Subatomic Physics II: Problem set 10

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10 Geoneutrinos

10.1 neutrino energy threshold in inverse β-decay

Here we have the process $\bar{\nu}_e p \to e^+ n$, looking at the system before the interaction in the lab frame we have:

$$s = (P^\mu_{\bar{\nu}_e} + P^\mu_p)^2 = 2m_p|\boldsymbol{p}_{\bar{\nu}_e}| + m_p^2 = 2m_p p_{\bar{\nu}_e} + m_p^2 \equiv 2m_p E_{\bar{\nu}_e} + m_p^2 \tag{1}$$

Where at the end the three-momentum of the anti-neutrino was chosen in the x-direction, now looking at the COM frame after the interaction and remembering that energy is minimal if the particles have no relative velocity:

$$s = (P^\mu_{e^+} + P^\mu_n)^2 = m_e^2 + m_n^2 + 2m_n m_e \tag{2}$$

So that we have a threshold energy for anti-neutrino detection via inverse β decay of:

$$s = s \implies 2m_p E_{\bar{\nu}_e} + m_p^2 = m_e^2 + m_n^2 + 2m_n m_e \implies E_{\bar{\nu}_e} = \frac{m_e^2 + m_n^2 - m_p^2 + 2m_n m_e}{2m_p} \tag{3}$$

Which is equal to 1.806 MeV.[1]

10.2 Uranium decay chain and geoneutrino flux

The uranium decay chain can take on many forms but to get from ^{238}U to ^{206}Pb the mass number should drop by 32 meaning in total there are 8 α-decays, after that though we're left with 76 protons while we should have 82 so there'll be 6 beta decays to get the amount of protons right (this was a crude analysis to get the number of beta decays and in no way how uranium decays stepwise). Every beta decay corresponds to the creation of an anti-neutrino so 1 decay chain produces 6 anti-neutrinos.
Assuming that uranium decays are responsible for 8TW of radiogenic heat, that the earth is a perfect sphere and that uranium is homogeneously distributed inside the earth we can calculate the geoneutrino flux as follows:
First off if uranium decays are responsible for 8 TW of radiogenic heat and every decay releases 52 MeV then there are

$$\frac{8\text{ TW}}{52\text{ MeV}} = \frac{8 \times 10^{12}}{52 \times 10^6 \times 1.602176634 \times 10^{-19}} \frac{1}{s} \approx 1 \times 10^{24}\text{ s}^{-1} \tag{4}$$

So every second 1×10^{24} uranium nuclei decay. As 6 anti-neutrino's are created in every decay that means 6×10^{24} geoneutrinos get released every second, and assuming the earth to be a perfect sphere we thus get a geoneutrino flux of:

$$f = \frac{N}{4\pi R_\oplus^2} = \frac{6 \times 10^{24}}{5.112 \times 10^{18}} \approx 1 \times 10^6 \frac{\text{geoneutrinos}}{cm^2 s} \tag{5}$$

Where $R_\oplus = 6.3781 \times 10^8$cm was used as the earth's radius [3].

10.3 Oscillation length

The probability that a $\bar{\nu}_e$ neutrino remains $\bar{\nu}_e$ after travelling a distance L is given by

$$P_{ee} = \sin^4\theta_{13} + \cos^4\theta_{13}\left[1 - \sin^2 2\theta_{12}\sin^2\left(\frac{\Delta m_{21}^2 L}{4E_\nu}\right)\right] \tag{6}$$

We can re-write this equation choosing Δm_{21}^2 in units [eV^2], L in [Km] and E_ν as:

$$P_{ee} = \sin^4\theta_{13} + \cos^4\theta_{13}\left[1 - \sin^2 2\theta_{12}\sin^2\left(1.267\frac{\Delta m_{21}^2[\text{eV}^2]L[\text{Km}]}{E_\nu[\text{GeV}]}\right)\right] \tag{7}$$

[1] The masses, as well as constants used in this work came from the PDG [1]

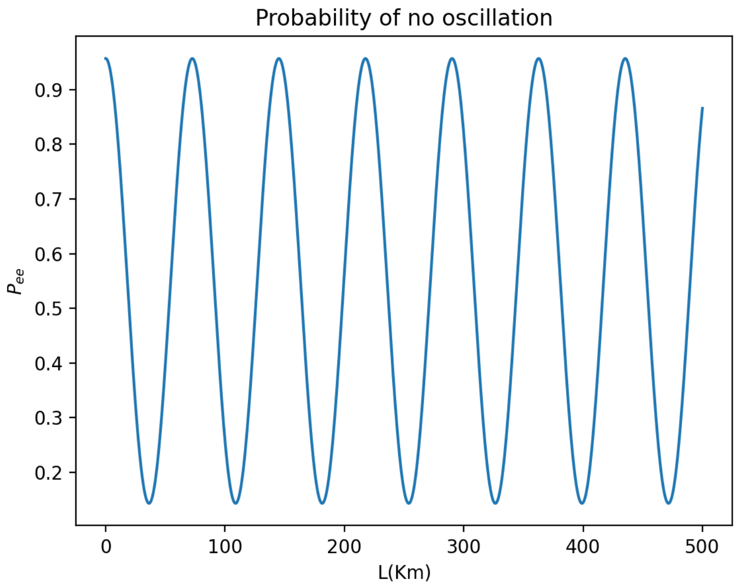

Figure 1

Filling in the values (again from the PDG) we get figure 1. The sine can be rewritten as $\sin(0.0433659 \times L)$ thus, assuming that with oscillation length one period is meant, we have an oscillation length of $\frac{2\pi}{0.0433659} \approx$ 144.89Km.
The earth's radius is $R_{\oplus} = 6.3781 \times 10^3$Km, if we assume the earth to be a sphere, then the average length a neutrino from anywhere within the earth will travel to a detector is the same as the length from the earth's core to the detector as everything perfectly cancels out. Thus if we fill in $R_{\oplus} = 6.3781 \times 10^3$Km into the equation we get $< P_{ee} > \approx 0.89$ or 89% survival probability of the $\bar{\nu}_e$ state.

10.4 Cross section for inverse β-decay

The cross section for $\bar{\nu}_e p \to e^+ n$ is given by:

$$\sigma = \frac{G_F^2}{\pi}(g_V^2 + 3g_A^2)(E_\nu - \Delta)\sqrt{(E_\nu - \Delta)^2 - m_e^2} \tag{8}$$

Filling in the values for an average geoneutrino energy of $E_\nu = 2.2$MeV we get $\sigma \approx 9.62 \times 10^{-18}\text{mb}^2 = 9.62 \times 10^{-45}\text{cm}^2$.

10.5 number of detected geoneutrino events per year

The total cross section is defined as:

$$\sigma = \frac{\text{number of reactions per unit time}}{\text{beam particles per unit time} \times \text{scattering centres per unit area}} \tag{9}$$

Assuming the detector to be a perfect cube then 1000 tons = 1000×10^3kg (assuming metric tons) of water at 1000 kg/m^3 the bottom surface area (where the geoneutrinos are coming from) can be calculated as follows:

$$V_{\text{cube}} = L^3 = \frac{1000 \times 10^3\text{Kg}}{1000\text{Kg}} m^3 = 10^3 m^3 \tag{10}$$

$$\implies \text{area} = L^2 = 10^2 m^2 \tag{11}$$

As water has a molar mass of 18.01528 g/mol, 1 mol=$6.02214076 \times 10^{23}$ and we have 1000 tons of water, there are

$$\frac{1000 \times 10^3}{18.01528 \times 10^{-3}} \times 6.02214076 \times 10^{23} = 3.34 \times 10^{31} \text{ molecules} \tag{12}$$

And thus $3.34 \times 10^{31} \times 10 = 3.34 \times 10^{32}$ protons to interact with. This implies $\frac{3.34\times10^{32}}{10^2} = 3.34 \times 10^{30}$ scattering centres per square meter or 3.34×10^{26} per cm^2. As we have previously calculated $f \approx 10^6$ geoneutrinos per unit area and time of which we'll detect a sixth ($\approx 1.66 \times 10^5$) as one in six has energy above the threshold and thus (surface in $\text{cm}^2\times$flux) $10^6 \times 1.66 \times 10^5 = 1.66 \times 10^{11}$ detectable geoneutrino's entering the volume every second. Using all this information we can calculate the number of reactions per unit time Ω:

$$\begin{aligned} \Omega &= \sigma \times \text{beam particles per unit time} \times \text{scattering centres per unit area} && (13) \\ &= (9.62 \times 10^{-45}\text{cm}^2) \times (1.66 \times 10^{11}\text{s}^{-1}) \times (3.34 \times 10^{26}\ \text{cm}^{-1}) && (14) \\ &= 5.33 \times 10^{-7}\ \text{s}^{-1} && (15) \end{aligned}$$

This implies $\approx 5.33 \times 10^{-7} \times 3600 \times 24 \times 365 \approx 17$ events per year.

10.6 How can we measure geonetrinos when the sun produces a huge neutrino flux?

The sun releases neutrinos, no antineutrinos, and it's only antineutrinos that can induce inverse β-decay. Terrestrial antineutrino observation relies on detecting both products of this inverse β reaction to know for sure that we're not measuring solar neutrino interactions. The spatial and temporal coincidence of a prompt positron and delayed neutron capture distinctly identifies the interaction of an electron antineutrino. [2]. Another important source of background would be reactor neutrinos (from nuclear fission) as these are antineutrinos, a way to control this would be to build the detector as far away from a nuclear reactor as possible.

References

[1] P.A. Zylaet al. (Particle Data Group). *Prog. Theor. Exp. Phys. 2020, 083C01*, 2020 and 2021 update.

[2] S. Dye, M. Alderman, Mikhail Batygov, J. Learned, S. Matsuno, J. Mahoney, William McDonough, S. Pakvasa, M. Rosen, S. Smith, and G. Varner. Geo-neutrino observation. *AIP Conference Proceedings*, 1182:48–51, 12 2009.

[3] E. E. Mamajek, A. Prsa, G. Torres, P. Harmanec, M. Asplund, P. D. Bennett, N. Capitaine, J. Christensen-Dalsgaard, E. Depagne, W. M. Folkner, M. Haberreiter, S. Hekker, J. L. Hilton, V. Kostov, D. W. Kurtz, J. Laskar, B. D. Mason, E. F. Milone, M. M. Montgomery, M. T. Richards, J. Schou, and S. G. Stewart. Iau 2015 resolution b3 on recommended nominal conversion constants for selected solar and planetary properties, 2015.