Final Assignment - Probability Theory

Arthur Rabello Oliveira[1]

19/06/2025

Abstract

coming soon

Contents

[1]Escola de Matemática Aplicada, Fundação Getúlio Vargas (FGV/EMAp), email: arthur.oliveira.1@fgv.edu.br

1. Introduction

2. User Parameters

3. Problem 1

3.1. Problem 1 (a)

3.2. Problem 1 (b)

We have 2 kinds of customers, so the waiting time S is a combination of distributions:

$$\mathbb{E}(S) = f\mathbb{E}(S_{\text{fast}}) + (1-f)\mathbb{E}(S_{\text{slow}}) \tag{1}$$

From Section 2 we have a $f \approx 0.35$ of fast customers and a PDF of:

$$g(\varphi) = 0.25 + c(\varphi - 4), 2 < \varphi < 6, c = 0.02 \tag{2}$$

The expected value is, therefore:

$$\begin{aligned}
\mathbb{E}(S_{\text{fast}}) &= \int_{-\infty}^{\infty} \varphi g(\varphi) d\varphi = \int_2^6 \varphi(0.25 + 0.02(\varphi - 4)) d\varphi \\
&= \int_2^6 0.25\varphi d\varphi + \int_2^6 0.02\varphi^2 d\varphi - \int_2^6 0.08\varphi d\varphi \\
&= 0.25\int_2^6 \varphi d\varphi + 0.02\int_2^6 \varphi^2 d\varphi - 0.08\int_2^6 \varphi d\varphi \\
&= 0.25\left[\frac{\varphi^2}{2}\right]_2^6 + 0.02\left[\frac{\varphi^3}{3}\right]_2^6 - 0.08\left[\frac{\varphi^2}{2}\right]_2^6 \\
&= 0.25(36-4) + 0.02(216-8) - 0.08(36-4) \\
&= \frac{32}{4} + 0.02 \cdot \frac{208}{3} - 0.08 \cdot \frac{32}{4} \approx 4.10
\end{aligned} \tag{3}$$

The remaining $S_{\text{slow}} \sim \text{Exp}(m = 5.5)$ customers have an expected value of $\mathbb{E}(S_{\text{slow}}) = 5.5$, so the total expected waiting time is:

$$\mathbb{E}(S) = 0.35 \cdot 4.10 + 0.65 \cdot 5.5 \approx 5.01 \text{ minutes} \tag{4}$$

.

On Section 3.1 our simulation shows an average waiting time of 5.010 min, close enough to the analytical expected value.

4. Problem 2

4.1. Problem 2 (a)

4.2. Problem 2 (b)

4.3. Problem 2 (c)

4.4. Problem 2 (d)