Subatomic Physics II

Problem Set 9

Due on December 9, 2021, 23:59

Problem 8.1: CKM parameters

- The branching fraction B for the decay $\Lambda \to p e^- \bar{\nu}_e$ and the lifetime τ of the Λ-baryon are measured to be $B = (8.32 \pm 0.14) \times 10^{-4}$ and $\tau = (2.632 \pm 0.020) \times 10^{-10}$ s, respectively. What interaction is responsible for the decay, and why? Draw the corresponding Feynman diagram and assign V_{ij} factors to the appropriate vertices. Neglecting other uncertainties, what is the resulting relative uncertainty $\delta|V_{us}|/|V_{us}|$ in the determination of $|V_{us}|$? (1.5pt)

- Within the Standard Model of particle physics, the mass difference Δm between K_L^0 and K_S^0 can be calculated. The dominant contribution arises from box diagrams (leading to $K^0 \leftrightarrow \bar{K}^0$ oscillations in the $K^0, \bar{K}^0$ basis) with virtual c quarks:

$$\Delta m = \frac{G_F^2}{4\pi^2} f_K^2 m_K m_c^2 |V_{cd}|^2 |V_{cs}|^2$$

  where G_F is the Fermi constant, $f_K \approx 0.16$ GeV is the so-called kaon decay constant, m_K is the average neutral kaon mass, and $m_c \approx 1.4$ GeV is the mass of the c quark. (Historically, this formula was used to predict the approximate mass of the c quark before it was discovered.)

  Draw the two box diagrams and assign V_{ij} factors to the appropriate vertices (1.0pt). Thus explain why the CKM elements V_{cd} and V_{cs} appear in the formula, and why in the combination $|V_{cd}|^2 |V_{cs}|^2$ (1.0pt). Why can the diagrams with virtual u or t quarks be neglected (1.0pt)? Using the Cabibbo approximation of the CKM matrix (2-by-2 submatrix describing first two fermion families with rotation angle $0 < \theta_C \ll 1$), determine the Cabibbo angle θ_C based on the measured mass difference of $\Delta m = 3.484 \times 10^{-6}$ eV (1.0pt).

- Based on the branching fractions for semi-leptonic D^0 decays, $B(D^0 \to K^- \mu^+ \nu_\mu) = 0.0341$ and $B(D^0 \to \pi^- \mu^+ \nu_\mu) = 0.00267$, determine the ratio $|V_{cd}|/|V_{cs}|$. Hints: draw the corresponding Feynman diagrams and assign the V_{ij} factors to the appropriate vertices. Neglect the mass difference between K^- and π^- relative to the D^0 meson (1.5pt).

- By what factor approximately would the lifetime of the B^0 meson change if the t quark had a mass similar to that of the c quark? Hint: $|V_{ub}| \ll |V_{cb}| \approx 0.04, |V_{tb}| \approx 1$ (1.5pt).

- Describe *briefly* a possible analysis to measure the CKM element $|V_{ub}|$, paying attention to the fact that $|V_{ub}| \ll |V_{cb}|$ (1.5pt).