
  iter 09: subdiag=[-0.011+0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.239e-01
  iter 10: subdiag=[-0.006+0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.239e-01
  iter 11: subdiag=[-0.004+0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 12: subdiag=[-0.002+0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 13: subdiag=[-0.001+0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 14: subdiag=[-0.001+0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 15: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 16: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 17: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 18: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 19: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
```

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iter 20: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
iter 21: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
iter 22: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
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iter 35: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
iter 36: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
iter 37: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
iter 38: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
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iter 41: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
iter 42: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
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iter 50: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
iter 51: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
iter 52: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
iter 53: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
iter 54: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
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iter 57: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
iter 58: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
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iter 64: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
iter 65: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
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  iter 66: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 67: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
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  iter 77: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 78: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 79: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 80: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 81: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 82: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 83: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 84: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 85: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 86: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 87: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 88: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 89: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 90: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 91: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 92: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 93: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 94: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 95: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 96: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 97: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 98: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  iter 99: subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
  after : subdiag=[-0.   +0.j  0.924+0.j  0.   +0.j],  ‖·‖₂=9.238e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.    0.924 0.   ]
└─ converged?    = no

┌─ Matrix 22/30  (size 4x4)
│  fixed shift μ = 0.833114 (|μ|=0.8331)
  before: subdiag=[-0.476+0.j  0.996+0.j  0.602+0.j],  ‖·‖₂=1.258e+00
  iter 00: subdiag=[-0.329+0.j -0.495+0.j -0.426+0.j],  ‖·‖₂=7.312e-01
  iter 01: subdiag=[-0.318+0.j  0.123+0.j  0.413+0.j],  ‖·‖₂=5.355e-01
  iter 02: subdiag=[-0.318+0.j -0.029+0.j -0.412+0.j],  ‖·‖₂=5.211e-01
```

```
  iter 03: subdiag=[-0.318+0.j  0.007+0.j  0.412+0.j],  ‖·‖₂=5.203e-01
  iter 04: subdiag=[-0.318+0.j -0.002+0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 05: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 06: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 07: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 08: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 09: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 10: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 11: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 12: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 13: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 14: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 15: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 16: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 17: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 18: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 19: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 20: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 21: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 22: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 23: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 24: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 25: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 26: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 27: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 28: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 29: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 30: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 31: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 32: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 33: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 34: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 35: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 36: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 37: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 38: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 39: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 40: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 41: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 42: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 43: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 44: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 45: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 46: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 47: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 48: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
```

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  iter 49: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 50: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 51: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 52: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 53: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 54: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 55: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 56: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 57: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 58: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 59: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 60: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 61: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 62: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 63: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 64: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 65: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 66: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 67: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 68: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 69: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 70: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 71: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 72: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 73: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 74: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 75: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 76: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 77: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 78: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 79: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 80: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 81: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 82: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 83: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 84: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 85: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 86: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 87: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 88: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 89: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 90: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 91: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 92: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 93: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 94: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
```

```
  iter 95: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 96: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 97: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  iter 98: subdiag=[-0.318+0.j -0.   +0.j -0.412+0.j],  ‖·‖₂=5.202e-01
  iter 99: subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
  after : subdiag=[-0.318+0.j  0.   +0.j  0.412+0.j],  ‖·‖₂=5.202e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.318 0.    0.412]
└─ converged?    = no

┌─ Matrix 23/30  (size 4x4)
│  fixed shift μ = 0.475111-0.570494j (|μ|=0.7424)
  before: subdiag=[0.907+0.j 0.834+0.j 0.79 +0.j],  ‖·‖₂=1.464e+00
  iter 00: subdiag=[ 0.727-0.j -0.911-0.j  0.447+0.j],  ‖·‖₂=1.248e+00
  iter 01: subdiag=[ 0.484-0.j -0.857-0.j -0.193-0.j],  ‖·‖₂=1.003e+00
  iter 02: subdiag=[-0.345+0.j  0.585+0.j  0.093+0.j],  ‖·‖₂=6.857e-01
  iter 03: subdiag=[ 0.253-0.j -0.345-0.j -0.048+0.j],  ‖·‖₂=4.310e-01
  iter 04: subdiag=[-0.185+0.j  0.195+0.j  0.026-0.j],  ‖·‖₂=2.705e-01
  iter 05: subdiag=[ 0.135-0.j -0.109-0.j -0.014-0.j],  ‖·‖₂=1.741e-01
  iter 06: subdiag=[-0.097+0.j  0.061+0.j  0.007+0.j],  ‖·‖₂=1.152e-01
  iter 07: subdiag=[ 0.07 -0.j -0.034-0.j -0.004-0.j],  ‖·‖₂=7.812e-02
  iter 08: subdiag=[-0.05 +0.j  0.019+0.j  0.002-0.j],  ‖·‖₂=5.393e-02
  iter 09: subdiag=[ 0.036-0.j -0.011-0.j -0.001+0.j],  ‖·‖₂=3.772e-02
  iter 10: subdiag=[-0.026+0.j  0.006+0.j  0.001+0.j],  ‖·‖₂=2.661e-02
  iter 11: subdiag=[ 0.019-0.j -0.003-0.j -0.   -0.j],  ‖·‖₂=1.888e-02
  iter 12: subdiag=[-0.013+0.j  0.002+0.j  0.   +0.j],  ‖·‖₂=1.345e-02
  iter 13: subdiag=[ 0.01 -0.j -0.001-0.j -0.   -0.j],  ‖·‖₂=9.598e-03
  iter 14: subdiag=[-0.007+0.j  0.001+0.j  0.   +0.j],  ‖·‖₂=6.862e-03
  iter 15: subdiag=[ 0.005-0.j -0.   -0.j -0.   -0.j],  ‖·‖₂=4.910e-03
  iter 16: subdiag=[-0.004+0.j  0.   +0.j  0.   +0.j],  ‖·‖₂=3.515e-03
  iter 17: subdiag=[ 0.003-0.j -0.   -0.j -0.   -0.j],  ‖·‖₂=2.517e-03
  iter 18: subdiag=[-0.002+0.j  0.   +0.j  0.   +0.j],  ‖·‖₂=1.803e-03
  iter 19: subdiag=[ 0.001-0.j -0.   -0.j -0.   -0.j],  ‖·‖₂=1.292e-03
  iter 20: subdiag=[-0.001+0.j  0.   +0.j  0.   +0.j],  ‖·‖₂=9.255e-04
  iter 21: subdiag=[ 0.001-0.j -0.   -0.j -0.   -0.j],  ‖·‖₂=6.631e-04
  iter 22: subdiag=[-0.+0.j  0.+0.j  0.+0.j],  ‖·‖₂=4.752e-04
  iter 23: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=3.405e-04
  iter 24: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=2.440e-04
  iter 25: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=1.748e-04
  iter 26: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=1.253e-04
  iter 27: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=8.975e-05
  iter 28: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=6.431e-05
  iter 29: subdiag=[ 0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=4.608e-05
  iter 30: subdiag=[-0.+0.j  0.+0.j  0.+0.j],  ‖·‖₂=3.302e-05
  iter 31: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=2.366e-05
```

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  iter 32: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=1.695e-05
  iter 33: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=1.215e-05
  iter 34: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=8.705e-06
  iter 35: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=6.238e-06
  iter 36: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=4.470e-06
  iter 37: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=3.203e-06
  iter 38: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=2.295e-06
  iter 39: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=1.645e-06
  iter 40: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=1.178e-06
  iter 41: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=8.444e-07
  iter 42: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=6.050e-07
  iter 43: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=4.335e-07
  iter 44: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=3.107e-07
  iter 45: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=2.226e-07
  iter 46: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=1.595e-07
  iter 47: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=1.143e-07
  iter 48: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=8.190e-08
  iter 49: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=5.868e-08
  iter 50: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=4.205e-08
  iter 51: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=3.013e-08
  iter 52: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=2.159e-08
  iter 53: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=1.547e-08
  iter 54: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=1.109e-08
  iter 55: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=7.944e-09
  iter 56: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=5.692e-09
  iter 57: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=4.079e-09
  iter 58: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=2.923e-09
  iter 59: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=2.094e-09
  iter 60: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=1.501e-09
  iter 61: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=1.075e-09
  iter 62: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=7.705e-10
  iter 63: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=5.521e-10
  iter 64: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=3.956e-10
  iter 65: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=2.835e-10
  iter 66: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=2.031e-10
  iter 67: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=1.455e-10
  iter 68: subdiag=[-0.+0.j  0.+0.j  0.-0.j],  ‖·‖₂=1.043e-10
  iter 69: subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=7.473e-11
  after : subdiag=[ 0.-0.j -0.-0.j -0.+0.j],  ‖·‖₂=7.473e-11
│  iterations    = 70/100
│  sub-diag magnitudes after last step:
│  [0. 0. 0.]
└─ converged?    = yes

┌─ Matrix 24/30  (size 4x4)
│  fixed shift μ = 0.921576-0.227859j (|μ|=0.9493)
```

```
   before: subdiag=[-0.916+0.j  0.433+0.j -0.245+0.j],  ‖·‖₂=1.042e+00
   iter 00: subdiag=[-0.888+0.j -0.146-0.j -0.052-0.j],  ‖·‖₂=9.016e-01
   iter 01: subdiag=[-0.756-0.j  0.066+0.j -0.006-0.j],  ‖·‖₂=7.589e-01
   iter 02: subdiag=[-0.595+0.j -0.031-0.j -0.001+0.j],  ‖·‖₂=5.955e-01
   iter 03: subdiag=[-0.446-0.j  0.015+0.j -0.   +0.j],  ‖·‖₂=4.459e-01
   iter 04: subdiag=[-0.325-0.j -0.007-0.j -0.   +0.j],  ‖·‖₂=3.251e-01
   iter 05: subdiag=[-0.234-0.j  0.003+0.j -0.   -0.j],  ‖·‖₂=2.336e-01
   iter 06: subdiag=[-0.167-0.j -0.002-0.j -0.   -0.j],  ‖·‖₂=1.666e-01
   iter 07: subdiag=[-0.118-0.j  0.001+0.j -0.   -0.j],  ‖·‖₂=1.184e-01
   iter 08: subdiag=[-0.084-0.j -0.   -0.j -0.   -0.j],  ‖·‖₂=8.395e-02
   iter 09: subdiag=[-0.059-0.j  0.   +0.j -0.   -0.j],  ‖·‖₂=5.947e-02
   iter 10: subdiag=[-0.042-0.j -0.   -0.j -0.   -0.j],  ‖·‖₂=4.211e-02
   iter 11: subdiag=[-0.03-0.j  0.  +0.j -0.  +0.j],  ‖·‖₂=2.981e-02
   iter 12: subdiag=[-0.021-0.j -0.   -0.j -0.   -0.j],  ‖·‖₂=2.110e-02
   iter 13: subdiag=[-0.015+0.j  0.   +0.j -0.   +0.j],  ‖·‖₂=1.493e-02
   iter 14: subdiag=[-0.011-0.j -0.   -0.j -0.   +0.j],  ‖·‖₂=1.057e-02
   iter 15: subdiag=[-0.007-0.j  0.   +0.j -0.   -0.j],  ‖·‖₂=7.480e-03
   iter 16: subdiag=[-0.005-0.j -0.   -0.j -0.   -0.j],  ‖·‖₂=5.294e-03
   iter 17: subdiag=[-0.004-0.j  0.   +0.j -0.   -0.j],  ‖·‖₂=3.747e-03
   iter 18: subdiag=[-0.003-0.j -0.   -0.j -0.   -0.j],  ‖·‖₂=2.652e-03
   iter 19: subdiag=[-0.002-0.j  0.   +0.j -0.   -0.j],  ‖·‖₂=1.877e-03
   iter 20: subdiag=[-0.001-0.j -0.   -0.j -0.   -0.j],  ‖·‖₂=1.328e-03
   iter 21: subdiag=[-0.001+0.j  0.   +0.j -0.   -0.j],  ‖·‖₂=9.400e-04
   iter 22: subdiag=[-0.001+0.j -0.   -0.j -0.   -0.j],  ‖·‖₂=6.652e-04
   iter 23: subdiag=[-0.+0.j  0.+0.j -0.+0.j],  ‖·‖₂=4.708e-04
   iter 24: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=3.332e-04
   iter 25: subdiag=[-0.+0.j  0.+0.j -0.-0.j],  ‖·‖₂=2.358e-04
   iter 26: subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=1.669e-04
   iter 27: subdiag=[-0.+0.j  0.+0.j -0.-0.j],  ‖·‖₂=1.181e-04
   iter 28: subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=8.359e-05
   iter 29: subdiag=[-0.+0.j  0.+0.j -0.-0.j],  ‖·‖₂=5.916e-05
   iter 30: subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=4.187e-05
   iter 31: subdiag=[-0.+0.j  0.+0.j -0.-0.j],  ‖·‖₂=2.963e-05
   iter 32: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=2.097e-05
   iter 33: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=1.484e-05
   iter 34: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=1.050e-05
   iter 35: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=7.434e-06
   iter 36: subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=5.261e-06
   iter 37: subdiag=[-0.+0.j  0.+0.j -0.-0.j],  ‖·‖₂=3.724e-06
   iter 38: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=2.635e-06
   iter 39: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=1.865e-06
   iter 40: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=1.320e-06
   iter 41: subdiag=[-0.+0.j  0.+0.j -0.-0.j],  ‖·‖₂=9.342e-07
   iter 42: subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=6.611e-07
   iter 43: subdiag=[-0.+0.j  0.+0.j -0.-0.j],  ‖·‖₂=4.679e-07
   iter 44: subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=3.311e-07
```

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  iter 45: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=2.344e-07
  iter 46: subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=1.659e-07
  iter 47: subdiag=[-0.+0.j  0.+0.j -0.-0.j],  ‖·‖₂=1.174e-07
  iter 48: subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=8.308e-08
  iter 49: subdiag=[-0.+0.j  0.+0.j -0.-0.j],  ‖·‖₂=5.880e-08
  iter 50: subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=4.161e-08
  iter 51: subdiag=[-0.+0.j  0.+0.j -0.-0.j],  ‖·‖₂=2.945e-08
  iter 52: subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=2.084e-08
  iter 53: subdiag=[-0.+0.j  0.+0.j -0.-0.j],  ‖·‖₂=1.475e-08
  iter 54: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=1.044e-08
  iter 55: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=7.388e-09
  iter 56: subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=5.229e-09
  iter 57: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=3.701e-09
  iter 58: subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=2.619e-09
  iter 59: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=1.854e-09
  iter 60: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=1.312e-09
  iter 61: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=9.284e-10
  iter 62: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=6.570e-10
  iter 63: subdiag=[-0.+0.j  0.+0.j -0.-0.j],  ‖·‖₂=4.650e-10
  iter 64: subdiag=[-0.-0.j -0.-0.j -0.-0.j],  ‖·‖₂=3.291e-10
  iter 65: subdiag=[-0.+0.j  0.+0.j -0.-0.j],  ‖·‖₂=2.329e-10
  iter 66: subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=1.648e-10
  iter 67: subdiag=[-0.+0.j  0.+0.j -0.-0.j],  ‖·‖₂=1.167e-10
  iter 68: subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=8.256e-11
  after : subdiag=[-0.+0.j -0.-0.j -0.-0.j],  ‖·‖₂=8.256e-11
│  iterations    = 69/100
│  sub-diag magnitudes after last step:
│  [0. 0. 0.]
└─ converged?    = yes

┌─ Matrix 25/30  (size 4x4)
│  fixed shift μ = -0.65739-0.393338j (|μ|=0.7661)
  before: subdiag=[ 0.282+0.j  0.81 +0.j -0.56 +0.j],  ‖·‖₂=1.024e+00
  iter 00: subdiag=[-0.423+0.j  0.699+0.j  0.406+0.j],  ‖·‖₂=9.128e-01
  iter 01: subdiag=[ 0.525-0.j  0.476+0.j -0.313+0.j],  ‖·‖₂=7.751e-01
  iter 02: subdiag=[ 0.389-0.j -0.366-0.j  0.245+0.j],  ‖·‖₂=5.879e-01
  iter 03: subdiag=[-0.207-0.j  0.318+0.j -0.189+0.j],  ‖·‖₂=4.242e-01
  iter 04: subdiag=[ 0.1  -0.j -0.287-0.j  0.143+0.j],  ‖·‖₂=3.359e-01
  iter 05: subdiag=[-0.047+0.j  0.259+0.j -0.108-0.j],  ‖·‖₂=2.846e-01
  iter 06: subdiag=[ 0.022+0.j -0.233-0.j  0.081+0.j],  ‖·‖₂=2.481e-01
  iter 07: subdiag=[-0.011-0.j  0.21 +0.j -0.061-0.j],  ‖·‖₂=2.187e-01
  iter 08: subdiag=[ 0.005-0.j -0.188+0.j  0.045+0.j],  ‖·‖₂=1.937e-01
  iter 09: subdiag=[-0.002-0.j  0.169+0.j -0.034-0.j],  ‖·‖₂=1.721e-01
  iter 10: subdiag=[ 0.001+0.j -0.151-0.j  0.025+0.j],  ‖·‖₂=1.531e-01
  iter 11: subdiag=[-0.001-0.j  0.135-0.j -0.019+0.j],  ‖·‖₂=1.363e-01
  iter 12: subdiag=[ 0.   +0.j -0.121-0.j  0.014+0.j],  ‖·‖₂=1.215e-01
```

```
  iter 13: subdiag=[-0.   +0.j  0.108+0.j -0.011+0.j],  ‖·‖₂=1.083e-01
  iter 14: subdiag=[ 0.   +0.j -0.096+0.j  0.008+0.j],  ‖·‖₂=9.653e-02
  iter 15: subdiag=[-0.   -0.j  0.086+0.j -0.006-0.j],  ‖·‖₂=8.606e-02
  iter 16: subdiag=[ 0.   +0.j -0.077-0.j  0.004+0.j],  ‖·‖₂=7.673e-02
  iter 17: subdiag=[-0.   -0.j  0.068+0.j -0.003-0.j],  ‖·‖₂=6.842e-02
  iter 18: subdiag=[ 0.   +0.j -0.061-0.j  0.003+0.j],  ‖·‖₂=6.100e-02
  iter 19: subdiag=[-0.   +0.j  0.054+0.j -0.002+0.j],  ‖·‖₂=5.438e-02
  iter 20: subdiag=[ 0.   -0.j -0.048-0.j  0.001+0.j],  ‖·‖₂=4.848e-02
  iter 21: subdiag=[-0.   -0.j  0.043-0.j -0.001+0.j],  ‖·‖₂=4.322e-02
  iter 22: subdiag=[ 0.   +0.j -0.039-0.j  0.001+0.j],  ‖·‖₂=3.853e-02
  iter 23: subdiag=[-0.   -0.j  0.034+0.j -0.001+0.j],  ‖·‖₂=3.434e-02
  iter 24: subdiag=[ 0.   +0.j -0.031-0.j  0.   +0.j],  ‖·‖₂=3.061e-02
  iter 25: subdiag=[-0.   -0.j  0.027+0.j -0.   +0.j],  ‖·‖₂=2.729e-02
  iter 26: subdiag=[ 0.   +0.j -0.024-0.j  0.   +0.j],  ‖·‖₂=2.432e-02
  iter 27: subdiag=[-0.   -0.j  0.022+0.j -0.   +0.j],  ‖·‖₂=2.168e-02
  iter 28: subdiag=[ 0.   -0.j -0.019-0.j  0.   -0.j],  ‖·‖₂=1.932e-02
  iter 29: subdiag=[-0.   +0.j  0.017-0.j -0.   +0.j],  ‖·‖₂=1.722e-02
  iter 30: subdiag=[ 0.   -0.j -0.015-0.j  0.   -0.j],  ‖·‖₂=1.535e-02
  iter 31: subdiag=[-0.   +0.j  0.014+0.j -0.   +0.j],  ‖·‖₂=1.368e-02
  iter 32: subdiag=[ 0.   +0.j -0.012-0.j  0.   +0.j],  ‖·‖₂=1.220e-02
  iter 33: subdiag=[-0.   -0.j  0.011+0.j -0.   +0.j],  ‖·‖₂=1.087e-02
  iter 34: subdiag=[ 0.  +0.j -0.01-0.j  0.  +0.j],  ‖·‖₂=9.688e-03
  iter 35: subdiag=[-0.   -0.j  0.009+0.j -0.   -0.j],  ‖·‖₂=8.635e-03
  iter 36: subdiag=[ 0.   +0.j -0.008-0.j  0.   +0.j],  ‖·‖₂=7.696e-03
  iter 37: subdiag=[-0.   -0.j  0.007+0.j -0.   -0.j],  ‖·‖₂=6.860e-03
  iter 38: subdiag=[ 0.   +0.j -0.006-0.j  0.   +0.j],  ‖·‖₂=6.114e-03
  iter 39: subdiag=[-0.   -0.j  0.005+0.j -0.   -0.j],  ‖·‖₂=5.449e-03
  iter 40: subdiag=[ 0.   +0.j -0.005-0.j  0.   +0.j],  ‖·‖₂=4.857e-03
  iter 41: subdiag=[-0.   -0.j  0.004+0.j -0.   +0.j],  ‖·‖₂=4.329e-03
  iter 42: subdiag=[ 0.   +0.j -0.004+0.j  0.   -0.j],  ‖·‖₂=3.858e-03
  iter 43: subdiag=[-0.   -0.j  0.003-0.j -0.   +0.j],  ‖·‖₂=3.439e-03
  iter 44: subdiag=[ 0.   +0.j -0.003+0.j  0.   -0.j],  ‖·‖₂=3.065e-03
  iter 45: subdiag=[-0.   -0.j  0.003+0.j -0.   +0.j],  ‖·‖₂=2.731e-03
  iter 46: subdiag=[ 0.   +0.j -0.002-0.j  0.   -0.j],  ‖·‖₂=2.434e-03
  iter 47: subdiag=[-0.   -0.j  0.002+0.j -0.   +0.j],  ‖·‖₂=2.170e-03
  iter 48: subdiag=[ 0.   +0.j -0.002-0.j  0.   -0.j],  ‖·‖₂=1.934e-03
  iter 49: subdiag=[-0.   -0.j  0.002+0.j -0.   +0.j],  ‖·‖₂=1.724e-03
  iter 50: subdiag=[ 0.   +0.j -0.002-0.j  0.   -0.j],  ‖·‖₂=1.536e-03
  iter 51: subdiag=[-0.   -0.j  0.001+0.j -0.   +0.j],  ‖·‖₂=1.369e-03
  iter 52: subdiag=[ 0.   +0.j -0.001-0.j  0.   -0.j],  ‖·‖₂=1.220e-03
  iter 53: subdiag=[-0.   -0.j  0.001-0.j -0.   +0.j],  ‖·‖₂=1.088e-03
  iter 54: subdiag=[ 0.   +0.j -0.001-0.j  0.   -0.j],  ‖·‖₂=9.694e-04
  iter 55: subdiag=[-0.   -0.j  0.001+0.j -0.   +0.j],  ‖·‖₂=8.640e-04
  iter 56: subdiag=[ 0.   +0.j -0.001-0.j  0.   -0.j],  ‖·‖₂=7.701e-04
  iter 57: subdiag=[-0.   -0.j  0.001+0.j -0.   +0.j],  ‖·‖₂=6.863e-04
  iter 58: subdiag=[ 0.   +0.j -0.001-0.j  0.   -0.j],  ‖·‖₂=6.117e-04
```

```
  iter 59: subdiag=[-0.   -0.j  0.001+0.j -0.   +0.j],  ‖·‖₂=5.452e-04
  iter 60: subdiag=[ 0.+0.j -0.-0.j  0.+0.j],  ‖·‖₂=4.859e-04
  iter 61: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=4.331e-04
  iter 62: subdiag=[ 0.+0.j -0.-0.j  0.+0.j],  ‖·‖₂=3.860e-04
  iter 63: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=3.440e-04
  iter 64: subdiag=[ 0.+0.j -0.-0.j  0.+0.j],  ‖·‖₂=3.066e-04
  iter 65: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=2.733e-04
  iter 66: subdiag=[ 0.+0.j -0.-0.j  0.+0.j],  ‖·‖₂=2.436e-04
  iter 67: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=2.171e-04
  iter 68: subdiag=[ 0.+0.j -0.-0.j  0.+0.j],  ‖·‖₂=1.935e-04
  iter 69: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=1.725e-04
  iter 70: subdiag=[ 0.+0.j -0.-0.j  0.+0.j],  ‖·‖₂=1.537e-04
  iter 71: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=1.370e-04
  iter 72: subdiag=[ 0.+0.j -0.-0.j  0.+0.j],  ‖·‖₂=1.221e-04
  iter 73: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=1.088e-04
  iter 74: subdiag=[ 0.+0.j -0.-0.j  0.+0.j],  ‖·‖₂=9.699e-05
  iter 75: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=8.644e-05
  iter 76: subdiag=[ 0.+0.j -0.-0.j  0.+0.j],  ‖·‖₂=7.704e-05
  iter 77: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=6.867e-05
  iter 78: subdiag=[ 0.+0.j -0.-0.j  0.+0.j],  ‖·‖₂=6.120e-05
  iter 79: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=5.455e-05
  iter 80: subdiag=[ 0.+0.j -0.-0.j  0.+0.j],  ‖·‖₂=4.862e-05
  iter 81: subdiag=[-0.-0.j  0.+0.j -0.+0.j],  ‖·‖₂=4.333e-05
  iter 82: subdiag=[ 0.+0.j -0.-0.j  0.-0.j],  ‖·‖₂=3.862e-05
  iter 83: subdiag=[-0.-0.j  0.+0.j -0.+0.j],  ‖·‖₂=3.442e-05
  iter 84: subdiag=[ 0.+0.j -0.-0.j  0.-0.j],  ‖·‖₂=3.068e-05
  iter 85: subdiag=[-0.-0.j  0.+0.j -0.+0.j],  ‖·‖₂=2.734e-05
  iter 86: subdiag=[ 0.+0.j -0.-0.j  0.+0.j],  ‖·‖₂=2.437e-05
  iter 87: subdiag=[-0.-0.j  0.+0.j -0.-0.j],  ‖·‖₂=2.172e-05
  iter 88: subdiag=[ 0.+0.j -0.-0.j  0.+0.j],  ‖·‖₂=1.936e-05
  iter 89: subdiag=[-0.-0.j  0.+0.j -0.+0.j],  ‖·‖₂=1.725e-05
  iter 90: subdiag=[ 0.+0.j -0.-0.j  0.-0.j],  ‖·‖₂=1.538e-05
  iter 91: subdiag=[-0.-0.j  0.+0.j -0.+0.j],  ‖·‖₂=1.371e-05
  iter 92: subdiag=[ 0.+0.j -0.-0.j  0.-0.j],  ‖·‖₂=1.222e-05
  iter 93: subdiag=[-0.-0.j  0.+0.j -0.+0.j],  ‖·‖₂=1.089e-05
  iter 94: subdiag=[ 0.+0.j -0.-0.j  0.-0.j],  ‖·‖₂=9.704e-06
  iter 95: subdiag=[-0.-0.j  0.-0.j -0.+0.j],  ‖·‖₂=8.649e-06
  iter 96: subdiag=[ 0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=7.708e-06
  iter 97: subdiag=[-0.-0.j  0.-0.j -0.+0.j],  ‖·‖₂=6.870e-06
  iter 98: subdiag=[ 0.+0.j -0.+0.j  0.-0.j],  ‖·‖₂=6.123e-06
  iter 99: subdiag=[-0.-0.j  0.-0.j -0.+0.j],  ‖·‖₂=5.458e-06
  after : subdiag=[-0.-0.j  0.-0.j -0.+0.j],  ‖·‖₂=5.458e-06
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0. 0. 0.]
└─ converged?    = no
```

```
┌─ Matrix 26/30  (size 4x4)
│  fixed shift μ = 0.755595 (|μ|=0.7556)
  before: subdiag=[-0.994+0.j -0.988+0.j -0.76 +0.j],  ‖·‖₂=1.594e+00
  iter 00: subdiag=[-0.882+0.j -0.998+0.j -0.19 +0.j],  ‖·‖₂=1.345e+00
  iter 01: subdiag=[-0.701+0.j -0.997+0.j -0.039+0.j],  ‖·‖₂=1.219e+00
  iter 02: subdiag=[-0.503+0.j -0.988+0.j -0.008+0.j],  ‖·‖₂=1.109e+00
  iter 03: subdiag=[-0.338+0.j -0.982+0.j -0.002+0.j],  ‖·‖₂=1.039e+00
  iter 04: subdiag=[-0.221+0.j -0.98 +0.j -0.   +0.j],  ‖·‖₂=1.004e+00
  iter 05: subdiag=[-0.142+0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.887e-01
  iter 06: subdiag=[-0.091+0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.821e-01
  iter 07: subdiag=[-0.058+0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.794e-01
  iter 08: subdiag=[-0.037+0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.783e-01
  iter 09: subdiag=[-0.024+0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.779e-01
  iter 10: subdiag=[-0.015+0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.777e-01
  iter 11: subdiag=[-0.01 +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.776e-01
  iter 12: subdiag=[-0.006+0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.776e-01
  iter 13: subdiag=[-0.004+0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.776e-01
  iter 14: subdiag=[-0.002+0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.776e-01
  iter 15: subdiag=[-0.002+0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 16: subdiag=[-0.001+0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 17: subdiag=[-0.001+0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 18: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 19: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 20: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 21: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 22: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 23: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 24: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 25: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 26: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 27: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 28: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 29: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 30: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 31: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 32: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 33: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 34: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 35: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 36: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 37: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 38: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 39: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 40: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 41: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
```

```
iter 42: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 43: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 44: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 45: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 46: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 47: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 48: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 49: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 50: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 51: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 52: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 53: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 54: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 55: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 56: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 57: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 58: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 59: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 60: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 61: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 62: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 63: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 64: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 65: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 66: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 67: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 68: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 69: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 70: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 71: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 72: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 73: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 74: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 75: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 76: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 77: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 78: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 79: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 80: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 81: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 82: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 83: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 84: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 85: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 86: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
iter 87: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
```

```
  iter 88: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 89: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 90: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 91: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 92: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 93: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 94: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 95: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 96: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 97: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 98: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  iter 99: subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
  after : subdiag=[-0.   +0.j -0.978+0.j -0.   +0.j],  ‖·‖₂=9.775e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.    0.978 0.   ]
└─ converged?    = no

┌─ Matrix 27/30  (size 4x4)
│  fixed shift μ = -0.995116 (|μ|=0.9951)
  before: subdiag=[-0.999+0.j  0.99 +0.j -0.085+0.j],  ‖·‖₂=1.409e+00
  iter 00: subdiag=[-0.804+0.j  0.902+0.j -0.   +0.j],  ‖·‖₂=1.208e+00
  iter 01: subdiag=[-0.466+0.j  0.855+0.j -0.   +0.j],  ‖·‖₂=9.735e-01
  iter 02: subdiag=[-0.234+0.j  0.841+0.j -0.   +0.j],  ‖·‖₂=8.733e-01
  iter 03: subdiag=[-0.113+0.j  0.838+0.j -0.   +0.j],  ‖·‖₂=8.458e-01
  iter 04: subdiag=[-0.054+0.j  0.838+0.j -0.   +0.j],  ‖·‖₂=8.392e-01
  iter 05: subdiag=[-0.026+0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.377e-01
  iter 06: subdiag=[-0.012+0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.374e-01
  iter 07: subdiag=[-0.006+0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 08: subdiag=[-0.003+0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 09: subdiag=[-0.001+0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 10: subdiag=[-0.001+0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 11: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 12: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 13: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 14: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 15: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 16: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 17: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 18: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 19: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 20: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 21: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 22: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 23: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 24: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
```

```
iter 25: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 26: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 27: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 28: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 29: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 30: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 31: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 32: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 33: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 34: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 35: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 36: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 37: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 38: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 39: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 40: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 41: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 42: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 43: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 44: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 45: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 46: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 47: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 48: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 49: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 50: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 51: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 52: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 53: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 54: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 55: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 56: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 57: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 58: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 59: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 60: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 61: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 62: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 63: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 64: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 65: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 66: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 67: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 68: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 69: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
iter 70: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
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  iter 71: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 72: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 73: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 74: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 75: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 76: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 77: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 78: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 79: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 80: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 81: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 82: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 83: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 84: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 85: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 86: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 87: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 88: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 89: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 90: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 91: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 92: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 93: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 94: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 95: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 96: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 97: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 98: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  iter 99: subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
  after : subdiag=[-0.   +0.j  0.837+0.j -0.   +0.j],  ‖·‖₂=8.373e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.    0.837 0.   ]
└─ converged?    = no

┌─ Matrix 28/30  (size 4x4)
│  fixed shift μ = -0.976473 (|μ|=0.9765)
  before: subdiag=[-0.799+0.j  0.936+0.j  0.149+0.j],  ‖·‖₂=1.239e+00
  iter 00: subdiag=[-0.683+0.j -0.093+0.j -0.118+0.j],  ‖·‖₂=6.991e-01
  iter 01: subdiag=[-0.682+0.j  0.006+0.j  0.118+0.j],  ‖·‖₂=6.923e-01
  iter 02: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 03: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 04: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 05: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 06: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 07: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
```

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  iter 08: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 09: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 10: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 11: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 12: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 13: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 14: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 15: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 16: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 17: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 18: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 19: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 20: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 21: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 22: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 23: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 24: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 25: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 26: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 27: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 28: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 29: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 30: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 31: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 32: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 33: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 34: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 35: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 36: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 37: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 38: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 39: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 40: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 41: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 42: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 43: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 44: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 45: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 46: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 47: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 48: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 49: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 50: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 51: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 52: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 53: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
```

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  iter 54: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 55: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 56: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 57: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 58: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 59: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 60: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 61: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 62: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 63: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 64: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 65: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 66: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 67: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 68: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 69: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 70: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 71: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 72: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 73: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 74: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 75: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 76: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 77: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 78: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 79: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 80: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 81: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 82: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 83: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 84: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 85: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 86: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 87: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 88: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 89: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 90: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 91: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 92: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 93: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 94: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 95: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 96: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 97: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
  iter 98: subdiag=[-0.682+0.j -0.   +0.j -0.118+0.j],  ‖·‖₂=6.922e-01
  iter 99: subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
```

```
  after : subdiag=[-0.682+0.j  0.   +0.j  0.118+0.j],  ‖·‖₂=6.922e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.682 0.    0.118]
└─ converged?    = no

┌─ Matrix 29/30  (size 4x4)
│  fixed shift μ = 0.790166 (|μ|=0.7902)
  before: subdiag=[ 0.811+0.j -0.867+0.j  0.948+0.j],  ‖·‖₂=1.519e+00
  iter 00: subdiag=[ 0.726+0.j -0.795+0.j  0.199+0.j],  ‖·‖₂=1.095e+00
  iter 01: subdiag=[ 0.664+0.j -0.832+0.j  0.027+0.j],  ‖·‖₂=1.064e+00
  iter 02: subdiag=[ 0.596+0.j -0.861+0.j  0.004+0.j],  ‖·‖₂=1.047e+00
  iter 03: subdiag=[ 0.525+0.j -0.883+0.j  0.   +0.j],  ‖·‖₂=1.028e+00
  iter 04: subdiag=[ 0.456+0.j -0.899+0.j  0.   +0.j],  ‖·‖₂=1.008e+00
  iter 05: subdiag=[ 0.391+0.j -0.909+0.j  0.   +0.j],  ‖·‖₂=9.899e-01
  iter 06: subdiag=[ 0.332+0.j -0.917+0.j  0.   +0.j],  ‖·‖₂=9.751e-01
  iter 07: subdiag=[ 0.28 +0.j -0.922+0.j  0.   +0.j],  ‖·‖₂=9.635e-01
  iter 08: subdiag=[ 0.234+0.j -0.925+0.j  0.   +0.j],  ‖·‖₂=9.547e-01
  iter 09: subdiag=[ 0.196+0.j -0.928+0.j  0.   +0.j],  ‖·‖₂=9.483e-01
  iter 10: subdiag=[ 0.163+0.j -0.93 +0.j  0.   +0.j],  ‖·‖₂=9.437e-01
  iter 11: subdiag=[ 0.135+0.j -0.931+0.j  0.   +0.j],  ‖·‖₂=9.405e-01
  iter 12: subdiag=[ 0.112+0.j -0.931+0.j  0.   +0.j],  ‖·‖₂=9.382e-01
  iter 13: subdiag=[ 0.093+0.j -0.932+0.j  0.   +0.j],  ‖·‖₂=9.366e-01
  iter 14: subdiag=[ 0.077+0.j -0.932+0.j  0.   +0.j],  ‖·‖₂=9.355e-01
  iter 15: subdiag=[ 0.064+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.348e-01
  iter 16: subdiag=[ 0.053+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.342e-01
  iter 17: subdiag=[ 0.044+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.339e-01
  iter 18: subdiag=[ 0.036+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.336e-01
  iter 19: subdiag=[ 0.03 +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.335e-01
  iter 20: subdiag=[ 0.025+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.334e-01
  iter 21: subdiag=[ 0.02 +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.333e-01
  iter 22: subdiag=[ 0.017+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.332e-01
  iter 23: subdiag=[ 0.014+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.332e-01
  iter 24: subdiag=[ 0.012+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.332e-01
  iter 25: subdiag=[ 0.01 +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 26: subdiag=[ 0.008+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 27: subdiag=[ 0.007+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 28: subdiag=[ 0.005+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 29: subdiag=[ 0.004+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 30: subdiag=[ 0.004+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 31: subdiag=[ 0.003+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 32: subdiag=[ 0.003+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 33: subdiag=[ 0.002+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 34: subdiag=[ 0.002+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 35: subdiag=[ 0.001+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 36: subdiag=[ 0.001+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
```

```
iter 37: subdiag=[ 0.001+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 38: subdiag=[ 0.001+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 39: subdiag=[ 0.001+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 40: subdiag=[ 0.001+0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 41: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 42: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 43: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 44: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 45: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 46: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 47: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 48: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 49: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 50: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 51: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 52: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 53: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 54: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 55: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 56: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 57: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 58: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 59: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 60: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 61: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 62: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 63: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 64: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 65: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 66: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 67: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 68: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 69: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 70: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 71: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 72: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 73: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 74: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 75: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 76: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 77: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 78: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 79: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 80: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 81: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
iter 82: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
```

```
  iter 83: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 84: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 85: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 86: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 87: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 88: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 89: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 90: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 91: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 92: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 93: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 94: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 95: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 96: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 97: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 98: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  iter 99: subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
  after : subdiag=[ 0.   +0.j -0.933+0.j  0.   +0.j],  ‖·‖₂=9.331e-01
│  iterations    = 100/100
│  sub-diag magnitudes after last step:
│  [0.    0.933 0.   ]
└─ converged?    = no

┌─ Matrix 30/30  (size 4x4)
│  fixed shift μ = -0.0383725-0.203948j (|μ|=0.2075)
  before: subdiag=[-0.744+0.j -0.999+0.j -0.997+0.j],  ‖·‖₂=1.596e+00
  iter 00: subdiag=[-0.752-0.j -0.99 +0.j -0.977+0.j],  ‖·‖₂=1.581e+00
  iter 01: subdiag=[0.757+0.j 0.957+0.j 0.928-0.j],  ‖·‖₂=1.533e+00
  iter 02: subdiag=[-0.758-0.j -0.894+0.j -0.87 +0.j],  ‖·‖₂=1.460e+00
  iter 03: subdiag=[0.76 +0.j 0.803-0.j 0.814-0.j],  ‖·‖₂=1.372e+00
  iter 04: subdiag=[-0.761-0.j -0.694-0.j -0.766+0.j],  ‖·‖₂=1.284e+00
  iter 05: subdiag=[0.761+0.j 0.582-0.j 0.729-0.j],  ‖·‖₂=1.204e+00
  iter 06: subdiag=[-0.758-0.j -0.477+0.j -0.699+0.j],  ‖·‖₂=1.136e+00
  iter 07: subdiag=[-0.751-0.j -0.385-0.j  0.674-0.j],  ‖·‖₂=1.080e+00
  iter 08: subdiag=[-0.738-0.j -0.309+0.j -0.653+0.j],  ‖·‖₂=1.033e+00
  iter 09: subdiag=[-0.719-0.j -0.247+0.j  0.635-0.j],  ‖·‖₂=9.907e-01
  iter 10: subdiag=[ 0.694+0.j -0.197-0.j -0.618+0.j],  ‖·‖₂=9.504e-01
  iter 11: subdiag=[-0.665-0.j -0.158-0.j  0.602-0.j],  ‖·‖₂=9.105e-01
  iter 12: subdiag=[ 0.631+0.j -0.126-0.j -0.586+0.j],  ‖·‖₂=8.704e-01
  iter 13: subdiag=[-0.594-0.j -0.102-0.j  0.571-0.j],  ‖·‖₂=8.301e-01
  iter 14: subdiag=[ 0.555+0.j -0.082-0.j -0.556+0.j],  ‖·‖₂=7.899e-01
  iter 15: subdiag=[-0.516+0.j -0.066-0.j  0.541+0.j],  ‖·‖₂=7.503e-01
  iter 16: subdiag=[ 0.476+0.j -0.054-0.j -0.526+0.j],  ‖·‖₂=7.118e-01
  iter 17: subdiag=[-0.438+0.j -0.044-0.j  0.511+0.j],  ‖·‖₂=6.747e-01
  iter 18: subdiag=[ 0.401-0.j -0.036-0.j -0.496-0.j],  ‖·‖₂=6.393e-01
  iter 19: subdiag=[-0.366-0.j -0.029+0.j  0.482+0.j],  ‖·‖₂=6.059e-01
```

```
  iter 20: subdiag=[ 0.333+0.j -0.024+0.j -0.467-0.j],  ‖·‖₂=5.745e-01
  iter 21: subdiag=[-0.303+0.j -0.019+0.j  0.453+0.j],  ‖·‖₂=5.451e-01
  iter 22: subdiag=[ 0.275+0.j -0.016+0.j -0.439+0.j],  ‖·‖₂=5.176e-01
  iter 23: subdiag=[-0.248+0.j -0.013+0.j  0.425-0.j],  ‖·‖₂=4.921e-01
  iter 24: subdiag=[ 0.224-0.j -0.011+0.j -0.411+0.j],  ‖·‖₂=4.682e-01
  iter 25: subdiag=[-0.203+0.j -0.009-0.j  0.397-0.j],  ‖·‖₂=4.460e-01
  iter 26: subdiag=[ 0.183+0.j -0.007-0.j -0.384+0.j],  ‖·‖₂=4.253e-01
  iter 27: subdiag=[-0.165-0.j -0.006-0.j  0.371-0.j],  ‖·‖₂=4.060e-01
  iter 28: subdiag=[ 0.148+0.j -0.005-0.j -0.358+0.j],  ‖·‖₂=3.878e-01
  iter 29: subdiag=[-0.134+0.j -0.004-0.j  0.346-0.j],  ‖·‖₂=3.708e-01
  iter 30: subdiag=[ 0.12 -0.j -0.003-0.j -0.334+0.j],  ‖·‖₂=3.548e-01
  iter 31: subdiag=[-0.108-0.j -0.003-0.j  0.322-0.j],  ‖·‖₂=3.397e-01
  iter 32: subdiag=[ 0.097+0.j -0.002-0.j -0.311+0.j],  ‖·‖₂=3.255e-01
  iter 33: subdiag=[-0.088+0.j -0.002-0.j  0.299-0.j],  ‖·‖₂=3.120e-01
  iter 34: subdiag=[ 0.079+0.j -0.002-0.j -0.289+0.j],  ‖·‖₂=2.992e-01
  iter 35: subdiag=[-0.071-0.j -0.001-0.j  0.278-0.j],  ‖·‖₂=2.870e-01
  iter 36: subdiag=[ 0.064-0.j -0.001+0.j -0.268+0.j],  ‖·‖₂=2.754e-01
  iter 37: subdiag=[-0.057+0.j -0.001+0.j  0.258-0.j],  ‖·‖₂=2.644e-01
  iter 38: subdiag=[ 0.051-0.j -0.001+0.j -0.249+0.j],  ‖·‖₂=2.538e-01
  iter 39: subdiag=[-0.046+0.j -0.001+0.j  0.239-0.j],  ‖·‖₂=2.437e-01
  iter 40: subdiag=[ 0.042-0.j -0.   +0.j -0.23 +0.j],  ‖·‖₂=2.341e-01
  iter 41: subdiag=[-0.037+0.j -0.   -0.j  0.222-0.j],  ‖·‖₂=2.249e-01
  iter 42: subdiag=[ 0.034+0.j -0.   -0.j -0.213+0.j],  ‖·‖₂=2.160e-01
  iter 43: subdiag=[-0.03 +0.j -0.   -0.j  0.205-0.j],  ‖·‖₂=2.075e-01
  iter 44: subdiag=[ 0.027-0.j -0.   -0.j -0.198+0.j],  ‖·‖₂=1.994e-01
  iter 45: subdiag=[-0.024+0.j -0.   -0.j  0.19 -0.j],  ‖·‖₂=1.916e-01
  iter 46: subdiag=[ 0.022-0.j -0.   -0.j -0.183+0.j],  ‖·‖₂=1.841e-01
  iter 47: subdiag=[-0.02 +0.j -0.   -0.j  0.176-0.j],  ‖·‖₂=1.769e-01
  iter 48: subdiag=[ 0.018-0.j -0.   -0.j -0.169+0.j],  ‖·‖₂=1.700e-01
  iter 49: subdiag=[-0.016+0.j -0.   -0.j  0.163-0.j],  ‖·‖₂=1.634e-01
  iter 50: subdiag=[ 0.014-0.j -0.   -0.j -0.156+0.j],  ‖·‖₂=1.570e-01
  iter 51: subdiag=[-0.013+0.j -0.   -0.j  0.15 -0.j],  ‖·‖₂=1.509e-01
  iter 52: subdiag=[ 0.012-0.j -0.   -0.j -0.145+0.j],  ‖·‖₂=1.450e-01
  iter 53: subdiag=[-0.01 +0.j -0.   -0.j  0.139+0.j],  ‖·‖₂=1.393e-01
  iter 54: subdiag=[ 0.009-0.j -0.   +0.j -0.134-0.j],  ‖·‖₂=1.339e-01
  iter 55: subdiag=[-0.008-0.j -0.   +0.j  0.128+0.j],  ‖·‖₂=1.286e-01
  iter 56: subdiag=[ 0.008-0.j -0.   +0.j -0.123-0.j],  ‖·‖₂=1.236e-01
  iter 57: subdiag=[-0.007+0.j -0.   +0.j  0.119+0.j],  ‖·‖₂=1.188e-01
  iter 58: subdiag=[ 0.006-0.j -0.   +0.j -0.114-0.j],  ‖·‖₂=1.141e-01
  iter 59: subdiag=[-0.005+0.j -0.   +0.j  0.11 +0.j],  ‖·‖₂=1.097e-01
  iter 60: subdiag=[ 0.005-0.j -0.   +0.j -0.105-0.j],  ‖·‖₂=1.054e-01
  iter 61: subdiag=[-0.004+0.j -0.   +0.j  0.101+0.j],  ‖·‖₂=1.013e-01
  iter 62: subdiag=[ 0.004-0.j -0.   -0.j -0.097-0.j],  ‖·‖₂=9.730e-02
  iter 63: subdiag=[-0.004+0.j -0.   -0.j  0.093+0.j],  ‖·‖₂=9.349e-02
  iter 64: subdiag=[ 0.003-0.j -0.   -0.j -0.09 -0.j],  ‖·‖₂=8.983e-02
  iter 65: subdiag=[-0.003+0.j -0.   -0.j  0.086+0.j],  ‖·‖₂=8.631e-02
```

```
3102   iter 66: subdiag=[ 0.003-0.j -0.   -0.j -0.083-0.j],  ‖·‖₂=8.292e-02
3103   iter 67: subdiag=[-0.002+0.j -0.   -0.j  0.08 +0.j],  ‖·‖₂=7.967e-02
3104   iter 68: subdiag=[ 0.002-0.j -0.   -0.j -0.077-0.j],  ‖·‖₂=7.655e-02
3105   iter 69: subdiag=[-0.002+0.j -0.   -0.j  0.074+0.j],  ‖·‖₂=7.355e-02
3106   iter 70: subdiag=[ 0.002-0.j -0.   -0.j -0.071-0.j],  ‖·‖₂=7.066e-02
3107   iter 71: subdiag=[-0.002+0.j -0.   -0.j  0.068+0.j],  ‖·‖₂=6.789e-02
3108   iter 72: subdiag=[ 0.001-0.j -0.   -0.j -0.065-0.j],  ‖·‖₂=6.522e-02
3109   iter 73: subdiag=[-0.001+0.j -0.   -0.j  0.063+0.j],  ‖·‖₂=6.266e-02
3110   iter 74: subdiag=[ 0.001-0.j -0.   -0.j -0.06 -0.j],  ‖·‖₂=6.020e-02
3111   iter 75: subdiag=[-0.001+0.j -0.   -0.j  0.058+0.j],  ‖·‖₂=5.783e-02
3112   iter 76: subdiag=[ 0.001-0.j -0.   -0.j -0.056-0.j],  ‖·‖₂=5.556e-02
3113   iter 77: subdiag=[-0.001+0.j -0.   -0.j  0.053+0.j],  ‖·‖₂=5.338e-02
3114   iter 78: subdiag=[ 0.001-0.j -0.   -0.j -0.051-0.j],  ‖·‖₂=5.128e-02
3115   iter 79: subdiag=[-0.001+0.j -0.   -0.j  0.049+0.j],  ‖·‖₂=4.926e-02
3116   iter 80: subdiag=[ 0.001-0.j -0.   -0.j -0.047-0.j],  ‖·‖₂=4.733e-02
3117   iter 81: subdiag=[-0.001+0.j -0.   -0.j  0.045+0.j],  ‖·‖₂=4.547e-02
3118   iter 82: subdiag=[ 0.   -0.j -0.   -0.j -0.044-0.j],  ‖·‖₂=4.368e-02
3119   iter 83: subdiag=[-0.   +0.j -0.   -0.j  0.042+0.j],  ‖·‖₂=4.196e-02
3120   iter 84: subdiag=[ 0.  -0.j -0.  -0.j -0.04-0.j],  ‖·‖₂=4.031e-02
3121   iter 85: subdiag=[-0.   +0.j -0.   -0.j  0.039+0.j],  ‖·‖₂=3.872e-02
3122   iter 86: subdiag=[ 0.   -0.j -0.   -0.j -0.037-0.j],  ‖·‖₂=3.720e-02
3123   iter 87: subdiag=[-0.   +0.j -0.   -0.j  0.036+0.j],  ‖·‖₂=3.574e-02
3124   iter 88: subdiag=[ 0.   -0.j -0.   -0.j -0.034-0.j],  ‖·‖₂=3.433e-02
3125   iter 89: subdiag=[-0.   +0.j -0.   -0.j  0.033+0.j],  ‖·‖₂=3.298e-02
3126   iter 90: subdiag=[ 0.   -0.j -0.   -0.j -0.032-0.j],  ‖·‖₂=3.168e-02
3127   iter 91: subdiag=[-0.  +0.j -0.  -0.j  0.03+0.j],  ‖·‖₂=3.043e-02
3128   iter 92: subdiag=[ 0.   -0.j -0.   -0.j -0.029-0.j],  ‖·‖₂=2.924e-02
3129   iter 93: subdiag=[-0.   +0.j -0.   -0.j  0.028+0.j],  ‖·‖₂=2.809e-02
3130   iter 94: subdiag=[ 0.   -0.j -0.   -0.j -0.027-0.j],  ‖·‖₂=2.698e-02
3131   iter 95: subdiag=[-0.   +0.j -0.   -0.j  0.026+0.j],  ‖·‖₂=2.592e-02
3132   iter 96: subdiag=[ 0.   -0.j -0.   -0.j -0.025-0.j],  ‖·‖₂=2.490e-02
3133   iter 97: subdiag=[-0.   +0.j -0.   -0.j  0.024+0.j],  ‖·‖₂=2.392e-02
3134   iter 98: subdiag=[ 0.   -0.j -0.   -0.j -0.023-0.j],  ‖·‖₂=2.298e-02
3135   iter 99: subdiag=[-0.   +0.j -0.   -0.j  0.022+0.j],  ‖·‖₂=2.207e-02
3136   after : subdiag=[-0.   +0.j -0.   -0.j  0.022+0.j],  ‖·‖₂=2.207e-02
3137 │  iterations    = 100/100
3138 │  sub-diag magnitudes after last step:
3139 │  [0.    0.    0.022]
3140 └─ converged?    = no
```

From this monstrosity we conclude that using one constant shift μ (taken from the trailing 2×2 block) attacks only the last sub-diagonal element. That entry shrinks quickly, but the higher sub-diagonals are influenced only through round-off-sized couplings, so they plateau at some non-zero value $\approx 10^{-2}$ Consequently the matrix rarely satisfies the stringent "$\left|H_{i,i-1}\right| < 10^{-10}$" test, and the script reports converged = no even after 100 iterations.

Using the Wilkinson shift would definitely force convergence, but since we are using one $\mu = \lambda_1$ of the trailing 2×2 block, we can expect convergence only for the last sub-diagonal element.

Bibliography

[1] L. N. Trefethen and D. Bau, *Numerical Linear Algebra*. 1997.