Subatomic Physics II: Problem set 9

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9 CKM parameters

9.1 Interactions in $\Lambda \to p e^- \bar{\nu}_e$

In this interaction we have the decay of a strange quark into an up quark by emittance of a W^- boson (as quarks can only change via weak decay):

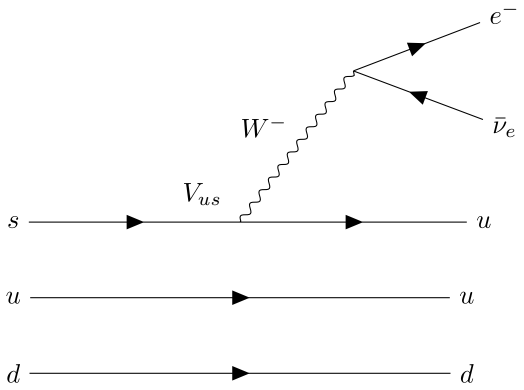


Figure 1: $\Lambda \to p e^- \bar{\nu}_e$

Now as

$$\Gamma\left(\Lambda \to p e^- \bar{\nu}_e\right) = \frac{G_F^2 m_\Lambda^5}{192\pi^3}\left(1+\delta_e^\Lambda\right)|V_{us}|^2 \tag{1}$$

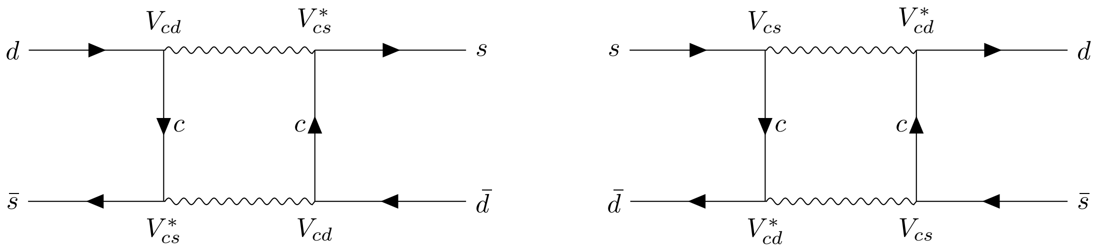
$$= \frac{B\left(\Lambda \to p e^- \bar{\nu}_e\right)}{\tau_\Lambda} \tag{2}$$

we have that

$$|V_{us}| \propto B^{\frac{1}{2}}\tau_\Lambda^{-\frac{1}{2}} \implies \frac{\delta|V_{us}|}{|V_{us}|} = \frac{1}{2}\left(\frac{\delta B}{B} + \frac{\delta\tau_\Lambda}{\tau_\Lambda}\right) = 0.0122 \tag{3}$$

I.e a 1.22% relative error.

9.2 $K^0 \leftrightarrow \bar{K}^0$



(a) $K^0 \to \bar{K}^0$

(b) $\bar{K}^0 \to K^0$

We can see in the left figure (right figure is completely analogous) that we have a term proportional to V_{cd} and V_{cs}^* in the 2 left-hand side vertices of the box-diagram which is also proportional to $V_{cd}^\dagger$ and $(V_{cs}^*)^\dagger$ as seen in the right-hand side of the diagram, yielding an interaction proportional to:

$$\propto V_{cd}V_{cs}^*V_{cd}^\dagger(V_{cs}^*)^\dagger = V_{cd}V_{cd}^\dagger V_{cs}^*(V_{cs}^*)^\dagger = |V_{cs}|^2|V_{cd}|^2 \approx 0.218 \tag{4}$$

The contribution of the virtual u quark can be neglected in the contribution of the oscillation as $\Delta m \propto m_q^2$ and $m_c^2 \gg m_u^2$. The contribution of the virtual t quark can be neglected as $|V_{td}|^2|V_{ts}|^2 \ll |V_{cs}|^2|V_{cd}|^2$, concretely, defining Δm_q as denoting the contribution of the virtual quark q:

$$\Delta m_q = m_q^2|V_{qd}|^2|^2V_{qs}|^2 \tag{5}$$

We have that

$$\begin{aligned} \Delta m_c &\approx 0.3 && (6)\\ &\gg \Delta m_t \approx 0.06 && (7)\\ &\gg \Delta m_u \approx 0.002 && (8)\\ & && (9) \end{aligned}$$

We'll now try to find the Cabbibo angle in the Cabibbo approximation of the CKM matrix:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \tag{10}$$

as we have that

$$\Delta m = 3.484 \times 10^{-6} eV = \frac{G_F^2}{4\pi^2} f_k^2 m_K m_c^2 |V_{cd}|^2|V_{cs}|^2 \approx \frac{G_F^2}{4\pi^2} f_k^2 m_K m_c^2 \cos^2\theta_c \sin^2\theta_c \tag{11}$$

Filling in $f_K \approx 0.16\text{GeV}$, $m_c \approx 1.4\text{GeV}$, $G_F = 1.166 \times 10^{-5}\ \text{GeV}^{-2}$ [1] and $m_K \approx 0.497611$ GeV we get:

$$3.484 \times 10^{-6} \approx 8.6 \times 10^{-5} \cos^2\theta_c \sin^2\theta_c = 8.6 \times 10^{-5}(1 - \sin^2\theta_c)\sin^2\theta_c \tag{12}$$

And solving for $\sin\theta_c$ yields $\theta_c \approx 0.207$ ($\approx 12°$).

9.3 Determine the ratio $\frac{|V_{cd}|}{V_{cs}}$

The relevant feynmann diagrams are:

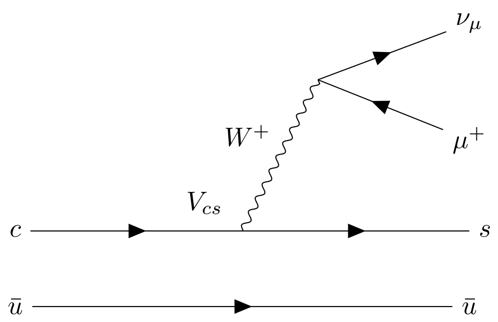


Figure 3: $D^0 \to K^-\mu^+\nu_\mu$

And

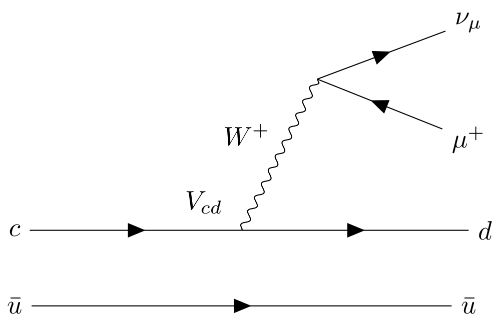


Figure 4: $D^0 \to \pi^-\mu^+\nu_\mu$

If we consider the pi-meson and kaon to have the same mass, they'll have the same τ meaning that $B \propto |V|^2$ and thus:

$$\frac{|V_{cd}|}{|V_{cs}|} = \sqrt{\frac{B(D^0 \to \pi^-\mu^+\nu_\mu)}{B(D^0 \to K^-\mu^+\nu_\mu)}} = 0.2798 \tag{13}$$

9.4 B^0 meson lifetime change if $m_t \approx m_c$

Considering the decay of B_0 we have $\tau \propto |V_{cb}|^{-2}$ in the primary decay mode but if the charm quark would have around the same mass as the top quark then we could expect a decay where the $\bar{b}$ quark becomes a $\bar{t}$, this would result in a $|V_{tb}|^2$ contribution and thus the lifetime would change by a factor $\frac{|V_{cb}|^2}{|V_{tb}|^2} = 0.0016$, i.e B^0 would live approximately 625 times shorter (it's assumed that the change in mass doesn't affect $|V_{tb}|^2$).

9.5 measuring V_{ub}

For this we can take a look at the ratio branching ratio's of B^0, as:

$$\frac{B\left(B^0 \to \pi^- e^+ \nu_e\right)}{\tau_B} = \frac{G_F^2 m_B^5}{192\pi^3}\left(1 + \delta_e^B\right)|V_{ub}|^2 \tag{14}$$

And

$$\frac{B\left(B^0 \to D^- e^+ \nu_e\right)}{\tau_B} = \frac{G_F^2 m_B^5}{192\pi^3}\left(1 + \delta_e^B\right)|V_{cb}|^2 \tag{15}$$

We have that

$$\frac{B\left(B^0 \to \pi^- e^+ \nu_e\right)}{B\left(B^0 \to D^- e^+ \nu_e\right)} = \frac{|V_{ub}|^2}{|V_{cb}|^2} \tag{16}$$

And thus if we measure the ratio of these branching ratio's and fill in $|V_{cb}|^2$ we'll get $|V_{ub}|$

References

[1] P.A. Zylaet al. (Particle Data Group). *Prog. Theor. Exp. Phys. 2020, 083C01*, 2020 and 2021 update.