Subatomic Physics II: Problem set 2

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1 The LHC accelerator

1.1 fraction of straight sections in the accelerator

Owing to the Lorentz force $\boldsymbol{v} \times \boldsymbol{B}$, a particle with charge $|q| = e$ has a path described by the formula

$$p \cos \lambda = 0.3\boldsymbol{B}R \tag{1}$$

where $\boldsymbol{B}$ is the magnetic flux density in tesla, p is the momentum in GeV/c, R is the radius of curvature in meters and λ is the angle between the particle path and the magnetic field. If we take this angle to be zero and rearrange the terms we get:

$$\frac{p}{0.3B} = R \tag{2}$$

Filling in the values for p and $\boldsymbol{B}$ we get R $\approx$ 2791m or a circumference of $\approx$ 17527m which is about 65% of the real circumference which implies that about 35% of the accelerator is straights.

1.2 Amount of collisions in 1 second

we can take in very good approximation $v \approx c$, if we do this a proton goes about 300000km every second. As the collider is 27km long the proton circles 11111 times every second. As there are 2808 bunches this implies $11111 \cdot 2808 = 31.2$MHz (31.2×10^6 collision per second).

1.3 New bunch every 25ns

Every 25ns implies a frequency of 40MHz which is not the same for the previous one as the accelerator is not operating at peak crossing rate (an example of a limiting factor is the time it takes for dump kickers to get up).

1.4 0.58A

Current is a measurement of charge over time with ampere being measured in coulomb a second. As a proton has an elementary charge which is equal to $e = 1.60217662 \times 10^{-19}$ coulomb, $0.58A = 0.36 \times 10^{19}$ elementary charges a second or $0.36 \times 10^{19} \frac{\text{protons}}{s}$. As there pass 31.2×10^6 bunches a second we get:

$$\frac{0.36 \times 10^{19}}{0.312 \times 10^8} \frac{\text{protons}}{\text{second}} \cdot \frac{\text{seconds}}{\text{bunch}} = 1.15 \times 10^{11} \frac{\text{protons}}{\text{bunch}} \tag{3}$$

1.5 Number of turns proton beam stays in LHC without focussing magnets

For this we'll take a look at the coulomb repulsion of the protons, the outermost proton of a bunch experiences approximately a force

$$F = 1.15 \times 10^{11} \frac{e^2}{4\pi\epsilon_0 r^2} \tag{4}$$

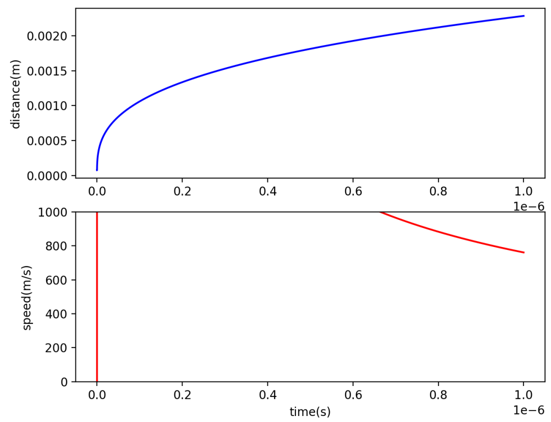
with r the distance to the other protons (which we assume stay grouped together for simplicity). Now if we take the center of the tube to be x=0 and name the distance to the proton x(t) then we can write:

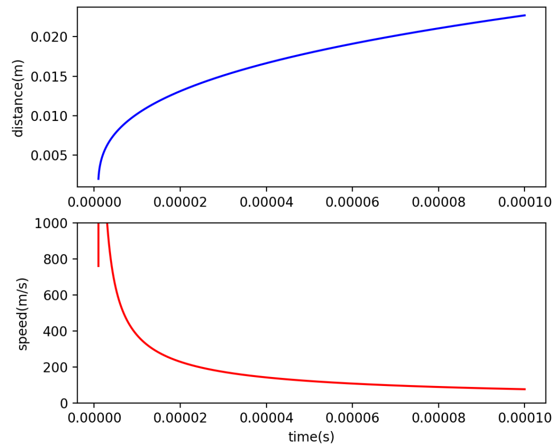
$$m_p \ddot{x}(t) = 1.15 \times 10^{11} \frac{e^2}{16\pi\epsilon_0 x(t)^2} \tag{5}$$

(As the distance to the other part of the bunch is 2x(t)). Now we can write this as

$$\ddot{x}(t) = \frac{A}{x(t)^2} \tag{6}$$

with A $\approx 3.97 \times 10^9 \text{s}^{-2}$. We now wish to solve for x, for this we can use euler's method and look at what point x becomes 18mm (1.8×10^{-2}):





Where I started at the width of a human hair as that's the quoted bunch width (0.75μm) resulting in 5×10^{-5}s or $300000 \cdot 5 \times 10^{-5} = 15$km so not even one rotation.

2 Mass of the muon-neutrino

$$P^{\mu}_{\pi^-} = P^{\mu}_{\mu^-} + P^{\mu}_{\bar{\nu}_\mu} \tag{7}$$

moving terms we get:

$$P^{\mu}_{\pi^-} - P^{\mu}_{\mu^-} = P^{\mu}_{\bar{\nu}_\mu} \tag{8}$$

Now multiplying this by itself and the minkowski metric $g_{\mu\nu}$ (using natural units):

$$m^2_{\pi^-} + m^2_\mu - 2P^{\mu}_{(\mu^-)}P^{(\pi)}_{\mu} = m^2_{\bar{\nu}_\mu} \tag{9}$$

The four-vectors of the particles in the stated system are the following:

$$\begin{aligned} P_{\pi^-} &= (m_\pi, 0, 0, 0) & (10)\\ P_\mu &= (E_\mu, p_\mu, 0, 0) & (11)\\ P_\nu &= (E_\nu, -p_\mu, 0, 0) & (12) \end{aligned}$$

As there is conservation of three-momentum. Inserting these in equation 9 gives:

$$m^2_{\pi^-} + m^2_\mu - 2m_\pi E_\mu = m^2_\nu \tag{13}$$

Now, as the muon has a momentum 29.79 ± 0.01MeV (0.034% error), the uncertainty on this value is way bigger than on any mass considered so we can just ignore the other uncertainties. Now as $E = \sqrt{m^2_\mu + p^2_\mu}$ we have that: $E_\mu \approx 109.78$MeV, now as to know the uncertainty on this value we first seek the uncertainty on p^2_μ, this simply follows from

$$\left(\frac{\sigma_f}{f}\right) = \left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2 \tag{14}$$

for xy, x/y and y/x. We thus get $\Delta p^2_\mu(\%) = \sqrt{2 * (3.36 \times 10^{-4})^2} = 0.048\%$ which gives $\Delta p^2_\mu = 0.42$ as this is 0.0035% of the error under the square root, we finally get that the error on E_μ is $0.0035\% \cdot 109.78 = 0.004$

$$\begin{aligned} m^2_\nu &= (139.57\text{ MeV})^2 + (105.66\text{ MeV})^2 - 2(139.57\text{ MeV})(109.78 \pm 0.004\text{ MeV}) & (15)\\ m^2_\nu &= 30643.59\text{ MeV}^2 - 30643.42 \pm 1.07\text{ MeV}^2 & (16)\\ m_\nu &= \sqrt{0.16 \pm 1.07}\text{ MeV} & (17)\\ m_\nu &= 402.71 \pm 2657\text{ eV} & (18) \end{aligned}$$