Subatomic Physics II

Problem set 3

Due on October 21, 2021. 11:59:59 PM

Problem 3.1: SU(2) algebra

The mathematical algebra of Lie group SU(2) is utilised several times in quantum mechanics and particle physics, describing spin (angular momentum), strong isospin of hadrons, and weak isospin of the electroweak interaction. Often one has to decompose a state $|J, M>$ into contributing direct-product states $|j_1, m_1> |j_2, m_2>$, with coefficients $C^{(J,j_1,j_2)}_{(M,m_1,m_2)}$:

$$|J, M >= \sum C^{(J,j_1,j_2)}_{(M,m_1,m_2)} |j_1, m_1 > |j_2, m_2 >$$

The coefficients C are called (for SU(2)) Clebsch-Gordan coefficients.

- Determine the ratio σ_1/σ_2 of the cross sections for the processes 1: $\pi^0 d \to pn$ and 2: $\pi^+ d \to pp$. Neglect mass-difference effects. (1pt)
- What are the relative probabilities of the strong decays of the $\Delta^+(1232)$ resonance into $n\pi^+$ and $p\pi^0$? Neglect mass-difference effects. (1pt)
- The $\Delta^+(1920)$ resonance decays into $p\rho^0$. What is the probability that the proton spin is oriented in the same direction as the Δ-spin? (1pt)
- Consider the following pion-nucleon scattering reactions:

$$\pi^+ p \to \pi^+ p \quad (1)$$
$$\pi^- p \to \pi^- p \quad (2)$$
$$\pi^- p \to \pi^0 n \quad (3)$$

  If at a certain energy the reaction proceeds through an intermediate $I = 3/2$ state, what are the relative cross sections? And if it proceeds through an intermediate $I = 1/2$ resonance? (3pt)

Problem 3.2: baryon structure

Baryons are $3q$ bound states, so in the rest frame of the baryon, corresponding to the centre-of-mass frame of the $3q$ system, there are two orbital angular momenta associated with the relative motion of the three quarks. The first is conveniently taken to be the orbital angular momentum $\vec{L}_{12}$ of a chosen pair of quarks in their mutual cms frame. The second is then the orbital angular momentum $\vec{L}_3$ of the third quark about the cms of the pair in the overall cms frame. The total orbital angular momentum is given by $\vec{L} = \vec{L}_{12} + \vec{L}_3$, while the spin is the sum of the spins of the three quarks $\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3$. What are the possible baryon states (total angular momentum J, spin S and parity P) for the ground state band ($L = 0$), and the next lowest-lying band ($L = 1$)? (4pt)