



Universidade de Brasília
Departamento de Engenharia Elétrica
Controle Digital

Exercício de Simulação 5

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Brasília
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Questão 1

Para um processo descrito pela função de transferência

$$G(z) = \frac{0.0125(z + 0.195)(z + 2.821)}{z(z - 1)(z - 0.368)(z - 0.8187)}$$

a)

$$G(z) = \frac{0.0125(z + 0.195)(z + 2.821)}{z(z - 1)(z - 0.368)(z - 0.8187)} = \frac{0.0125z^{-2}(1 + 0.195z^{-1})(1 + 2.821z^{-1})}{(1 - z^{-1})(1 - 0.368z^{-1})(1 - 0.8187z^{-1})}$$

$$p = \max\{1, 1\} = 1$$

$$n = 4 - 2 = 2 \implies k \geq n = 2$$

Para tempo mínimo, $k = 2$

$$M(z) = (1 + 2.821z^{-1})(?)_1$$

$$(?)_1 = (M_2z^{-2} + \dots) \implies M(z) = (1 + 2.821z^{-1})(M_2z^{-2} + \dots)$$

$$1 - M(z) = (1 - z^{-1})(?)_z$$

$$(?)_z = (1 + a_1z^{-1} + \dots) \implies 1 - M(z) = (1 - z^{-1})(1 + a_1z^{-1} + \dots)$$

$$1 - (1 + 2.821z^{-1})(M_2z^{-2} + \dots) = (1 - z^{-1})(1 + a_1z^{-1} + \dots)$$

Para manter o maior grau da equação como 3 (z^{-3}),

$$1 - (1 + 2.821z^{-1})M_2z^{-2} = (1 - z^{-1})(1 + a_1z^{-1} + a_2z^{-2})$$

$$M(z) = (1 + 2.821z^{-1})M_2z^{-2}$$

$$1 - M(z) = (1 - z^{-1})(1 + a_1z^{-1} + a_2z^{-2})$$

$$1 - M_2z^{-2} - 2.821M_2z^{-3} = 1 + a_1z^{-1} + a_2z^{-2} - z^{-1} - a_1z^{-2} - a_2z^{-3}$$

$$1 - M_2z^{-2} - 2.821M_2z^{-3} = 1 + (a_1 - 1)z^{-1} + (a_2 - a_1)z^{-2} - a_2z^{-3}$$

$$\begin{cases} a_1 - 1 = 0 \\ M_2 = a_1 - a_2 \\ 2.821M_2 = a_2 \end{cases} \implies \begin{cases} a_1 = 1 \\ a_2 = 0.7383 \\ M_2 = 0.2617 \end{cases}$$

$$M(z) = (1 + 2.821z^{-1})0.2617z^{-2} = \frac{0.2617(z + 2.821)}{z^3}$$

$$1 - M(z) = (1 - z^{-1})(1 + z^{-1} + 0.7383z^{-2}) = \frac{(z - 1)(z^2 + z + 0.7383)}{z^3}$$

$$G_D(z) = \frac{1}{G(z)} \frac{M(z)}{1 - M(z)} = \frac{z(z - 1)(z - 0.368)(z - 0.8187)}{0.0125(z + 0.195)(z + 2.821)} \frac{\frac{0.2617(z + 2.821)}{z^3}}{\frac{(z - 1)(z^2 + z + 0.7383)}{z^3}}$$

$$G_D(z) = \frac{20,936z(z - 0.368)(z - 0.8187)}{(z + 0.195)(z^2 + z + 0.7833)}$$

b)

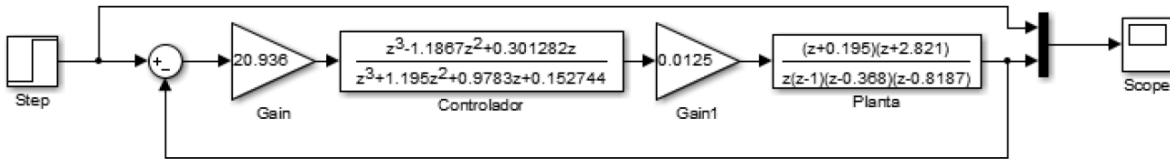


Figura 1: Diagrama do sistema.

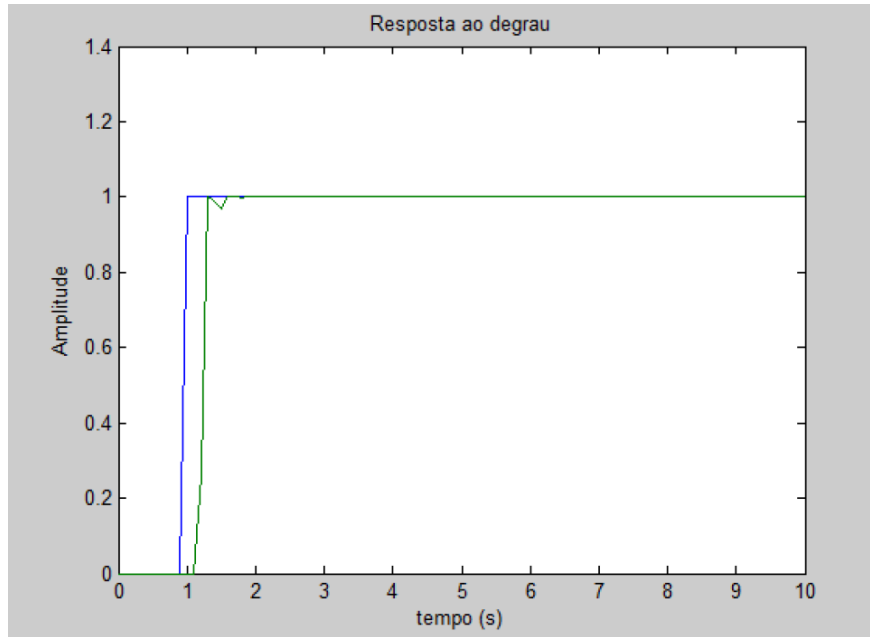


Figura 2: Resposta ao degrau.

c)

$$p = \max\{2, 1\} = 2$$

$$n = 4 - 2 = 2 \implies k \geq n = 2$$

$$1 - (1 + 2.821z^{-1})(M_1z^{-1} + M_2z^{-2}) = (1 - z^{-1})^2(1 + a_1z^{-1})$$

$$M(z) = (1 + 2.821z^{-1})(M_1z^{-1} + M_2z^{-2})$$

$$1 - M(z) = (1 - z^{-1})^2(1 + a_1z^{-1})$$

$$1 - M_1z^{-1} - M_2z^{-2} - 2.821M_1z^{-2} - 2.821M_2z^{-3} = 1 + a_1z^{-1} - 2z^{-1} - 2a_1z^{-2} + z^{-2} + a_1z^{-3}$$

$$1 - M_1z^{-1} - (M_2 + 2.821M_1)z^{-2} - 2.821M_2z^{-3} = 1 + (a_1 - 2)z^{-1} + (1 - 2a_1)z^{-2} + a_1z^{-3}$$

$$\begin{cases} a_1 - 2 = -M_1 \\ 1 - 2a_1 = -M_2 - 2.821M_1 \\ a_1 = 2.821M_2 \end{cases} \implies \begin{cases} a_1 = 1.2833 \\ M_1 = 0.7167 \\ M_2 = -0.46493 \end{cases}$$

$$M(z) = (1 + 2.821z^{-1})(0.7167z^{-1} - 0.46493z^{-2}) = \frac{0.7167(z + 2.821)(z - 0.63475)}{z^3}$$

$$1 - M(z) = (1 - z^{-1})^2(1 + 1.2833z^{-1}) = \frac{(z - 1)^2(z + 1.2833)}{z^3}$$

$$G_D(z) = \frac{1}{G(z)} \frac{M(z)}{1 - M(z)} = \frac{z(z - 1)(z - 0.368)(z - 0.8187)}{0.0125(z + 0.195)(z + 2.821)} \frac{\frac{0.7167(z + 2.821)(z - 0.63475)}{z^3}}{\frac{(z - 1)^2(z + 1.2833)}{z^3}}$$

$$G_D(z) = \frac{57.336z(z - 0.368)(z - 0.8187)(z - 0.63475)}{(z - 1)(z + 0.195)(z + 1.2833)}$$

d)

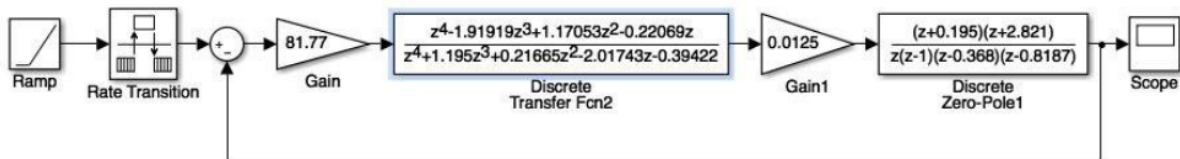


Figura 3: Diagrama do sistema.

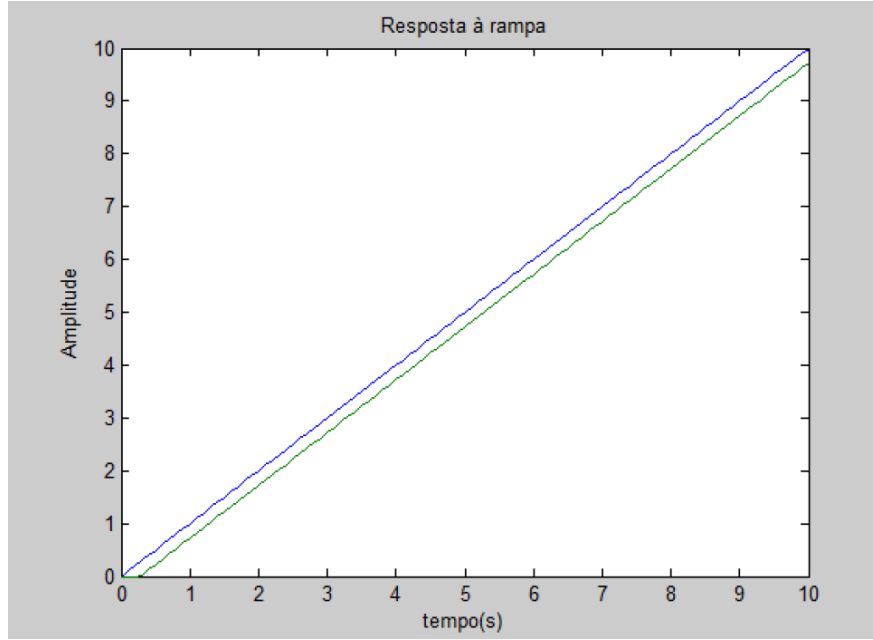


Figura 4: Resposta à rampa.

Questão 2

Para um processo descrito pela função de transferência

$$G(z) = \frac{0.0003916(z + 2.8276)(z + 0.19)}{(z - 1)^2(z - 0.2865)}$$

a)

$$p = \max\{2, 1\} = 2$$

$$n = 3 - 2 = 1 \implies k \geq n = 1$$

$$M(z) = (1 + 2.8276z^{-1})(M_1z^{-1} + \dots)$$

$$1 - M(z) = (1 - z^{-1})^2(1 + a_1z^{-1} + \dots)$$

$$1 - (1 + 2.8276z^{-1})(M_1z^{-1} + \dots) = (1 - z^{-1})^2(1 + a_1z^{-1} + \dots)$$

Para manter o maior grau da equação como 3 (z^{-3}),

$$1 - (1 + 2.8276z^{-1})(M_1z^{-1} + M_2z^{-2}) = (1 - z^{-1})^2(1 + a_1z^{-1})$$

$$M(z) = (1 + 2.8276z^{-1})(M_1z^{-1} + M_2z^{-2})$$

$$1 - M(z) = (1 - z^{-1})^2(1 + a_1 z^{-1})$$

$$1 - M_1 z^{-1} - M_2 z^{-2} - 2.8276 M_1 z^{-2} - 2.8276 M_2 z^{-3} = 1 + a_1 z^{-1} - 2z^{-1} - 2a_1 z^{-2} + z^{-2} + a_1 z^{-3}$$

$$1 - M_1 z^{-1} - (M_2 + 2.8276 M_1) z^{-2} - 2.8276 M_2 z^{-3} = 1 + (a_1 - 2) z^{-1} + (1 - 2a_1) z^{-2} + a_1 z^{-3}$$

$$\begin{cases} a_1 - 2 = -M_1 \\ 1 - 2a_1 = -M_2 - 2.8276 M_1 \\ a_1 = 2.8276 M_2 \end{cases} \implies \begin{cases} a_1 = 1.2845 \\ M_1 = 0.7155 \\ M_2 = -0.4543 \end{cases}$$

$$M(z) = (1 + 2.8276 z^{-1})(0.7155 z^{-1} - 0.4543 z^{-2}) = \frac{0.7155(z + 2.8276)(z - 0.6349)}{z^3}$$

$$1 - M(z) = (1 - z^{-1})^2(1 + 1.2845 z^{-1}) = \frac{(z - 1)^2(z + 1.2845)}{z^3}$$

$$G_D(z) = \frac{1}{G(z)} \frac{M(z)}{1 - M(z)} = \frac{(z - 1)^2(z - 0.2865)}{0.0003916(z + 2.8276)(z + 0.19)} \frac{\frac{0.7155(z + 2.8276)(z - 0.6349)}{z^3}}{\frac{(z - 1)^2(z + 1.2845)}{z^3}}$$

$$G_D(z) = \frac{1827.17(z - 0.6349)(z - 0.2865)}{(z + 0.19)(z + 1.2845)}$$

b)

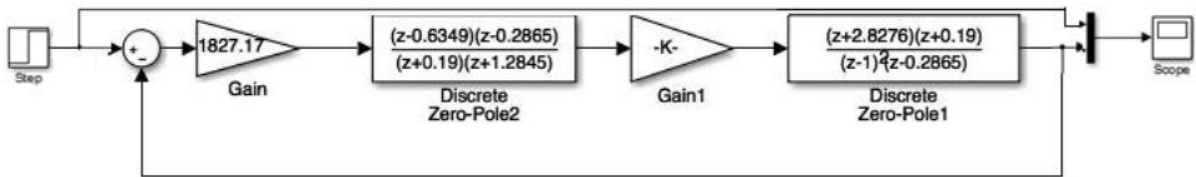


Figura 5: Diagrama do sistema.

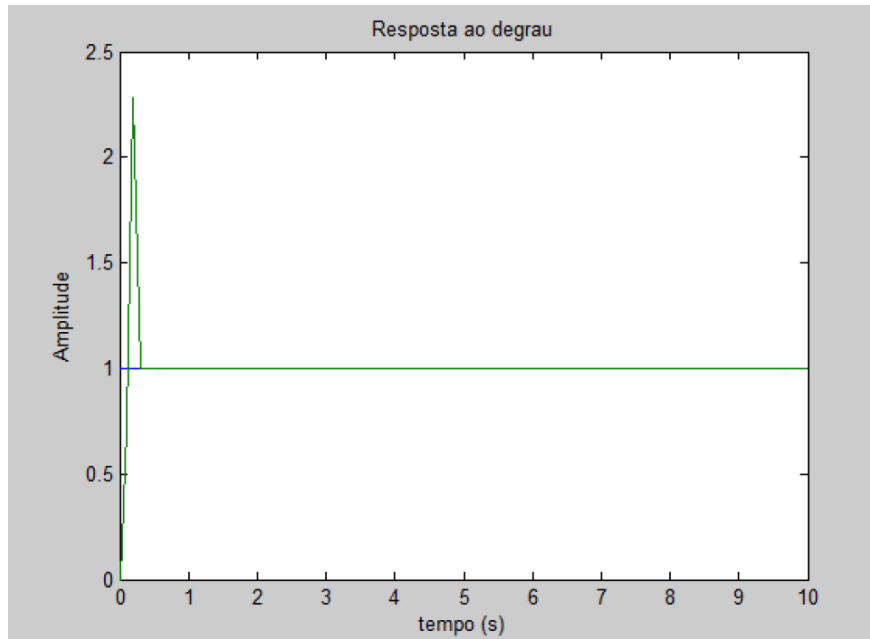


Figura 6: Resposta ao degrau.

c)

$$p = \max\{2, 2\} = 2$$

$$n = 3 - 2 = 1 \implies k \geq n = 1$$

Como k e p não sofrem alteração, o controlador permanece o mesmo, sendo

$$G_D(z) = \frac{1827.17(z - 0.6349)(z - 0.2865)}{(z + 0.19)(z + 1.2845)}$$

d)

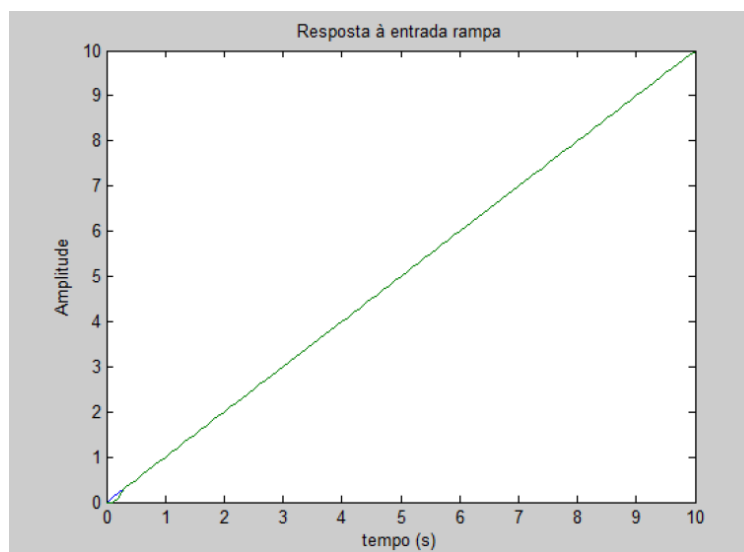


Figura 7: Resposta à rampa.

Questão 3

d)

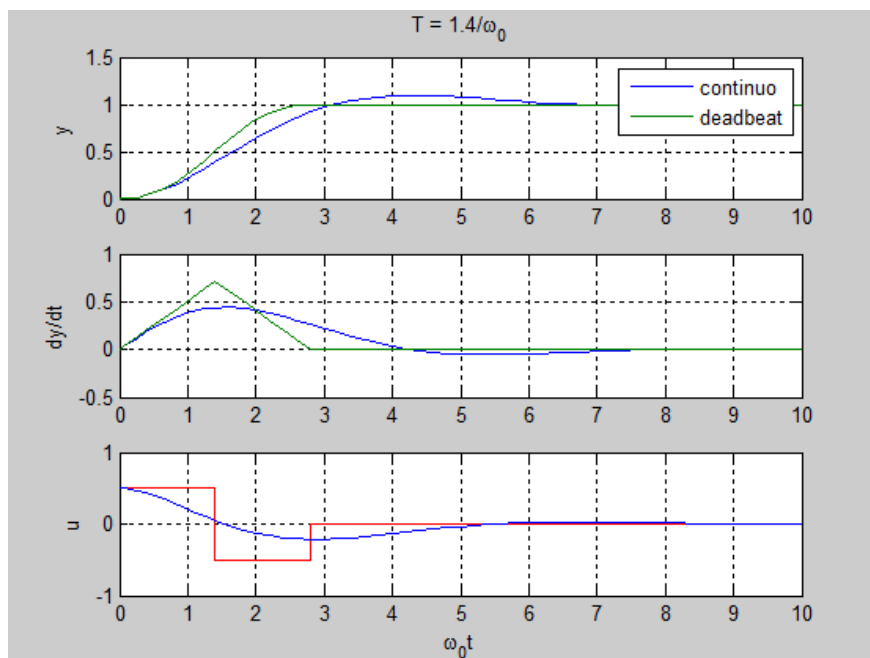


Figura 8: Comparação das ações de controle para $T = 1.4s$.

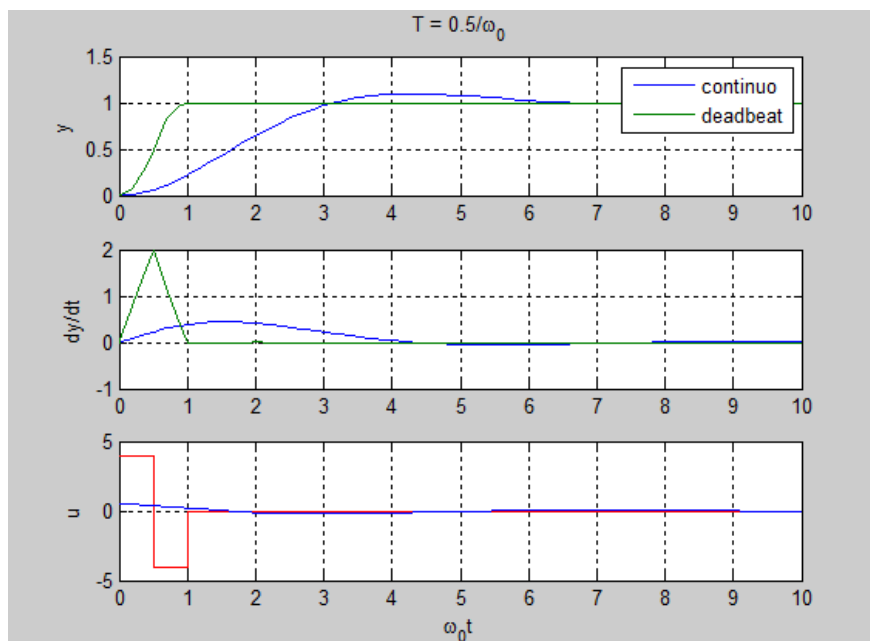


Figura 9: Comparação das ações de controle para $T = 0.5s$.

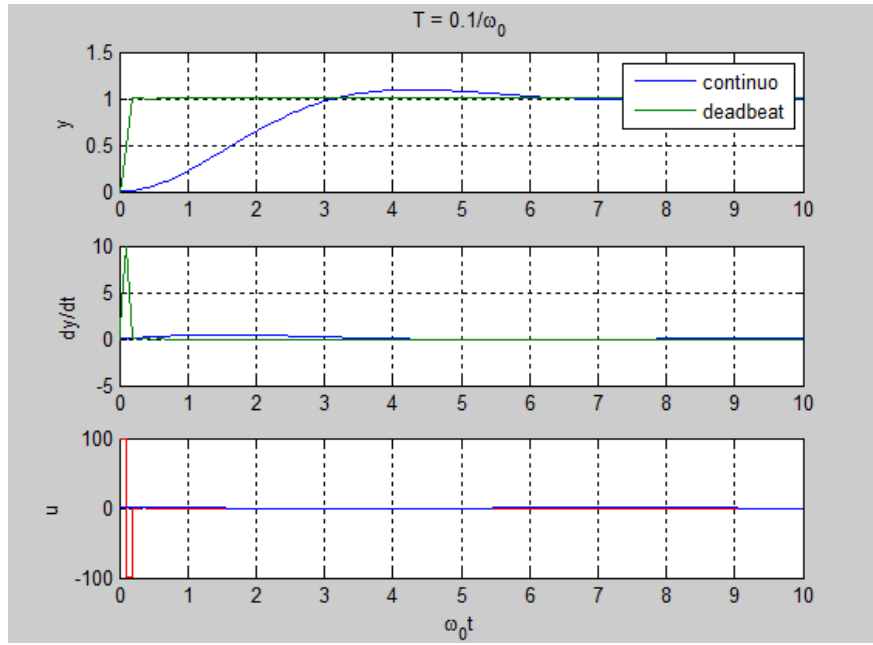


Figura 10: Comparação das ações de controle para $T = 0.1s$.