Subatomic Physics II: Problem set 7

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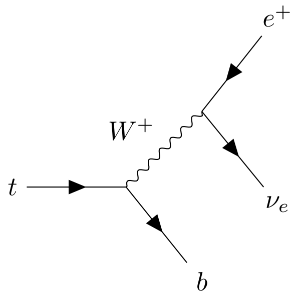
7.1 Feynman rules

7.1.1 $Ze\mu$

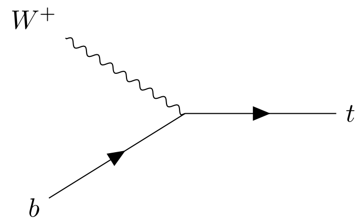
We can say that this is not possible as lepton flavour violation would occur.

7.1.2 Wtb

This is a possible vertex with the following examples for feynmann diagrams:



(a) single particle decay



(b) collision

Single particle decay
Here we have (COM frame):

$$
\begin{align}
s &= (P^\mu_{W^+} + P^\mu_b)^2 g_{\mu\nu} \tag{1}\\
s &= m^2_{W^+} + m^2_b + 2(E_{W^+}E_b - p_{W^+}\cos(\theta)p_b\cos(\theta') + p_{W^+}\sin(\theta)p_b\sin(\theta')) \tag{2}\\
s &\geq m^2_{W^+} + m^2_b + 2(E_{W^+}E_b)|_{\vec{p}_{w,b}=0} = m^2_{W^+} + m^2_b + 2(m_{W^+}m_b) \tag{3}\\
\sqrt{s} &\geq 84.56\text{GeV} \tag{4}\\
& \tag{5}
\end{align}
$$

Where $m_b = 4.18GeV$ and $m_{W^+} = 80.379GeV$[1], now as $m_t = 172.76GeV$ we see that we definitely have the required center of mass energy, even if the W^+ boson were on-shell (which it doesn't have to be).

Collision
Here we have the following four-vectors (COM frame):

$$
\begin{align}
P^\mu_{W^+} &= (E_{W^+}, 0, -p, 0) \tag{6}\\
P^\mu_b &= (E_b, 0, p, 0) \tag{7}
\end{align}
$$

Doing analogues calculations as the previous one:

$$
\begin{align}
s &= (P^\mu_{W^+} + P^\mu_b)^2 g_{\mu\nu} \tag{8}\\
s &= m^2_{W^+} + m^2_b + 2(E_{W^+}E_b + p^2) \tag{9}\\
s &= m^2_{W^+} + m^2_b + 2\left(\sqrt{m^2_{W^+} + p^2}\sqrt{m^2_b + p^2} + p^2\right) \tag{10}\\
& \tag{11}
\end{align}
$$

Now as $m^2_\tau \leq s$ we have that $p \geq 67.6$ GeV for the t-quark to be on shell, in which W^+ has to be on-shell.

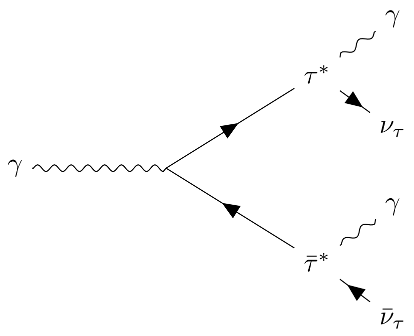
[1] All the masses used in this work are obtained from the PDG [2]

7.1.3 $H\gamma\gamma$

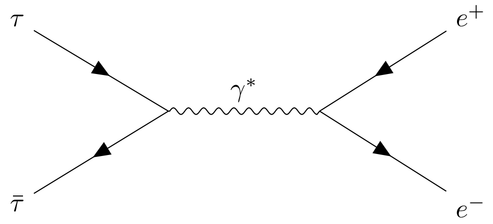
Although this is in principle 'possible' as "the Higgs boson can decay to all Standard Model particles" [1], the coupling strenght of the higgs boson is proportional to the mass of the particles involved and thus this will tend to zero for the photon.

7.1.4 $\gamma\tau\tau$

The decay isn't possible for on-shell particles as we could always Lorentzboost to a frame of reference where 3-momentum $\mathbf{p}$ isn't conserved (e.g the τ particles' COM frame). It is however possible for the photon to decay into 2 off-shell τ leptons (denoted τ^* and $\bar{\tau}^*$) as then we can momentarily violate 3-momentum conservation:



(a) single particle decay

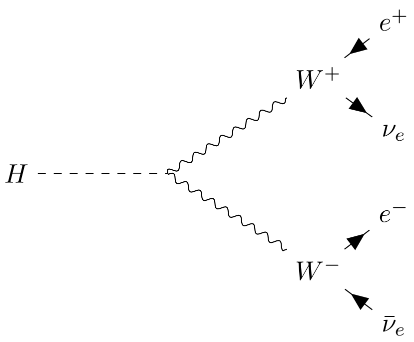


(b) collision

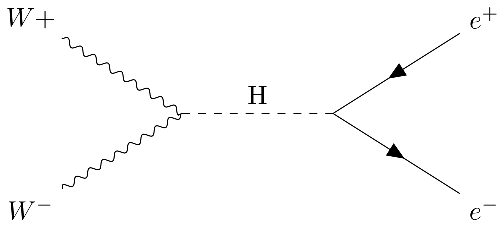
And, as seen from the figure, it's also possible for 2 on-shell tau leptons to decay into a virtual photon which decays into an electron-positron pair, as $m_\tau \gg m_e$ there's no need to concern ourselves with the COM calculations.

7.1.5 HWW

This is a possible vertex with the following examples:



(a) single particle decay



(b) collision

Single particle decay
This is possible but as $m_H < 2m_W$ (125.25GeV<160.758GeV) One of the W bosons produced is off-shell.

Collision
As $m_H < 2m_W$ at least one of the W bosons has to be virtual, we can understand this by looking at the COM frame. before the interaction we have:

$$s = (P^\mu_{W^+} + P^\mu_{W^-})^2 g_{\mu\nu} \tag{12}$$

$$s = m^2_{W^+} + m^2_{W^-} + 2(E_{W^+}E_{W^-} + pp) \tag{13}$$

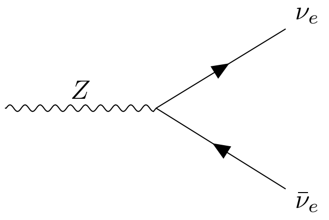
$$s_{\min} \overset{p\to 0}{=} m^2_{W^+} + m^2_{W^-} + 2m_{W^+}m_{W^-} \tag{14}$$

$$\tag{15}$$

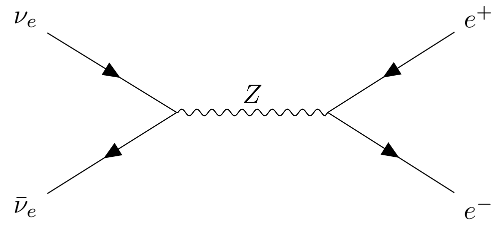
Which implies a minimal COM energy way bigger than 125.25GeV if both bosons are on-shell.

7.1.6 $Z\nu_e\nu_e$

This is a possible vertex with example diagrams:



(a) single particle decay



(b) collision

Single particle decay
As we're working with the tree-level Feynman rules of the Standard Model, we assume the neutrino to be massless (not that this matters much as $m_{\nu_e} < 1.1\text{eV}$). In the COM frame we thus have:

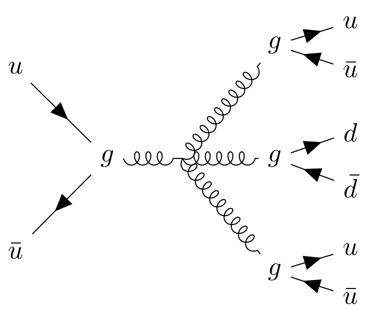
$$m_Z^2 = s = (P^{\mu}_{\nu_e} + P^{\mu}_{\bar{\nu}_e})^2 g_{\mu\nu} \tag{16}$$
$$= 2E^2 \tag{17}$$

I.e the neutrinos have energies of $\frac{1}{\sqrt{2}} m_Z = \frac{1}{\sqrt{2}} 91.1876\text{GeV} = 64.48\text{GeV}$.

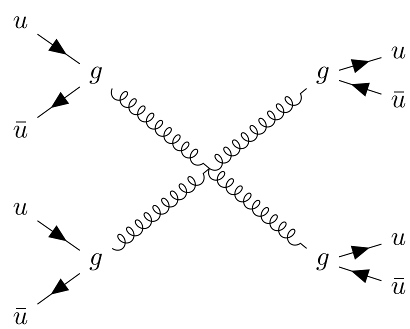
Collision
If the neutrinos in this interaction have energies of 64.48GeV, then this is a possible on-shell Z-boson production mechanism. It's also possible to produce off-shell Z-bosons with less energetic neutrinos.

7.1.7 $gggg$

This is a possible vertex with the following feynmann diagrams:



(a) Particle decay



(b) gg $\to$ gg

As all the gluons are off-shell (due to the specific nature of gluons and color confinement), the kinematic condition is reduced to if there's enough COM energy to produce the observed pions:

Decay
Here we have that the end COM energy $\sqrt{s} \geq 3 \times m_{\pi^0} = 404.91$ MeV, the com energy on the left is:

$$\sqrt{s} = \sqrt{2m_u^2 + 2P_u P_{u'}} = \sqrt{2m_u^2 + 2m_u^2 + |\boldsymbol{p}|^2} \approx |\boldsymbol{p}| \tag{18}$$

And thus $|\boldsymbol{p}| \geq 405$ MeV.
Collision
In the collision, the incoming and outgoing particles are the same and so because of conservation of energy there are no kinematic terms that need to be computed.

References

[1] Mark Thomson. *Modern Particle Physics*. 2019.

[2] P.A. Zyla et al. Review of Particle Physics. *PTEP*, 2020(8):083C01, 2020.