
Subatomic Physics II: Problem set 6

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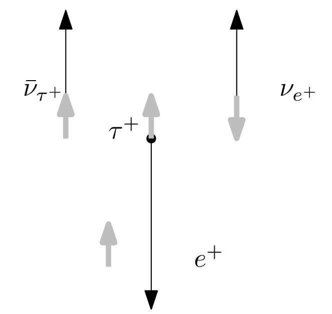
6.1 Helicity

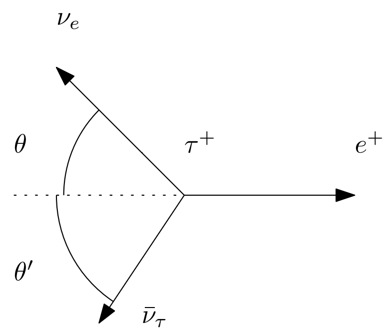
As parity violation is maximal in the V-*minus*-A *theory*, helicity defined as

$$h = \frac{\boldsymbol{s} \cdot \boldsymbol{p}}{|\boldsymbol{s}| \cdot |\boldsymbol{p}|} \tag{1}$$

will change sign during a charged weak interaction, this is only possible if either $\boldsymbol{s}$ or $\boldsymbol{p}$ changes sign. We therefore have that if we consider the decay of a τ^+ lepton with spin up this will preferably decay into a positron with spin up, antiparallel to the momentum. With the neutrino's traveling the other direction with positive helicity for the $\bar{\nu}_{\tau^+}$ and negative for the ν_{e^+} neutrino as (anti)neutrino's are always (right)left handed. For maximal positron momentum we thus have:



With the grey arrows representing the spins and the black arrows representing the momenta. Considering the decay in the rest system we choose the electron to align with the x-axes:



We have the following four-vector equation:

$$P_{\tau^+} = P_{e^+} + P_{\nu_e} + P_{\bar{\nu}_\tau} \tag{2}$$

And thus

$$P_{\tau^+} - P_{\bar{\nu}_\tau} = P_{e^+} + P_{\nu_e} \tag{3}$$

which we can square to:

$$m_\tau^2 - 2m_\tau|\boldsymbol{p}_{\bar{\nu}_\tau}| = m_{e^+}^2 + 2\left(E_{e^+}|\boldsymbol{p}_{\nu_e}| - |\boldsymbol{p}_{\nu_e}| \cdot |\boldsymbol{p}_{e^+}|\cos(\theta)\right) \tag{4}$$

And thus the energy of the positron has a spectrum following

$$E_{e^+} = \frac{m_\tau^2 - 2m_\tau|\boldsymbol{p}_{\bar{\nu}_\tau}| - m_{e^+}^2}{2|\boldsymbol{p}_{\nu_e}|} + |\boldsymbol{p}_{e^+}|\cos(\theta) \tag{5}$$

Which is maximal if $\theta = 0$ implying the neutrino's traveling parallel (conservation of three-momentum) directly away from the positron as we expect.

6.2 Particle hunting using the Dalitz plot

Bevatron

The unit in the plot is in squared BeV which stands for 10^9 elektronvolts, equivalent to today's GeV. The name comes from the Bevatron particle accelerator at Lawrence Berkeley National Laboratory, U.S., which began operating in 1954 [1]. This was a proton synchrotron which accelerated protons into a fixed target with energies up to billions of electronvots (from which the name of the accelerator, **B**illions of **eV** Synchro**tron**, came and thus the unit).

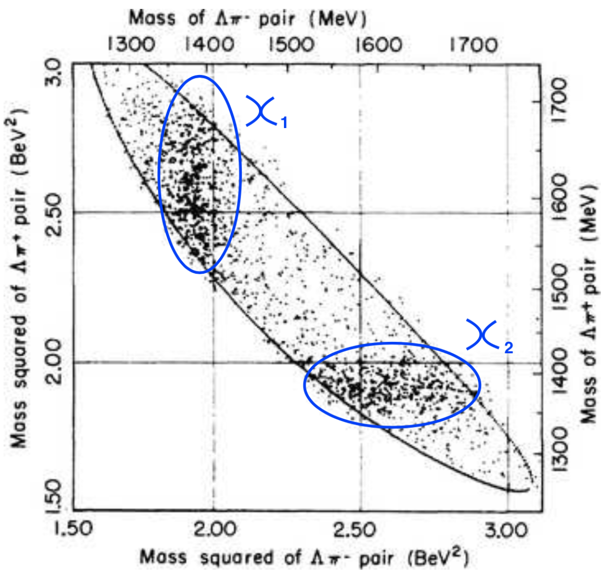
Electric charge, I(J^P) and strangeness

Mostly from appendix C of "Modern particle physics" [2]:

Hadron	Q(e)	I(J^P)	S
p	+1	$\frac{1}{2}(\frac{1}{2}^+)$	0
π^+	+1	$1(0^-)$	0
K$^-$	-1	$\frac{1}{2}(0^-)$	-1
π^-	-1	$1(0^-)$	0
Λ	0	$0(\frac{1}{2}^+)$	-1

Number of intermediate resonances

From the plot we can read off 2 resonances, X_1 at a $\Lambda\pi^-$ pair mass of ≈1380MeV with the $\Lambda\pi^+$ pair mass ranging between ≈1500 and 1700MeV and X_2 at a $\Lambda\pi^+$ pair mass of ≈1380MeV with the $\Lambda\pi^-$ pair mass between ≈1500 and 1700MeV. X_1 decays into $\Lambda\pi^-$ and X_2 decays into $\Lambda\pi^+$:



Mesons, baryons or anti-baryons?

For this we can take a look at the strictly conserved baryon number of the involved particles, the initial state has $B_{tot} = 1$ (also from appendix C, knowing that mesons have B=0 and baryons B=1). As the intermediate state has $\pi^\pm$ with B=0, the resonances X_i have to be baryons.

Electric charges and strangeness quantum numbers

As charge and strangeness are conserved quantities in the strong interaction we can just look at the difference in these numbers for the final and initial particles:

For $pK^- \to X_1\pi^+$ on the left hand side we have $Q_{tot} = 0$ and $S_{tot} = -1$, as π^+ has S=0 and a charge of 1 this means that X_1 must have a charge of -1 and a strangeness of -1.

For $pK^- \rightarrow X_2\pi^-$ on the left hand side we have $Q_{tot} = 0$ and $S_{tot} = -1$, as π^- has S=0 and a charge of -1 this means that X_2 must have a charge of 1 and a strangeness of -1.

masses

The masses of the X_i-baryons must be around ≈1380MeV as this is the mass of the $\Lambda\pi^\pm$ pair at which the resonances occur.

Strong isospin I and I_z

As the resonances decay into $\Lambda\pi^\pm$ we can couple their isospin ($|0,0\rangle \otimes |1,\pm1\rangle$), to arrive at $|1,\pm1\rangle$. X_1 thus has $|I,I_z\rangle = |1,-1\rangle$ and X_2 has $|I,I_z\rangle = |1,1\rangle$.

possibilities for J^P restricting l to 0 or 1

As X_i decays into particles $\pi^\pm$ and Λ with J^P respectively 0^- and $\frac{1}{2}^+$, the expected J^P of the X_i baryons is either $\frac{1}{2}^-$ (if Δl =0) or $\frac{3}{2}^+$ (if Δl =1 $\implies P = (-1) \times (1) \times (-1)^l = 1$).

identify X_i if J=3/2

The Resonances are Σ^* (sigma) baryons as they have a mass of 1385MeV ≈ 1380MeV. Their main decay modes are $\Lambda\pi$ and $\Sigma\pi$. In particular, looking at the charges, we can see that X_1 corresponds to $\Sigma(1385)^-$ with decay modes $\Lambda\pi^-$ and $\Sigma^0\pi^-$ and X_2 to $\Sigma(1385)^+$ with decay modes $\Lambda\pi^+$ and $\Sigma^0\pi^+$

References

[1] Berkeley Lab. Bumper crop, 1981. Lawrence and his laboratory.

[2] Mark Thomson. *Modern Particle Physics*. 2019.