Subatomic Physics II: Problem set 5

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5.1 Strong coupling constant

In first order approximation we are given that:

$$\alpha_s(Q^2) = \frac{\alpha_s(m_Z^2)}{1+\beta_0\alpha_s(m_Z^2)ln\left(\frac{Q^2}{m_Z^2}\right)}, \text{ where } \beta_0 = \frac{33-2N_q}{12\pi} \tag{1}$$

With N_q the number of active quark flavours and $\alpha_s(m_Z^2) \approx 0.12$.

Planck scale value of α_s

For this we'll Assume $N_q = 6$ (up,down,strange,charm,bottom and top), if we fill in $Q^2 = m_{Pl}^2 = (10^{19}$ GeV$)^2$ and $m_Z = 91.1876 \pm 0.0021$GeV [1]$\approx$ 91.19 GeV (we neglect the uncertainty as it's just an approximation) in equation 1 we get that $\alpha_s(m_{Pl}^2) \approx 0.02$.

This value is lower compared to $\alpha_s(m_Z^2)$ as Q^2 is higher ($10^{38} \gg 91.19^2$), α_s behaves this way because gluons can carry colour themselves, and therefore can also couple to other gluons. The higher Q^2 is, the smaller the distances between interacting particles (quarks) and thus the gluons have less space to couple to each other making it so the interacting particles see less charge and thus decreasing α_s (also called antiscreening). There's also an effect which causes screening but this is much weaker than the antiscreening.

Find Λ in MeV

A more compact way of writing 1 is:

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)} \tag{2}$$

If we assume $N_q = 3$ (i.e only the 3 lowest energy quarks) we find Λ by plugging in $\alpha_s(m_z^2) \approx 0.12$ (as the original formula is expanded around this point) giving:

$$\begin{aligned}
\alpha_s(Q^2) &= \frac{1}{\beta_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)} && (3)\\
\ln\left(\frac{Q^2}{\Lambda^2}\right) &= \frac{1}{\alpha_s(Q^2)\beta_0} && (4)\\
\ln\left(\frac{\Lambda^2}{Q^2}\right) &= -\frac{1}{\alpha_s(Q^2)\beta_0} && (5)\\
\frac{\Lambda^2}{Q^2} &= \exp\left(-\frac{1}{\alpha_s(Q^2)\beta_0}\right) && (6)\\
\Lambda^2 &= Q^2\exp\left(-\frac{1}{\alpha_s(Q^2)\beta_0}\right) && (7)\\
&\overset{Q^2\approx 8315.18\text{ GeV}^2}{\approx} 73.6\cdot 10^3 \text{ MeV}^2 && (8)\\
\Lambda &\approx 271.2 \text{ MeV} && (9)
\end{aligned}$$

So we get that $\Lambda \approx 271.2$ MeV.

5.2 Running Quark masses

Quark masses are also running quantities, in the leading logarithmic QCD approximation we have:

$$m_q = m_q(m_Z^2)\left[\frac{\alpha_s(Q^2)}{\alpha_s(m_Z^2)}\right]^c, \text{ where } c = \frac{1}{\pi\beta_0} \tag{10}$$

Bottom quark mass at Z-boson mass scale

The Bare quark masses found in Particles and Nuclei [3] are:

Quark	Mass (MeV)
Down	≈ 300
Up	≈ 300
Strange	≈ 450
Charm	$\approx 1,250 - 1,300$
Bottom	$\approx 4,150 - 4,210$
Top	$\approx 172.5 \cdot 10^3 - 174.5 \cdot 10^3$

Now due to the fact that the b quark has a mass of about $\approx 4.2 \cdot 10^3$ MeV at this center-of-mass energy it won't be possible to produce any top quarks, and thus we'll take $N_q = 5$.

We can rewrite equation 10 as:

$$m_q(m_Z^2) = m_q \left[\frac{\alpha_s(Q^2)}{\alpha_s(m_Z^2)}\right]^{-c} = m_q \left[\frac{\alpha_s(m_Z^2)}{\alpha_s(Q^2)}\right]^{c} \tag{11}$$

With which we can calculate the mass of the bottom quark at the Z-boson mass scale, we'll start by calculating $\alpha_s(Q^2 = (4.2 \text{ GeV})^2)$ which is ≈ 0.22, plugging this, as well as $m_b = 4.2$ GeV into equation 11 yields $m_b(m_Z^2) \approx 3.07$ GeV.

Comparing values

The DELPHI Collaboration measured [4]:

$$m_b(m_Z^2) = 2.85 \pm 0.32\text{GeV} \tag{12}$$

Which means that our calculation falls inside the confidence interval of the measurement.

Experimental determination of c-quark mass running and consistency

A. Gizhko et al. have experimentally determinated the running of the c-quark mass from HERA deep-inelastic scattering data [2].

Here it's assumed that with 'are the measurements for both c and b quarks consistent' it's asked to look at the consistency of the measurements with the theory, the measurements of the charm running mass is said to be consistent with the theory in the previously mentioned paper: "The scale dependence of the mass is found to be consistent with QCD expectations". As well as that of the botton (beauty) quark: "The results are found to agree with theoretical predictions treating mass corrections at next-to-leading order" [4].

References

[1] P.A. Zylaet al. (Particle Data Group). *Prog. Theor. Exp. Phys. 2020, 083C01*, 2020 and 2021 update.

[2] A. Gizhko et al. Running of the charm-quark mass from hera deep-inelastic scattering data. *Physics Letters B*, 775:233–238, 2017.

[3] B. Povh, M. Lavelle, K. Rith, C. Scholz, and F. Zetsche. *Particles and Nuclei: An Introduction to the Physical Concepts*. Springer Berlin Heidelberg, 2015.

[4] The DELPHI Collaboration. Determination of the b quark mass at the mz scale with the delphi detector at lep. *Eur. Phys. J. C*, 46(3):569–583, 2006.