Nuclear instrumentation: Exercise 2

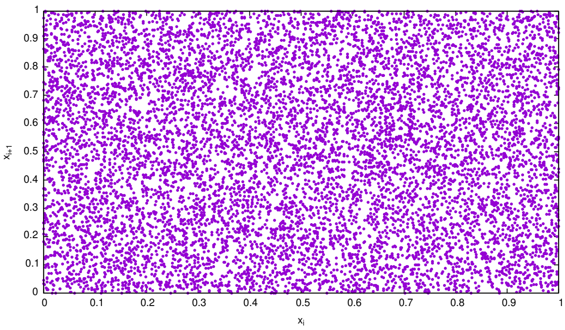
Arthur Adriaens

Ghent University

1. Random Numbers

Truly Random?

I'll be using the Mersenne Twister pseudorandom number generator, a general test to see if this is in fact a good random number generator is plotting the next number to the previous number and see if there's a pattern or not:

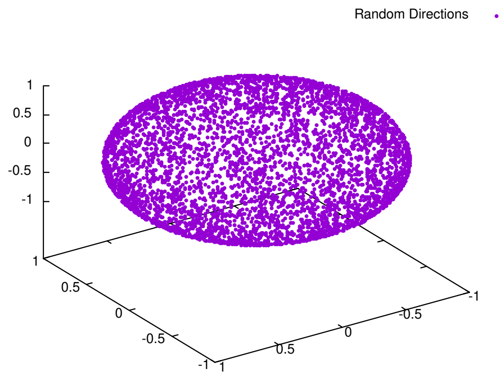


As there doesn't seem to be a pattern we"ll continue on with this number generator.

Random direction

If we wish to compensate against the clustering at the poles we would get if we took a unifor θ distribution, we'll have to take a non-uniform distribution, scaling with $\sin(\theta)$. We'll be using inverse Transform Sampling which is a method that allows us to sample a general probability distribution using a uniform random number. After some mathematics we find that, say we have random number u between 0 and 1, we find θ distributed correctly with:

$$\theta = arccos(1 - 2u)$$



In the above figure it is clear that this gives the right distribution.

Random Origin

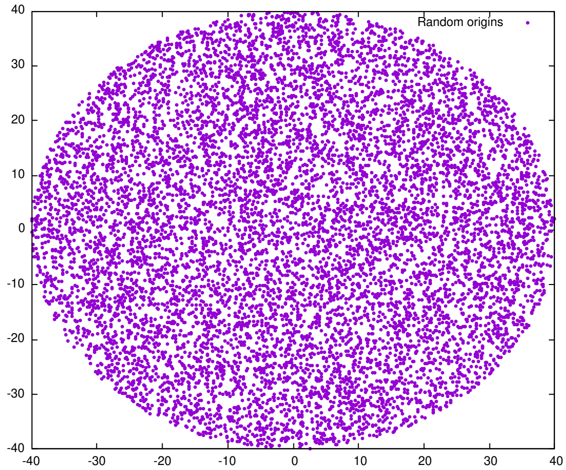
As with the random direction, we can't just uniformly take r to be between 0 and 40mm and theta uniformly between 0 and 2π. We'll be using the following algorithm:

Generate v_1 uniformly between -1 and +1 and v_2 between 0 and 1. Calculate $r^2 = v_1^2 + v_2^2$, if $r^2 > 1$, start over. Otherwise, the sine (S) and cosine (C) of a random angle are given by:

$$S = 2 v_1 v_2 / r^2$$

$$C = (v_1^2 - v_2)/r^2$$

And the radius is given by r*40 (in millimeters), this gives the following:



2. How to solve this problem

We've now generated the origin coordinate and the direction, the way we're going to check whether the particle arrives on the detector is by first proportionally scaling the direction vector. I.e if z is 0.6 we check the factor with which we'll have to scale z to get to 70 (mm) and scale both x and y with this same factor. We then add on the offset due to the origin and convert x and y into circular coordinates to check if the radius is smaller than 60mm. We'll only do this for directions with z positive as the particles going the opposite way can never reach the detector.

3. What is the solid angle? (a)

The solid angle of the detector is given by $4\pi N_{hit}/N_{tot}$, 5 quick simulations of 10^6 particles each give the following values:

$$N_{hit} = 109888, 109215, 109062, 109533, 110262$$

Corresponding to a solid angle of about 1.377 ± 0.006 steradians.

4. Check to find point source (b)

Taking the start position to always be (0,0,0) we find, after 5 calculations of 10^6 points, a solid angle of 1.511 ± 0.0018 steradians. Let's now analytically calculate the solid angle:

Of the encapsulating sphere the detector blocks a certain θ:

$$tan(\theta) = \frac{60}{70} \rightarrow \theta = \tan^{-1}\frac{60}{70} = 0.7086$$

So the solid angle is:

$$\Omega = \int\int_S sin(\theta) d\theta d\phi = \int_0^{0.7086} \sin(\theta) \int_0^{2\pi} d\phi = 1.5126$$

Which is perfectly in line with our estimate.

5. Partially covered detector (c)

We have the detector covered with a foil except for an opening in the middle in the shape of a square with a diagonal of 75% the diameter of the detector. I.e a sidelength of 63.64mm. This can be added to the program by checking if the particle falls within this square or not. Doing this gives:

$$\Omega = 0.591 \pm 0.004$$

6. Typical amount of particles (d)

as mentioned before, 10^6 particles is chosen. Looking back at the previous answers, 5 of these generally gives an accuracy of about 0.5%. This thus means that $5 * 10^6$ is a good number of particles.

7. Efficiency (e)

The efficiency of the simulation was improved by recognizing the fact that no particles with a z direction smaller than 0.1 were going to end up on the detector. So not only the ones going the opposite way, but also the ones going almost completely to a combination of the x and y axes were skipped and set to "missed". This limitation could be further improved upon by manually calculating the angles with which the detector would or wouldn’t be reached but as the program ran good enough this wasn’t tested119923.