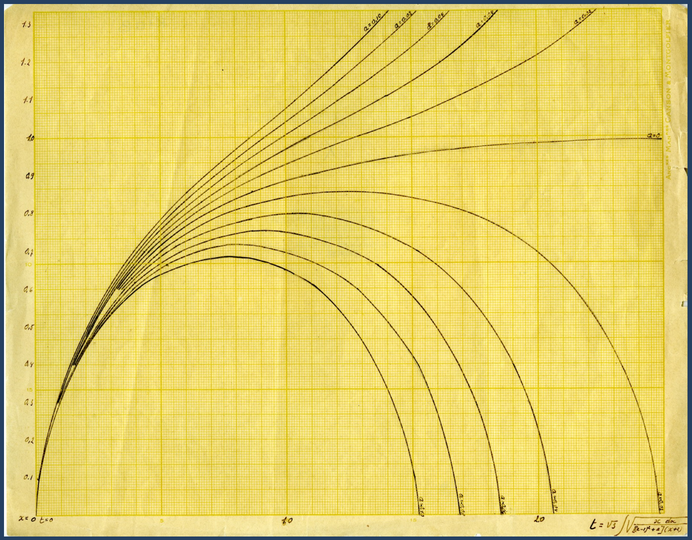

Quantum Cosmology

ANNALES

DE LA

SOCIÉTÉ SCIENTIFIQUE

DE BRUXELLES

EXTRAIT

Un univers homogène de masse constante et de rayon croissant, rendant compte de la vitesse radiale des nébuleuses extra-galactiques

Note de M. l'Abbé G. LEMAITRE

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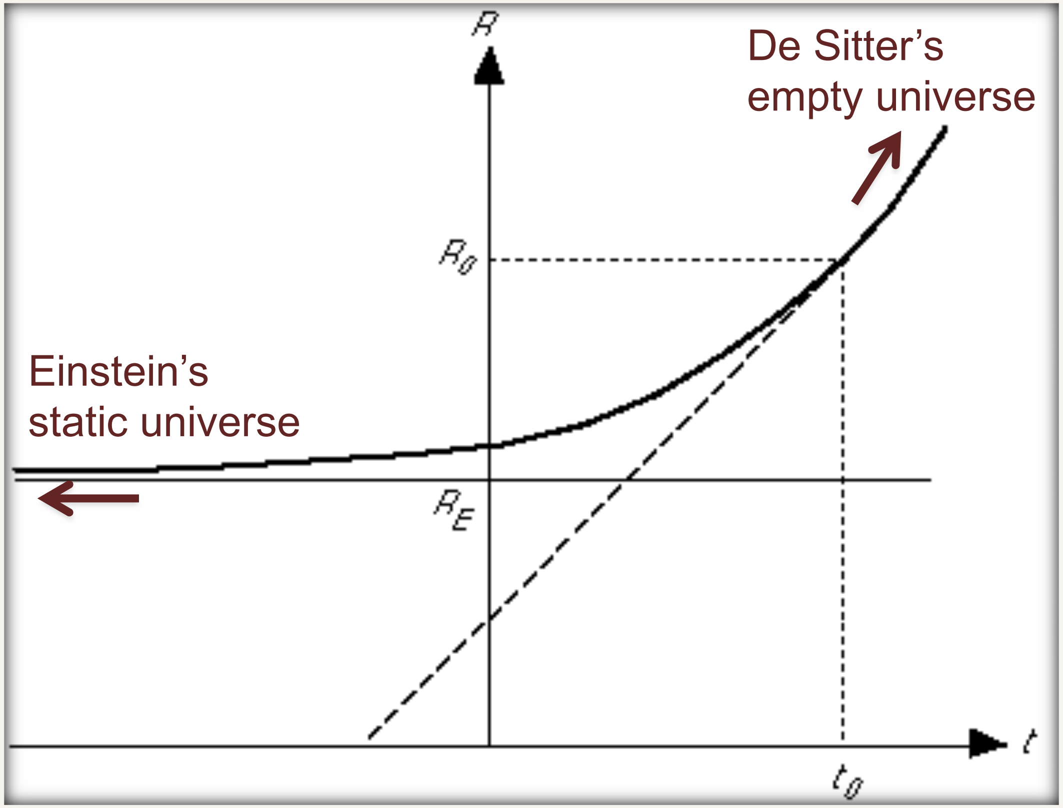
Compte chèques postaux 392-33

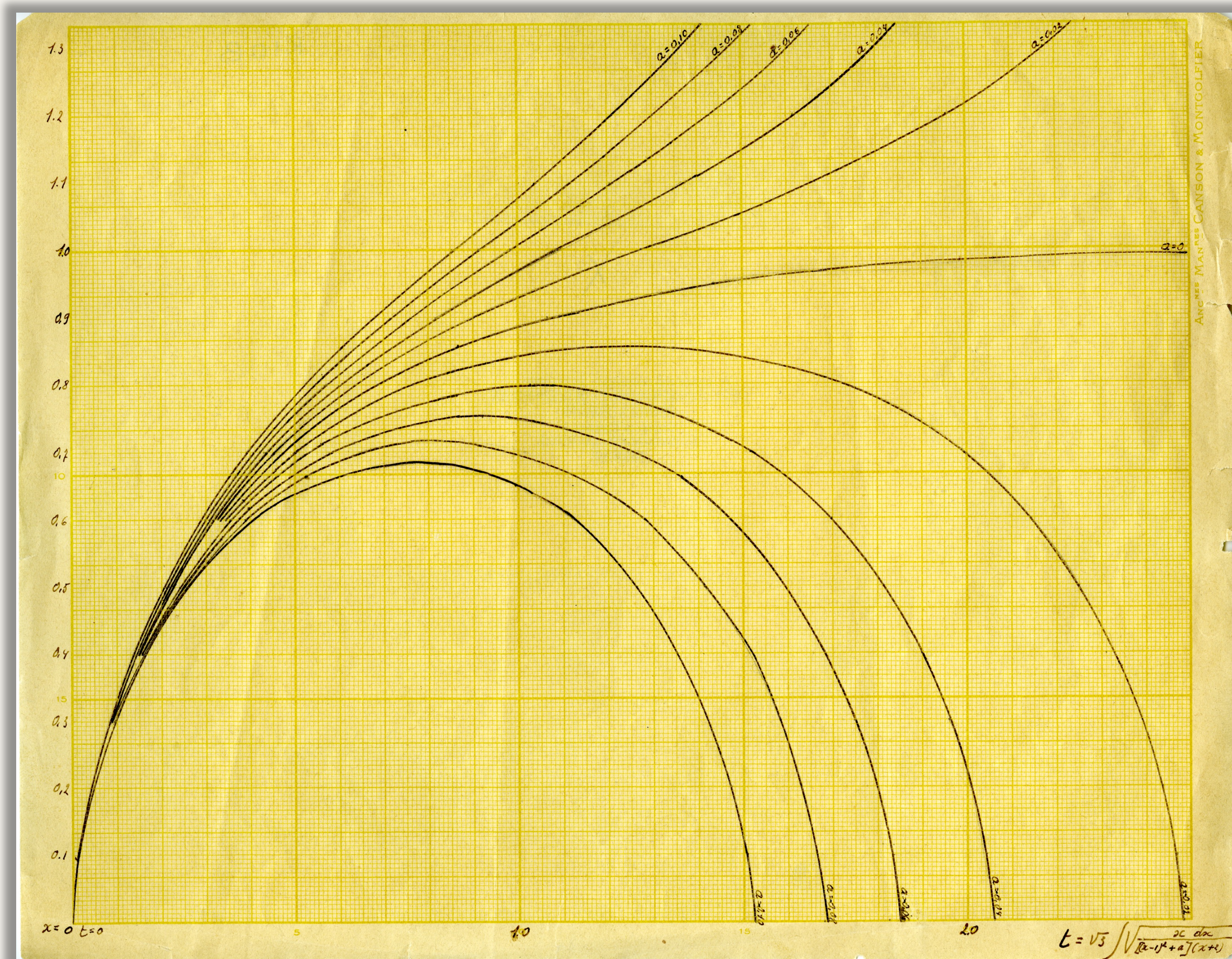
1927



Utilisant les 42 nébuleuses figurant dans les listes de Hubble et de Strömberg (1), et tenant compte de la vitesse propre du soleil (300 Km. dans la direction $\alpha = 315°$, $\delta = 62°$), on trouve une distance moyenne de 0,95 millions de parsecs et une vitesse radiale de 600 Km./sec, soit 625 Km./sec à 10^6 parsecs (2).

Nous adopterons donc

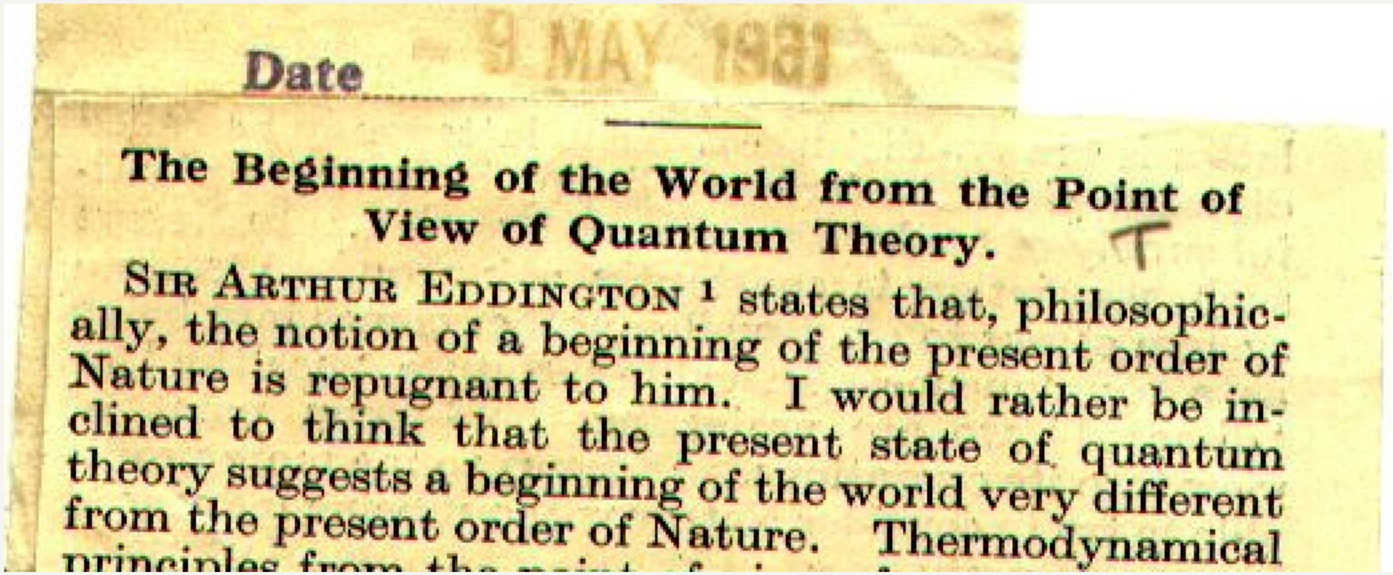
$$\boxed{\frac{R'}{R} = \frac{v}{rc} = \frac{625 \times 10^5}{10^6 \times 3{,}08 \times 10^{18} \times 3 \times 10^{10}} = 0{,}68 \times 10^{-27}\ \mathrm{cm}^{-1}} \qquad (24)$$



G. Lemaître, 1929

Primeval atom

G. Lemaître, Nature, 1931



Date 9 MAY 1931

The Beginning of the World from the Point of View of Quantum Theory.

SIR ARTHUR EDDINGTON [1] states that, philosophically, the notion of a beginning of the present order of Nature is repugnant to him. I would rather be inclined to think that the present state of quantum theory suggests a beginning of the world very different from the present order of Nature. Thermodynamical

"*If we go back in the course of time … the notions of space and time would altogether fail to have any meaning ...*

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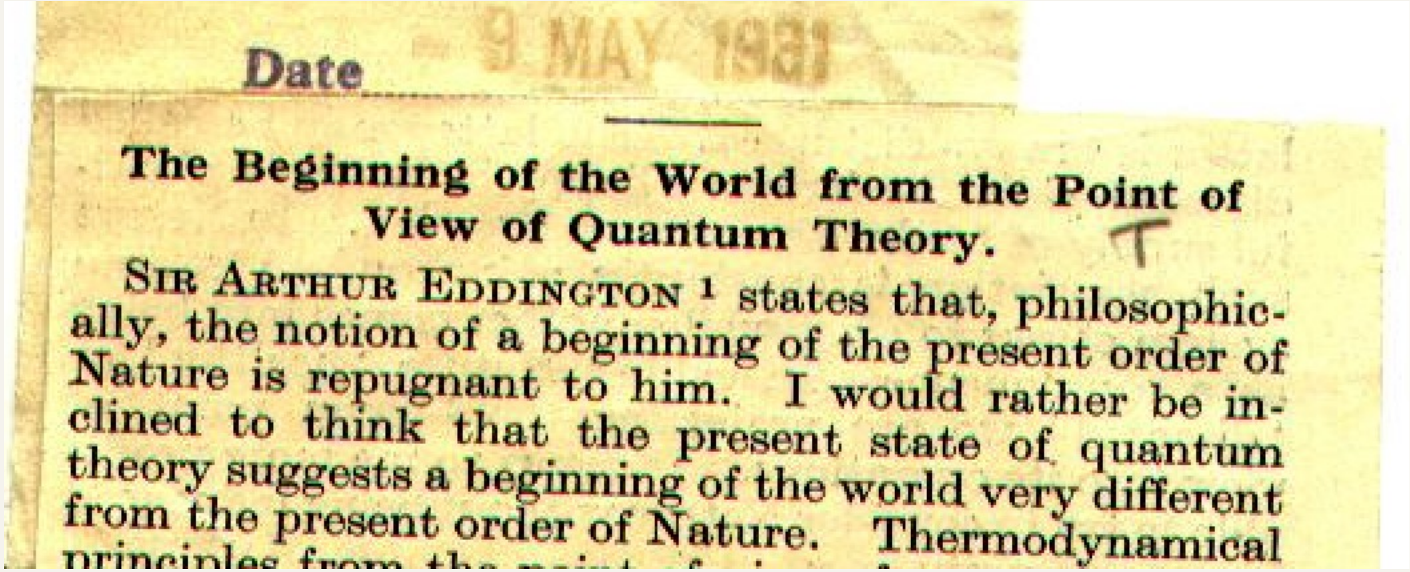

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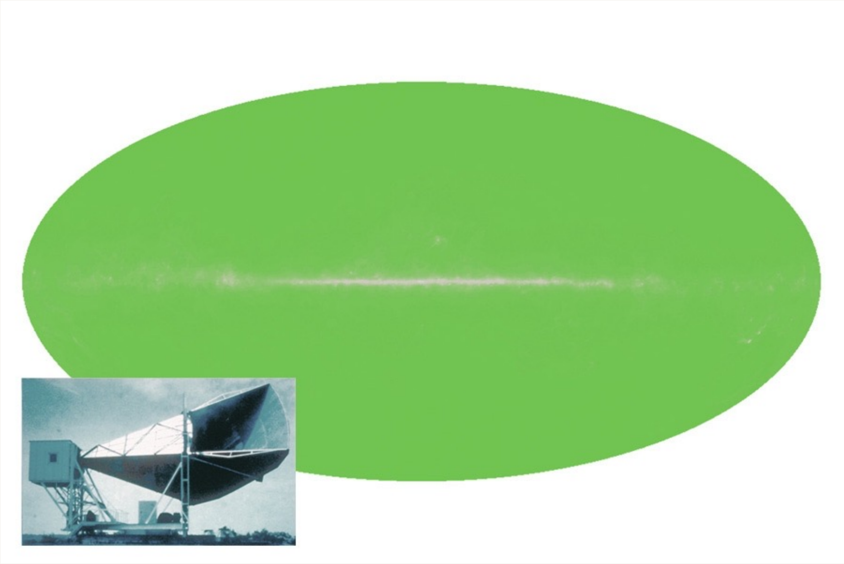
"If we go back in the course of time … the notions of space and time would altogether fail to have any meaning ...

Clearly the initial quantum could not conceal in itself the whole complex course of evolution. The story of the world need not have been written down in the first quantum like the song on a disc of a phonograph ...

Instead from the same beginning widely different universe could have evolved."

1965

- Penzias and Wilson discover microwave background radiation at temperature 3 Kelvin
- Dicke, Peebles, Roll & Wilkinson provide cosmological interpretation as fossil radiation in the framework of `Big Bang’ theory



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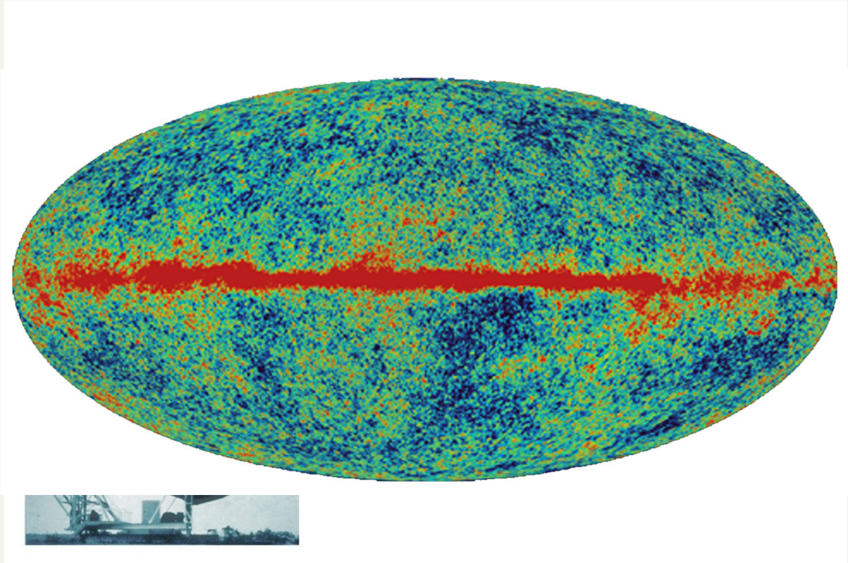
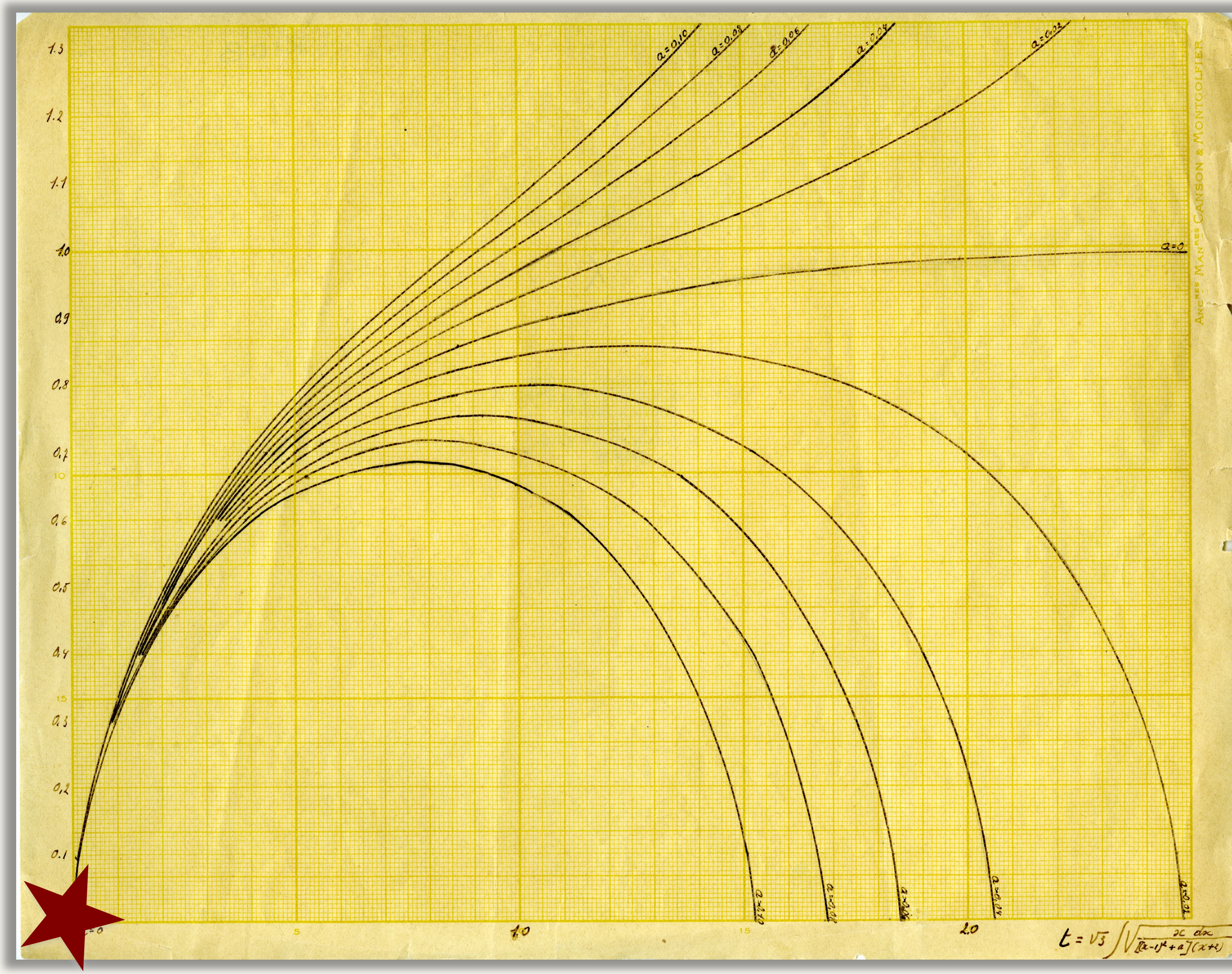


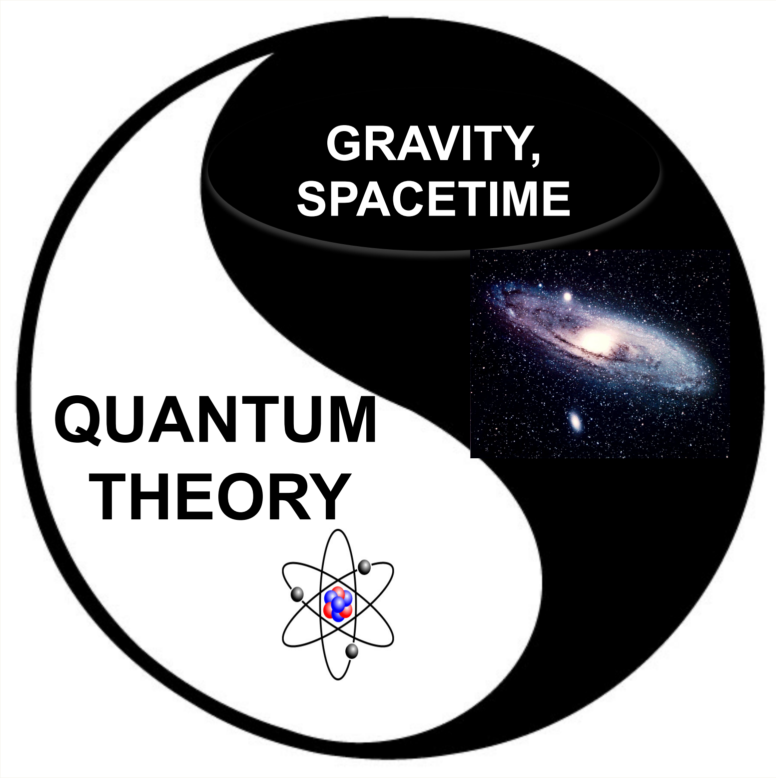


G. Lemaître, 1929

Why bother with Quantum Cosmology?

- critical expansion rate
- low entropy
- small Λ
- $\delta T/T \sim 10^{-5}$,...
- relative strength of forces
- weak gravitation
- three large dimensions,...
- classical space-time itself!

From Quantum to Cosmos



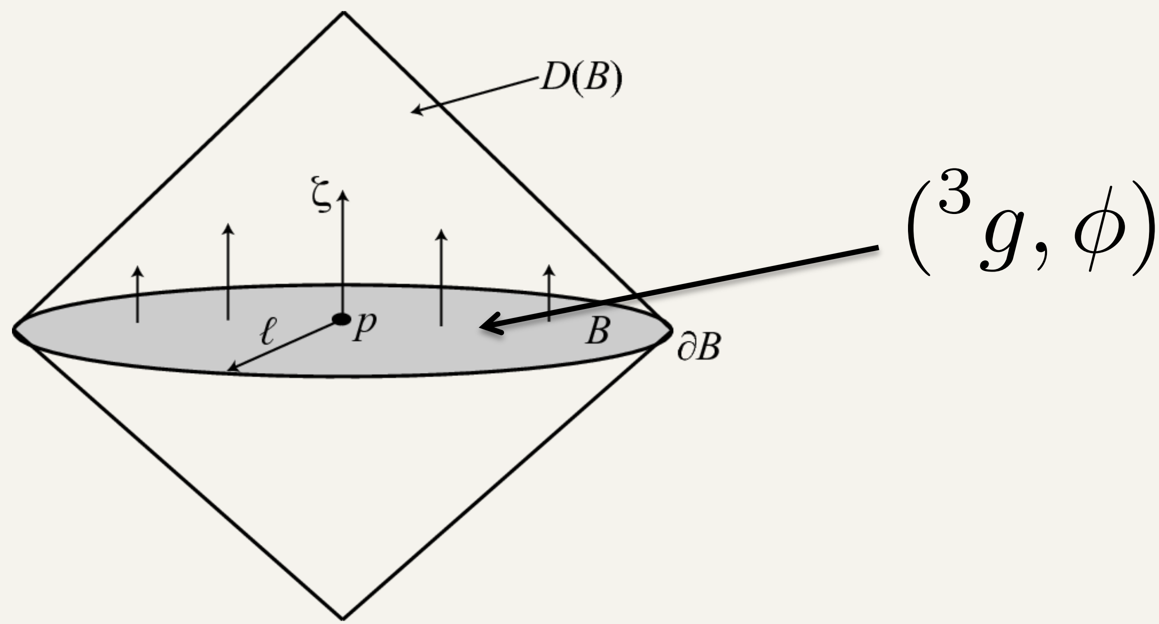
$$g \rightarrow \Psi[g]$$

Outline

- **Wave function of the Universe**
- How to put the wave function to use?!
- Recent developments -> holographic cosmology

Primeval Atom 2.0

$$\Psi[{}^3g, \phi, \cdots]$$



`No-Boundary' Wave Function

[Hartle & Hawking, 1983]

``Ground state' analogue

$$\Psi[{}^3g,\phi] = \int_C \delta g \delta\phi \exp\left(-I_E[{}^3g,\phi]/\hbar\right)$$

"The amplitude of a configuration $({}^3g,\phi)$ *on a spacelike surface Σ is given by a Euclidean path integral over the class C of all regular four-geometries and matter field histories that match* $({}^3g,\phi)$ *on their only boundary."*

Ground state

Ground state in Quantum Mechanics

Euclidean PI: $\psi(x_0) = \int \delta x \exp\{-I[x(\tau)]/\hbar\}$

with $I[x(\tau)] = \frac{1}{2} \int d\tau [\dot{x}^2 + \omega^2 x^2]$

Saddle pt appr: $\psi(x_0) \propto \exp\{-\omega x_0^2/2\}$

`No-Boundary' Wave Function

``Ground state' analogue

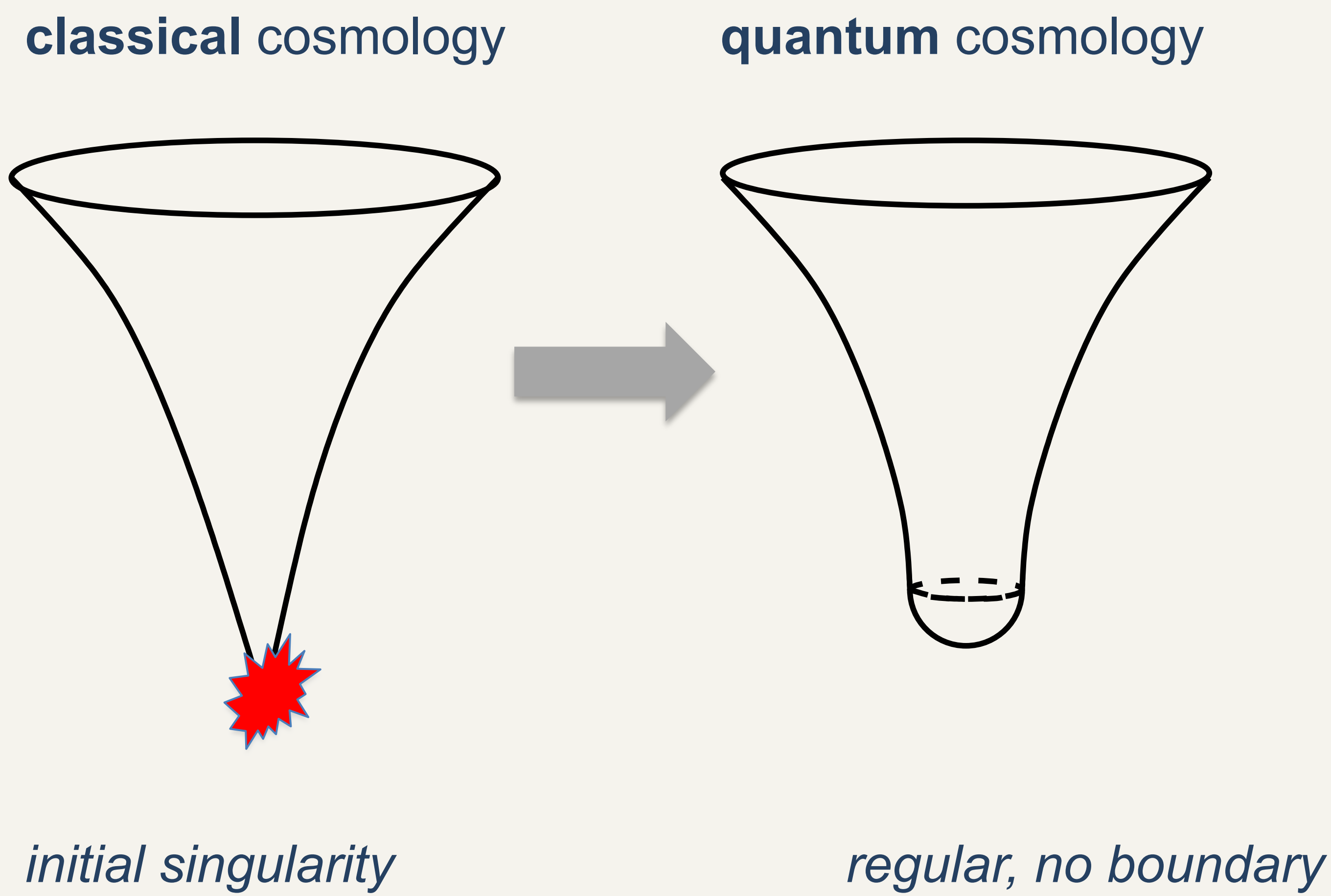
$$\Psi[{}^3g, \phi] = \int_C \delta g \delta\phi \exp\left(-I_E[{}^3g, \phi]/\hbar\right)$$

"The boundary condition of the universe is that it has no boundary."

[Hawking, Pont. Ac. Sci.,1982]

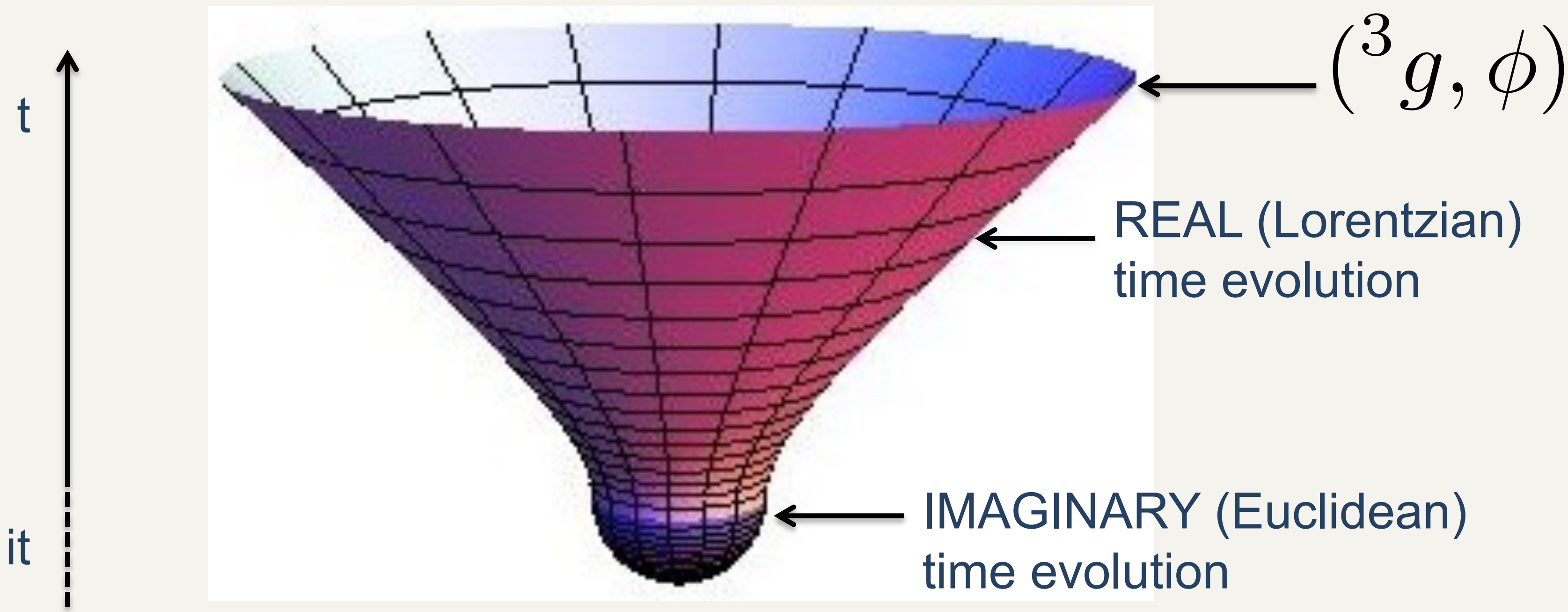
$$I[g, \phi] = -\tfrac{1}{2}\int \sqrt{g}(R - 2\Lambda) + \int \sqrt{g}\left[(\nabla\phi)^2 + V(\phi)\right]$$

`No-Boundary' Wave Function



Example: creation from nothing of de Sitter space

`No-Boundary' wave function



$$\Psi[{}^3g, \phi] \approx \exp\left(-I_E[{}^3g, \phi]/\hbar\right) = A\exp(iS)$$

$$I({}^3g, \chi) = -I_R({}^3g, \chi) + iS({}^3g, \chi)$$

`No-Boundary' wave function

WKB approximation:

$$\Psi[{}^3g, \phi] \approx A \exp(iS)$$

$$|\nabla A| \ll |\nabla S| \longrightarrow$$ **classical cosmological evolution**

$$p_q = \nabla_q S$$

$$P_{hist} \propto A^2 = \exp(-2I_R/\hbar)$$

What universes emerge?

Consider homogeneous/isotropic multiverse:

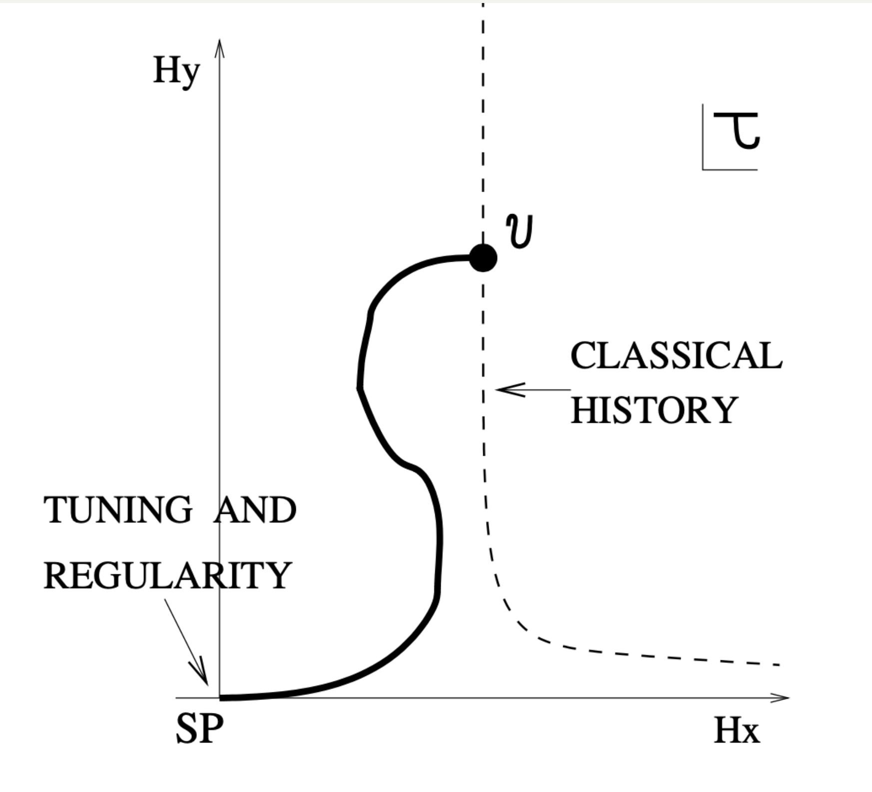
$$ds^2 = d\tau^2 + a^2(\tau)d\Omega_3, \quad \phi(\tau), \quad \Psi[b, \chi]$$

$$V(\phi) = \Lambda + \tfrac{1}{2}m^2\phi^2$$

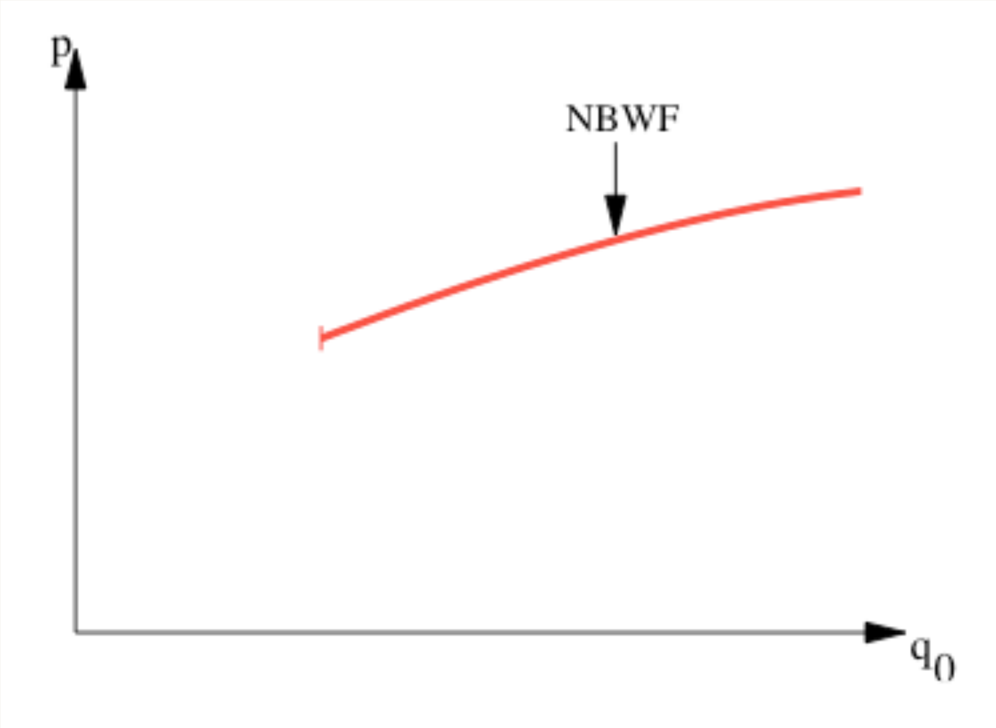
Regularity at SP: $g_{ij}(0) \to 0, \quad \dot{\phi}(0) \to 0$

At final boundary: $g_{ij}(\upsilon) = {}^3g, \quad \phi(\upsilon) = \chi$

Tuning at SP: $\phi(0) = \phi_0 e^{i\gamma}, \ldots$



Multiverse

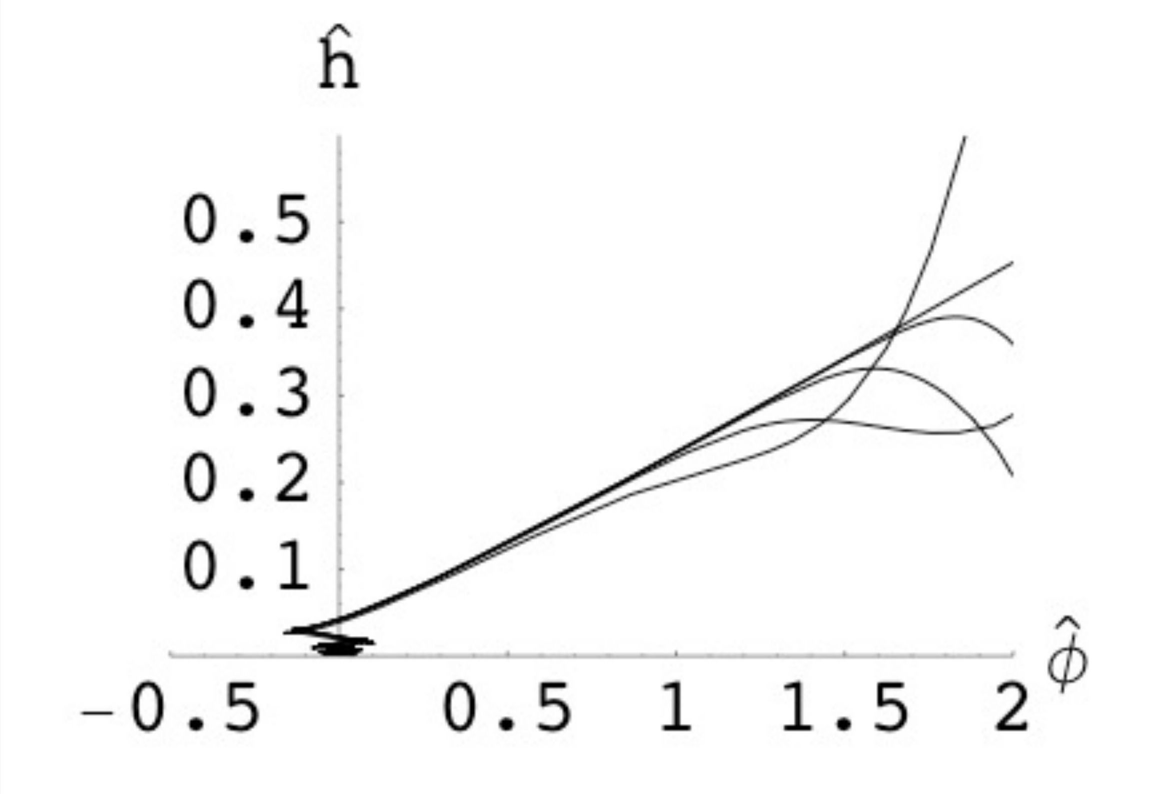


→ Measure on phase space of FRW cosmologies

Inflation!

[Hartle, Hawking, TH 2008]

$$p_q = \nabla_q S$$

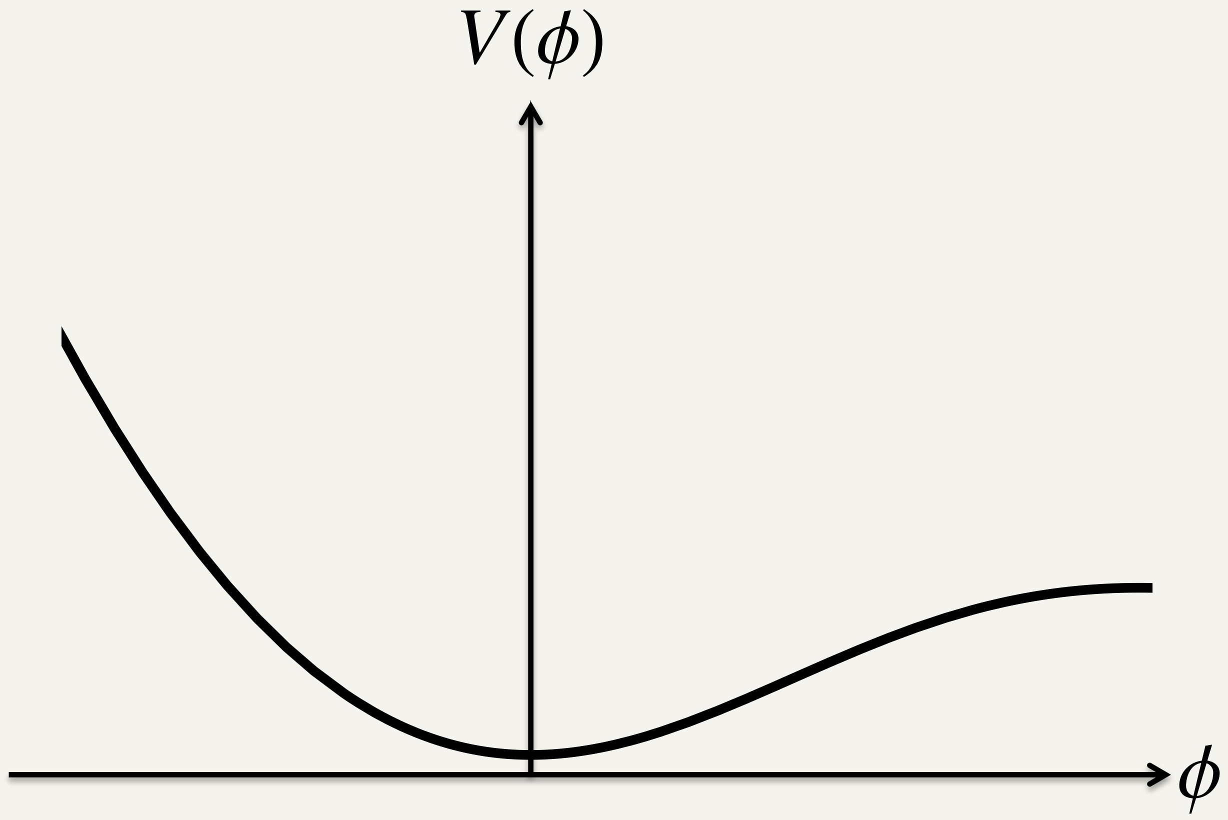


Early period of inflation emerges as a *prediction*

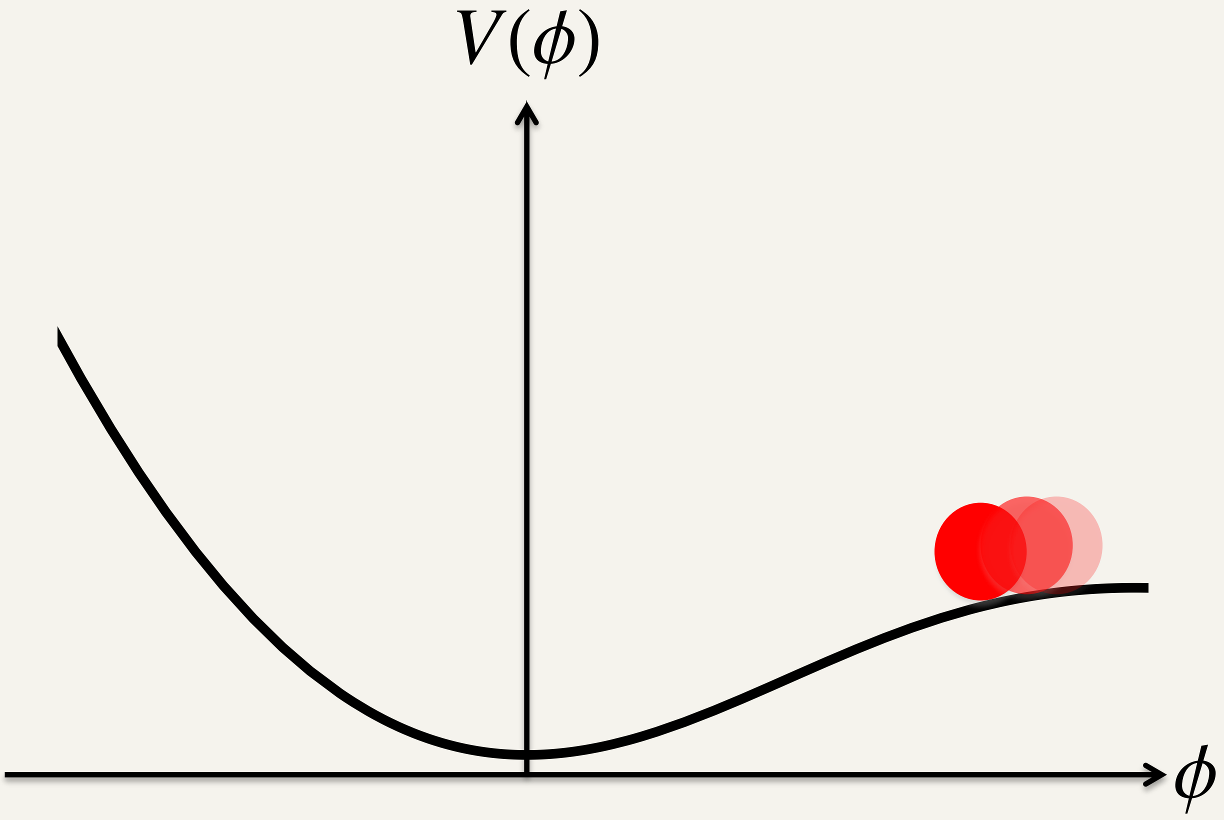
Outline

- Wave function of the Universe
- **How to put the wave function to use?!**
- Recent developments -> holographic cosmology

Example: a toy model of inflation



Example: a toy model of inflation



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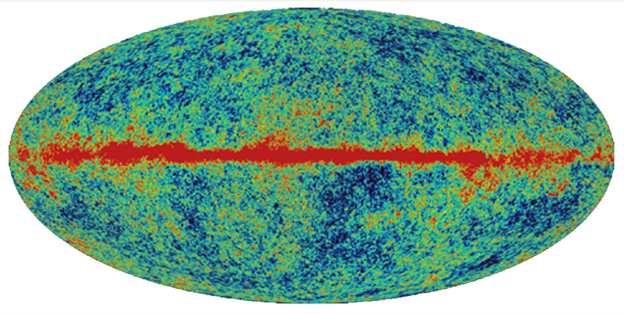

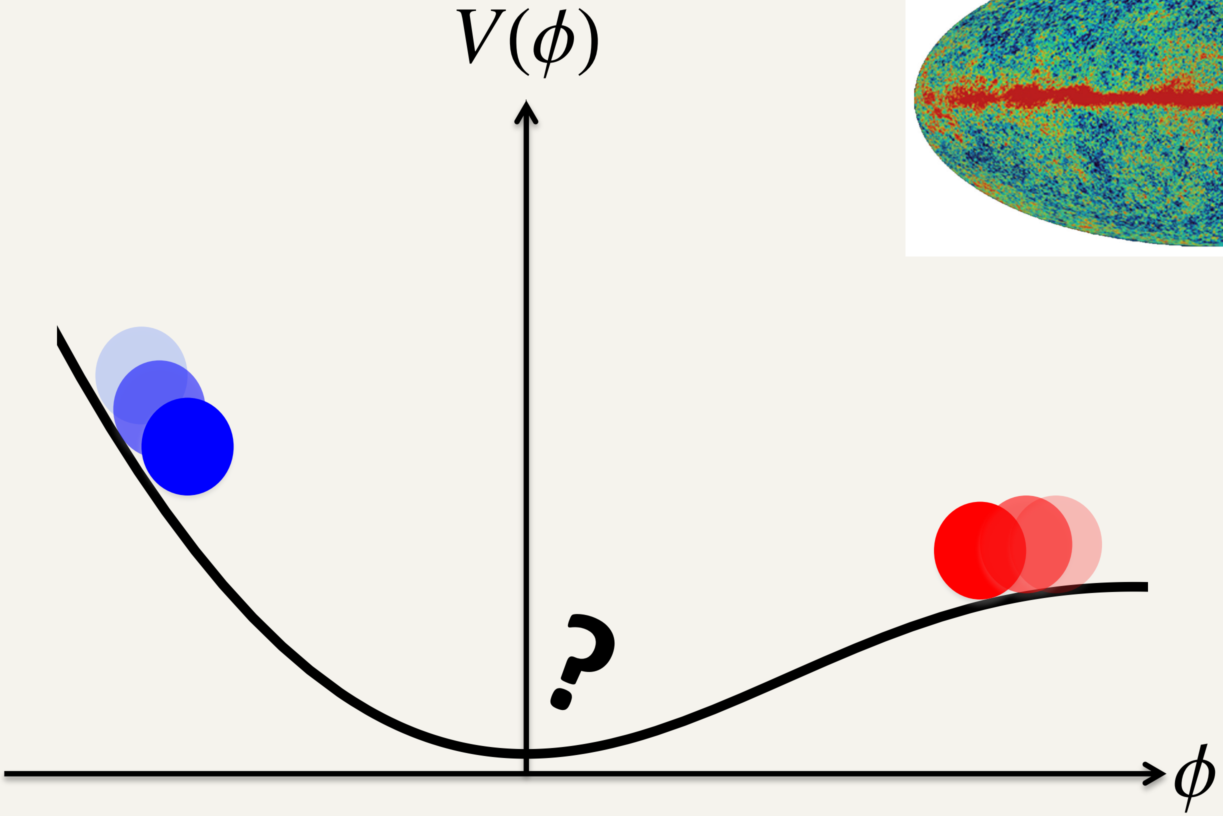

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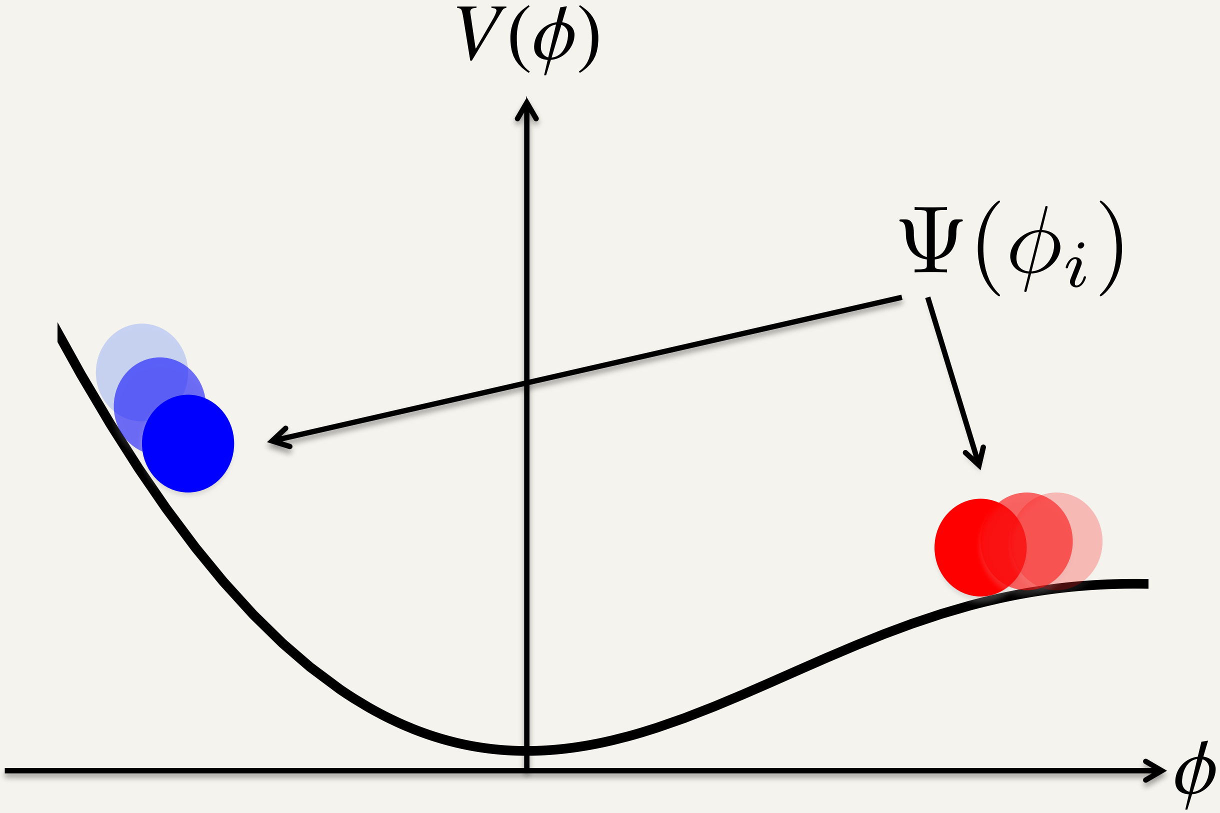
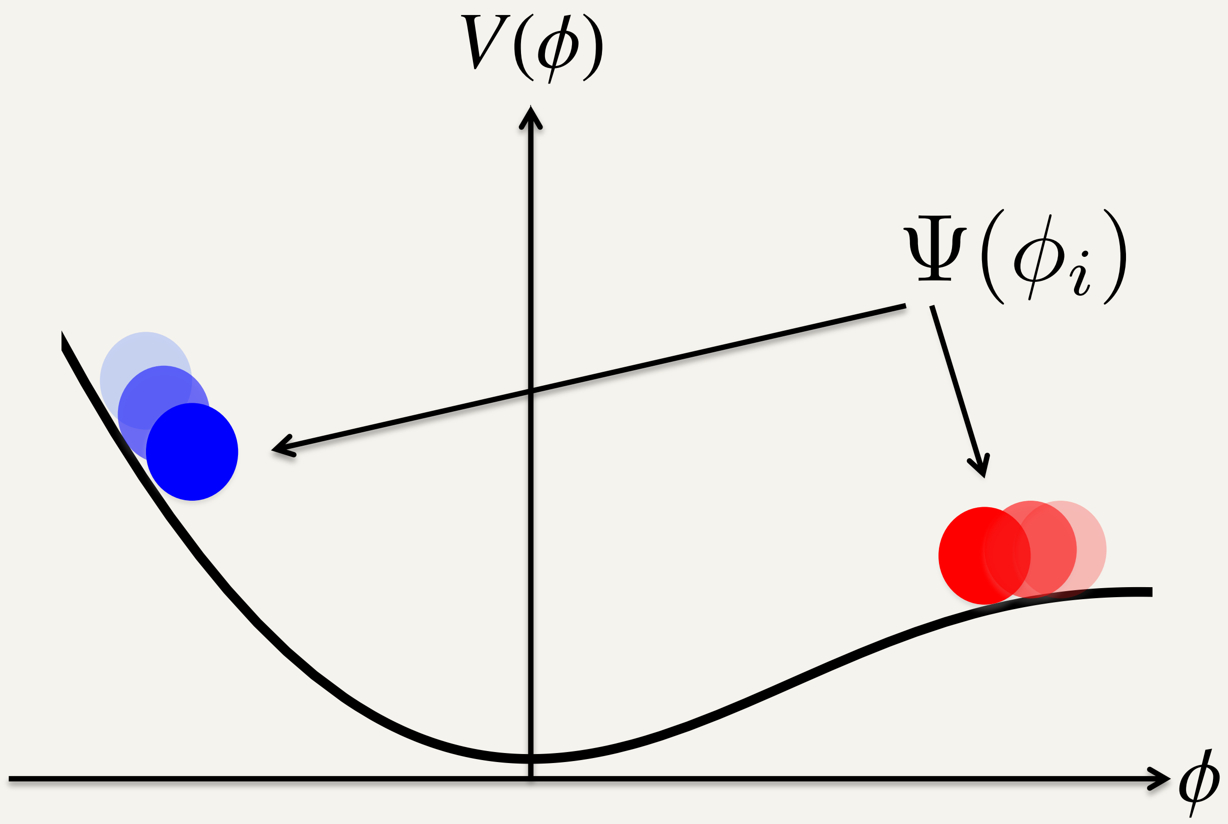


$$|\Psi|^2 \propto \exp\left(\frac{\pi}{4V(\phi_i)}\right)$$

Example: a toy model of inflation

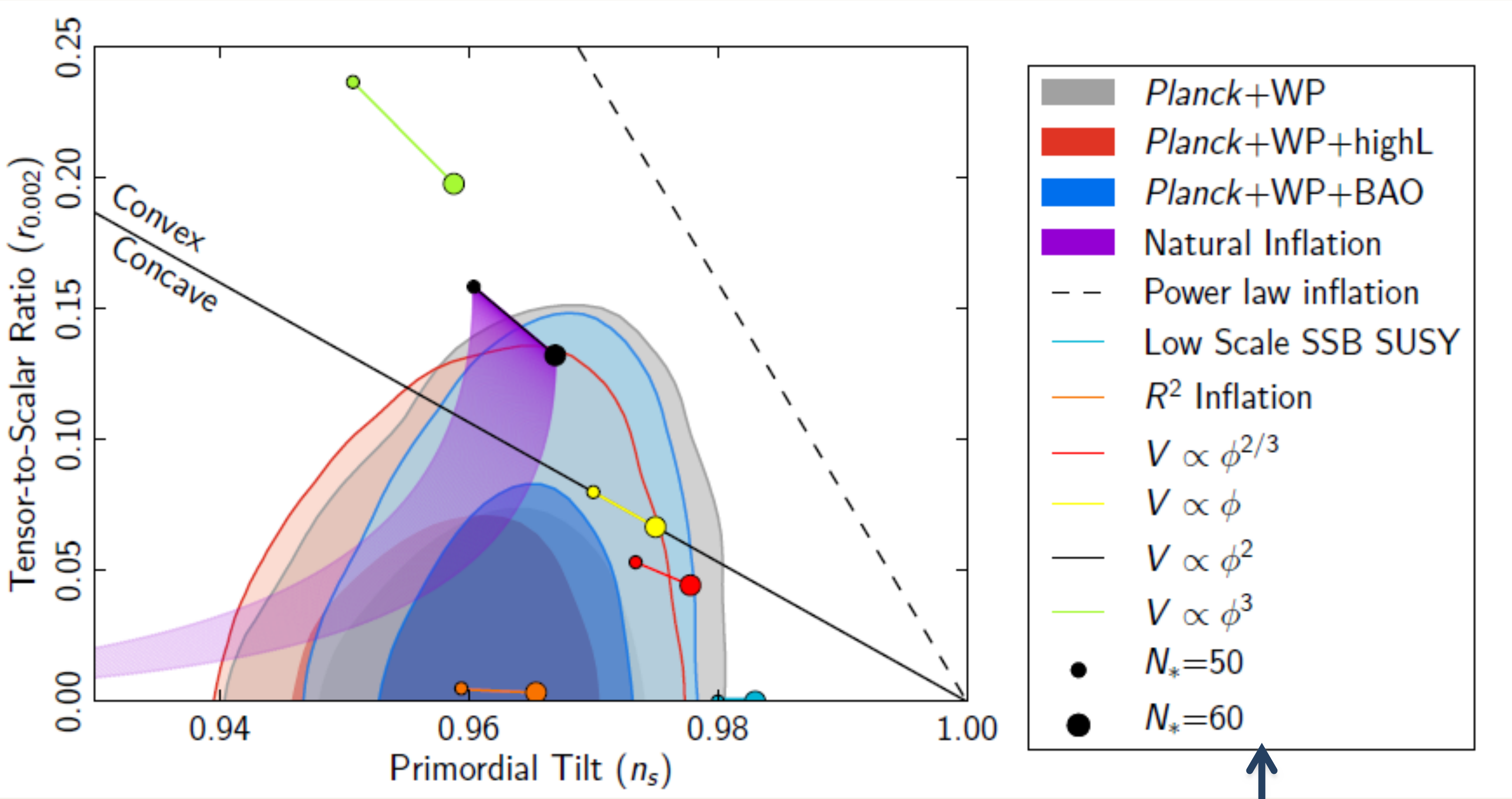


$$|\Psi|^2 \propto \exp\left(\frac{\pi}{4V(\phi_i)}\right) \longrightarrow$$

Relative weighting between different `models of inflation'
-> selection principle

Application: a Prior for Planck

[Planck XXII 2013, TH 2013]

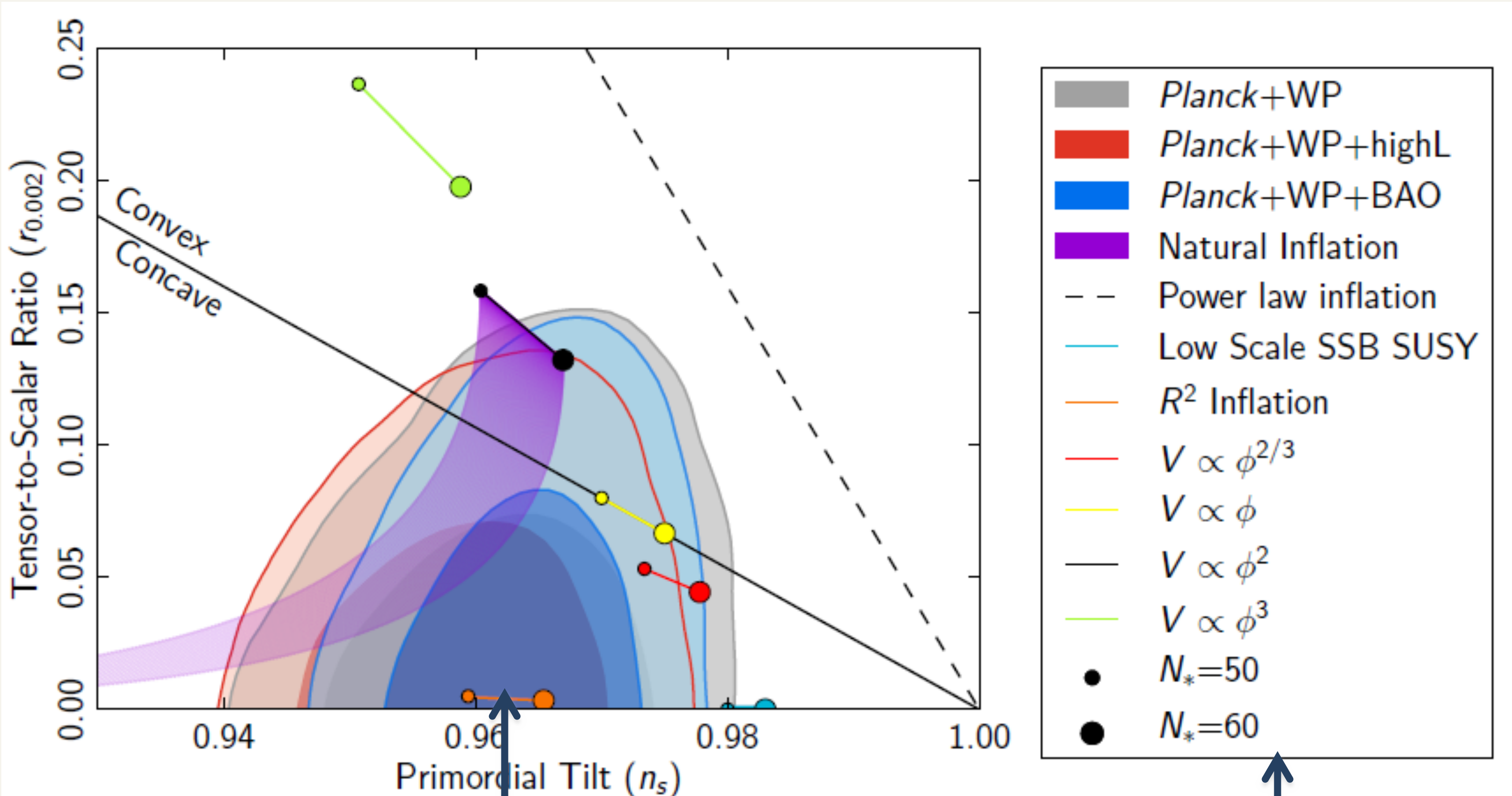


flat prior

`Landscape'

Application: a Prior for Planck

[Planck XXII 2013, TH 2013]



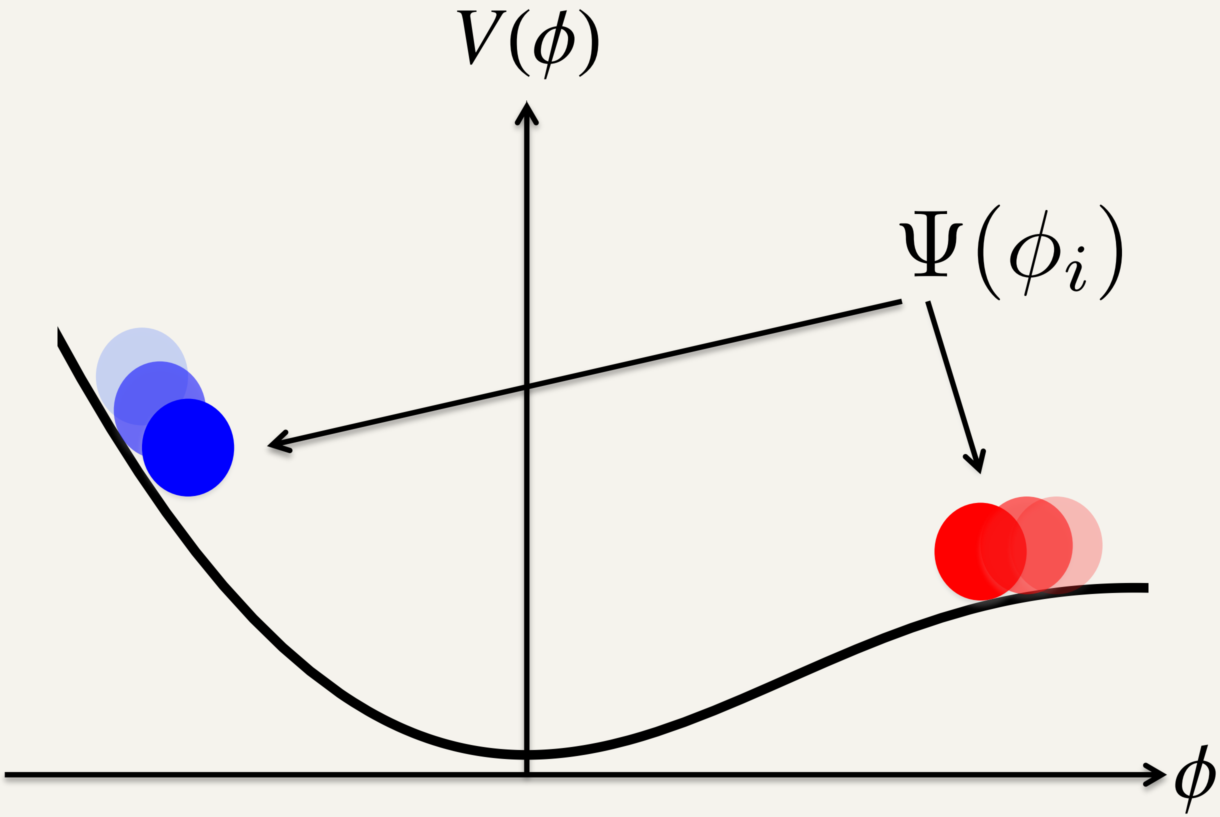
Prior: $\Psi_{HH} \propto e^{-I_E}$

`Landscape'

!

$|\Psi|^2 \longrightarrow$ Probabilities

Example: a toy model of inflation



$$|\Psi|^2 \propto \exp\left(\frac{\pi}{4V(\phi_i)}\right) \longrightarrow$$

Relative weighting between different `models of inflation'
-> selection principle

Predictions for Observations

[Everett 1967; Hartle, TH 2013]

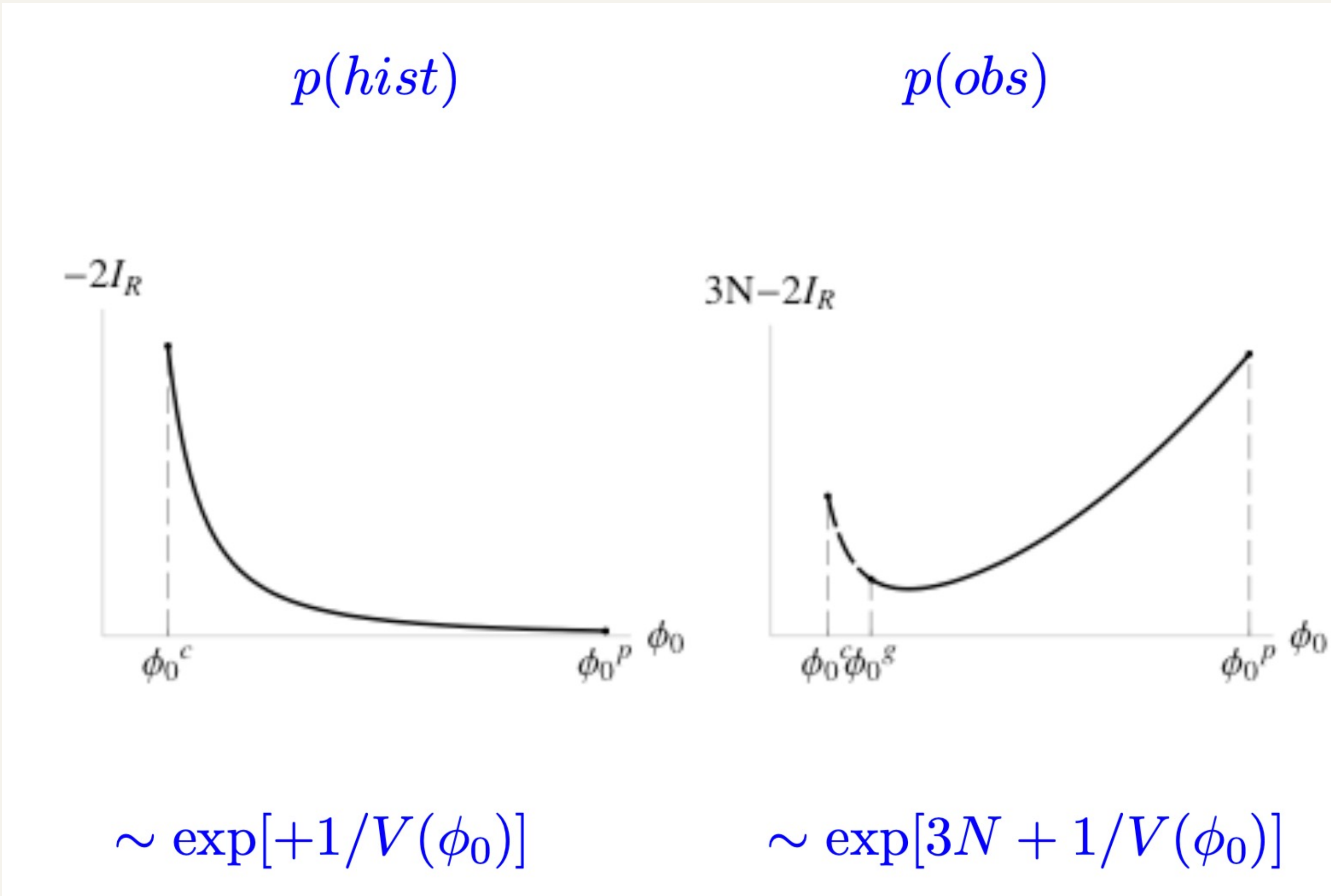
Predictions for observations are conditional probabilities for observables *O*, given our physical observational situation *D* within the universe.

$$P(\mathcal{O}|D)$$

$\longrightarrow$ "Anthropic" selection automatic in quantum cosmology

$$P(\mathcal{O}|D) \propto \sum_J \int P(D|\mathcal{O}, \phi_i^J) P(\mathcal{O}, \phi_i^J)$$

Predictions for Observations



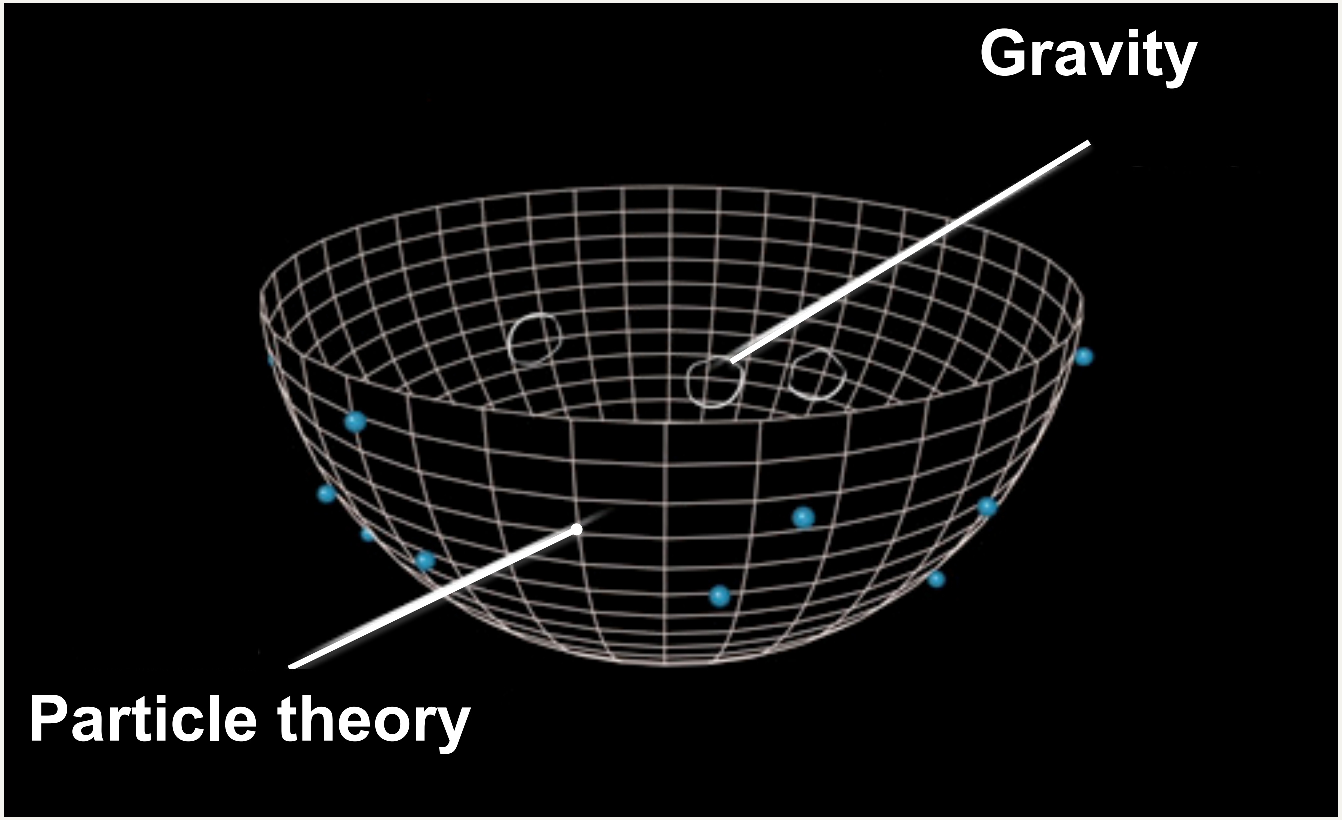
Outline

- Wave function of the Universe
- How to put the wave function to use?!
- **Recent developments -> holographic cosmology**

$$\Psi[^3g,\phi] = \int_C \delta g \delta\phi \exp\left(-I_E[^3g,\phi]/\hbar\right)$$

Holography

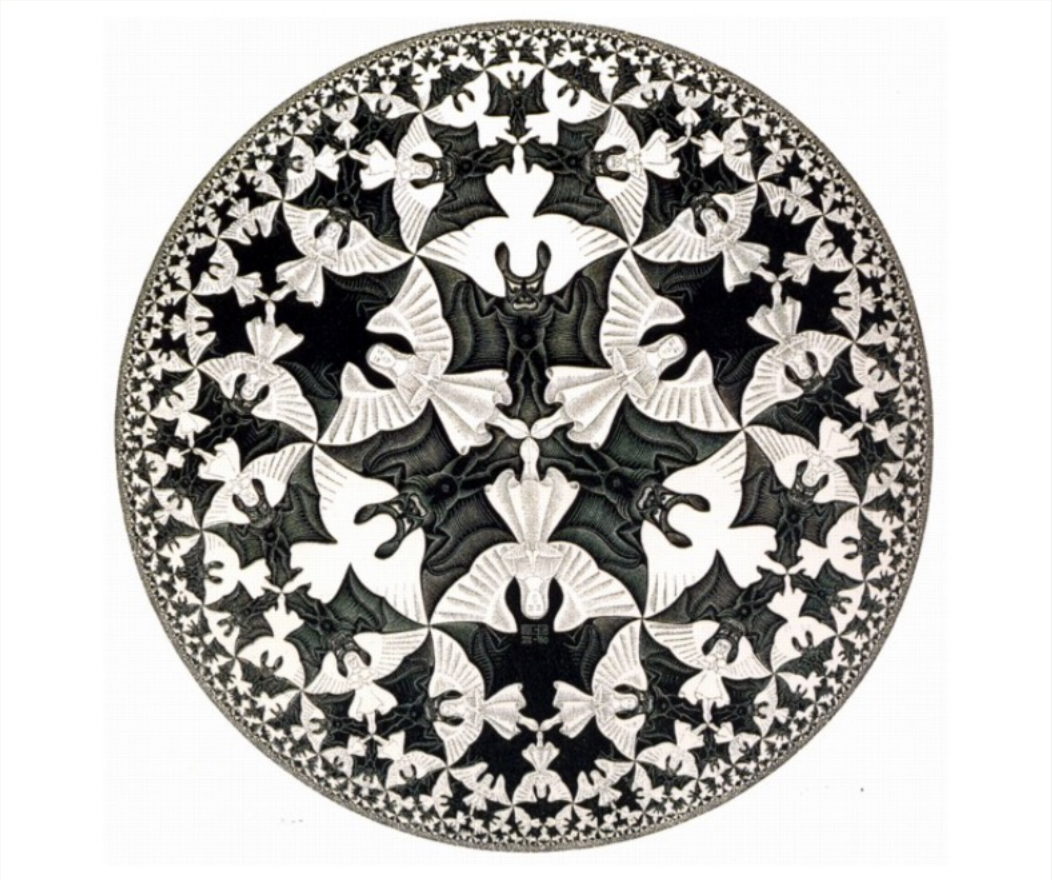
[Maldacena 1997; Witten; Gubser etal]



$$\Psi_{\mathrm{GRAVITY}} \longleftrightarrow Z_{\mathrm{QFT}}$$

Holography

[Maldacena 1997; Witten; Gubser etal]



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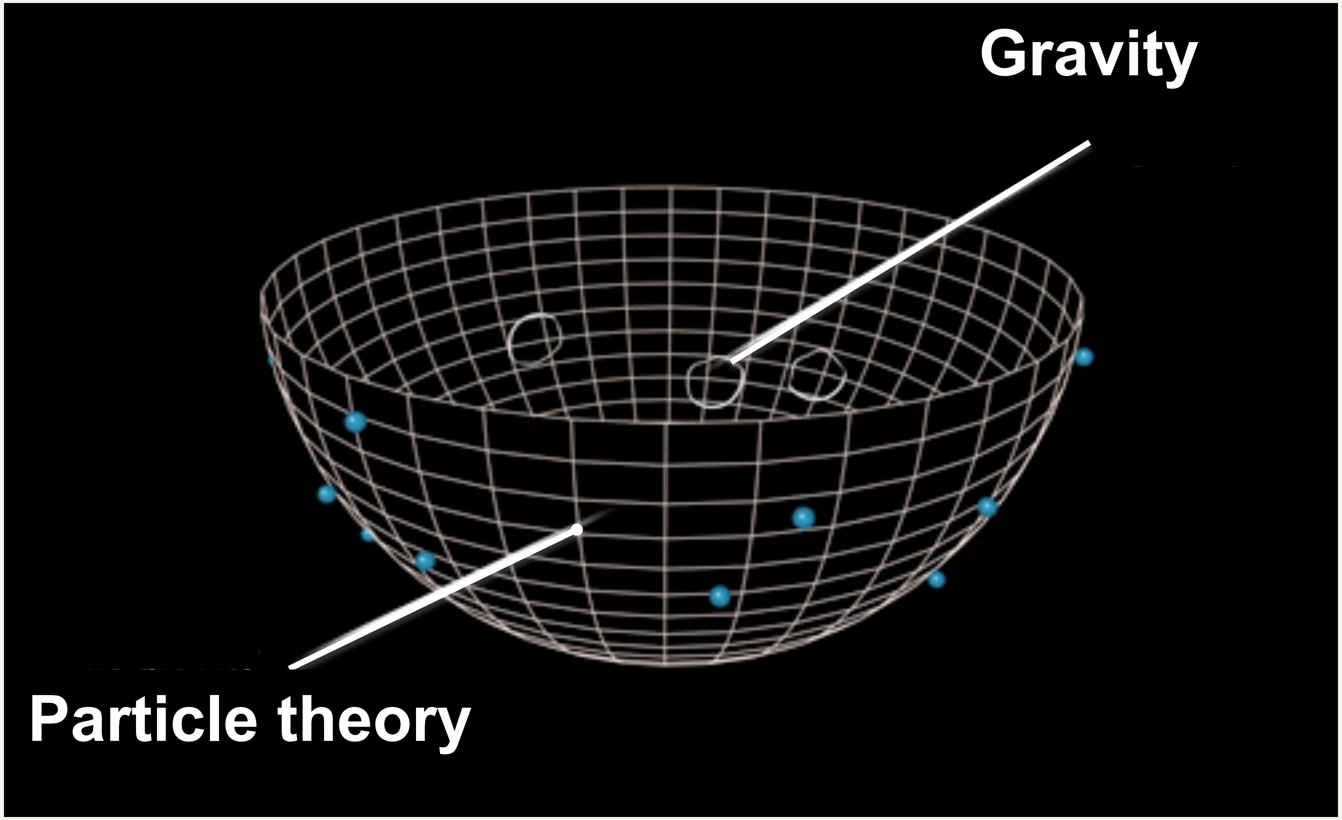
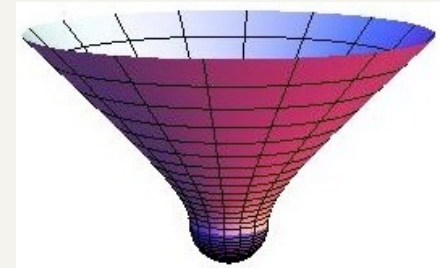
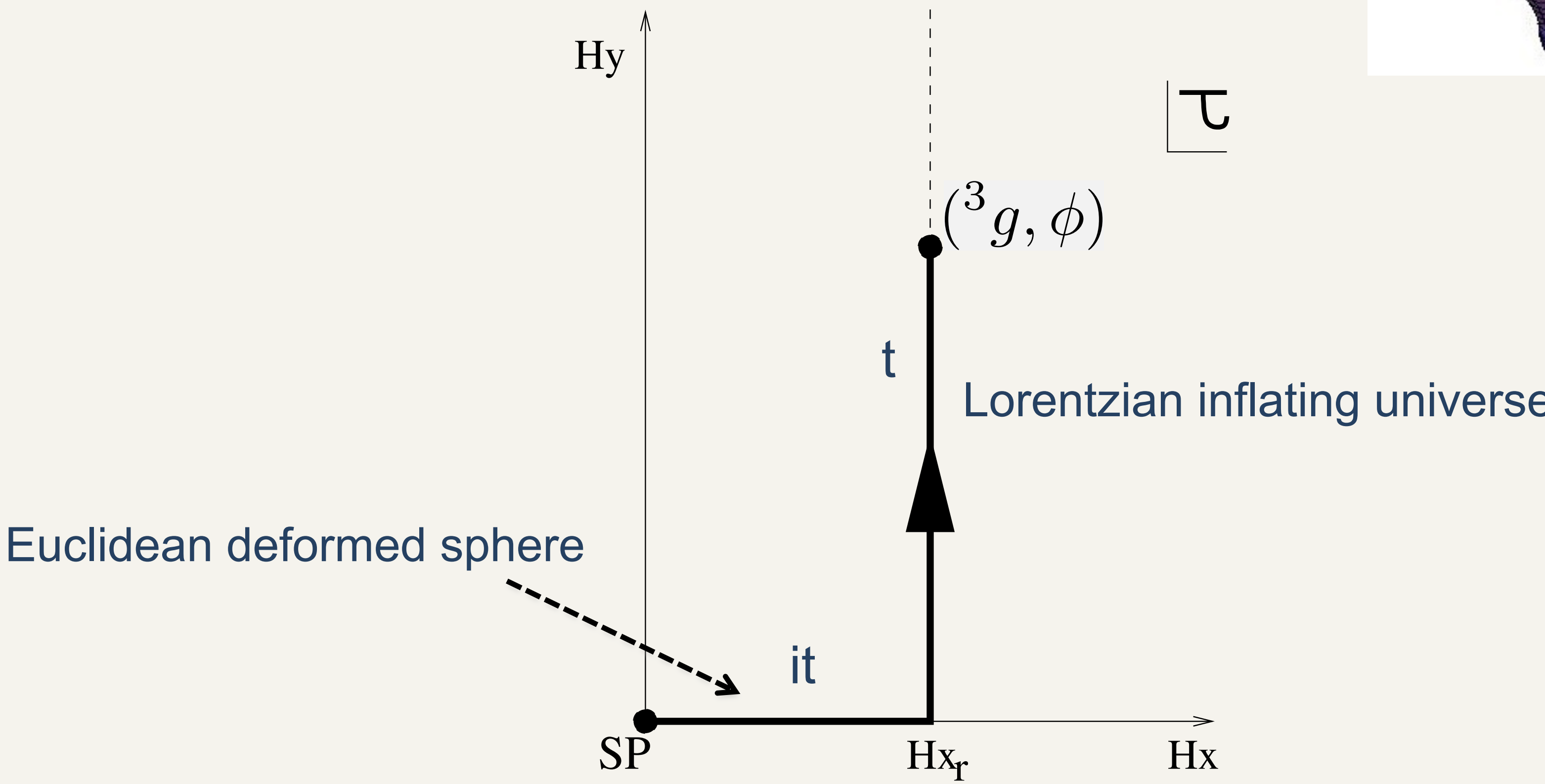


$$\exp(-I^R_{ADS}[h,\chi]/\hbar) = Z_{QFT}[h,\chi]$$

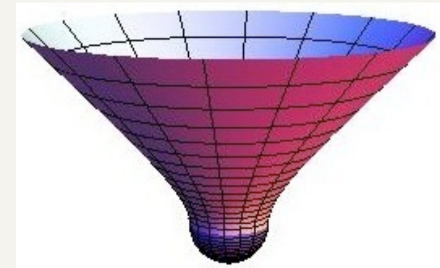
Complex saddle points

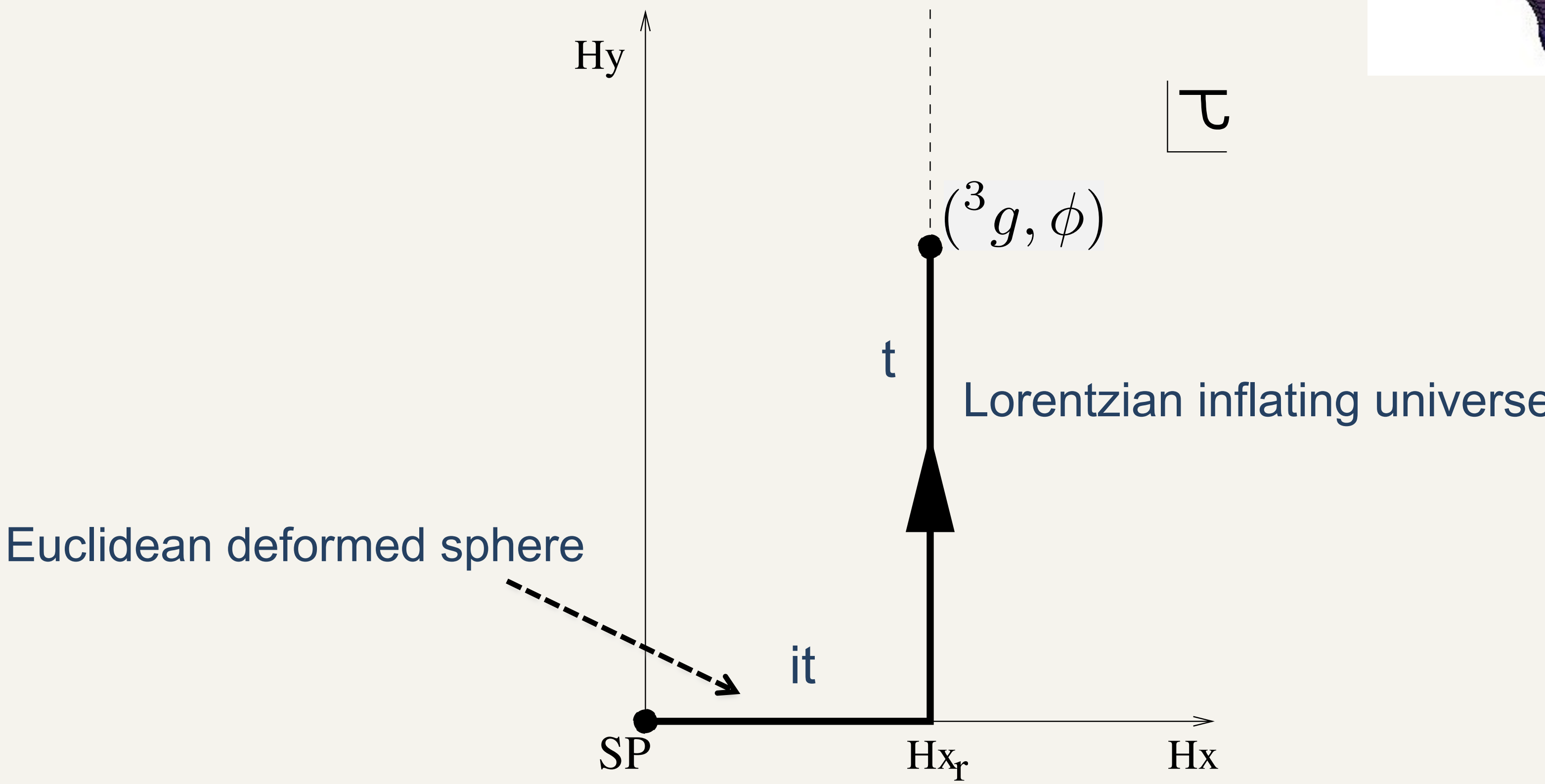




$$\Psi_{HH} \approx \exp\left(-I_E[{}^3g,\phi]\right) = \exp\left(-I_R + iS\right)$$

Complex saddle points



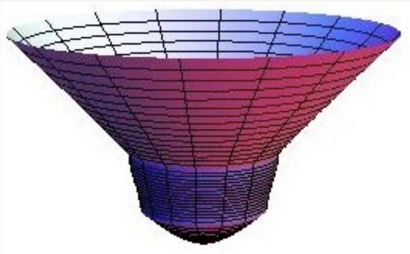


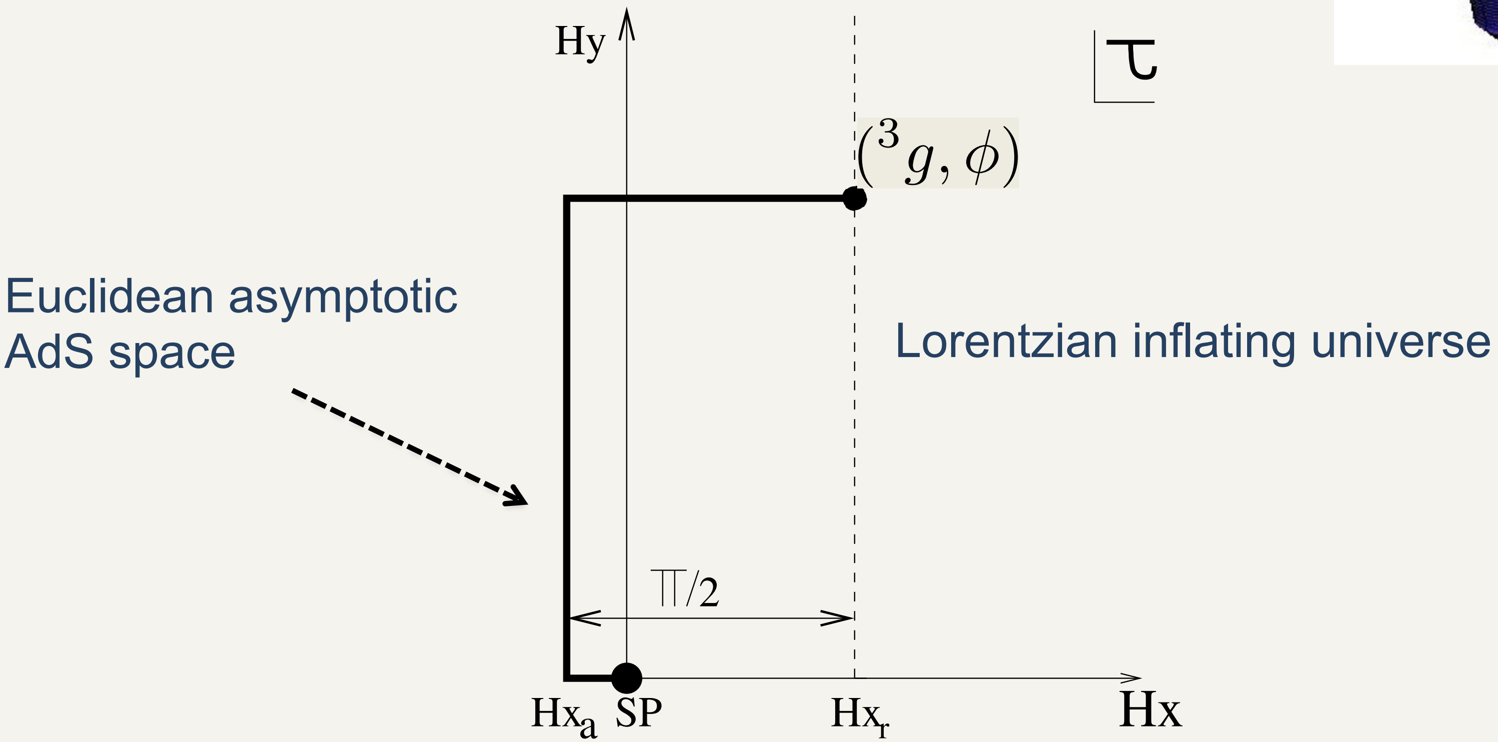
horizontal part: $ds^2 = d\tau^2 + \frac{1}{H^2}\sin^2(H\tau)d\Omega_3^2$

vertical part: $ds^2 = -dy^2 + \frac{1}{H^2}\cosh^2(Hy)d\Omega_3^2$

Alternative representation

[Hartle & TH, 2011]

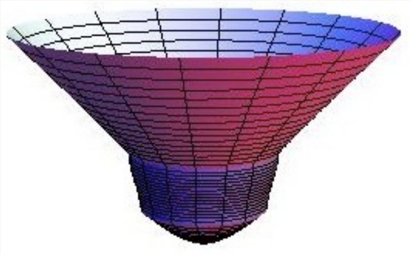




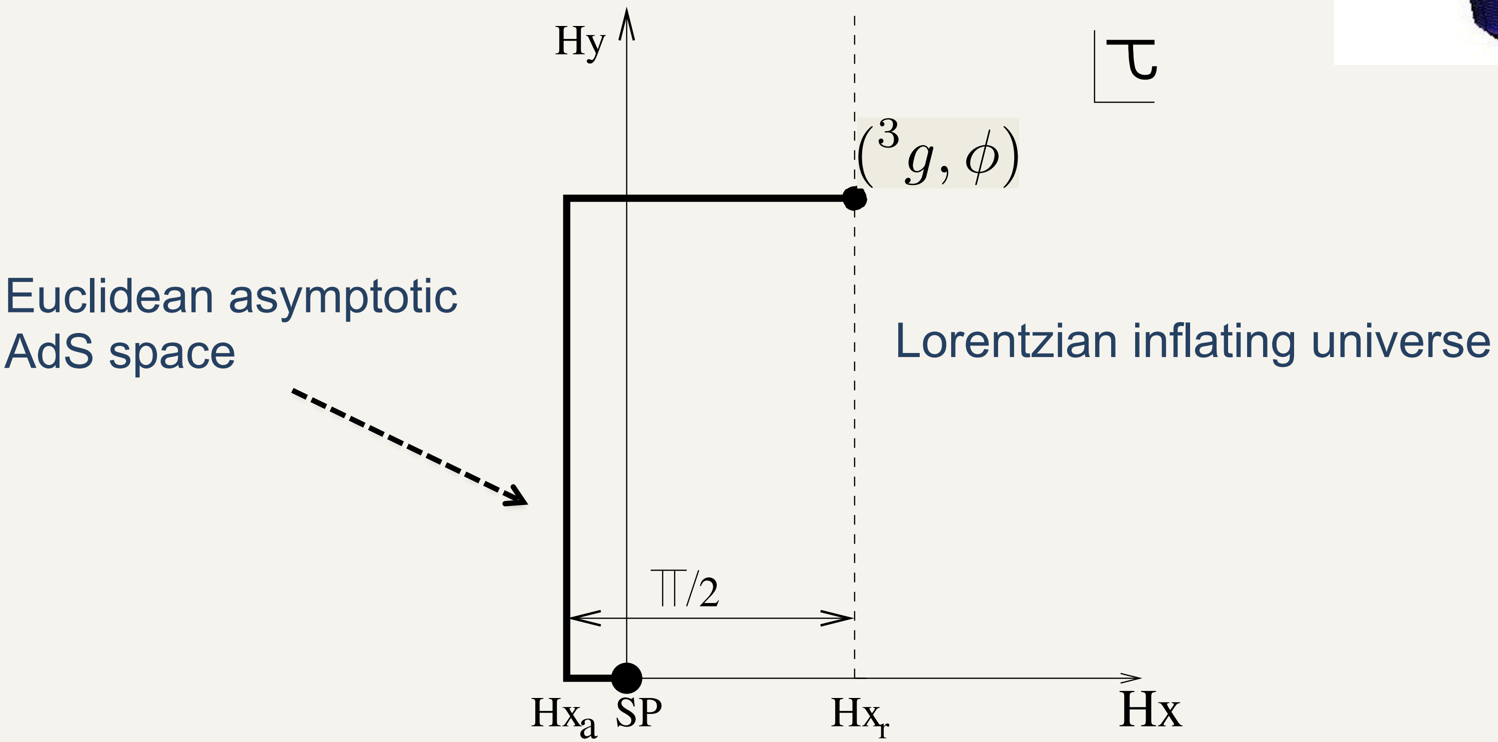
vertical part: Euclidean ADS

$$ds^2 = -dy^2 - \frac{1}{H^2}\sinh^2(Hy)d\Omega_3^2$$

Alternative representation



[Hartle & TH, 2011]



Ψ “connects” Euclidean AdS and Lorentzian de Sitter

Alternative representation

- Action integral along vertical part:

$$I_v = \int_v I[g,\phi] = -I^R_{AdS}({}^3\tilde{g},\chi) - S_{ct}({}^3g,\chi)$$

where I^R_{AdS} is finite when $a \to \infty$.

- Action integral along horizontal part:

$$I_h = \int_h I[g,\phi] = +S_{ct}({}^3g,\chi) - iS_{ct}({}^3g,\chi)$$

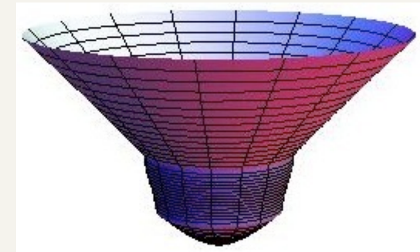
and no finite contribution.

$$S_{ct}(h,\phi) = a_0 \int \sqrt{h} + a_1 \int \sqrt{h} R^{(3)} + a_2 \int \sqrt{h}\phi^2$$

- Total action:

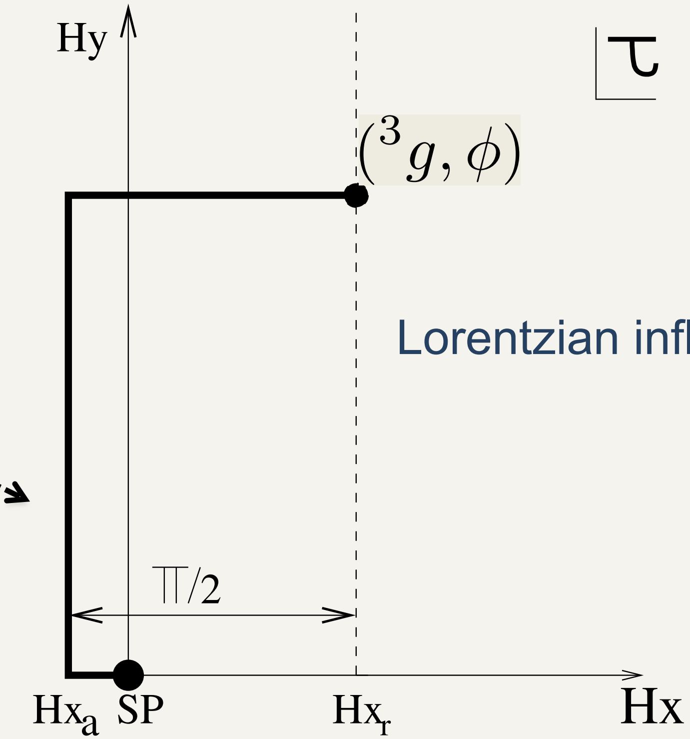
$$I_{dS}({}^3g,\chi) = I_v + I_h = -I^R_{AdS}({}^3\tilde{g},\chi) - iS_{ct}({}^3g,\chi)$$

Alternative representation



[Hartle & TH, 2011]

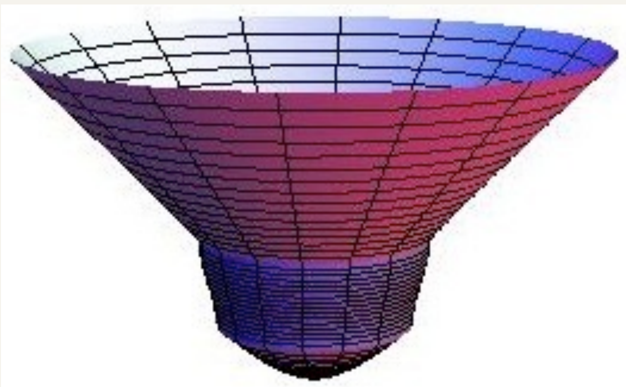
Euclidean asymptotic AdS space



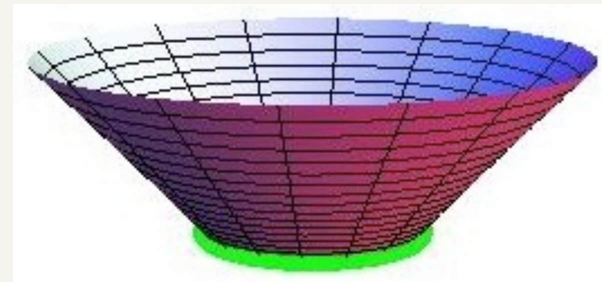
Lorentzian inflating universe

$$\Psi[\,{}^3g, \phi] = \exp\left(+I^{Reg}_{AdS}/\hbar\right)\exp\left(-iS_{ct}/\hbar\right)$$

Holographic Wave Function



Euclidean
AdS/CFT



AdS/CFT dual QFT
with UV cutoff

$$\Psi_{HH}[{}^3g,\phi] = Z^{-1}_{QFT}[{}^3\tilde{g},\tilde{\phi}]\exp\left(-iS_{ct}[{}^3g,\phi]\right)$$

$$Z_{QFT}[{}^3\tilde{g},\tilde{\phi}] = \langle \exp \int d^3x\sqrt{\tilde{g}}\tilde{\phi}\mathcal{O}\rangle$$