Subatomic Physics II: Problem set 3

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1 SU(2)

1.1 $\frac{\sigma(\pi^0 d\to pn)}{\sigma(\pi^+ d\to pp)}$

For this we can take a look at the angular momenta [1]:

	J	I	I_3
π^0	0	1	0
π^+	0	1	1
p	1/2	1/2	1/2
n	1/2	1/2	-1/2

Now taking a look at the isospins of the interactions we have the following:
If we couple p and n we have $|\frac{1}{2},\frac{1}{2}\rangle \otimes |\frac{1}{2},-\frac{1}{2}\rangle = \sqrt{\frac{1}{2}}\,|1,0\rangle + \sqrt{\frac{1}{2}}\,|0,0\rangle$ and if we couple p and p we have $|\frac{1}{2},\frac{1}{2}\rangle \otimes |\frac{1}{2},\frac{1}{2}\rangle = |1,1\rangle$.
Now deuterium is build up of a proton and a neutron, thus it should have $|\frac{1}{2},\frac{1}{2}\rangle \otimes |\frac{1}{2},-\frac{1}{2}\rangle = \sqrt{\frac{1}{2}}\,|1,0\rangle + \sqrt{\frac{1}{2}}\,|0,0\rangle$ Coupling π^0 and deuterium gives:

$$\langle 1,0| \otimes \left(\sqrt{\frac{1}{2}}\,|1,0\rangle + \sqrt{\frac{1}{2}}\,|0,0\rangle\right) = \sqrt{\frac{1}{3}}\,|2,0\rangle + \sqrt{\frac{1}{2}}\,|1,0\rangle - \sqrt{\frac{1}{6}}\,|0,0\rangle \tag{1}$$

And coupling π^+ and deuterium:

$$\langle 1,1| \otimes \left(\sqrt{\frac{1}{2}}\,|1,0\rangle + \sqrt{\frac{1}{2}}\,|0,0\rangle\right) = \sqrt{\frac{1}{4}}\,|2,1\rangle + \sqrt{\frac{1}{4}}\,|1,1\rangle + \sqrt{\frac{1}{2}}\,|1,1\rangle \tag{2}$$

So we have that:

$$\frac{\sigma(\pi^0 d\to pn)}{\sigma(\pi^+ d\to pp)} \propto \frac{|(\sqrt{\frac{1}{3}}\,\langle 2,0| + \sqrt{\frac{1}{2}}\,\langle 1,0| - \sqrt{\frac{1}{6}}\,\langle 0,0|)\cdot(\sqrt{\frac{1}{2}}\,|1,0\rangle + \sqrt{\frac{1}{2}}\,|0,0\rangle)|^2}{|(\sqrt{\frac{1}{4}}\,\langle 2,1| + \sqrt{\frac{1}{4}}\,\langle 1,1| + \sqrt{\frac{1}{2}}\,\langle 1,1|)\cdot|1,1\rangle\,|^2} = \frac{\frac{1}{3} - \sqrt{\frac{1}{12}}}{\frac{3}{4} + \sqrt{\frac{1}{2}}} \tag{3}$$

1.2 $\Delta^+(1232)$-decays

$\Delta^+(1232)$ has an isospin and I_3 described by $\langle\frac{3}{2},\frac{1}{2}|$. Coupling n and π^+ gives: $|\frac{1}{2},-\frac{1}{2}\rangle \otimes |1,1\rangle = \sqrt{\frac{1}{3}}\,|\frac{3}{2},\frac{1}{2}\rangle + \sqrt{\frac{2}{3}}\,|\frac{1}{2},\frac{1}{2}\rangle$ and coupling p and π^0 gives: $|\frac{1}{2},\frac{1}{2}\rangle \otimes |1,0\rangle = \sqrt{\frac{2}{3}}\,|\frac{3}{2},\frac{1}{2}\rangle - \sqrt{\frac{1}{3}}\,|\frac{1}{2},\frac{1}{2}\rangle$.

We hereby see that there is a 33.33% chance that $\Delta^+(1232)$ decays into nπ^+:

$$\left|\langle\frac{3}{2},\frac{1}{2}|\left(|\frac{1}{2},-\frac{1}{2}\rangle\otimes|1,1\rangle\right)\right|^2 = \left|\langle\frac{3}{2},\frac{1}{2}|\left(\sqrt{\frac{1}{3}}\,|\frac{3}{2},\frac{1}{2}\rangle + \sqrt{\frac{2}{3}}\,|\frac{1}{2},\frac{1}{2}\rangle\right)\right|^2 = \frac{1}{3}$$

and a 66.67% chance that it decays into pπ^0:

$$\left|\langle\frac{3}{2},\frac{1}{2}|\left(|\frac{1}{2},\frac{1}{2}\rangle\otimes|1,0\rangle\right)\right|^2 = \left|\langle\frac{3}{2},\frac{1}{2}|\left(\sqrt{\frac{2}{3}}\,|\frac{3}{2},\frac{1}{2}\rangle - \sqrt{\frac{1}{3}}\,|\frac{1}{2},\frac{1}{2}\rangle\right)\right|^2 = \frac{2}{3} \tag{4}$$

1.3 propability proton spin parallel to Δ in $\Delta^+(1920) \to p\rho^0$

When a $\Delta^+(1920)$ decays to a proton and ρ_0-meson it can do this in 2 ways: sending out the meson parallel to Δ and leaving the proton anti-parallel or sending the meson out anti-parallel to Δ. As the spin of the ρ_0-meson is 1 a decay in which ρ_0 is parallel to the spin of the proton results in a proton with spin 1/2 anti-parallel to the original Δ while an anti-parallel decay results in a proton with spin 3/2 parallel to the Δ which is not possible, as such $\Delta^+(1920)$ only decays with the resulting proton being anti-parallel.

1.4 Pion-nucleon scattering

$$\begin{aligned}\pi^+ p &\to \pi^+ p\\ \pi^- p &\to \pi^- p\\ \pi^- p &\to \pi^0 n\end{aligned}$$

If at a certain energy the reaction proceeds through an intermediate $I = 3/2$ state i.e a Δ state then we have to select the coupling with $I = 3/2$:

- Coupling π^+ and p gives: $|1,1\rangle \otimes |\frac{1}{2},\frac{1}{2}\rangle = |\frac{3}{2},\frac{3}{2}\rangle$
- Coupling π^- and p gives: $|1,-1\rangle \otimes |\frac{1}{2},\frac{1}{2}\rangle = \sqrt{\frac{1}{3}}\,|\frac{3}{2},-\frac{1}{2}\rangle - \sqrt{\frac{2}{3}}\,|\frac{1}{2},-\frac{1}{2}\rangle$
- Coupling π^0 and n gives: $|1,0\rangle \otimes |\frac{1}{2},-\frac{1}{2}\rangle = \sqrt{\frac{2}{3}}\,|\frac{3}{2},-\frac{1}{2}\rangle + \sqrt{\frac{1}{3}}\,|\frac{1}{2},-\frac{1}{2}\rangle$

Now we can see from the feynmann diagrams that:

$$\begin{aligned}\pi^+ p &\to \Delta^{++} \to \pi^+ p\\ \pi^- p &\to \Delta^0 \to \pi^- p\\ \pi^- p &\to \Delta^0 \to \pi^0 n\end{aligned}$$

And thus:

- $\frac{\sigma(\Delta^{++}\to\pi^+ p)}{\sigma(\Delta^0\to\pi^- p)} \propto \frac{|\langle\frac{3}{2},\frac{3}{2}|\cdot(|1,1\rangle\otimes|\frac{1}{2},\frac{1}{2}\rangle)|^2}{|\langle\frac{3}{2},-\frac{1}{2}|\cdot(|1,-1\rangle\otimes|\frac{1}{2},\frac{1}{2}\rangle)|^2} = \frac{|\langle\frac{3}{2},\frac{3}{2}|\cdot(|\frac{3}{2},\frac{3}{2}\rangle)|^2}{|\langle\frac{3}{2},-\frac{1}{2}|\cdot(\sqrt{\frac{1}{3}}|\frac{3}{2},-\frac{1}{2}\rangle-\sqrt{\frac{2}{3}}|\frac{1}{2},-\frac{1}{2}\rangle)|^2} = \frac{1}{1/3} = 3$
- Analogously $\frac{\sigma(\Delta^0\to\pi^+ p)}{\sigma(\Delta^0\to\pi^0 n)} = \frac{1}{2/3} = \frac{3}{2}$
- And $\frac{\sigma(\Delta^0\to\pi^- p)}{\sigma(\Delta^0\to\pi^0 n)} = \frac{1/3}{2/3} = \frac{1}{2}$

Now for a nucleon state ($I = 1/2$) we can do the same procedure but project to $\langle\frac{1}{2},\pm\frac{1}{2}|$, this gives us:

- $\frac{\sigma(N\to\pi^+ p)}{\sigma(N\to\pi^- p)} = 0$
- $\frac{\sigma(N\to\pi^+ p)}{\sigma(N\to\pi^0 n)} = 0$
- $\frac{\sigma(N\to\pi^- p)}{\sigma(N\to\pi^0 n)} = \frac{\sigma(n\to\pi^- p)}{\sigma(n\to\pi^0 n)} = \frac{|\langle\frac{1}{2},-\frac{1}{2}|(\sqrt{\frac{1}{3}}|\frac{3}{2},-\frac{1}{2}\rangle-\sqrt{\frac{2}{3}}|\frac{1}{2},-\frac{1}{2}\rangle)|^2}{|\langle\frac{1}{2},-\frac{1}{2}|(\sqrt{\frac{2}{3}}|\frac{3}{2},-\frac{1}{2}\rangle+\sqrt{\frac{1}{3}}|\frac{1}{2},-\frac{1}{2}\rangle)|^2} = 2$

2 Baryon structure

In the ground state band, the total angular momentum J is entirely dependent on de spins of the quarks. If we name $\vec{S}_{12} = \vec{S}_1 + \vec{S}_2$ (analogously to $\vec{L}_{12}$), we have that:

$$\vec{S}_{12} = \vec{S}_1 \otimes \vec{S}_2 = |\frac{1}{2}, m_1\rangle \otimes |\frac{1}{2}, m_2\rangle$$

Now we have that, if you couple two angular momenta j_1 and j_2 ($j = j_1 + j_2$), j can take on the values[1]:

$$j = j_1 + j_2, j_1 + j_2 - 1, ..., |j_1 - j_2| \tag{5}$$

And the z-projection m can be:

$$m = -j, -j+1, ..., j-1, j \tag{6}$$

Now applying this to $\vec{S}_1 \otimes \vec{S}_2$ we get that S_{12} can take on the values 1 and 0, and thus the states

$$|0,0\rangle\,;|1,-1\rangle\,;|1,0\rangle\,|1,1\rangle$$

Now coupling $\vec{S}_{12}$ and $\vec{S}_3$ to $\vec{S}$ we get that S can take the values $\frac{1}{2}$ and $\frac{3}{2}$, giving us the following possible states:

$$|\frac{1}{2},\pm\frac{1}{2}\rangle\,;|\frac{3}{2},\pm\frac{1}{2}\rangle \text{ and } |\frac{3}{2},\pm\frac{3}{2}\rangle \tag{7}$$

[1] This is only true if pauli's exclusion principle is satisfied, which is the case here because of color charge

As L=0 we have that the parity of these states is $(-1)^L = 1$, i.e a positive parity $+$. We thus have the states

$$^2S^+_{\frac{1}{2}} \text{ and } ^4S^+_{\frac{3}{2}} \tag{8}$$

Written as $^{(2S+1)}\boldsymbol{L}^p_J$.

Now for L=1 (negative parity) we have to couple one more time resulting in $J = \frac{3}{2}, \frac{1}{2}$ for $S = \frac{1}{2}$ and $J = \frac{5}{2}, \frac{3}{2}, \frac{1}{2}$ for $S = \frac{3}{2}$, or equivalently the states:

$$^2P^-_{3/2} \; ; \; ^2P^-_{1/2} \; ; \; ^4P^-_{5/2} \; ; \; ^4P^-_{3/2} \; ; \; ^4P^-_{1/2} \tag{9}$$

References

[1] B. Povh, M. Lavelle, K. Rith, C. Scholz, and F. Zetsche. *Particles and Nuclei: An Introduction to the Physical Concepts*. Springer Berlin Heidelberg, 2015.