Radio detection of high energy neutrinos in the Greenland icecap

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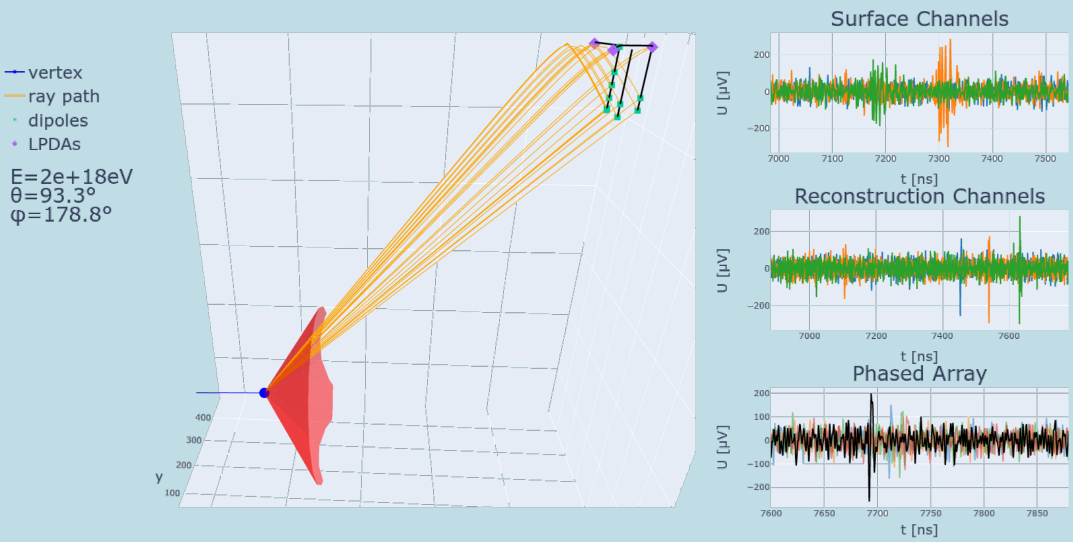

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1 NEUTRINOS

1.1 Discovery

1.2 Standard model

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1.3.1 Cosmic neutrinos

To estimate the temperature of the neutrinos who decoupled at the start of the universe, we can take a look at conservation of entropy [1] (...) The entropy before and after decoupling are:

$$s(a_1) = \frac{2\pi^2}{45}(2 + \frac{7}{8}(2 + 2 + 3 + 3))T_1^3 \tag{1.1}$$

$$= \frac{2\pi^2}{45}\frac{86}{8}T_1^3 \tag{1.2}$$

$$s(a_2) = \frac{2\pi^2}{45}(2T_\gamma^3 + \frac{7}{8}(6)T_\nu^3) \tag{1.3}$$

$$\tag{1.4}$$

Conservation of entropy:

$$s(a_1)a_1^3 = s(a_2)a_2^3 \tag{1.5}$$

$$\frac{86}{8}(T_1 a_1)^3 = \left(2\left(\frac{T_\gamma}{T_\nu}\right)^3 + \frac{42}{8}\right)(T_\nu a_2)^3 \tag{1.6}$$

$$\frac{86}{8} = 2\left(\frac{T_\gamma}{T_\nu}\right)^3 + \frac{42}{8} \tag{1.7}$$

$$\frac{44}{16} = \left(\frac{T_\gamma}{T_\nu}\right)^3 \tag{1.8}$$

$$\left(\frac{T_\gamma}{T_\nu}\right) = \left(\frac{11}{4}\right)^{1/3} \tag{1.9}$$

i.e

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \tag{1.10}$$

Φ

1.3.2 Oscillations

1.3.3 Majorana

CHAPTER

2 RADIO DETECTION

Here I'll first give a short overview of the equations governing radiation emitted by moving charges, more specifically of Cherenkov radiation. The reader who wants a thorough explanation and derivation is advised to check out *Chapter 14: Radiation by Moving Charges* from the book *Classical Electrodynamics* by Jackson.

2.1 Spectral distribution of radiation

We wish to know the emitted energy per elementary unit solid angle over a certain frequency interval for a moving charge far away from the source. For this we have that the vectorpotential **A**, defined as

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{2.1}$$

takes the form

$$\mathbf{A}(\omega) = \frac{q}{4\pi\sqrt{2\pi}}\sqrt{\frac{\mu}{\epsilon}}\frac{e^{ikr}}{r}\boldsymbol{\alpha} \tag{2.2}$$

with q the charge, r the distance from the charge to the observer and

$$\boldsymbol{\alpha} = \int_{\infty}^{\infty} \boldsymbol{\beta}(t) e^{i\omega(t - \boldsymbol{e}_r \cdot \boldsymbol{r}_0(t)/c)} \mathrm{d}t \tag{2.3}$$

With $\boldsymbol{\beta} := \boldsymbol{u}/c$ and $\boldsymbol{u}$ the speed of the particle, the integration is along the path of the moving charged particle. The energy emitted per unit solid angle is given by

$$\frac{\mathrm{d}\mathscr{P}}{\mathrm{d}\Omega} = R'^2 \mathbf{S}(t) \cdot \mathbf{n}' \tag{2.4}$$

Defining $\mathscr{E}$ to be the time integral of this, we can reformulate this into (standard practice to integrate over the frequencies)

$$\frac{\mathrm{d}\mathscr{E}}{\mathrm{d}\Omega} = r^2 \int_{\infty}^{\infty} \mathrm{d}\omega (\mathbf{E}(\omega) \times \mathbf{H}(-\omega)) \cdot \mathbf{e}_r = \int_0^{\infty} \frac{\mathrm{d}^2 \mathscr{I}(\omega)}{\mathrm{d}\omega \mathrm{d}\Omega} \tag{2.5}$$

i.e $\frac{\mathrm{d}^2\mathscr{I}}{\mathrm{d}\omega\mathrm{d}\Omega}$ is the energy radiated per elementary unit solid angle and per elementary unit frequency interval, re-writing gives

$$\frac{\mathrm{d}^2\mathscr{I}(\omega)}{\mathrm{d}\omega\mathrm{d}\Omega} = 2r^2\Re\{\mathbf{E}(\omega)\times\mathbf{H}^*(\omega))\}\cdot\mathbf{e}_r \tag{2.6}$$

up to $\mathcal{O}(r^{-2})$ we get

$$\frac{\mathrm{d}^2\mathscr{I}(\omega)}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{q^2\omega^2}{16\pi^3}\sqrt{\frac{\mu}{\epsilon}}|\mathbf{e}_r\times(\mathbf{e}_r\times\boldsymbol{\alpha})|^2 \tag{2.7}$$

2.2 Cherenkov radiation

Cherenkov radiation is like the elektromagnetic equivalent of a sonic boom, a sonic boom happens when something goes faster than the sounds speed in the medium; A particle emits Cherenkov radiation if it goes faster than the light speed in the medium. Choosing the particle trajectory to lie along the z axis we can approximate equation 2.7 as

$$\frac{\mathrm{d}^2\mathscr{I}(\omega)}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{q^2}{4\pi}\sqrt{\frac{\mu}{\epsilon}}\beta^2\omega^2\delta^2[\omega(1-\beta\mathbf{e}_r\cdot\mathbf{e}_z)]|\mathbf{e}_r\times\mathbf{e}_z|^2 \tag{2.8}$$

or, in spherical coordinates, $1-\beta\mathbf{e}_r\cdot\mathbf{e}_z = 1-\beta\cos(\theta_c)$ in the delta function. We thus only expect radiation if

$$\cos(\theta_c) = \frac{1}{\beta} = \frac{c}{u} \tag{2.9}$$

I.e if $u > c$ Cherenkov radiation will be emitted along a cone surface with half angle $\frac{\pi}{2}-\theta_c$ as illustrated in figure 2.2. Integrating equation 2.8 over the solid angle and formally deviding by the time interval we get:

$$\frac{\mathrm{d}^2\mathscr{I}}{\mathrm{d}\omega\mathrm{d}t} = \frac{q^2}{4\pi}\sqrt{\frac{\mu}{\epsilon}}\beta\omega\left(1-\frac{1}{\beta^2}\right) \tag{2.10}$$

We see that the energy is proportional to ω, so we expect that most radiation will be emitted ”in blue”, as seen in figure 2.1.

Figure 2.1: Cherenkov radiation in a nuclear reactor

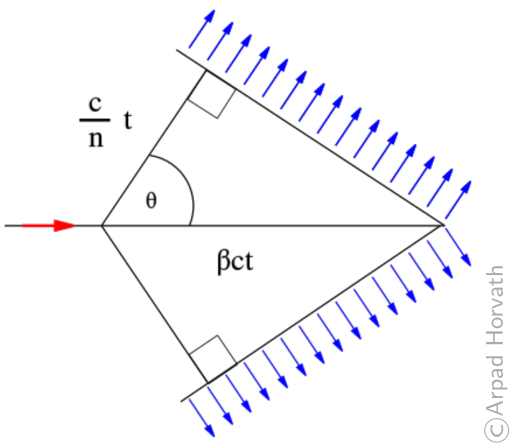

Figure 2.2: Diagrammatic representation of Cherenkov radiation

BIBLIOGRAPHY

[1] Scott Dodelson. Modern Cosmology. Academic Press, Amsterdam, 2003.