Contact with CMB observations

Slow-roll inflation $\Rightarrow$ Gaussian fluctuations

1) adiabatic density perturbations (scalar)

$$\delta = \delta_b = \frac{3}{4}\delta_\nu = \frac{3}{4}\delta_\gamma \qquad \delta_x = \frac{\delta\rho_x}{\rho_x^{(0)}}$$

δ: dark matter

For super-Hubble scales during

- radiation domination : $\Psi = -\frac{1}{2}\delta_\gamma = \text{const}$
- matter domination : $\Psi = -\frac{1}{2}\delta = \text{const}$

Power spectrum : $k^3 P_\Psi(k) \sim k^{n-1}$

n: tilt / spectral index / slope

$$n - 1 = -4\varepsilon - 2\delta$$

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2) gravitational waves (tensor modes):

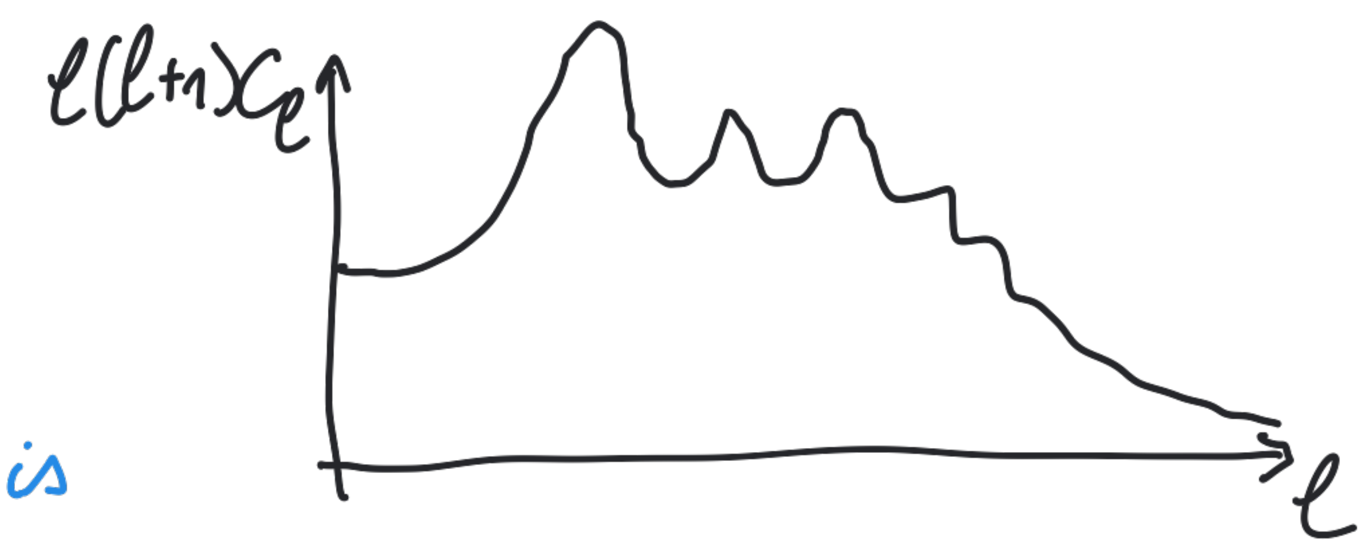
$$k^3 P_h(k) \sim k^{n_T}$$

$$n_T = -2\varepsilon$$

these set initial conditions for further cosmological evolution

CMB observable

Goal : understand physics behind this



Photon scattering

photons $\longleftrightarrow$ electrons $\longleftrightarrow$ baryons

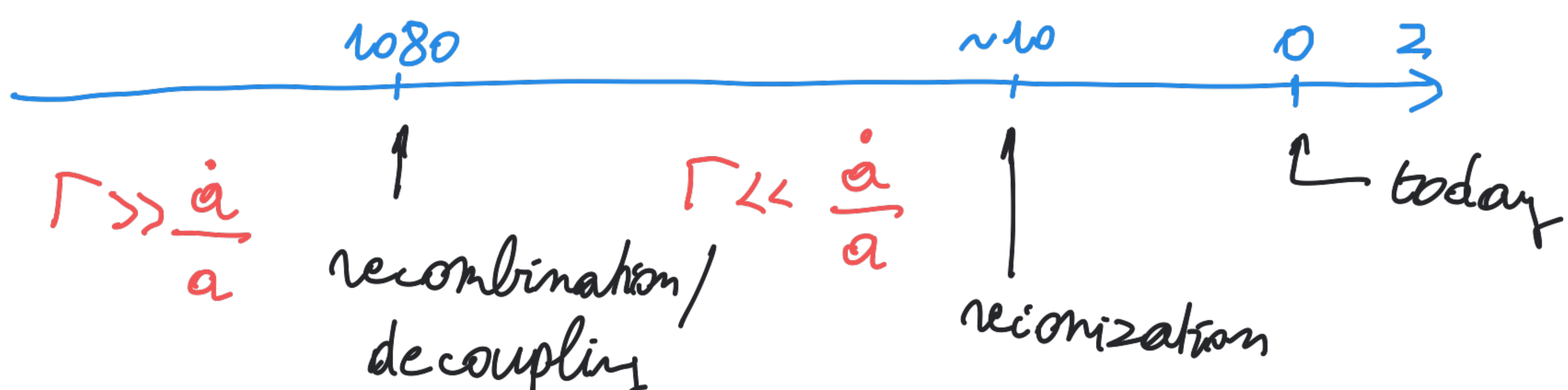
Thomson — Coulomb (very efficient)

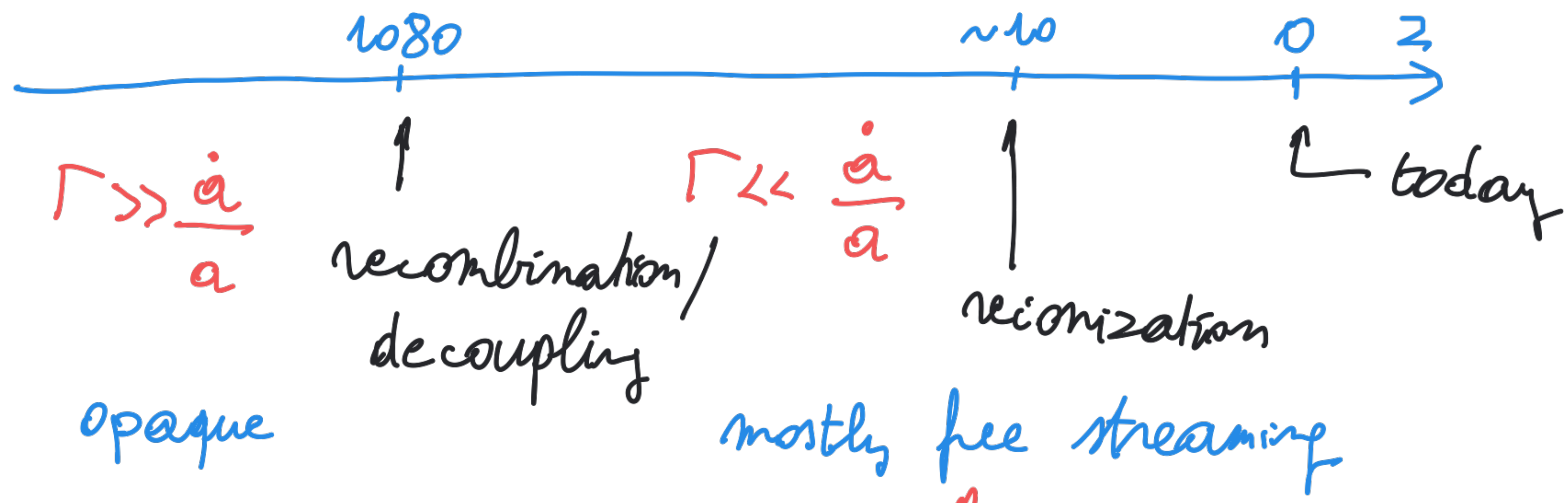
- Thomson scattering rate (w.r.t. conf. time):

$$\Gamma = \sigma_T \, a \, n_e$$

σ_T: cross section

n_e: number density of ionized electrons





- Optical depth $\quad \tau(\eta) = \int_{\eta}^{\eta_0} d\tilde\eta \; \Gamma(\tilde\eta)$

opacity at time η when seen from today ($\eta = \eta_0$)

expected # of scatterings of a photon since time η

$\tau \to \infty$ as $\eta \to 0$

$\tau \approx 0{,}1$ between recombination and reionization

$\tau = 0$ today

$\tau \approx 0{,}1$ between recombination and reionization

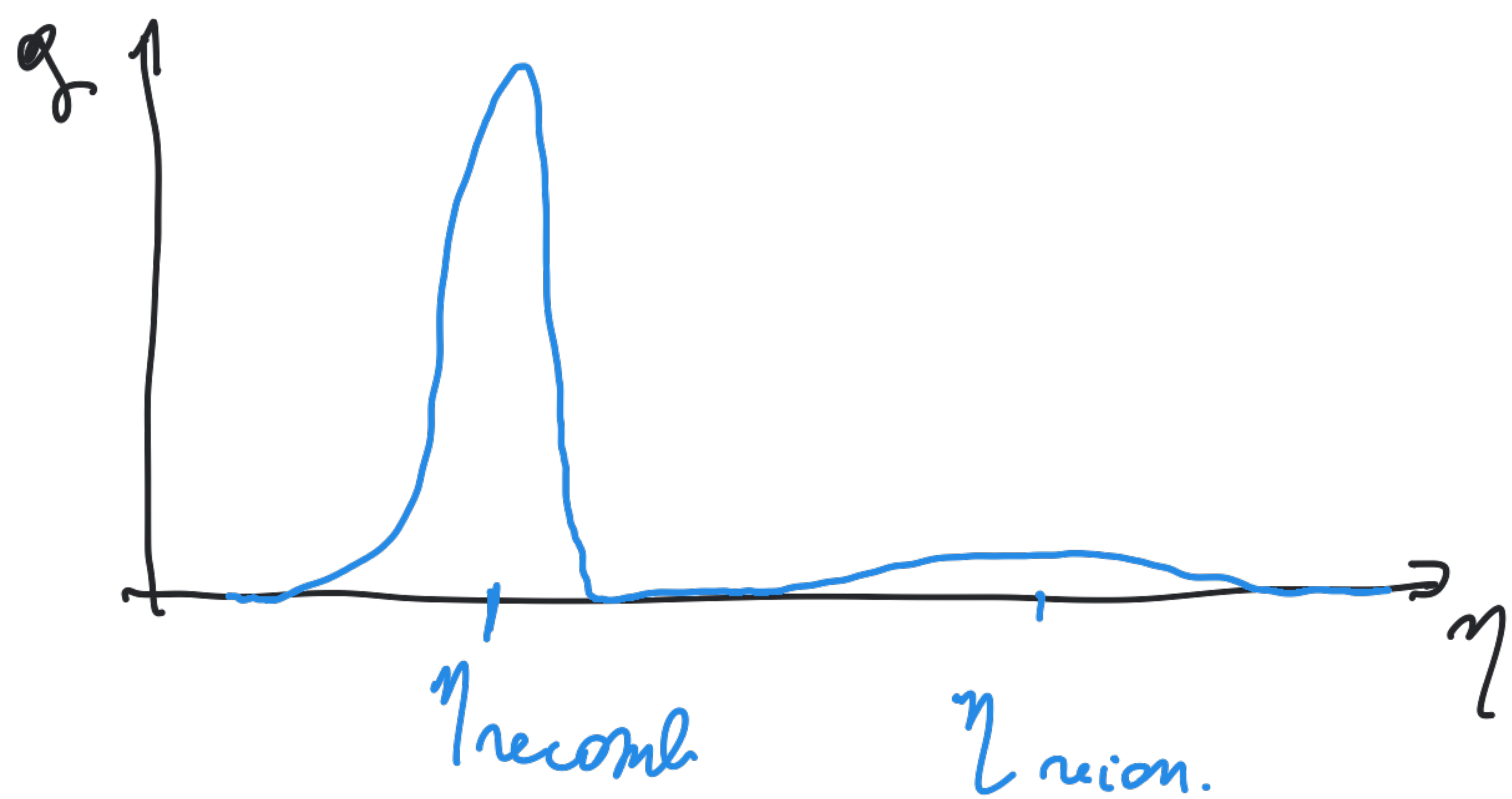
$\tau = 0$ today

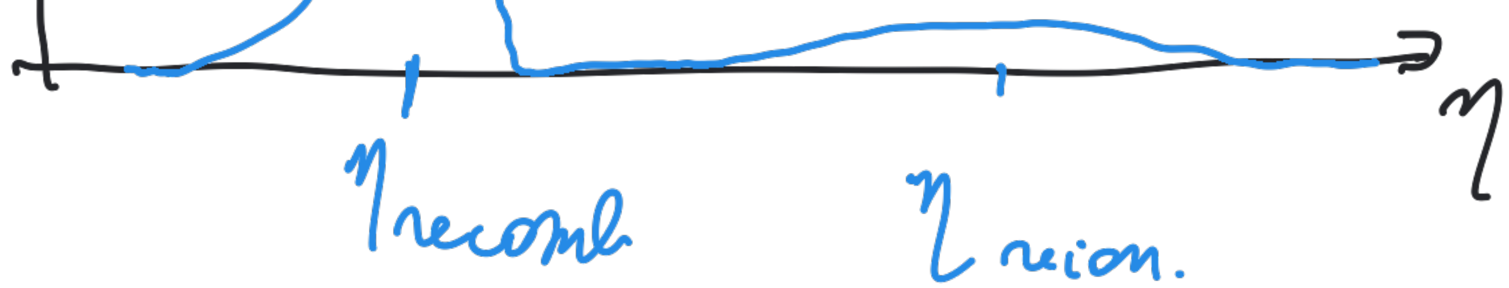
Planck 2018:

$\tau = 0{,}054 \pm 0{,}007$

- Visibility function $g(\eta) = -\dot{\tau}\, e^{-\tau}$

probability density that CMB photon seen today experienced last scattering at time η





- Diffusion length $\lambda_d = a\, r_d$

Comoving mean free path of photons: $r_{mfp} = \dfrac{1}{\Gamma(\eta)}$

$(c=1)$

Random walk: $$r_d(\eta) \sim \left[\int_0^\eta d\tilde{\eta}\, \Gamma\, r_{mfp}^2\right]^{1/2}$$
$$= \left[\int_0^\eta d\tilde{\eta}\, \frac{1}{\Gamma(\tilde{\eta})}\right]^{1/2}$$

Photon diffusion before recombination will damp small-scale CMB anisotropies

Boltzmann equation for photons

Expand photon distribution f'ion about Bose-Einstein:

$$f(\eta, \vec{x}, \vec{p}) = \left[\exp\left\{ \frac{p}{T(\eta)\left[1 + \Theta(\eta, \vec{x}, p)\right]} \right\} - 1 \right]^{-1}$$

Early times: photons in local thermal equilibrium with e^-: $\Theta(\eta, \vec{x})$

After decoupling: geod. eq. implies that Bose-Einstein shape preserved, but with "temperature" depending on propagation direction: $\Theta(\eta, \vec{x}, \hat{p})$

Boltzmann eq: $\frac{df}{d\eta} = C[f, f_e]$

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$\llcorner$ collision term due to Thomson scattering

$$\Leftrightarrow \dot{\Theta} + \hat{p}\cdot\vec{\nabla}\Theta + \dot{\Phi} + \hat{p}\cdot\vec{\nabla}\Psi = -\Gamma\left(\Theta - \Theta_0 - \hat{p}\cdot\vec{V}_e\right)$$

$$\Theta_0 = \frac{1}{4\pi}\int d\Omega'\,\Theta(\eta, \vec{x}, \hat{p}')$$

$$\vec{V}_e = \vec{V}_b$$

Early times: Γ huge $\Rightarrow$ Θ driven to $\Theta = \Theta_0 + \hat{p}\cdot\vec{V}_b$ (*)

cf. perfect fluid: determined by energy density and velocity

Will assume irrotational velocity: gradient of scalar velocity potential

Fourier transform $\Rightarrow$ $\tilde{\Theta}(\eta, \vec{k}, \hat{p})$

Claim: only depends on $\eta, \vec{k}$ and $\hat{k}\cdot\hat{p}$

$\Rightarrow$ $\tilde{\Theta}(\eta, \vec{k}, \hat{k}\cdot\hat{p})$

Proof: 1) Irrotational velocity $\Rightarrow$ $\vec{v} \parallel \vec{k}$

Only dependence on $\hat{p}$ in (*) is on $\hat{p}\cdot\vec{v}_b$

$$\hat{p}\cdot\vec{v}_b = v_b\, \hat{p}\cdot\hat{k}$$

$\Rightarrow$ claim holds at early times

2) Evolution eq. only depends on $\hat{p}$ via $\hat{p}\cdot\hat{k}$ (isotropy) $\Rightarrow$ claim continues to hold

Can expand in Legendre polynomials:

$$\tilde{\Theta}(\eta, \vec{k}, \hat{k}\cdot\hat{p}) = \sum_\ell (-i)^\ell (2\ell+1)\, \Theta_\ell(\eta, \vec{k})\, P_\ell(\hat{k}\cdot\hat{p})$$

$$\Theta(\eta, k, \hat{k}\cdot\hat{p}) = \sum_\ell (-i)^\ell (2\ell+1)\,\Theta_\ell(\eta, k)\, P_\ell(\hat{k}\cdot\hat{p})$$

Early times: only monopole ($\ell=0$) and dipole ($\ell=1$).

Temperature anisotropy in a given direction

Temperature anisotropy observed today when looking in a direction $\hat{p}$:

$$\frac{\delta T}{T}(\hat{p}) = \Theta(\eta_0, \vec{0}, -\hat{p})$$

now here photon moves in direction $-\hat{p}$

Integrate Boltzmann eq along line of sight and use instantaneous decoupling approximation $g(\eta) = \delta(\eta - \eta_{dec})$

$$\Rightarrow (\Theta + \psi)\big|_{obs} = (\Theta_0 + \psi + \hat{p}\cdot\vec{v}_b)\big|_{dec} + \int_{\eta_{dec}}^{\eta_0} d\eta\, (\dot{\psi} - \dot{\phi})$$

temp. pert.

$$\Rightarrow \quad (\Theta+\Psi)\big|_{obs} = (\Theta_0+\Psi+\hat{p}\cdot\vec{v}_b)\big|_{dec} + \int_{\eta_{dec}}^{\eta_0} d\eta\,(\dot{\Psi}-\dot{\Phi})$$

$\frac{\delta T}{T}(\hat{p})$ today

at observer location, along line of sight

temp. pert. at recomb.

on last scattering surface, along line of sight

Doppler correction due to velocity of baryon-photon fluid along line of sight

Grav. redshift/blueshift would depend on $\Psi|_{obs}-\Psi|_{dec}$ for potentials constant in time

tiny isotropic correction
$\Rightarrow$ drop (unobservable)

non-conservative correction

$$\Rightarrow \quad \Theta\big|_{obs} = \underbrace{(\Theta_0+\Psi)\big|_{dec}}_{\text{"Sachs-Wolfe" (SW)}} + \underbrace{\hat{p}\cdot\vec{v}_b\big|_{dec}}_{\text{Doppler}} + \underbrace{\int_{\eta_{dec}}^{\eta_0} d\eta\,(\dot{\Psi}-\dot{\Phi})}_{\text{"integrated Sachs-}}$$

$$\Rightarrow \Theta\big|_{obs} = \underbrace{(\Theta_0 + \psi)\big|_{dec}}_{\text{"Sachs-Wolfe" (SW)}} + \underbrace{\hat{p}\cdot\vec{v}_e\big|_{dec}}_{\text{Doppler}} + \underbrace{\int_{\eta_{dec}}^{\eta_0} d\eta\,(\dot{\psi} - \dot{\phi})}_{\text{"integrated Sachs-Wolfe" (ISW)}}$$

Large scale anisotropy

CMB maps smoothed over small scales (cf. COBE) $\Longleftrightarrow$ contributions to Sachs-Wolfe term from super-Hubble modes at time of decoupling (Doppler negligible, ISW small)

First consider $\Theta_0\big|_{dec}$

averaged over all directions

Thermodyn: $\rho_\gamma \sim T^4 \Rightarrow \delta_\gamma = \frac{\delta\rho_\gamma}{\rho} = \frac{4\,\delta T}{T} = 4\Theta_0$

First consider $\Theta_0|_{dec}$

averaged over all directions

Thermodyn: $\rho_\gamma \sim T^4 \Rightarrow \delta_\gamma = \frac{\delta\rho_\gamma}{\rho_\gamma} = \frac{4\,\delta T}{T} = 4\Theta_0$

Next consider $\psi|_{dec}$

On super-Hubble scales for adiabatic initial cond:

$$\delta_\gamma = \frac{4}{3}\delta_b$$

For super-Hubble scales during matter domination:

$$-2\psi = \delta_{tot} = \delta_b = \delta$$

$\Rightarrow$ on last scattering surface:

$$\Theta_0 + \psi = \frac{1}{4}\delta_\gamma + \psi = \frac{1}{4}(-2)\frac{4}{3}\psi + \psi = -\frac{2}{3}\psi + \psi = \frac{1}{3}\psi$$

$\Rightarrow$

neglect Doppler + ISW

$$\Theta|_{obs,\ large\ scales} \approx \frac{1}{3}\psi|_{dec} = -\frac{1}{8}\delta_\gamma|_{dec}$$

Note: ψ dominates Θ_0:

$$\text{overdensity on last scattering surface } (\delta_\gamma > 0)$$
$$\Updownarrow$$
$$\text{cold spot in observed map } (\Theta < 0)$$

Spectrum of temperature anisotropies

Expand in spherical harmonics:

$$\frac{\delta T}{T}(\hat{p}) = \Theta(\eta_0, \vec{0}, -\hat{p}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{p})$$

Using the addition theorem for spherical harmonics,

$$P_\ell(-\hat{p}\cdot\hat{k}) = \frac{4\pi}{2\ell+1} \sum_{m=-\ell}^{\ell} Y_{\ell m}(\hat{p})\, Y^*_{\ell m}(-\hat{k}),$$

we can write

Using the addition theorem for spherical harmonics,

$$P_\ell(-\hat{p}\cdot\hat{k}) = \frac{4\pi}{2\ell+1}\sum_{m=-\ell}^{\ell} Y_{\ell m}(\hat{p})\, Y^*_{\ell m}(-\hat{k}),$$

we can write

$$\Theta(\eta_0, \vec{0}, -\hat{p}) = \int \frac{d^3k}{(2\pi)^3}\, \Theta(\eta_0, \vec{k}, -\hat{p})$$

$$= \int \frac{d^3k}{(2\pi)^3} \sum_\ell (-i)^\ell (2\ell+1)\, \Theta_\ell(\eta_0, \vec{k})\, P_\ell(-\hat{p}\cdot\hat{k})$$

$$= \int \frac{d^3k}{2\pi^2} \sum_{\ell m} (-i)^\ell\, \Theta_\ell(\eta_0, \vec{k})\, Y^*_{\ell m}(-\hat{k})\, Y_{\ell m}(\hat{p})$$

so

$$a_{\ell m} = (-i)^\ell \int \frac{d^3k}{2\pi^2}\, Y^*_{\ell m}(-\hat{k})\, \Theta_\ell(\eta_0, \vec{k})$$

Write

$$\Theta_\ell(\eta_0, \vec{k}) = \delta(\vec{k}) \times \frac{\Theta_\ell(\eta_0, \vec{k})}{\delta(\vec{k})} \equiv \Theta_\ell(\eta_0, k)$$

Write $\Theta_\ell(\eta_0,\vec{k}) = \zeta(\vec{k}) \times \dfrac{\Theta_\ell(\eta_0,\vec{k})}{\zeta(\vec{k})} \equiv \Theta_\ell(\eta_0,k)$

"initial condition", chosen during inflation from Gaussian distribution

"transfer function" indep. of init. cond. (linear evolution eq) only depends on $|\vec{k}|$

$$\langle a_{\ell m}\, a^*_{\ell' m'} \rangle = (-i)^{\ell-\ell'} \int \frac{d^3k}{2\pi^2}\, Y^*_{\ell m}(-\hat{k})\, \Theta_\ell(\eta_0,k) \times$$

$$\times \int \frac{d^3k'}{2\pi^2}\, Y_{\ell' m'}(-\hat{k}')\, \Theta^*_{\ell'}(\eta_0,k') \times$$

$$\times \langle \zeta(\vec{k})\, \zeta^*(\vec{k}') \rangle$$

$$\ell' \int \frac{d^3k'}{2\pi^2}\, Y_{\ell' m'}(-\hat{k}')\, \Theta^*_{\ell'}(\eta_0, k') \times$$

$$\times \underbrace{\langle \zeta(\vec{k})\, \zeta^*(\vec{k}') \rangle}_{=\,(2\pi)^3 P_\zeta(k)\, \delta(\vec{k}-\vec{k}')}$$

$$= \delta_{\ell,\ell'}\, \delta_{m,m'}\, C_\ell$$

homogeneity

indep. of m: isotropy

with $$C_\ell = \frac{2}{\pi} \int dk\, k^2\, |\Theta_\ell(\eta_0, k)|^2\, P_\zeta(k)$$

Best estimator for C_ℓ : $$C_\ell^{obs} \equiv \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}^{obs}|^2$$

Cosmic variance important for small ℓ.

Line-of-sight integral in momentum space $\Rightarrow$

$$\Theta_\ell(\eta_0,k) \simeq \left[\Theta_0(\eta_{dec},k) + \Psi(\eta_{dec},k)\right] j_\ell\left[k(\eta-\eta_{dec})\right] \quad \underline{\text{SW}}$$

$$+\, i\, v_b\, j_\ell'\left[k(\eta_0-\eta_{dec})\right] \quad \underline{\text{Doppler}}$$

$$+ \int_{\eta_{dec}}^{\eta_0} d\eta \left[\dot\Psi(\eta,k) - \dot\Phi(\eta,k)\right] j_\ell\left[k(\eta_0-\eta)\right] \quad \underline{\text{ISW}}$$

$\Rightarrow$ C_ℓ consists of six terms: C_ℓ^{SW}, $C_\ell^{Doppler}$, C_ℓ^{ISW}, cross terms

For large ℓ: $j_\ell(x)$ and $j_\ell'(x)$ peaked near $x \approx \ell$

$\Rightarrow$ SW and Doppler contributions dominated by $k \approx \dfrac{\ell}{\eta_0-\eta_{dec}}$

spatial scale at decoupling $\Leftrightarrow$ angular scale today

$$C_\ell^{SW} \simeq \left| \Theta_0(\eta_{\dots}, k) + \Psi(\eta_{\dots}, k) \right|^2 P(k) \Big|$$

$$C_\ell^{SW} \sim \left| \Theta_0(\eta_{dec}, k) + \Psi(\eta_{dec}, k)\right|^2 P_\zeta(k) \Big|_{k \simeq \frac{\ell}{\eta_0 - \eta_{dec}}}$$

Acoustic oscillations

Before decoupling: single fluid with sound speed

$$c_s^2 = \frac{\delta p_\gamma + \delta p_b}{\delta \rho_\gamma + \delta \rho_b}$$

$$\left.\begin{array}{l} \rho_b \sim T^3 \\ \rho_\gamma \sim T^4 \end{array}\right\} \Rightarrow \quad \delta_\gamma = \frac{4}{3}\delta_b$$

$$|\delta p_b| \ll |\delta p_\gamma|$$

$$c_s^2 = \frac{1}{3(1+R)}$$

with $R \equiv \dfrac{3\rho_b^{(0)}}{4\rho_\gamma^{(0)}}$ $(\sim a)$

E.o.m. for photon temp. fluctuations $\Theta_0(\eta, \vec{x})$ in

E.o.m. for photon temp. fluctuations $\Theta_0(\eta, \vec{x})$ in tightly coupled regime:

$$\ddot{\Theta}_0 + \frac{\dot{R}}{1+R}\dot{\Theta}_0 + k^2 c_s^2 \Theta_0 = -\frac{k^2}{3}\Psi - \frac{\dot{R}}{1+R}\dot{\Phi} - \ddot{\Phi}$$

$$\left(R \sim a \Rightarrow \dot{R} = \frac{\dot{a}}{a} R = aHR \right)$$

will ignore → damping (stronger if more baryons) ≠ diffusion damping (unrelated)

pressure forces ⇓ acoustic oscillations

grav. force

dilation effects

Sound horizon: $r_s(\eta) \equiv \int_0^{\eta} c_s(\tilde{\eta})\, d\tilde{\eta}$

Ignore damping and approximate R as slowly varying:

Ignore damping and approximate R as slowly varying:

$$\theta_0(\eta) = \theta_0(0)\cos[k\, r_s(\eta)] + \text{grav. corrections}$$

"acoustic oscillations"

phase fixed by adiabatic initial cond: $\theta_0 = \text{const}$ for $k\eta \ll 1$

$\Rightarrow$ all modes are in phase!

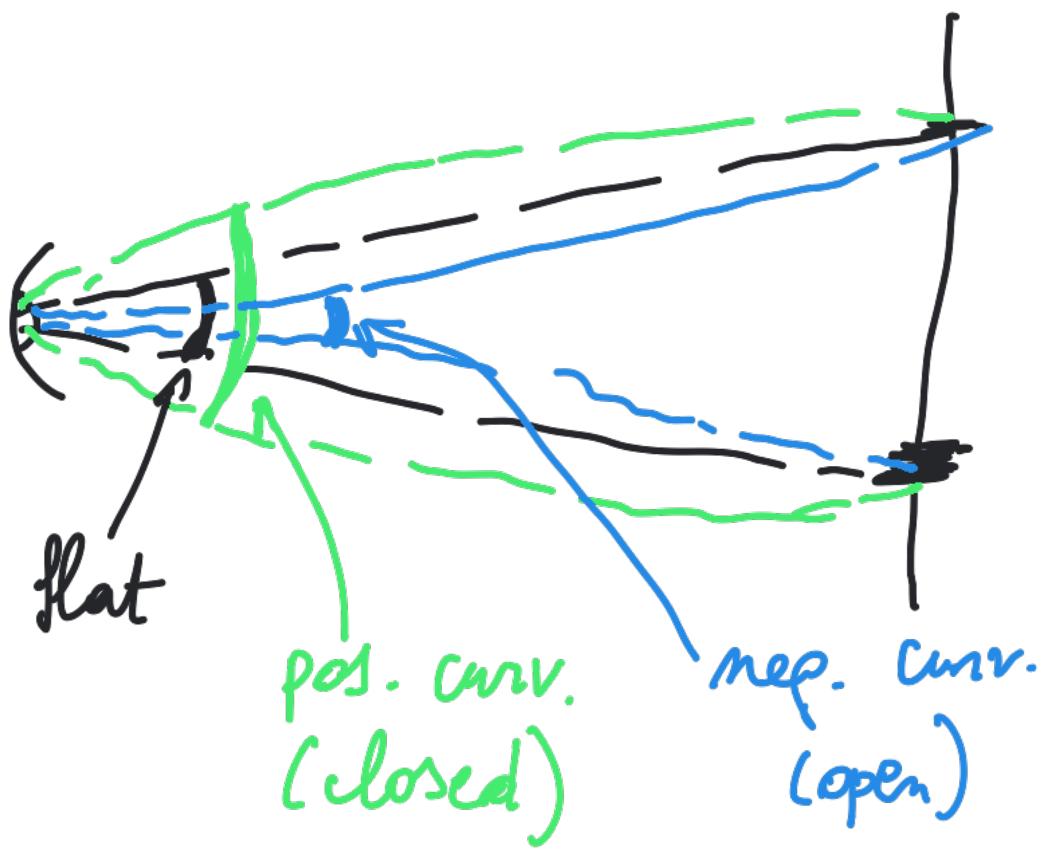
Important for CMB: modes k that are extremal at η_{dec}

Peaks: $\quad k\, r_s(\eta_{dec}) = n\pi$

$\Rightarrow$ peaks on corresponding angular scales in CMB power spectrum!

Relation between spatial and angular scales depends on

Relation between spatial and angular scales depends on spatial curvature of universe!



$\Rightarrow$ measure curvature!

(location of 1st peak)

Gravitational potential Ψ shifts zero point of oscillations:

$$\ddot{\Theta}_0 + \frac{\dot{R}}{1+R}\dot{\Theta}_0 + k^2 c_s^2 \Theta_0 = -\frac{k^2}{3}\Psi$$

equilibrium: $\ddot{\Theta}_0 = \dot{\Theta}_0 = 0 \Rightarrow \Theta_0^{eq} = -\frac{1}{3c_s^2}\Psi = -(1+R)\Psi$

Gravitational potential ψ shifts zero point of oscillations:

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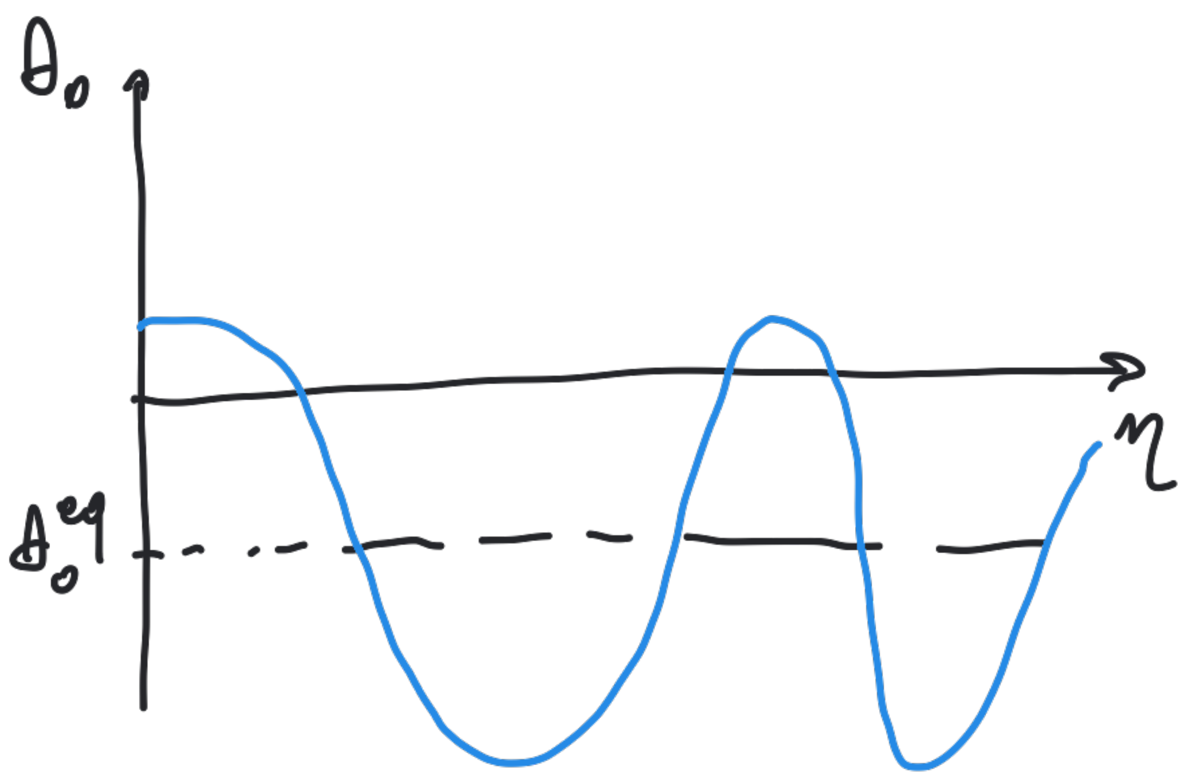
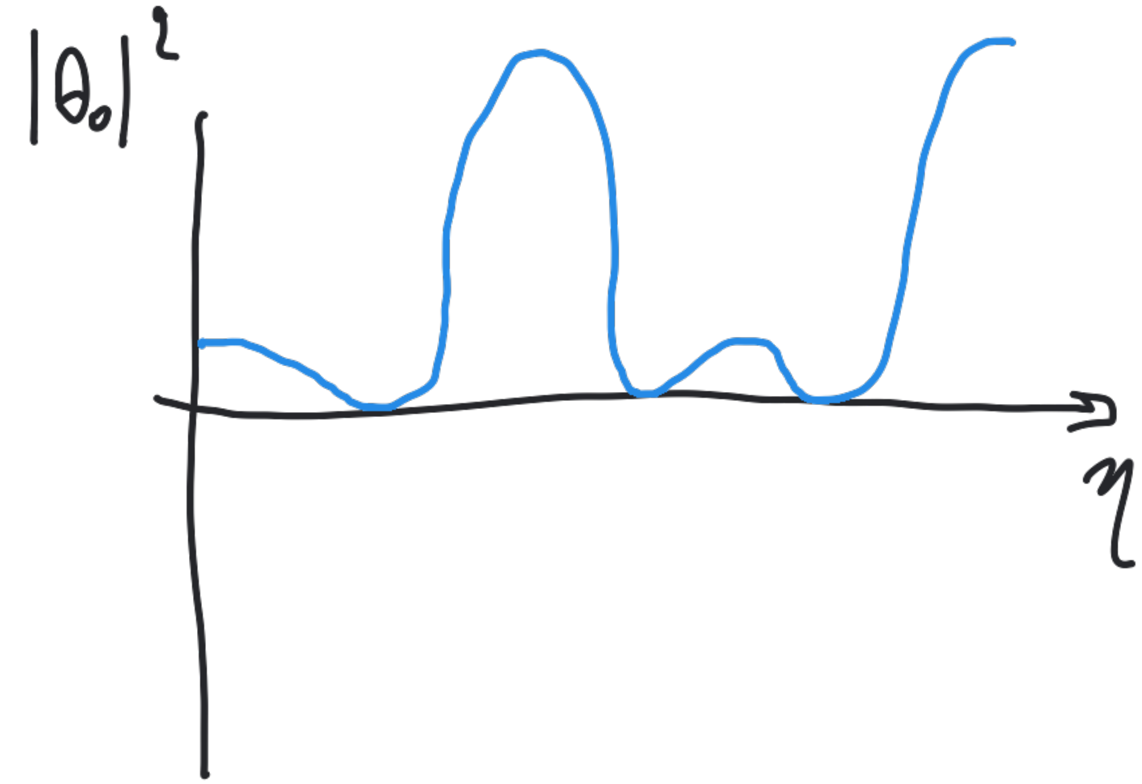
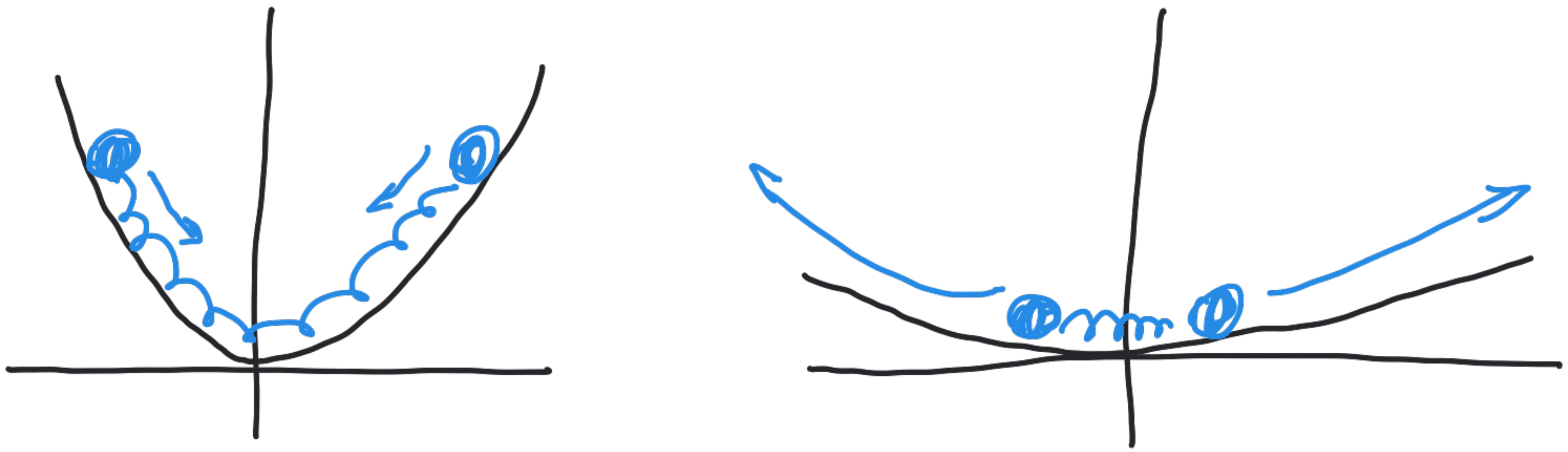




Enhances odd over even peaks!

Effect stronger when c_s is small, i.e. when baryon fraction is high $\Rightarrow$ measure baryons!
(height of second vs first peak)

Decaying potential : strong driving force



This happens for modes entering the horizon during radiation domination, when grav. potentials decay inside the horizon.

More dark matter ⇒ radiation domination stopped earlier ⇒ less driving force ⇒ lower peaks

⇒ measure dark matter ! (from first 3 peaks)

Diffusion damping : $\exp\left(\frac{-\ell^2}{}\right)$ suppression of small-

Diffusion damping: $\exp\left(-\frac{\ell^2}{\ell_D^2}\right)$ suppression of small-angle anisotropies, i.e. of high peaks

related to diffusion length

$\Downarrow$

measure baryons!
(consistency check)

ISW contributions: from regions in (k,η) space where metric fluctuations vary in time

- shortly after decoupling, on sub-soundhorizon scales (EISW) $\longrightarrow$ increases first peak
  early
- during Λ domination (LISW) $\longrightarrow$ increases lowest multipoles
  late

late multipoles

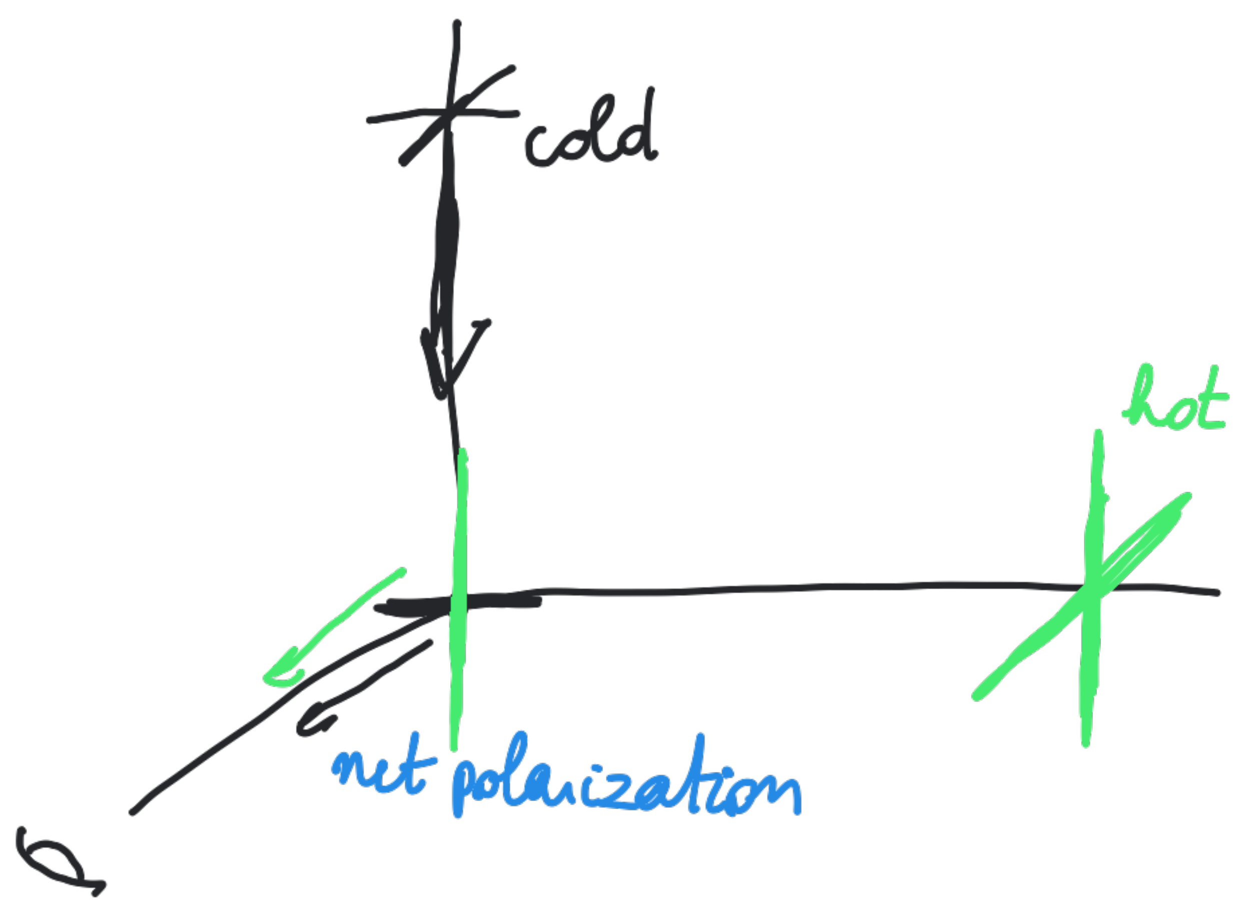
More dark matter $\Rightarrow$ less radiation left after decoupling

$\Rightarrow$ metric fluctuations decay less

$\Rightarrow$ EISW contributes less $\Rightarrow$ lower first peak

Reionization: rescattering at small redshift ($z \lesssim 10$) tends to smooth out anisotropy $\Rightarrow$ decrease of C_ℓ for $\ell \gtrsim 40$.

Polarization



Thomson scattering in presence of quadrupole

Polarization represented by sticks of different size and orientation in different points of map, related to size and orientation of quadrupole at each point of last scattering surface.

Can be decomposed in E and B modes

Scalar only produce E, tensor produce E and B

C_ℓ^{TT}, C_ℓ^{EE}, C_ℓ^{TE}, C_ℓ^{BB}

Only produced by scattering

- recombination $\longrightarrow$ small scales
  $\sim 10^{-6}\,K$ (diffusion scale peaks at $\ell \approx 1000$)
- reionization $\longrightarrow$ large scales
  $\sim 10^{-7}\,K$ $\Rightarrow$ measure τ_{reio}

Scalar only produce E, tensor produce E and B

C_ℓ^{TT}, C_ℓ^{EE}, C_ℓ^{TE}, C_ℓ^{BB}

Only produced by scattering

- recombination $\sim 10^{-6}$ K → small scales (diffusion scale peaks at $\ell \approx 1000$)
- reionization $\sim 10^{-7}$ K → large scales
  ⇒ measure τ_{reio} (optical depth to recombination)

Tensor modes

- C_ℓ^{TT} for $\ell \lesssim 100$ (grav. waves decay quickly inside Hubble radius)
  → cosmic variance
- C_ℓ^{BB} big challenge!

Cosmological parameters

Free parameters of ΛCDM

- curvature density $\Omega_k \equiv 1 - \Omega_m - \Omega_\Lambda$
- baryon density $\Omega_b h^2$
- matter density $\Omega_m h^2$
- cosmological constant energy density Ω_Λ
- normalization C_{10}
- primordial tilt / slope n
- optical depth to recombination
- tensor modes $r \equiv \frac{C_2^T}{C_2^S}$ ("tensor-to-scalar ratio")

tilt/slope: $n < 1$ (decreases with T

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- primordial tilt / slope n
- optical depth to recombination
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tilt / slope : $n < 1$ significantly (degeneracy with τ_{reio} broken by large-scale polarization data)

Planck 2018 : $n = 0{,}965 \pm 0{,}004$