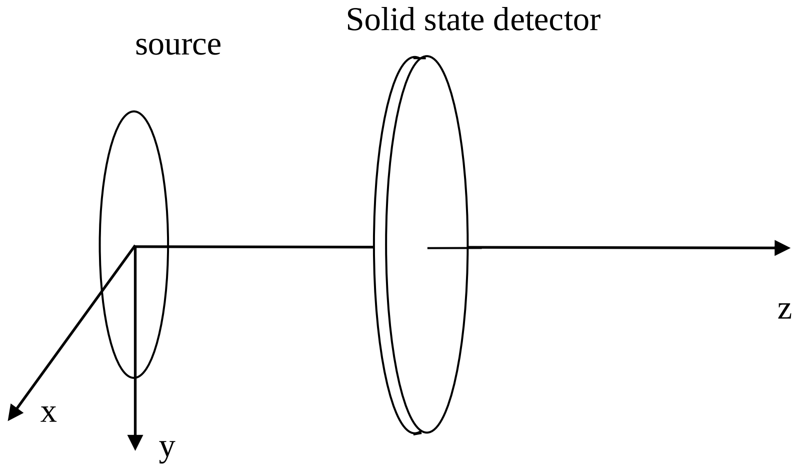
Nuclear Instrumentation, partim radiation detectors: exercise 2

Consider the following source-detector system:

- A circular source with a radius of 40 mm is present in the x-y plane, centered at the origin. Each point of the source emits alpha particles isotropically and the intensity over the surface of the source is constant.

- A circular solid state detector has a radius of 60 mm and is positioned such that its circular entrance window is parallel with the circular source, and at a distance of 70 mm from the source. The detector is centered on the z-axis.



Task:

Develop a Monte Carlo simulation to determine the solid angle of the system.
(a) What is the solid angle (in steradians) of the geometry described above?
(b) Check that, if you make the radius of the source very small in your code, the point source value of the solid angle is found.
(c) What is the value of the solid angle if the detector is partially covered by a thin plate which stops alpha-particles, having a square opening with a diagonal of 75% of the diameter of the detector and of which the centre coincides with the centre of the detector?
(d) What typical amount of particles do you have to emit in your simulation to obtain an accurate result (i.e. standard deviation of the order of 0,5 %) ? How did you determine this amount?
(e) Did you improve the efficiency of the calculation and how? (see last paragraph below)

This problem can be solved by writing a computer program that simulates this setup. What is the problem really about? One is dealing with a radioactive source, which is emitting particles which will or will not reach the detector. The source is emitting particles isotropically, which means that any direction a particle can go within the complete 4π solid angle has an equal probability. The source has a given surface, and its intensity is constant over its surface, meaning that all points on the surface are radiating particles with equal intensity. In the computer program, this source will emit N_{tot} particles isotropically in 4π, each of the particles is tracked and one checks whether it reaches the detector surface or not. In case N_{hit} particles out of N_{tot} reach the detector, the solid angle of the detector is given by $4\pi . N_{\mathrm{hit}}/N_{\mathrm{tot}}$

How can one generate in a realistic way initial positions and directions for a particle? Each position and direction is randomly determined. To simulate this, one uses a random number generator. A uniform random number generator generates numbers between 0 and 1, 0 included, 1 just excluded (mathematically noted as [0,1[ ), and this with an equal probability for each number in the range [0,1[. After sampling a lot of times, one will obtain a uniform distribution of values between 0 and 1. Such a generator is available in many mathematical packages or even as a predefined function in programming languages (in FORTRAN one has for example the RAN function), or it can be found in books such as "Numerical Recepies". Using such a generator, it is for example possible to simulate the throwing of a dice: evaluating (int(RAN x 6) + 1) many times (whereby RAN stands for the random number between 0 and 1 and int is the integer function which removes all numbers after the decimal point) will yield the numbers from 1 to 6, each with equal probability.

A random number generator needs a seed which is an initializing number for the generator. Each sequence of random numbers is started using a seed, and re-using the same seed will give rise to exactly the same sequence of random numbers. This actually demonstrates that the generator is not really random, it only mimics randomness and one calls this a pseudo-random number generator. Sometimes, the seed is determined internally in the computer by using for example the time indicated by the clock at the moment the program starts. As such, each run will use another seed and thus another sequence of random numbers will be obtained.

For this particular problem, there are two items which have to be generated randomly for each particle to be considered: the position on the surface of the source from which the particle emerges and the direction with which it moves away from the source.

To determine the particle direction, it is most useful to use the spherical coordinates θ and ϕ, each direction is indeed determined by a unique θ and ϕ. However, in case one wants to generate directions in an isotropical way, it is not correct to simply generate θ and ϕ uniformly between 0 and π and 0 and 2π, respectively. For the ϕ-angle, a uniform generation between 0 and 2π is correct, but for θ there is a problem: in case one generates each θ with equal probability, one will obtain, when looking where the particle directions cut through a spherical surface with the source at its center, a much larger point density close to the pole of the coordinate system (where the z-axis cuts through the sphere) than at the equator (where the plane containing the x- and y-axis cuts through the sphere). As such, θ will have to be generated with another distribution than a uniform distribution. The question is which one (see also the file Monte_Carlo.pdf in the exercises directory).

With regard to position generation, there is a similar problem. One is dealing with a circular source, as such the use of polar coordinates r and φ is handy. φ can again be generated uniformly between 0 and 2π, but for r a uniform distribution does not work. In case one generates r uniformly, the density of generated points on the source surface will be much larger for small radii than for large radii.

It is clear that the accuracy of the result for this kind of simulations will depend on the amount of particles that is tracked. The larger this amount, the more accurate the result will be. As such, it is interesting to investigate the stability of the result as a function of the number of samples. One way to do this is perform a number of simulations with the same number of samples, while using a different seed for each run. The result will statistically fluctuate around a mean value, and the magnitude of the fluctuations will indicate the statistical accuracy. One can make a rough estimate of the statistical error on the results by looking at the amount of particles that reach the detector and calculating the standard deviation on this number.

While solving the problem, consider also the efficiency of the calculation. For this simple problem, which can be simulated very fast on present day computers, the efficiency is not a serious problem, but for more complex simulations the efficiency can start to play a role. For the present problem, it is immediately clear that it is of no use to emit particles in the half-sphere at the backside of the source, since they will never hit the detector anyway. As such, the necessary calculating time can immediately be reduced by a factor of two by taking this into account. It is possible to increase the efficiency even more. What would be the most efficient way of working for this particular situation?