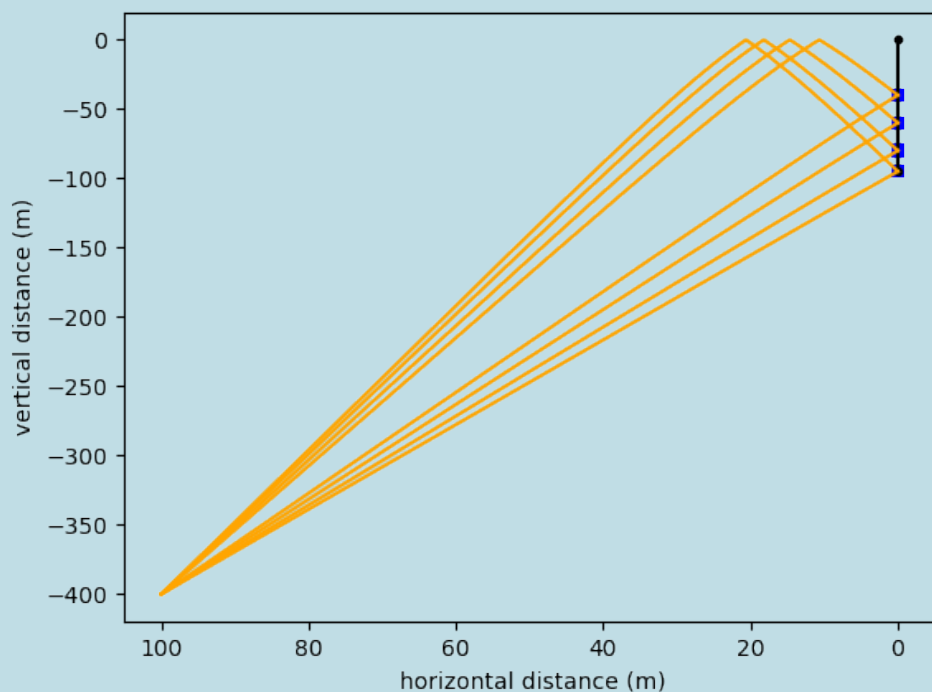


Radio detection of high energy neutrinos in the Greenland icecap

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CHAPTER

1

FOREWORD

1.1 Abstract: English

The Radio Neutrino Observatory in Greenland - RNO-G - is under construction at Summit Station in Greenland to search for neutrinos of several PeV energy up to the EeV range. It's a mid-scale, discovery phase, extremely high-energy neutrino telescope that will probe the astrophysical neutrino flux at energies beyond the reach of IceCube. More particularly it will make it possible to reach the next major milestone in astroparticle physics: the discovery of cosmogenic neutrinos. All simulations carried out within this work were made with the three

programs

- NuRadioMC
- radiotools
- RadioPropa

whom are free to download on [github](#). The projects built over the course of this thesis were the "Hybrid minimizer": a new algorithm for finding the path of a neutrino interaction to the detector in complex ice models and (...)

1.2 Abstract: Nederlands

De *Radio Neutrino Observatory in Greenland* - RNO-G - is een detector die momenteel onder constructie is in Groenland. Deze detector heeft als doel het vinden van neutrino's met energieën van enkele PeV tot in het EeV gebied. Deze extreem hoge energy neutrino telescoop zal astrofysische neutrinos zoeken op energieën waar IceCube te klein voor is. Het zal ook het vinden van cosmogenische neutrinos mogelijk maken. Alle simulaties werden mogelijk gemaakt door de volgende programmas:

- NuRadioMC

- radiotools
- RadioPropa

Dewelke te downloaden zijn op [github](#) De projecten die tot completie zijn gebracht over deze master thesis waren de *Hybrid minimizer*: een algoritme gemaakt met als doel het snel vinden van een pad vanuit de interactievertex tot de detector in complexe ijsmodellen en (...)

CHAPTER

2

NEUTRINOS

When looking outside into the night sky you might see various stars, but invisible to the naked eye various kinds of particles are also transversing light-years to end up where you stand.

When looking at phenomena outside our earth the astronomer will turn to electromagnetic radiation, as a lot of the most interesting events emit photons in some kind of way this is very useful but he's missing out on a big part of the full picture. That's where the astroparticle physicist comes in who takes a look at muons, nuclei,... and even neutrinos.

Neutrinos are ideal messengers to identify sources of ultra high energy (UHE) cosmic rays in the universe. They also interact very weakly making it possible for them to traverse large distances unhindered. To fully grasp how little they interact consider that on average 100 trillion neutrinos pass through your body per second, none of them having any effect. You'd also need a light year of lead to give you a 50% chance of stopping a neutrino.

2.1 Discovery

When researching beta decay, the decay of a neutron, researcher detected a proton and an electron coming from the neutron. However on closer inspection it became apparent that energy was lost somewhere in violation with the conservation law of energy, and angular momentum wasn't conserved. The solution postulated by Wolfgang Pauli was to introduce a new, really hard to detect particle: the neutrino. The neutrino comes in three flavours: electron, muon and tau neutrinos, each corresponding to their respective lepton and each also having an anti-particle. Now with the introduction of the neutrino the full beta decay becomes

$$n \rightarrow p^+ + e^- + \nu_e \quad (2.1)$$

The inverse can then also be used to detect neutrinos:

$$n \rightarrow p^+ + e^- + \nu_e \quad (2.2)$$

Which is called beta capture and was first experimentally detected in 1956 [4] also making it the first experimental detection of a neutrino

2.2 Neutrino sources

2.3 Do neutrinos have mass?

Neutrinos get produced through the following ways in the nucleus of our star:

pp – cycle

$$p + p \rightarrow D + e^+ + \nu_e \quad (2.3)$$

$$D + p \rightarrow {}^3\text{He} + \gamma \quad (2.4)$$

$${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + p + p \quad (2.5)$$

boron – cycle

$${}^4\text{He} + {}^3\text{He} \rightarrow {}^7\text{Be} + \gamma \quad (2.6)$$

$${}^7\text{Be} + p \rightarrow {}^8\text{B} + \gamma \quad (2.7)$$

$${}^8\text{B} \rightarrow {}^8\text{Be}^* + e^+ + \nu_e \quad (2.8)$$

$${}^8\text{Be}^* \rightarrow {}^4\text{He} + {}^4\text{He} \quad (2.9)$$

Be – capture

$${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e \quad (2.10)$$

pep

$$p + e^- + p \rightarrow D + \nu_e \quad (2.11)$$

Now with this and some information about the sun like the pressure and mass, the so-called "standard solar model" was made. This model predicted a certain amount of neutrinos to be hitting the earth from the previously mentioned thermonuclear fusion, it was however 3 times higher than the observed amount of neutrinos back at our planet. Through various experiments it became apparent that this was due to the various kind of neutrinos oscillating into each other, i.e 2/3 of the original electron neutrinos had oscillated into mu and tau neutrinos but for them to oscillate into each other, they require mass:

2.3.1 neutrino oscillations

So the transition probability is

$$P(\nu_e \rightarrow \nu_\mu) = |\langle \nu_\mu | \psi(L, T) \rangle|^2 = c_\mu c_\mu^* = \sin^2(2\theta) \sin^2\left(\frac{\Delta\phi_{12}}{2}\right) \quad (2.12)$$

with

$$\Delta\phi_{12} \approx \frac{m_1^2 - m_2^2}{2p} L \quad (2.13)$$

I.e not only mass but difference in mass of the eigenstates is a requirement for oscillations to occur. In general

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle \quad (2.14)$$

With $U_{\alpha i}$ e.g the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. This phenomenon has been observed e.g from the discrepancy from the observed and expected neutrino events coming from a nuclear reactor [6]. Now the question remains, how do neutrinos get their mass (theoretically)?

2.3.2 See-saw mechanism

We can give neutrino's mass as with any other spinor through Yukawa couplings and an extra term:

$$\mathcal{L} = i\bar{\nu}\not{\partial}\nu - m\bar{\psi}\psi - \frac{M}{2}(\bar{\nu}_R\nu_{Rc} + \bar{\nu}_{Rc}\nu_R) \quad (2.15)$$

More general, given a spinor ψ with the following Lagrange density:

$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi - \frac{M}{2}(\bar{\psi}_R\psi_{Rc} + \bar{\psi}_{Rc}\psi_R) \quad (2.16)$$

We can re-write this Lagrangian in terms of

$$\chi := \frac{1}{\sqrt{2}}(\psi_R + \psi_{Rc}) \quad (2.17)$$

$$\omega := \frac{1}{\sqrt{2}}(\psi_L + \psi_{Lc}) \quad (2.18)$$

by first identifying that

$$-\frac{M}{2}(\bar{\psi}_R\psi_{Rc} + \bar{\psi}_{Rc}\psi_R) \quad (2.19)$$

is equal to

$$-M\bar{\chi}\chi = -\frac{M}{2}(\bar{\psi}_R + \bar{\psi}_{Rc})(\psi_R + \psi_{Rc}) = -\frac{M}{2}(\bar{\psi}_R\psi_{Rc} + \bar{\psi}_{Rc}\psi_R) \quad (2.20)$$

Where we've used

$$\bar{\psi}_R\psi_R \equiv \psi_R^\dagger\gamma^0\psi_R \equiv P_R\psi^\dagger\gamma^0P_R\psi = P_RP_L\psi^\dagger\gamma^0\psi = 0 \quad (2.21)$$

And next that in

$$i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi = i\bar{\psi}_R\not{\partial}\psi_R + i\bar{\psi}_L\not{\partial}\psi_L - m\bar{\psi}_R\psi_L - m\bar{\psi}_L\psi_R \quad (2.22)$$

We can identify that

$$\bar{\chi}\not{\partial}\chi = \frac{1}{2}(\bar{\psi}_R\not{\partial}\psi_R + \bar{\psi}_{Rc}\not{\partial}\psi_{Rc}) = \bar{\psi}_R\not{\partial}\psi_R \quad (2.23)$$

$$\bar{\omega}\not{\partial}\omega = \bar{\psi}_L\not{\partial}\psi_L \quad (2.24)$$

Where we've done an integration by parts and used $\bar{\psi}_c\gamma_\mu\chi_c = -\bar{\chi}\gamma_\mu\psi$. We thus get:

$$\mathcal{L} = i\bar{\chi}\not{\partial}\chi + i\bar{\omega}\not{\partial}\omega - m\bar{\psi}_R\psi_L - m\bar{\psi}_L\psi_R - M\bar{\chi}\chi \quad (2.25)$$

And finally we notice that

$$\bar{\chi}\omega = \frac{1}{2}(\bar{\psi}_R\psi_L + \bar{\psi}_R\psi_{Lc} + \bar{\psi}_{Rc}\psi_L + \bar{\psi}_{Rc}\psi_{Lc}) \quad (2.26)$$

$$= \frac{1}{2}(\bar{\psi}_R\psi_L + \bar{\psi}_R\psi_{Lc} + \bar{\psi}_{Rc}\psi_L + \bar{\psi}_L\psi_R) \quad (2.27)$$

$$\bar{\omega}\chi = \frac{1}{2}(\bar{\psi}_L\psi_R + \bar{\psi}_{Lc}\psi_R + \bar{\psi}_L\psi_{Rc} + \bar{\psi}_{Lc}\psi_{Rc}) \quad (2.28)$$

$$= \frac{1}{2}(\bar{\psi}_L\psi_R + \bar{\psi}_{Lc}\psi_R + \bar{\psi}_L\psi_{Rc} + \bar{\psi}_R\psi_L) \quad (2.29)$$

As $\bar{\psi}_c\chi_c = \bar{\chi}\psi$. Summing these, we get:

$$\bar{\chi}\omega + \bar{\omega}\chi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L \quad (2.30)$$

$$+ \frac{1}{2}(\bar{\psi}_R\psi_{Lc} + \bar{\psi}_{Rc}\psi_L + \bar{\psi}_{Lc}\psi_R + \bar{\psi}_L\psi_{Rc}) \quad (2.31)$$

$$= \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L \quad (2.32)$$

Where we've used the handedness flips a charge conjugation entails, at last we now have:

$$\mathcal{L} = i\bar{\chi}\not{\partial}\chi + i\bar{\omega}\not{\partial}\omega - m(\bar{\chi}\omega + \bar{\omega}\chi) - M\bar{\chi}\chi \quad (2.33)$$

Now to determine the mass eigenstates we'll re-write this equation into the form

$$\mathcal{L} = i\bar{\chi}\not{\partial}\chi + i\bar{\omega}\not{\partial}\omega - \begin{pmatrix} \bar{\chi} & \bar{\omega} \end{pmatrix} \cdot \begin{pmatrix} M & m \\ m & 0 \end{pmatrix} \cdot \begin{pmatrix} \chi \\ \omega \end{pmatrix} \quad (2.34)$$

lets now find the eigenvalues:

$$\begin{vmatrix} M - \lambda & m \\ m & -\lambda \end{vmatrix} = (\lambda - M)\lambda - m^2 = 0 \implies \lambda_{\pm} = \frac{M \pm \sqrt{M^2 + 4m^2}}{2} \quad (2.35)$$

i.e

$$\lambda_{\pm} = \frac{M}{2} \left(1 \pm \sqrt{1 + 4m^2/M^2} \right) \quad (2.36)$$

Now we'll determine the eigenvectors:

$$\begin{bmatrix} M - \lambda_{\pm} & m \\ m & -\lambda_{\pm} \end{bmatrix} \begin{pmatrix} x_{\pm} \\ y_{\pm} \end{pmatrix} = \vec{0} \quad (2.37)$$

this gives

$$Mx_{\pm} - \lambda_{\pm}x_{\pm} + my_{\pm} = 0 \quad (2.38)$$

$$mx_{\pm} - \lambda_{\pm}y_{\pm} = 0 \implies \frac{mx_{\pm}}{\lambda_{\pm}} = y_{\pm} \quad (2.39)$$

inserting the second equation in the first we get:

$$Mx_{\pm} - \lambda_{\pm}x_{\pm} + m\frac{mx_{\pm}}{\lambda_{\pm}} = 0 = M\lambda_{\pm}x_{\pm} - \lambda_{\pm}^2x_{\pm} + m^2x_{\pm} \quad (2.40)$$

$$= x_{\pm}(\lambda_{\pm}^2 - M\lambda_{\pm} + m^2) \quad (2.41)$$

which is true for every x_{\pm} (see equation 2.35) i.e we can freely choose x_{\pm} so let's opt for $x_{\pm}^2 + y_{\pm}^2 = 1$:

$$x_{\pm}^2 \left(1 + \frac{m^2}{\lambda_{\pm}^2} \right) = 1 \implies x_{\pm}^2 = \frac{1}{1 + \frac{m^2}{\lambda_{\pm}^2}} \implies x_{\pm} = \frac{\lambda_{\pm}}{\sqrt{\lambda_{\pm}^2 + m^2}} \quad (2.42)$$

$$\& \quad y_{\pm} = \frac{m}{\sqrt{\lambda_{\pm}^2 + m^2}} \quad (2.43)$$

And thus, summing up, we get the eigenvectors ϕ_{\pm} with eigenvalues m_{\pm} :

$$\phi_{\pm} := \begin{pmatrix} x_{\pm} \\ y_{\pm} \end{pmatrix} = \begin{pmatrix} \frac{\lambda_{\pm}}{\sqrt{\lambda_{\pm}^2 + m^2}} \\ \frac{m}{\sqrt{\lambda_{\pm}^2 + m^2}} \end{pmatrix} \quad \text{and} \quad m_{\pm} := \lambda_{\pm} = \frac{M}{2} \left(1 \pm \sqrt{1 + 4m^2/M^2} \right) \quad (2.44)$$

Now we'll consider the limit $M \gg m$:

$$m_{\pm} = \frac{M}{2} \left(1 \pm \sqrt{1 + 4m^2/M^2} \right) \approx \frac{M}{2} \left(1 \pm 1 \pm \frac{2m^2}{M^2} \right) \quad (2.45)$$

Which gives:

$$m_{+} \approx M \quad (2.46)$$

$$m_{-} \approx -\frac{m^2}{M} \quad (2.47)$$

and:

$$\phi_+ \approx \begin{pmatrix} \frac{M}{m} \\ \frac{m}{M} \end{pmatrix} = \begin{pmatrix} 1 \\ m/M \end{pmatrix} \quad (2.48)$$

$$\phi_- \approx \begin{pmatrix} -\frac{m}{M} \\ \frac{m}{m} \end{pmatrix} = \begin{pmatrix} -m/M \\ 1 \end{pmatrix} \quad (2.49)$$

i.e, in the χ, ω basis:

$$m_+ \approx M \quad \text{with eigenstate} \quad \chi + \frac{m}{M}\omega \quad (2.50)$$

$$m_- \approx -\frac{m^2}{M} \quad \text{with eigenstate} \quad \omega - \frac{m}{M}\chi \quad (2.51)$$

$$(2.52)$$

This is what's called the See-Saw mechanism, it's both a theory of giving mass to neutrinos and explaining their small mass. For a certain m if we choose a big M we'll get a big m_+ and small m_- (and vice versa). We also see that there's only a very small mixing of states, i.e the m_- mass state is almost purely ω (and m_+ almost purely χ). The parameter m in the original matrix is forbidden by electroweak gauge symmetry, and can only appear after the symmetry has been spontaneous broken by a Higgs mechanism; for this reason a good estimate of the order of m is the vacuum expectation energy: $m \approx v = 246 \approx 10^2 \text{ GeV}$. In grand unified theories it's theorised that $M \approx 10^{15} \text{ GeV}$ after symmetry breaking, using these values we get

$$m_- \approx 10^{-11} \text{ GeV} \approx 10^{-2} \text{ eV} \quad (2.53)$$

which seems [2] to be a reasonable order of magnitude estimate for the observed neutrino mass. This mechanism would also lead to supermassive neutrinos, which are a possible dark matter candidate.

2.3.3 Majorana fermions

A theory that's also quite interesting is that neutrinos are majorana, a dirac fermion has the following density:

$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi \quad (2.54)$$

$$= i\bar{\psi}_L\not{\partial}\psi_L + i\bar{\psi}_R\not{\partial}\psi_R - m\bar{\psi}_R\psi_L - m\bar{\psi}_L\psi_R \quad (2.55)$$

Now assume that we only have right-handed particles:

$$\mathcal{L} = i\bar{\psi}_R\not{\partial}\psi_R - \frac{M}{2}\bar{\psi}_R\psi_{Rc} - \frac{M}{2}\bar{\psi}_{Rc}\psi_R \quad (2.56)$$

A majorana fermion is a fermion which is it's own anti-particle, i.e:

$$\chi = \frac{1}{\sqrt{2}}(\psi_R + \psi_{Rc}) = \chi_c \quad (2.57)$$

With this we have that

$$\bar{\chi}\chi = \frac{1}{2}(\bar{\psi}_R\psi_{Rc} + \bar{\psi}_{Rc}\psi_R) \quad (2.58)$$

Which we can recognize in 2.56 and

$$\bar{\chi}\not{\partial}\chi = \frac{1}{2}(\bar{\psi}_R + \bar{\psi}_{Rc})\not{\partial}(\psi_R + \psi_{Rc}) \quad (2.59)$$

$$= \frac{1}{2}\bar{\psi}_R\not{\partial}\psi_R + \frac{1}{2}\bar{\psi}_{Rc}\not{\partial}\psi_{Rc} \quad (2.60)$$

$$= \bar{\psi}_R\not{\partial}\psi_R \quad (2.61)$$

where we've used integration by parts at the end, with these we can re-write 2.56 as:

$$\mathcal{L} = i\bar{\chi}\not{\partial}\chi - M\bar{\chi}\chi \quad (2.62)$$

Thus arriving at a density for Majorana fermions with mass M , a possible detection mechanism to find out if neutrinos are Majorana is "Neutrinoless double beta decay":



Figure 2.1: normal and neutrinoless double beta decay

This is (at the time of writing) actively investigated.

2.4 Why study neutrinos?

Neutrinos are ideal messengers to identify the UHE (Ultra High Energy) sources in the universe. Unlike cosmic rays, which are deflected by magnetic fields and interact with matter and radiation on their way to us, neutrinos point back to sources and can reach Earth unperturbed from the most distant corners of the universe. Neutrinos can be generated in 2 ways: either they're generated in interactions at the sources, termed *astrophysical neutrinos*. Or they're created through the interaction of ultra-high energy cosmic rays during propagation with the cosmic microwave or other photon backgrounds termed *cosmogenic neutrinos*.

2.4.1 astrophysical neutrinos: general

As cosmic rays get accelerated they will occasionally interact with matter and photon fields near the source. These will then produce new particles, of which the most common is pions due to their low mass (a lot of available phase space). As seen in chapter ?? pions can decay into neutrino's as follows:

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \rightarrow e^- + \bar{\nu}_e + \nu_\mu + \bar{\nu}_\mu \quad (2.63)$$

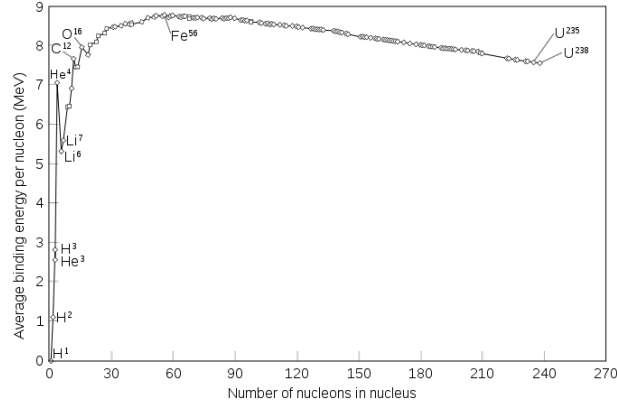
$$\pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu \quad (2.64)$$

Another possibility is direct production of neutrino's at the source, of which Supernovae are a prime example:

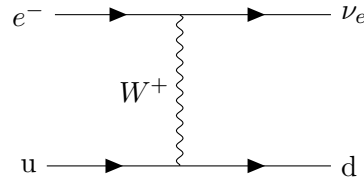
2.4.2 Supernovae

A star starts its life as a ball of pure hydrogen. At the core, due to the gravitational pressure of the outside plasma, fusion of hydrogen into deuterium and helium happens. Thus converting mass into energy. The pressure of this energy counteracts the pressure of gravity and the star is stable.

When the hydrogen at the core runs out no more hydrogen can be fused. For stars with masses between $8M_\odot$ and $30M_\odot$ the fusion of heavier elements starts, this can't keep going on however



as at some point the star starts to form the most stable element: iron. It costs energy to both make lighter elements than iron and heavier ones. As the iron core builds up the outside pressure from the core starts to decrease as no new energy is released. This goes on until the threshold of an iron core with a mass of $1.4M_{\odot}$ known as the Chandrasekhar limit and the the inwards pressure becomes too large, making the electrons surrounding the iron core fuse with the protons (uud), creating neutrons (udd) and neutrino's, diagrammatically:



This last part happens in a split second as the collapse goes at 25% the speed of light, creating a very dense neutron star (3000km in diameter iron core to 30km in diameter neutron star) and up to 10^{52} ultra-relativistic neutrinos, carrying up to 99% of the released energy [12]¹. As the density has suddenly increased so much there's a huge distance of pure vacuum between the plasma outer layer and the (now) neutron star, this plasma starts free-falling inwards, also at 25% the speed of light whilst the neutrinos carrying tremendous amounts of energy start going outwards from the neutron stars core.

The neutrinos then collide with the plasma resulting in what we observe as a "supernova", wrongly thought of by Kepler as being a "new (nova) star" rather being a violent death of an old star.

This is quite unexpected as neutrinos rarely interact, they only do because the incoming plasma is so dense and due to the tremendous amount of neutrinos that collisions happen at all. Some, however, escape and will be visible on earth in our neutrino detectors ≈ 18 h before the light escapes the exploding star.

Neutrino observatories are thus useful to know where to point our various telescopes before the supernova is actually visible in the night sky.

2.4.3 primordial neutrinos

Aside from all the previously mentioned sources, there's also a less spectacular source of neutrinos: the neutrino version of the CMB. We can estimate the temperature of the neutrinos

¹1% is released as kinetic energy, only 0.001% as electromagnetic radiation

who decoupled at the start of the universe, for this we can take a look at conservation of entropy [5]

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \quad (2.65)$$

2.4.4 How do they fit into the full detector spectrum?

The origin of the most energetic cosmic rays is still not conclusively identified. One approach to solving this problem is *multi-messenger astrophysics*, where several types of cosmic particles are used to identify the sources of these ultra-high energy cosmic rays (UHECRs). E.g we simultaneously measure gravitational waves (gravitons?) with the Einstein telescope, neutrino's with IceCube (or eventually RNO-G), photons with various telescopes and muons with a muon detector.

CHAPTER

3

RADIO DETECTION OF NEUTRINOS

3.1 Neutrino interactions in ice

As neutrinos propagate through ice they can interact with nuclei in the following ways [7]:



Figure 3.1: Most prominent ways of neutrino-nucleus interaction

With the produced leptons in the W boson mediated interaction being either electrons, resulting in an electromagnetic shower, muons which typically go undetected as they live too long or taus which will decay via

$$\tau^- \rightarrow e^- + \bar{\nu}_e + \nu_\tau \quad (3.1)$$

or, less ideally

$$\tau^- \rightarrow \mu^- + \bar{\nu}_\mu + \nu_\tau \quad (3.2)$$

In both of the possible interactions the resulting nucleus will result in a hadronic shower, for the neutral current interaction (mediated by the Z boson) the fraction of the neutrino energy that gets transferred to the nucleon is described by the inelasticity y and is heavily shifted towards small values of y [1]. This causes a big, irreducible uncertainty when trying to estimate the original neutrino energy from these kinds of events. With the charged current interaction (mediated by the W^\pm bosons) this isn't a problem however as the full neutrino energy ends up in the resulting cascades.

3.2 Askaryan effect

For a particle shower to emit strong radio signals, two conditions have to be met:

- There needs to be a separation of positive and negative charges in the shower front
- The signals produced over the length of the shower profile need to overlap coherently.

The *Askaryan* [3] effect, also known as Askaryan radiation describes the effect at radio frequencies which abides by both of these conditions, in general it's a quite difficult effect but we'll give a crude overview. The above described interactions create a shower of secondary charged particles containing a charge anisotropy, this charge imbalance is a result of medium electrons either Compton scattering into the advancing shower or annihilating with shower positrons. In the end you have a moving charge anisotropy, propagating faster than the speed of light in the medium, creating Cherenkov radiation. Cherenkov radiation is like the electromagnetic equivalent of a sonic boom, a sonic boom happens when something goes faster than the sound speed in the medium; A particle emits Cherenkov radiation if it goes faster than the light speed in the medium¹. Choosing the particle trajectory to lie along the z axis we can approximately find an equation for $\frac{d^2 \mathcal{J}}{d\omega d\Omega}$: the energy radiated per elementary unit solid angle and per elementary unit frequency interval

$$\frac{d^2 \mathcal{J}(\omega)}{d\omega d\Omega} = \frac{q^2}{4\pi} \sqrt{\frac{\mu}{\epsilon}} \beta^2 \omega^2 \delta^2[\omega(1 - \beta \mathbf{e}_r \cdot \mathbf{e}_z)] |\mathbf{e}_r \times \mathbf{e}_z|^2 \quad (3.3)$$

Now we can re-write this equation in spherical coordinates, which gives $1 - \beta \mathbf{e}_r \cdot \mathbf{e}_z = 1 - \beta \cos(\theta_c)$ in the delta function. We thus only expect radiation if

$$\cos(\theta_c) = \frac{1}{\beta} = \frac{c'}{u} = \frac{c}{n} \cdot \frac{1}{u} \quad (3.4)$$

I.e if $u > \frac{c}{n}$ with n the index of refraction, Cherenkov radiation will be emitted along a cone surface with half angle $\frac{\pi}{2} - \theta_c$ as illustrated in figure 3.3. Integrating equation 3.3 over the solid angle and formally dividing by the time interval we get:

$$\frac{d^2 \mathcal{J}}{d\omega dt} = \frac{q^2}{4\pi} \sqrt{\frac{\mu}{\epsilon}} \beta \omega \left(1 - \frac{1}{\beta^2}\right) \quad (3.5)$$

We see that the energy is proportional to ω , so we expect that most radiation will be emitted "in blue", as seen in figure 3.2. For ice the index of refraction is roughly 1.78 in deep ice, so we expect an ultra-relativistic particle to produce the most radiation at around 56° opening as

$$\cos(\theta_c) \approx \frac{1}{n} \implies \cos^{-1}\left(\frac{1}{1.78}\right) \approx 56^\circ \quad (3.6)$$

Of course this is just an estimate, as the actual index of refraction is depth-dependent which we'll get to in section (...). Now this explains how the signals get generated but logically, from only knowing this we'd expect radio waves to almost be non-existent due to the "in blue" nature of Cherenkov radiation. This isn't the full story however as we'll need to talk about coherent overlap to fully understand the Askaryan effect. This can be intuitively explained as follows: generally the shower is of length $\mathcal{O}(10\text{cm})$ [11], over this length the radiation gets emitted, most frequencies decoherently interfering, but radio waves with wavelengths of $\approx 10\text{cm}$ coherently interfere, and it's these waves we then wish to detect.

¹The reader who wants a thorough explanation and derivation is advised to check out *Chapter 14: Radiation by Moving Charges* from the book *Classical Electrodynamics* by Jackson.

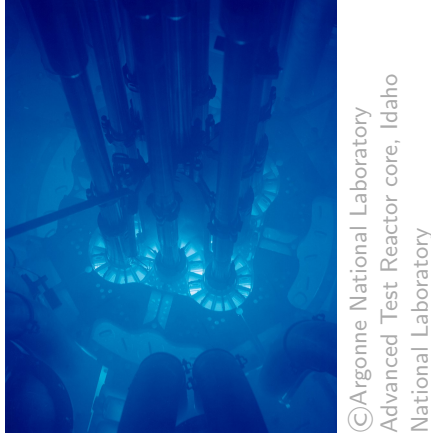


Figure 3.2: Cherenkov radiation in a nuclear reactor

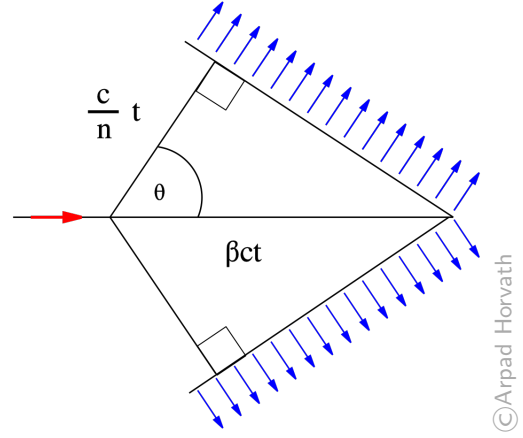
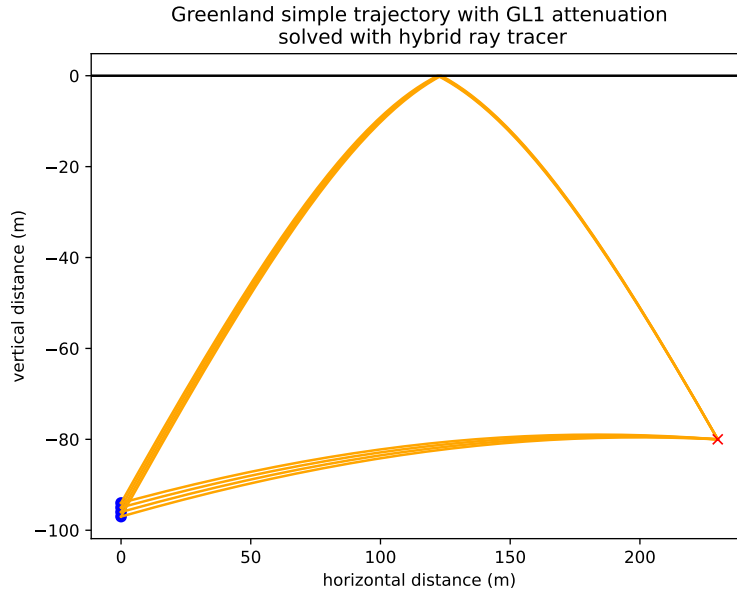


Figure 3.3: Diagrammatic representation of Cherenkov radiation

3.3 Wave propagation

Waves propagating through ice to a detector generally looks something like this:



Here the interaction of the neutrino happens somewhere in the neighbourhood of the red cross, the orange rays represent the resulting radio wave paths to the detectors which are signaled by the blue dot. Notice that reflection at the surface is also a possible path to the detector. In a homogeneous dielectric medium a ray propagates straight with it's signal wave-speed determined by the local index of refraction as $v = c/n$, the dependence of the index of refraction on density for ice is given by the Schytt equation:

$$n(x, y, z) \approx 1 + 0.78\rho(x, y, z)/\rho_0 \quad (3.7)$$

Where $\rho(x, y, z)$ is the local ice density and ρ_0 is the density for solid ice (917 kg/m^3). This isn't the only part of the index of refraction with which we'll need to concern ourselves however. If a ray propagates towards a boundary dividing 2 media with different indexes of refraction, the

ray will refract and the refracted angle can be found from Snell's law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (3.8)$$

Where n is the index of refraction and θ is the angle with respect to the surface. The system we'll consider however, isn't homogeneous with some specified boundary, it's continuous: ice in greenland has a continuously varying density. Because of this we can't work with Snell's law but we'll have to take a look at the continuous version to figure out how these rays propagate. The "continuous version of Snell's law" is the eikonal equation: a path of a ray $\mathbf{r}(s)$ with path parameter s in a medium with index of refraction $n(\mathbf{r})$ is described by:

$$\frac{d}{ds} \left(n(\mathbf{r}) \frac{d\mathbf{r}}{ds} \right) = \nabla n \quad (3.9)$$

The software we'll be using for ray reconstruction is called "radiopropa" [16] and in radiopropa the local paraxial approximation is used, i.e. if we assume that in any individual step of the algorithm the change of the refractive index along the path ds is small it's possible to re-write the equation as:

$$n(\mathbf{r}) \frac{d^2 \mathbf{r}}{ds^2} \approx \nabla n \quad (3.10)$$

Which is then easily iteratively solved. If there are boundaries (such as defects or the surface) these are treated separately using Snell's law.

3.4 Ice model

Equation 3.10, and thus the path, depends on the index of refraction on a given location. Purely from classical gravity and density considerations it can be derived that the index of refraction abides by

$$n(z) = n_{ice} - \Delta n e^{z/z_0} \quad (3.11)$$

with n_{ice} the refractive index of solid ice and $\Delta n = n_{ice} - n_s$ with n_s the index of refraction of snow. This is called "the exponential model".

From the measured density with depth variation this seems to roughly hold but it could be better (as will be explained in chapter (TBC)). This exponential model has a huge advantage as it's analytically solvable, meaning that we don't have to iteratively search for the path to the detector but know it instantly after the location of the neutrino interaction and the detector are specified, this "model" is called the *analytic ray tracer*.

3.5 Reconstruction

It can pose a challenge to reconstruct the radio signals produced by the Cherenkov radiation as they are often obscured by background noise. A solution used in RadioReco is Information Field Theory (IFT) implemented in RadioReco by Welling et al. [14] which uses Bayesian inference to calculate the most likely radio signal, given recorded data. The full reconstruction will then work as follows: events and their propagation are generated in a monte-carlo simulation using NuRadioMC [9] [8], how the detector signals would look like is then simulated with RadioReco and saved in a database. If we then detect a neutrino event in the real detector it can be compared to the database, thus finding the origin.

CHAPTER

4

THE DETECTOR

Both cosmic ray and neutrino detectors face the same main problem at the highest energies: the steeply falling flux (as can be seen on figure 4.1) requires large effective areas, which leads to the construction of neutrino detectors with volumes on the cubic kilometer scale: IceCube. As we wish to detect neutrino's with even higher energies we turn to look at an array of detectors spanning multiple cubic kilometers: RNO-G.

One such detector is illustrated in figure 4.2.

The deep component of the detector can be split up in three parts: Two *helper strings*, one *power string* and the surface components. The helper strings are the 2 vertical cables shown on the right of the figure and each one houses 2 vertically polarized antennas (Vpols), one quadslot antenna for the horizontal polarization component (Hpol) and one radio pulser on each helper string which can be used to generate calibration signals.

The power string (the leftmost vertical cable) is more densely instrumented: At the bottom it houses a set of four Vpol and two Hpol antennas with a spacing of 1m and further up the string, with a spacing of 20m are three more Vpol antennas.

The signal from each of these antennae are fed into a low-noise amplifier directly above it, from there the signal is send to the data acquisition (DAQ) system at the surface via a Radio Frequency over Fiber (RFoF) cable. There it's again amplified, digitized and saved onto an SD card. This data is then transmitted via a Long Term Evolution (LTE) telecommunications network to a local server¹, from where it is sent via a satellite link.

There are solar panels as a power source who charge up battery banks, but as there isn't enough light during the Greenland winters, there're plans to build wind turbines (with the problem being the possibly detectable RF noise the 'engine' produces)

¹There is additionally a Long Range Wide Area Network (LoRaWAN) antenna as backup in case of problems with the LTE network

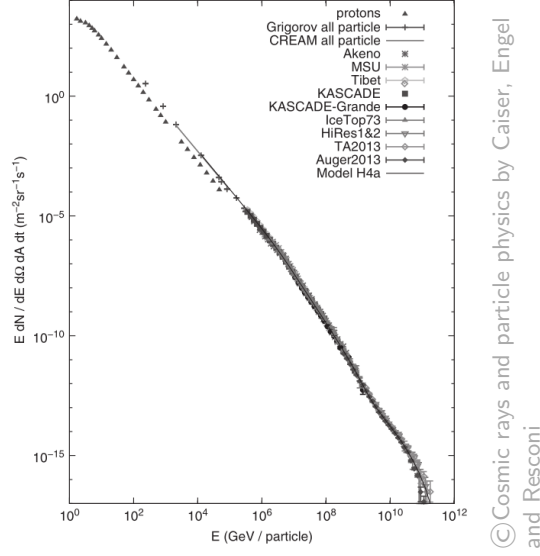


Figure 4.1: Falling cosmic ray flux with energy

The radio signal from a neutrino often travels along both direct and refracted paths (designated DnR) to the deep array, this happens because the upper ice layer is a non-uniform medium where the signal trajectory is bent, as illustrated by a simulation in figure 4.3.

This double pulse characteristic would be a smoking-gun signature of an in-ice source. The two helper strings are needed for a full direction reconstruction. Three independent measurements are needed for azimuthal information, which is provided by the Vpol (Vertical polarization) antennas and placing the Hpol (Horizontal polarization) antennas at different depths on every string, both zenith and azimuth information will be provided for those signals. The helper strings' calibration pulsers, as well as one on the surface, will ensure regular monitoring of the performance of the station and provide information useful for precise calibration of the antenna geometry.

The plan is to construct these detectors in an array as shown in figure 4.4, note that all the individual detectors are named after various species living in greenland (in the native tongue).

Christoph Welling did an investigation into energy reconstruction from the received signals [15] for air showers in one single station (as the RNO-G stations are so far apart this is the case here aswell) and he noticed that it is necessary to know if the detector who observes an event falls inside or outside the Cherenkov cone to accurately reconstruct the primary particle energy as most over-estimated energies in his simulations are caused by events viewed from within the Cherenkov ring being mistaken for events outside of it. He went on to show that, if we somehow know if the shower was seen from inside or outside the ring from some extra source, that most outliers in the energy disappeared. It is shown by Hiller et al. [10] that the combination of a muon detector with the radio detector might make the issue of confusion between being within or outside of the Cherenkov-ring disappear. Because of this the RNO-G stations are fitted with surface Log Periodic Dipole Antennas (LPDA), capable of detecting muons. Note that this is for air showers, the radio signal from neutrinos show additional complexities.

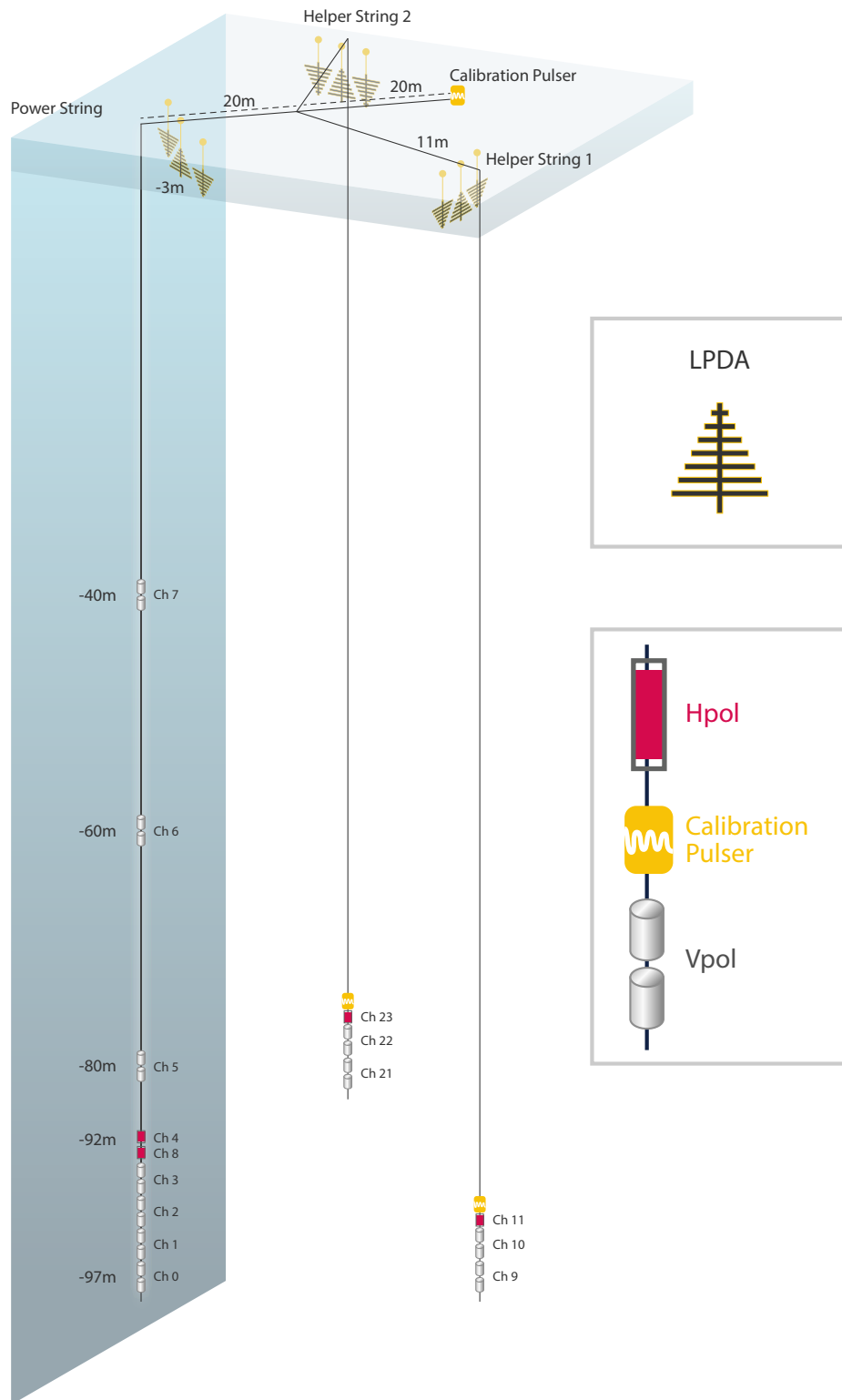


Figure 4.2: illustration of the detector

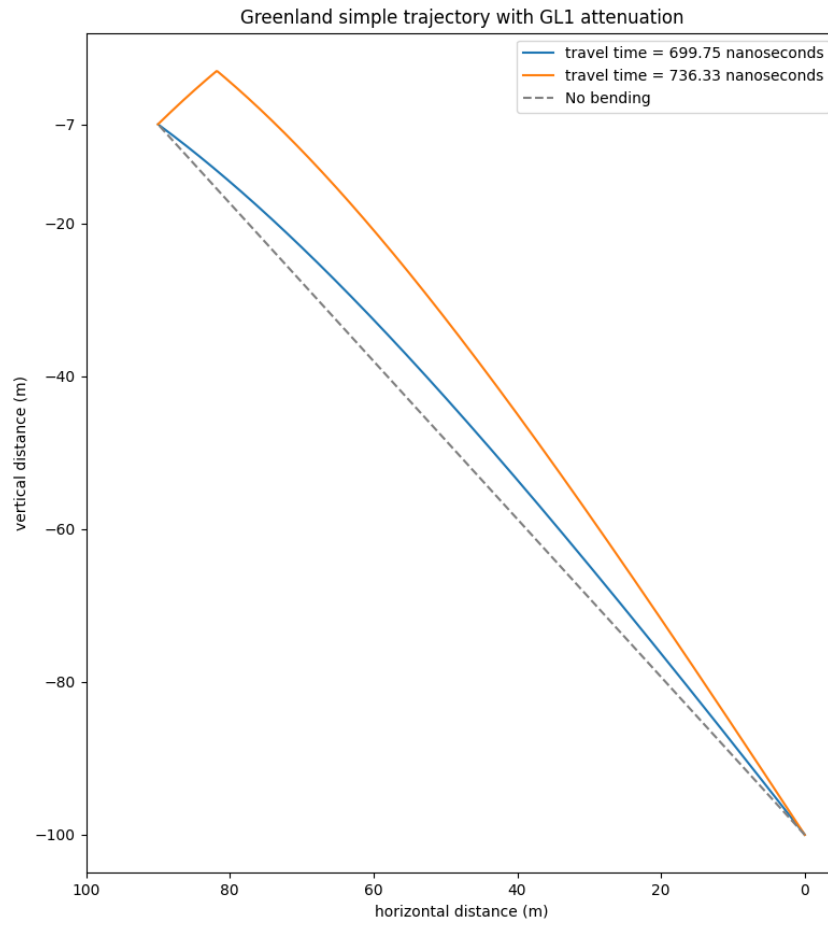
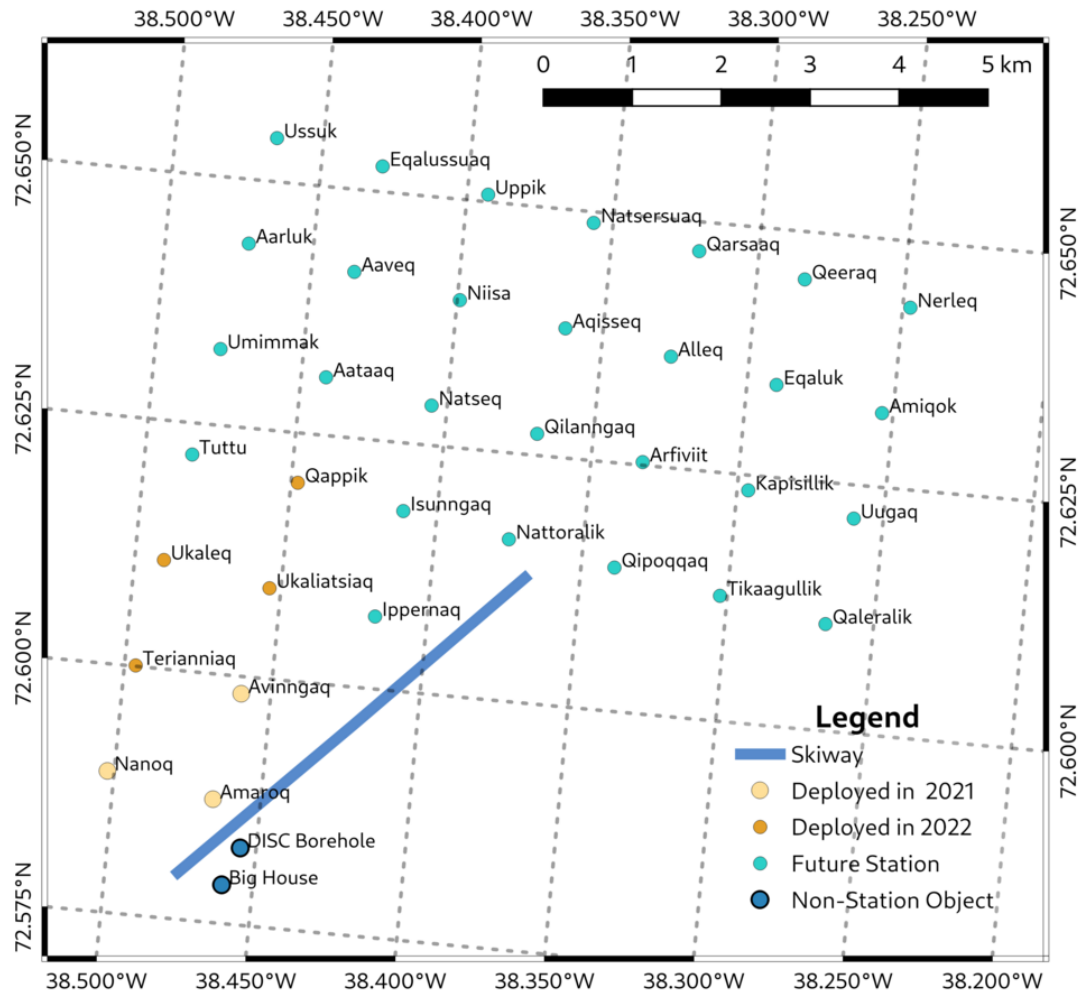


Figure 4.3: illustration of a neutrino signal path, the neutrino interaction being located at (0,-100) and the detector at (90,-7)

RNO-G Planned Layout



Notes:

- Station numbering follows a grid, where the first numeral is in increasing W-E and the second numeral is in increasing S-N, skipping non-existent stations (the Seckel method).
- Station spacing is 1.25 km in map coordinates (but really 1.23 km due to projection, which creates a 2% scale difference.)
- Projection is Greenland Polar Stereographic (EPSG:5938). True north indicated by Rose, offset from grid north by 5.37°.
- Magnetic Declination, for August 1 2022, is -25.2° according to the WMM.
- In list below, all future stations labeled as 2023.



v 0.5.1
2022-08-26
68000:1
Greenland Polar Stereographic Projection (EPSG:5938)

Figure 4.4: map of the station

CHAPTER

5

HYBRID RAY TRACER

5.1 Shortcomings of the exponential ice model

It has become apparent (TBC) that complex ice models will be necessary moving forward as the exponential ice model fails to fit the density curve. The ideal software for radio wave propagation through ice is radioprop [16], but due to the way it works you'll have to know the start point, the end point and the launch angle of your ray to work out the path. This isn't difficult for the analytic model as it's exactly solvable but for a general ice model you'll somehow have to find where to shoot the ray. Work has been done on finding the launch angle in the case of complex ice models by B. Oeyen et al. [13], where they created a ray tracer which iteratively finds the solution, called the "iterative ray tracer". The full explanation of how their algorithm works can be found in the mentioned paper. This is however a sub-optimal solution in python as an optimisation library will generally work faster, work had been done on trying to implement such an algorithm but this attempt failed. As we saw this work the idea came to mind to combine the iterative ray tracer and the code using the optimisation libraries (a so called "minimizer"), to come up with the algorithm that will be discussed in this chapter: The hybrid ray tracer, in the source code called the "hybrid minimizer".

It succeeds in more rapidly tracing the path from the event to the detector, is more accurate and also arrives closer to the detector as the final result is not limited by the final drawn sphere size but by a given tolerance.

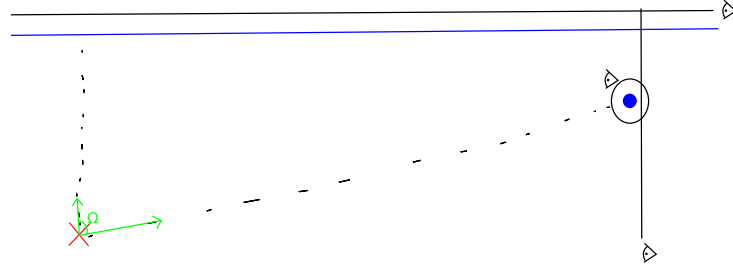
5.2 How it works

The full code can be found in appendix A, here we'll walk you through the algorithm. The hybrid minimizer can be seen as an extension of the iterative raytracer [13] as it starts out the same way: Say our source of radiation is at position \mathbf{X}_1 and our detector is located at position \mathbf{X}_2 , we start by defining the vector $\mathbf{v} = \mathbf{X}_2 - \mathbf{X}_1$, then we clone it as a new vector \mathbf{u} and set \mathbf{u} 's z coordinate to 0, making it a normal vector of a plane parallel to the z direction. we now wish to know where we'd actually be able to find possible paths, looking at figure 4.3 we see that no solutions below the direct path are possible as there would need to be upwards reflection,

so we convert our vector \mathbf{v} representing the path from the source to the detector to spherical coordinates, giving us a polar angle (zenith angle) "theta-direct". With this we know that the, at the source, the ray should propagate with an initial zenith angle within the angle interval 0° to theta-direct $^\circ := \Omega$.

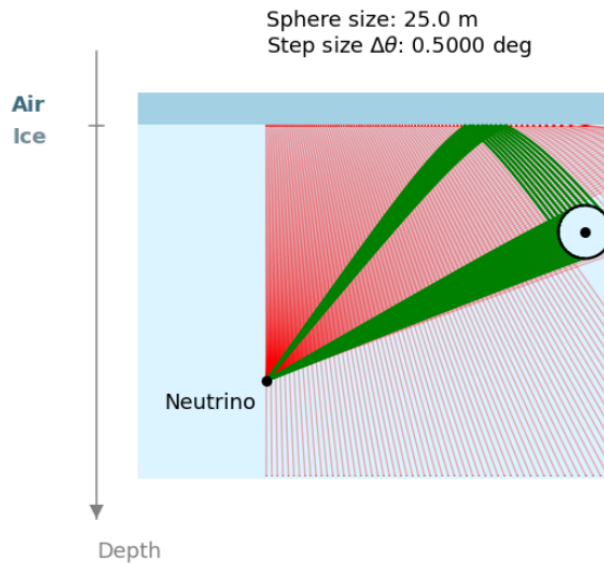
Next we need to define our "observers", if you shoot a ray with the radiopropa module from a certain point at a certain angle the ray path will get simulated until it interacts with this "observer". Ideally we would like to a priori know where to shoot our ray and have the detector be an infinitesimally small observer in our simulation, but as we'll be working with general ice models this can't be done.

The algorithm of finding the possible paths is then as follows: We define a spherical observer at the location of the detector, with a radius of fair size. We place an observer plane directly behind the detector (with normal vector \mathbf{u} , no rays can propagate back after crossing) and an observer above the surface (as no rays could make it back after escaping the ice) our full setup is then what's illustrated in the figure below

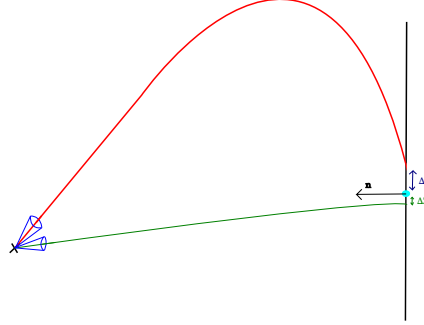


Here the red cross is the radio source, the blue horizontal line is the ice-air boundary surface, the blue dot to the right is the detector and the green Ω indicates the range over which solutions to the problem are possible.

We start off by just iteratively guessing: given a certain angle stepsize $\Delta\theta$ shoot rays at the angles $\{0, \Delta\theta, 2\Delta\theta, \dots, \Omega\}$ And see which ones get detected at the sphere around the detector, this process is illustrated in a modified version of B. Oeyen et al. their figure below



if there are 2 distinct launch regions, it will start the so called "minimization", using scipy's module `optimize.minimize`. First we get rid of the spherical observer and place the vertical observer exactly at the detector, now to be able to use the minimize module we'll need a function to minimize, for this reason we define the function `delta_z` as, given a certain launch angle, returning the distance from the point where it lands on the plane to the detector as illustrated below



The function we'll minimize is then `delta_z_squared` which is just the square of `delta_z` as we wish it to be as close as possible to 0, it gets minimized within the angle boundaries found from the previous step. With this our algorithm is done, it does have a fail-safe as well for if the first step, finding the launch regions, doesn't work namely it reverts back to being the iterative ray tracer.

5.3 Performance Optimisation

To test the hybrid minimizer the numpy random module was used to generate random coordinates, the considered square (as there is only a z component to the ice model the 3D problem is essentially only a 2D problem) is $x:0.1\text{km}, 4\text{km}$ and $z:-0.1\text{km}, -3\text{km}$.¹ Every test point shown in the following subsections consists of at least 500 random initial positions. As the speed of the algorithm is computer dependent the algorithm's speed is always plotted relative to the iterative ray tracer's speed, simulated with the same coordinates at the same time.

5.3.1 Length of the normal vector

As visually explained in figure 5.1, the size of the normal vector seems to influence how big the ray tracer's step size is taken close to the detector. This thus influences the convergence and time taken. The results of varying this are shown in figures 5.4 and 5.5. Looking at these figures the first optimization conclusion is as expected: take the normal vector length to be 1 meter.

5.3.2 ztol

We'll now change the tolerance on the vertical distance away from the detector which is deemed accepted i.e in figure 5.1 if Δz is below this threshold it's accepted. The results are shown in figures 5.6 and 5.7. From which we can conclude the second optimization conclusion: take `ztol` to be 0.05 m.

¹This start at 100m depth was to get around issues concerning events that won't even trigger in a full simulation

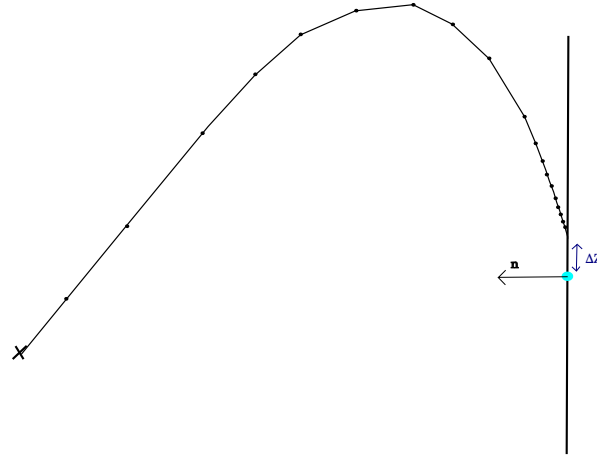


Figure 5.1: how normal vector size influences the stepsize

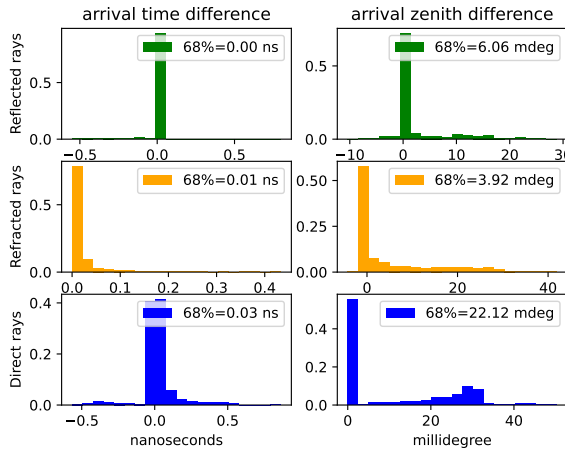


Figure 5.2: Hybrid

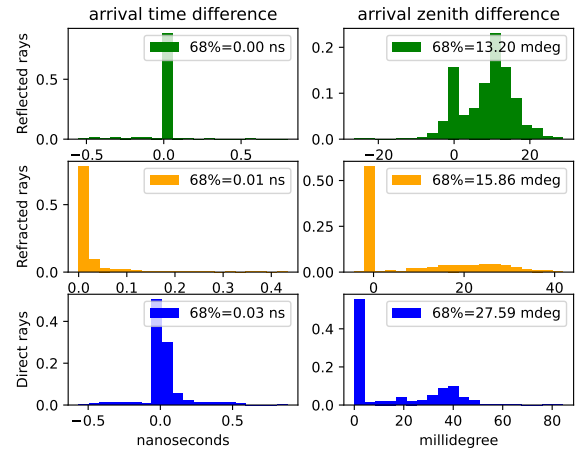


Figure 5.3: Iterative

5.3.3 Sphere Size & Step Size

The initial rays are sent out in steps of a certain angle and with a sphere around the detector of a certain size. As this initial search for launch angle regions is the slowest step in the hybrid ray tracer it's imperative to optimize this. The procedure was: change the sphere size and loop over various step sizes, recording the speed. The results are shown in figure 5.8 the lower on the chart the better, zooming in onto the lowest point as is shown in a combined plot on figure 5.9, we see that an optimum seems to be around a spheresize of 45m and a stepsize of 0.7° .

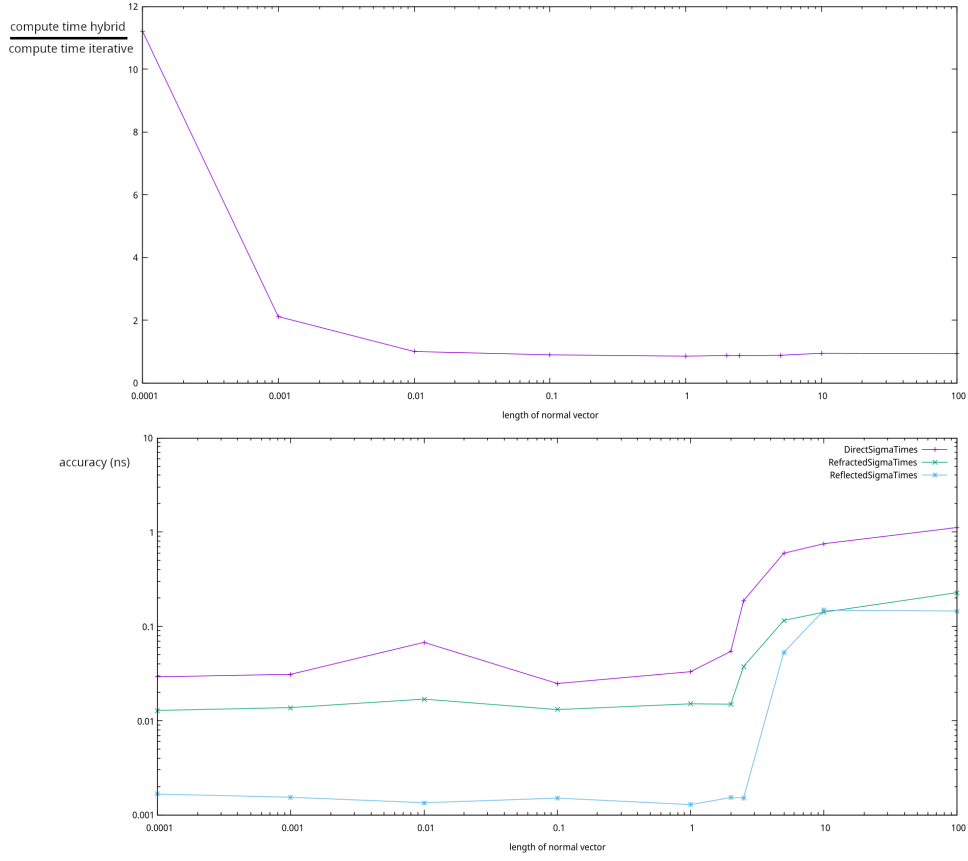


Figure 5.4: influence of the length of the normal vector

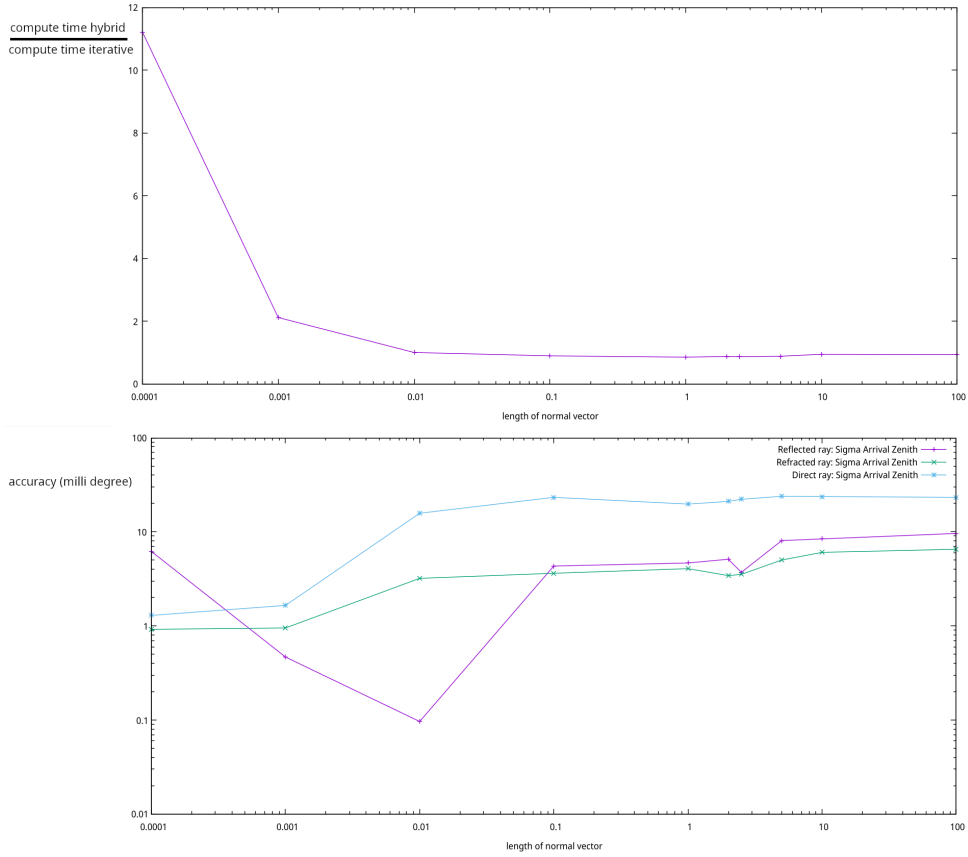


Figure 5.5: influence of the length of the normal vector

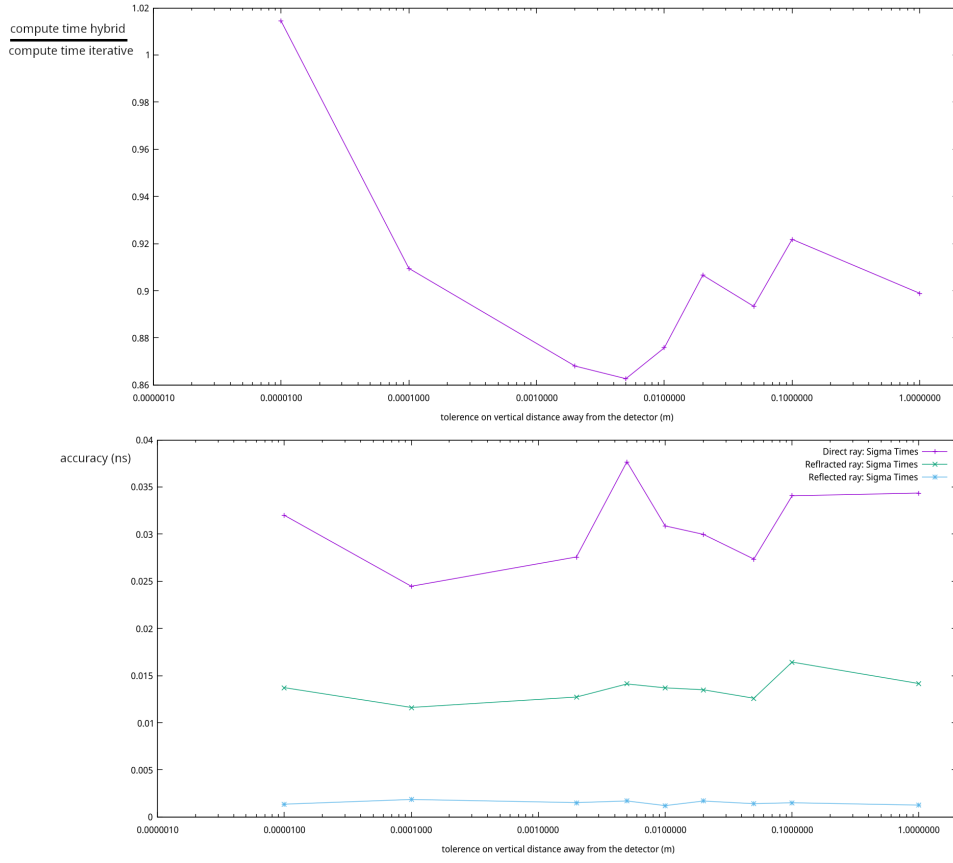


Figure 5.6: influence of the tolerance on vertical distance

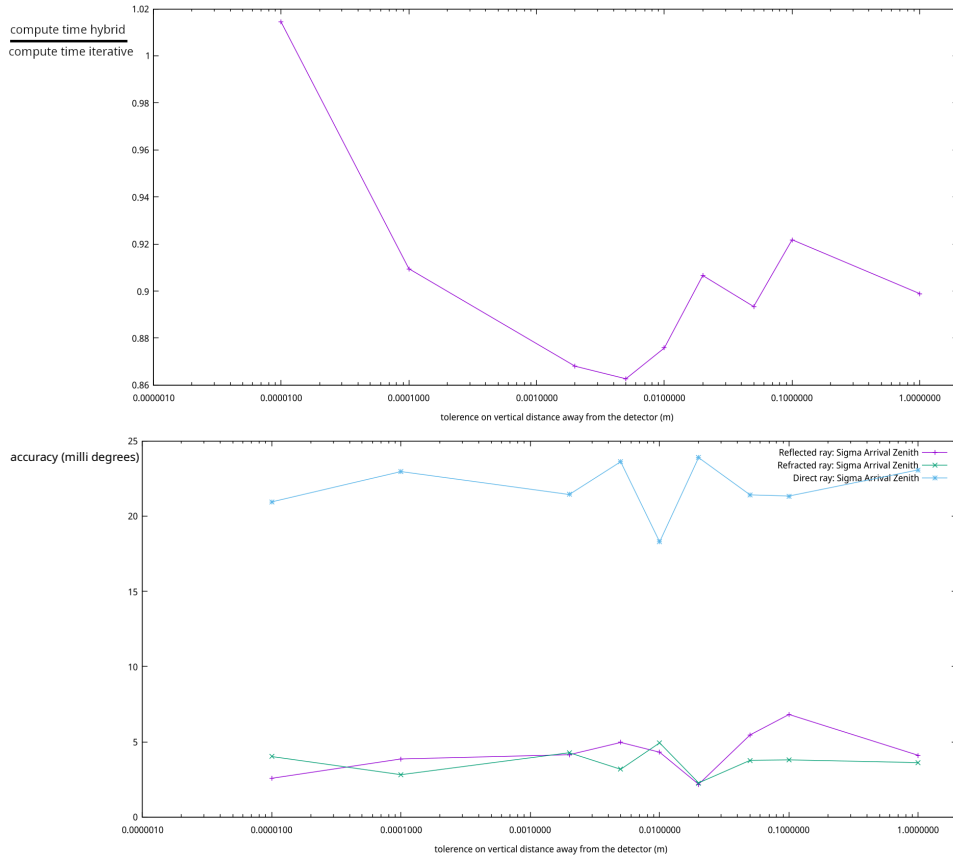


Figure 5.7: influence of the tolerance on vertical distance

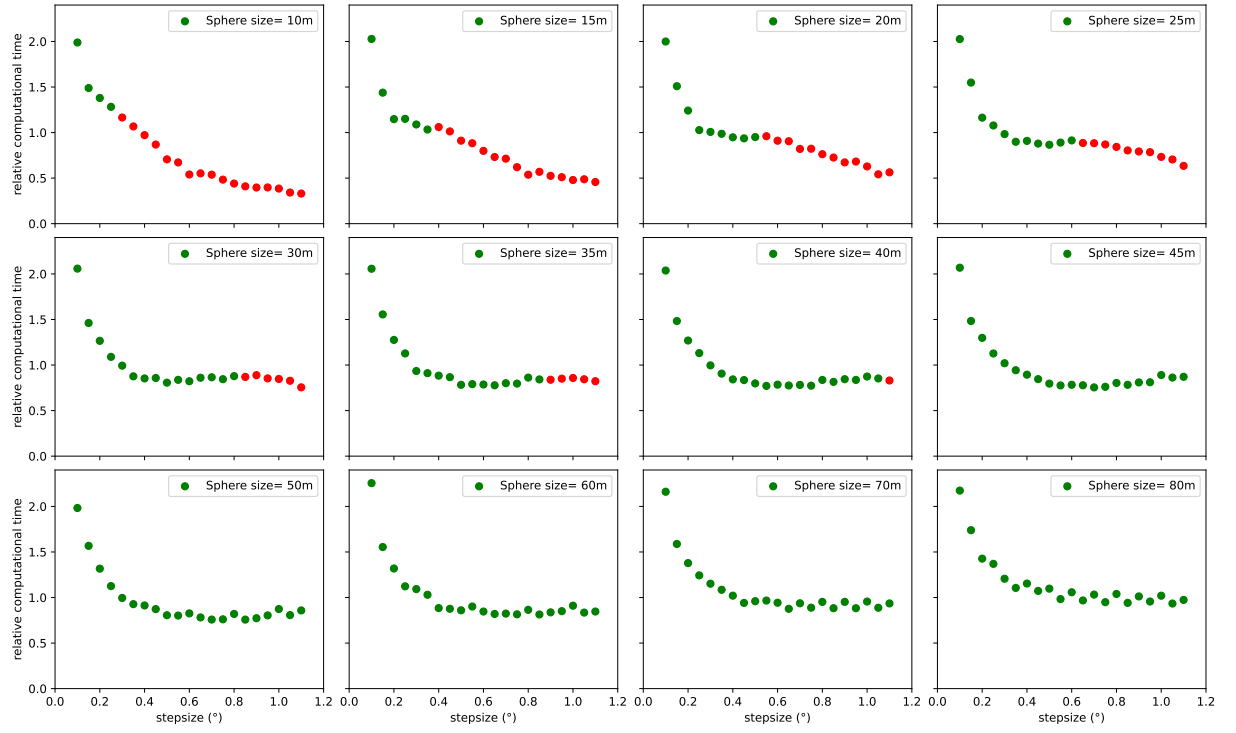


Figure 5.8: Variation in Sphere and angle step size with report on relative time.

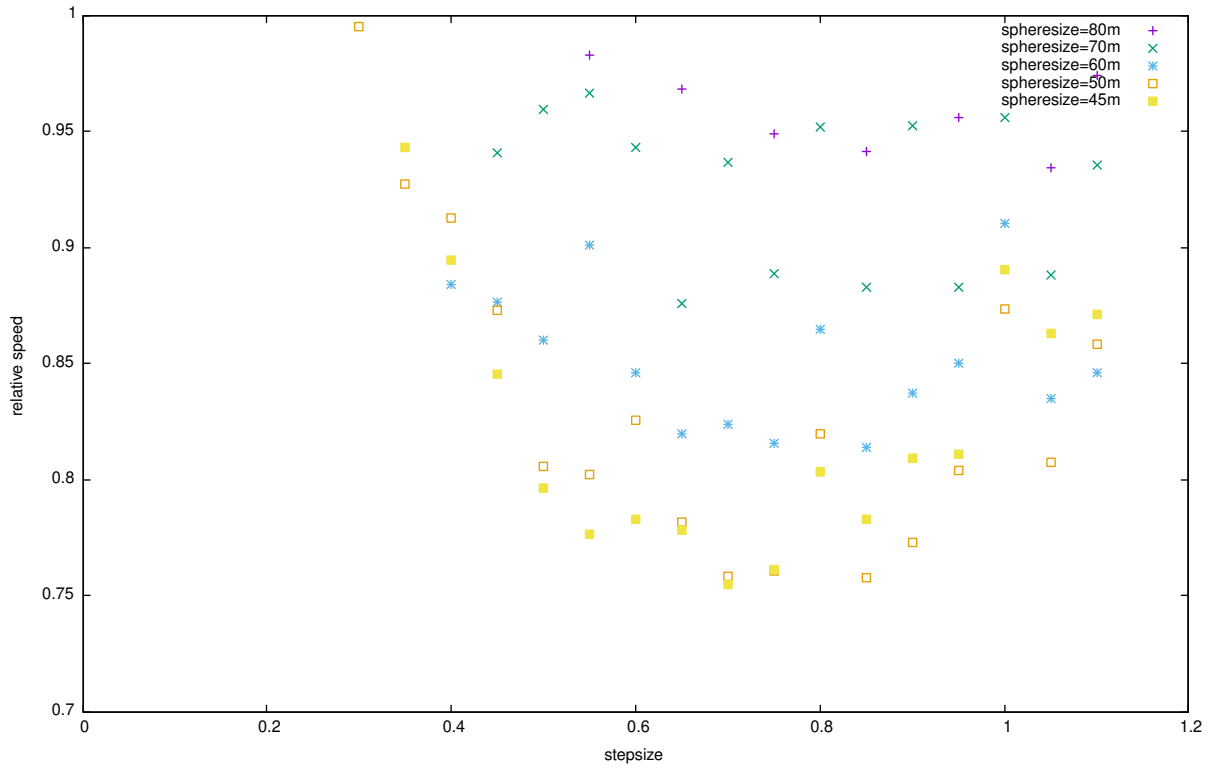


Figure 5.9: Green values in variation in sphere and angle step size with report on relative time.

CHAPTER

6

WEATHER BALLOON

In this chapter we'll simulate radio signals coming from a weather balloon flying over the stations. Our goal is to use the plane waves method to reconstruct the position of the weather balloon with the timing information inferred from the detected radio signals in the detectors. If this is somewhat succesfull it can be used (as we'll get to shortly) to find out the local index of refraction in the ice.

There's a little change that needs to happen first however for us to be able to simulate this: the air observers needs to be hard removed as to make a ray tracing possible from within the air to the detector. Now another problem that seemed to arise after removing the air observer in the radioproparaytracing.py (where the hybrid ray tracer is stored) file is that, even though the simulation seems to work, only the path IN the ice and the timing for that path is recorded, so the timing was later added by drawing a line from the balloon to the beginning of the recorded path and setting the speed of radio waves in air to c .

Now having established this the first problem we'll consider is plane wave reconstruction of the original position, an example path to some of the sensors is given in figure 6.1 The plane wave reconstruction can easily be understood using figure 6.2, the waves coming in are drawn in blue and make a certain angle with the detectors. the top detector (top box) detects the wave at a certain time t_1 , the bottom detector detects it at a time t_2 . In our database, after decoding the signal we'd thus see that these two detectors got a signal $\Delta t = t_1 - t_2$ seconds apart from eachother. Now ideally this is equal to the time $\Delta t'$ which is the time it took the wave to propagate that distance which we can calculate from basic trigonometry and dimensional analysis:

$$\Delta t' = \frac{m}{(m/s)} = (s/m) * m = v^{-1} * m = v^{-1} \cos \theta \Delta z \quad (6.1)$$

With $v = c/n$ the local speed of light. As previously discussed this n is depth-dependent and we'll be using the local index of refraction, say we have 4 detectors, at depths -94,-95,-96,-97 this thus would mean at position -95.5:

```
1 position = np.array([0,0,-95.5])
2 n=medium.greenland_firn.get_index_of_refraction(medium.greenland_firn(),position)
```

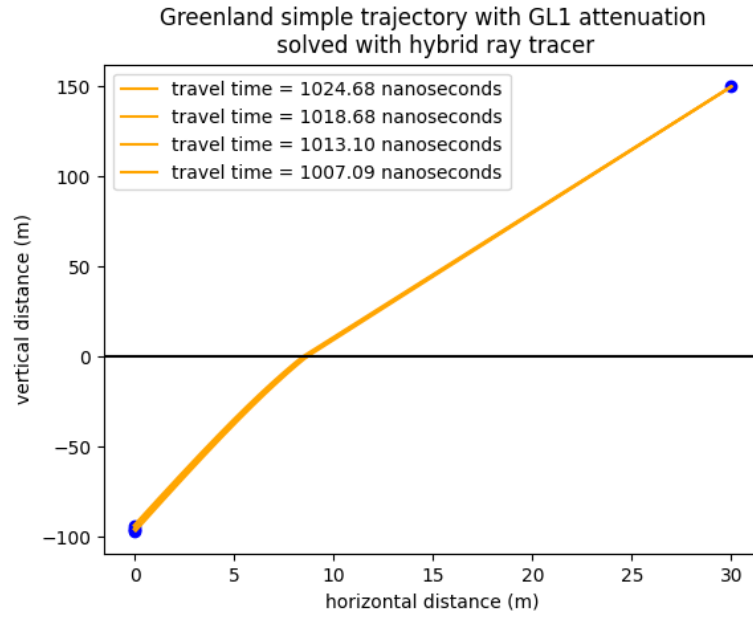


Figure 6.1: Example trajectory of rays coming from a weather balloon (blue dot top right) and going through the ice to the various detectors (blue dots bottom left)

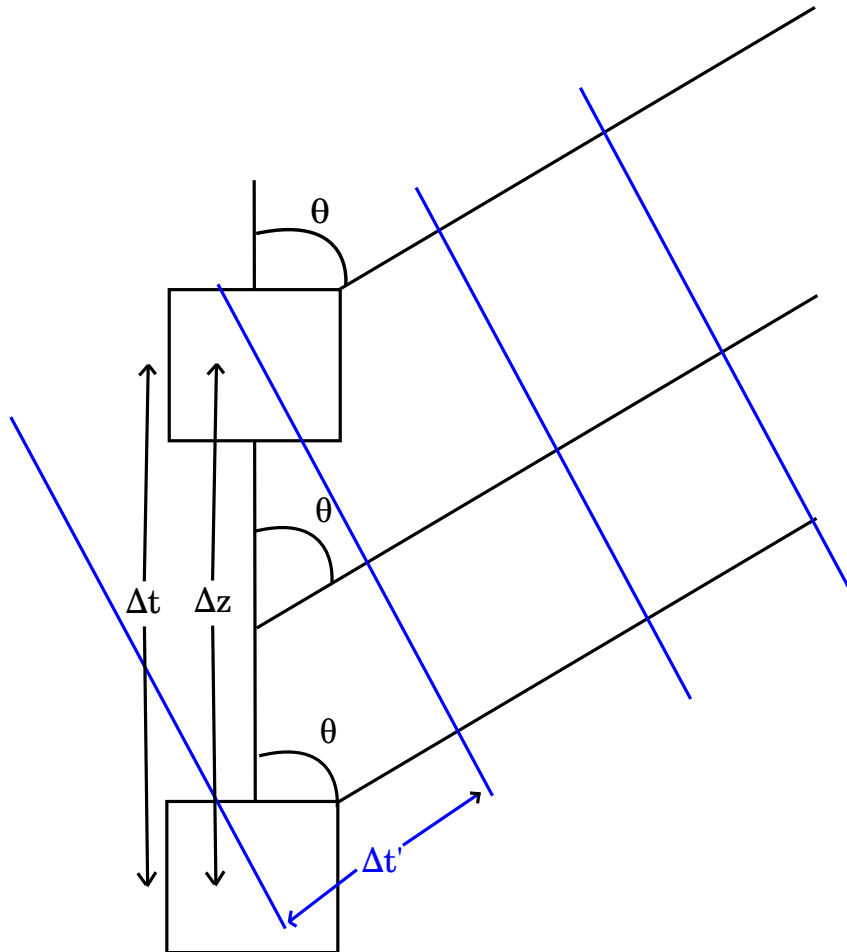
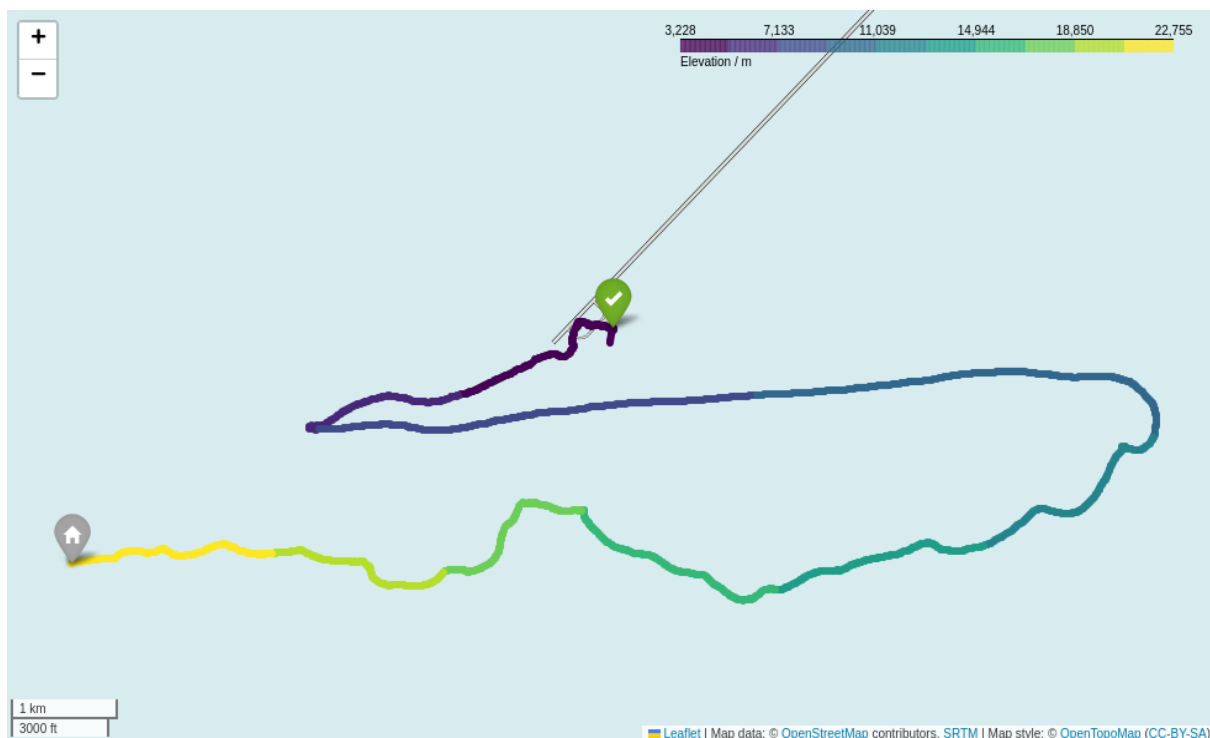


Figure 6.2: Illustration of Plane waves



Now in reality we don't know the angle a priori so we'll perform a scan by minimizing the correlation function:

$$Correlation(\theta) = \Delta t - \frac{\cos \theta \Delta z}{v} \quad (6.2)$$

If we have more than 1 station however, than we need to specify various correlation functions. E.g if we have four detectors labeled 0 to 3 we'll have to construct correlation functions between detectors 0&1, 0&2, 0&3, 1&2, 1&3 and 2&3 . As all of these correlation functions will have different sizes we'll norm them as follows:

$$Correlation_{Normed}(\theta) = \frac{Correlation(\theta)}{\int Correlation(\theta) \Delta \theta} \quad (6.3)$$

After this we can sum them and take the minimum of the sum, in practice the various correlation functions look like those shown in figure

APPENDIX

A

HYBRID MINIMIZER CODE

```
1 def raytracer_hybrid_minimizer(self,n_reflections=0):
2     """
3     Uses RadioPropa to first find all the numerical ray tracing solutions
4     ↪ between sphere x1
5     and sphere x2 for a big sphere. After which it uses the scipy
6     ↪ optimize.minimize module
7     to find the best path in these angle intervals.
8     Tracer does not work for reflective bottoms or secondary creation at the
9     ↪ moment
10    """
11
12    try:
13        X1 = self._X1 * (radiopropa.meter/units.meter)
14        X2 = self._X2 * (radiopropa.meter/units.meter)
15    except TypeError:
16        self.__logger.error('NoneType: start or endpoint not initialized')
17        raise TypeError('NoneType: start or endpoint not initialized')
18
19    def get_ray(theta,phi):
20        ray_dir = hp.spherical_to_cartesian(theta,phi)
21        if ray_dir.shape==(3,1): ray_dir = ray_dir.T[0] #doesn't always give
22        ↪ the right shape
23        source = radiopropa.Source()
24        source.add(radiopropa.SourcePosition(radiopropa.Vector3d(*X1)))
25        source.add(radiopropa.SourceDirection(radiopropa.Vector3d(*ray_dir)))
26        ray = source.getCandidate()
27        return ray
28
29    def shoot_ray(theta):
30        ray = get_ray(theta,phi_direct)
31        sim.run(ray, True)
32        return ray
```

```

30     def cot(x):
31         return 1/np.tan(x)
32
33     def arccot(x):
34         return np.arctan(-x) + np.pi/2
35
36     def delta_z(cot_theta):
37         theta = arccot(cot_theta)
38         ray = shoot_ray(theta)
39         ray_endpoint = self.get_path_candidate(ray)[-1]
40         return (ray_endpoint-self._X2)[2]
41
42     def delta_z_squared(cot_theta):
43         return delta_z(cot_theta)**2
44
45     def MinimizeAble(lower,upper):
46         #This checks if there are 2 distinct regions, takes about 10~-6% of
47         ↪ the total time
48         if ((len(lower) == 2) and (len(upper)==2)):
49             return ((lower[1] > upper[0]) and (lower[0] < upper[0]) and
50                     ↪ (lower[1] < upper[1]))
51         else:
52             return False
53
54     LetsMinimize = False
55
56     v = (self._X2 - self._X1)
57     u = copy.deepcopy(v)
58     u[2] = 0
59     theta_direct, phi_direct = hp.cartesian_to_spherical(*v) # zenith and
60     ↪ azimuth for the direct linear ray solution (radians)
61     cherenkov_angle = np.arccos(1. /
62     ↪ self._medium.get_index_of_refraction(self._X1))
63
64     if np.linalg.norm(u) != 0:
65         delta_theta =
66         ↪ 2*abs(self.delta_theta_direct(dz=self._sphere_sizes[0]))
67     else:
68         delta_theta = self._step_sizes[0]/units.radian
69
70     ## regions of theta with possible solutions (radians)
71     launch_lower = [0]
72     launch_upper = [theta_direct + delta_theta] # below theta_direct no
73     ↪ solutions are possible without upward reflections
74
75     if n_reflections > 0:
76         if self.medium.reflection is None:
77             self.__logger.error("a solution for {:d} reflection(s) off the
78             ↪ bottom reflective layer is requested,"
79                                + "but ice model does not specify a reflective
80                                ↪ layer".format(n_reflections))
81             raise AttributeError("a solution for {:d} reflection(s) off the
82             ↪ bottom reflective layer is requested,"

```

```

74         + "but ice model does not specify a reflective
75         ↪ layer".format(n_reflections))
76     else:
77         z_refl = self._medium.reflection
78         rho_channel = np.linalg.norm(u)
79         if self._X2[2] > self._X1[2]:
80             z_up = self._X2[2]
81             z_down = self._X1[2]
82         else:
83             z_up = self._X1[2]
84             z_down = self._X2[2]
85         rho_bottom = (rho_channel * (z_refl - z_down)) / (2*z_refl - z_up
86         ↪ - z_down)
87         alpha = np.arctan((z_down - z_refl)/rho_bottom)
88         ## when reflection on the bottom are allowed, a initial region
89         ↪ for theta from 180-alpha to 180 degrees is added
90         launch_lower.append(((np.pi/2 + alpha) -
91         ↪ 2*abs(self.delta_theta_bottom(dz=self._sphere_sizes[0],
92         ↪ z_refl=z_refl) / units.radian)))
93         launch_upper.append(np.pi)
94     #we first do the same as in the iterative solver
95     for s,sphere_size in enumerate(self._sphere_sizes):
96
97         sphere_size = sphere_size * (radiopropa.meter/units.meter)
98         detected_rays = []
99         results = []
100
101         ##define module list for simulation
102         sim = radiopropa.ModuleList()
103         sim.add(radiopropa.PropagationCK(self._ice_model.get_scalar_field(),
104         ↪ 1E-8, .001, 1.)) ## add propagation to module list
105         for module in self._ice_model.get_modules().values():
106             sim.add(module)
107         sim.add(radiopropa.MaximumTrajectoryLength(self._max_traj_length *
108         ↪ (radiopropa.meter/units.meter)))
109
110         ## define observer for detection (channel)
111         obs = radiopropa.Observer()
112         obs.setDeactivateOnDetection(True)
113         channel =
114         ↪ radiopropa.ObserverSurface(radiopropa.Sphere(radiopropa.Vector3d(*X2),
115         ↪ sphere_size)) ## when making the radius larger than 2 meters,
116         ↪ sometimes three solution times are found
117
118         obs.add(channel)
119         sim.add(obs)
120
121         ## define observer for stopping simulation (boundaries)
122         obs2 = radiopropa.Observer()
123         obs2.setDeactivateOnDetection(True)
124         w = (u / np.linalg.norm(u)) * 2*sphere_size
125         boundary_behind_channel =
126         ↪ radiopropa.ObserverSurface(radiopropa.Plane(radiopropa.Vector3d(*X2
127         ↪ + w)), radiopropa.Vector3d(*w))

```

```

116     obs2.add(boundary_behind_channel)
117     boundary_above_surface =
118     ↪ radiopropa.ObserverSurface(radiopropa.Plane(radiopropa.Vector3d(0,
119     ↪ 0, 1*radiopropa.meter), radiopropa.Vector3d(0, 0, 1)))
120     obs2.add(boundary_above_surface)
121     sim.add(obs2)
122
123     #create total scanning range from the upper and lower thetas of the
124     ↪ bundles
125     step = self._step_zeniths[s] / units.radian
126     theta_scanning_range = np.array([])
127     for iL in range(len(launch_lower)):
128         new_scanning_range = np.arange(launch_lower[iL],
129         ↪ launch_upper[iL]+step, step)
130         theta_scanning_range = np.concatenate((theta_scanning_range,
131         ↪ new_scanning_range))
132
133     for theta in theta_scanning_range:
134         ray_dir = hp.spherical_to_cartesian(theta, phi_direct)
135
136         def delta(ray_dir, shower_dir):
137             viewing = np.arccos(np.dot(shower_dir, ray_dir)) *
138             ↪ units.radian
139             return viewing - cherenkov_angle
140
141         if (self._shower_axis is None) or
142         ↪ (abs(delta(ray_dir, self._shower_axis)) <
143         ↪ self._cut_viewing_angle):
144             source = radiopropa.Source()
145
146             ↪ source.add(radiopropa.SourcePosition(radiopropa.Vector3d(*X1)))
147
148             ↪ source.add(radiopropa.SourceDirection(radiopropa.Vector3d(*ray_dir)))
149             sim.setShowProgress(True)
150             ray = source.getCandidate()
151             sim.run(ray, True)
152
153             current_rays = [ray]
154             while len(current_rays) > 0:
155                 next_rays = []
156                 for ray in current_rays:
157                     if channel.checkDetection(ray.get()) ==
158                     ↪ radiopropa.DETECTED:
159                         detected_rays.append(ray)
160                         result = {}
161                         if n_reflections == 0:
162                             result['reflection']=0
163                             result['reflection_case']=1
164                         elif self._ice_model.get_modules()["bottom
165                         ↪ reflection"].get_times_reflectedoff(ray.get())
166                         ↪ <= n_reflections:
167
168                             ↪ result['reflection']=self._ice_model.get_modules()["b
169                             ↪ reflection"].get_times_reflectedoff(ray.get())

```

```

155         ↪ result['reflection_case']=int(np.ceil(theta/np.deg2ra
156         results.append(result)
157         for secondary in ray.secondaries:
158             next_rays.append(secondary)
159             current_rays = next_rays
160         #loop over previous rays to find the upper and lower theta of each
161         ↪ bundle of rays
162         #uses step, but because step is initialized after this loop this ios
163         ↪ the previous step size as intended
164         if len(detected_rays) > 0:
165             launch_lower.clear()
166             launch_upper.clear()
167             launch_theta_prev = None
168             for iDC,DC in enumerate(detected_rays):
169                 launch_theta = DC.getLaunchVector().getTheta()/radiopropa.rad
170                 if iDC == (len(detected_rays)-1) or iDC == 0:
171                     if iDC == 0:
172                         launch_lower.append(launch_theta-step)
173                     if iDC == (len(detected_rays)-1):
174                         launch_upper.append(launch_theta+step)
175                     elif abs(launch_theta - launch_theta_prev) > 1.1*step: ##take
176                         ↪ 1.1 times the step to be sure the next ray is not in the
177                         ↪ bundle of the previous one
178                         launch_upper.append(launch_theta_prev+step)
179                         launch_lower.append(launch_theta-step)
180                     else:
181                         pass
182                     launch_theta_prev = launch_theta
183             isMinimizeAble = MinimizeAble(launch_lower,launch_upper)
184             #check if we have two distinct launch regions so we can minimize
185             #accuracy is chosen above time duration, so this will take a
186             ↪ while
187             if isMinimizeAble and (s == 0): # speed hack: Only check on first
188                 ↪ iteration
189                 LetsMinimize = True
190                 break
191             else:
192                 #if detected_rays is empty, no solutions where found and the
193                 ↪ tracer is terminated
194                 break
195         if LetsMinimize:
196             iterative = False
197             ##define module list for simulation, this needs to be redone to get
198             ↪ rid of the spherical observer
199             sim = radiopropa.ModuleList()
200             sim.add(radiopropa.PropagationCK(self._ice_model.get_scalar_field(),
201             ↪ 1E-8, .001, 1.)) ## add propagation to module list
202             for module in self._ice_model.get_modules().values():
203                 sim.add(module)
204             ↪ sim.add(radiopropa.MaximumTrajectoryLength(self._max_traj_length*(radiopropa.

```

```

198
199     ## define observer for detection (channel)
200     obs = radiopropa.Observer()
201     obs.setDeactivateOnDetection(True)
202     #a bigger normal value makes the calculation faster, a smaller one
203     ↪ more precise
204     #NormalScale = 2.5
205     w = (u / np.linalg.norm(u)) ## NormalScale
206     plane_channel =
207     ↪ radiopropa.ObserverSurface(radiopropa.Plane(radiopropa.Vector3d(*X2),
208     ↪ radiopropa.Vector3d(*w)))
209     obs.add(plane_channel)
210     sim.add(obs)
211
212     # handy?
213     boundary_above_surface =
214     ↪ radiopropa.ObserverSurface(radiopropa.Plane(radiopropa.Vector3d(0,
215     ↪ 0, 1*radiopropa.meter), radiopropa.Vector3d(0, 0, 1)))
216     obs2.add(boundary_above_surface)
217     sim.add(obs2)
218
219     detected_rays = []
220     detected_theta = []
221     #we minimize the cotangens of the zenith to reflect the same
222     ↪ resolution in z to the different angles (vertical vs horizontal)
223
224     for i in range(len(launch_lower)):
225         ll = launch_lower[i]
226         lu = launch_upper[i]
227         cotll = cot(ll)
228         cotlu = cot(lu)
229         InitGuess = cot((ll+lu)/2)
230         if cotlu > cotll:
231             bounds = scipy.optimize.Bounds(lb=cotll, ub=cotlu,
232             ↪ keep_feasible=False)
233         else:
234             bounds = scipy.optimize.Bounds(lb=cotlu, ub=cotll,
235             ↪ keep_feasible=False)
236
237         root =
238         ↪ optimize.minimize(delta_z_squared,x0=InitGuess,bounds=bounds,options={'xa
239         theta = arccot(root.x)
240         detected_theta.append(theta)
241         detected_rays.append(shoot_ray(theta))
242
243     self._rays = detected_rays
244     self._results = [{'reflection':0,'reflection_case':1} for ray in
245     ↪ detected_rays]
246     launch_bundles = None
247     self.__used_method = 'hybrid minimizer'
248     return launch_bundles,iterative
249
250 else:
251     iterative = True

```

```
242         self._rays = detected_rays
243         self._results = results
244         self.__used_method = 'iterator'
245         launch_bundles = np.transpose([launch_lower, launch_upper])
246         return launch_bundles, iterative
```

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