Subatomic Physics II

Problem Set 5

Due on November 4, 2021, 11:59 PM

Problem 5.1: Strong coupling constant

In QCD, the coupling α_s is not a true constant but depends on the energy scale: $\alpha_s(Q^2)$. For example, if we know its value at the $Z-$boson pole, $\alpha_s(Q^2 = m_Z^2) \approx 0.12$, then, in the first approximation,

$$\alpha_s(Q^2) = \frac{\alpha_s(m_Z^2)}{1 + \beta_0 \alpha_s(m_Z^2) \ln\left(\frac{Q^2}{m_Z^2}\right)}, \quad \text{where} \quad \beta_0 = \frac{33 - 2N_q}{12\pi}, \tag{1}$$

and N_q is the number of active quark flavours.

- Assuming $N_q = 6$, find the value of α_s at the Planck scale, $\sqrt{Q^2} = m_{Pl} = 10^{19}$ GeV. Why does this value go up/down compared to $\alpha_s(m_Z^2)$? (1pt)
- A more compact way of writing Eq. (1) is $\alpha_s(Q^2) = 1/\left[\beta_0 \ln(Q^2/\Lambda^2)\right]$. Assuming $N_q = 3$, find the value of Λ in MeV. (1.5pt)

Problem 5.2: Running quark masses

Quark masses are also running quantities: $m_q(Q^2)$. The deeper reason for this is that they are not observable as free particles, and one must always take into account their coupling to the strong field. In the leading logarithmic QCD approximation, they are given by

$$m_q = m_q(m_Z^2) \left[\frac{\alpha_s(Q^2)}{\alpha_s(m_Z^2)}\right]^c, \quad \text{where} \quad c = \frac{1}{\pi\beta_0}, \tag{2}$$

with the same β_0 as in (1). The b-quark mass evaluated at its own mass scale is $m_b(Q^2 = m_b^2) = 4.2$ GeV.

- Calculate its value at the mass scale of the Z-boson, $m_b(M_Z^2)$, and motivate your choice of N_q in this calculation. (3pt)
- Compare your calculated value to an experimental result. Do they agree? (1.5pt)
- Find a publication on the experimental determination of the c-quark mass running. Are the measurements for both c and b quarks consistent? (3pt)