Knowledge Tracing

Machine Learning for Behavioral Data

March 28, 2022

Today’s Topic

Week	Lecture/Lab
1	Introduction
2	Data Exploration
3	Regression
4	Classification
5	Model Evaluation
6	Knowledge Tracing
7	Knowledge Tracing
8	Time Series Prediction

Supervised learning on time series:
- **Probabilistic graphical models**
- **Neural networks: LSTM, GRU, etc.**

Getting ready for today's lecture…

- **If not done yet**: clone the repository containing the Jupyter notebook and data for today's lecture into your Noto workspace
- SpeakUp room for today's lecture:

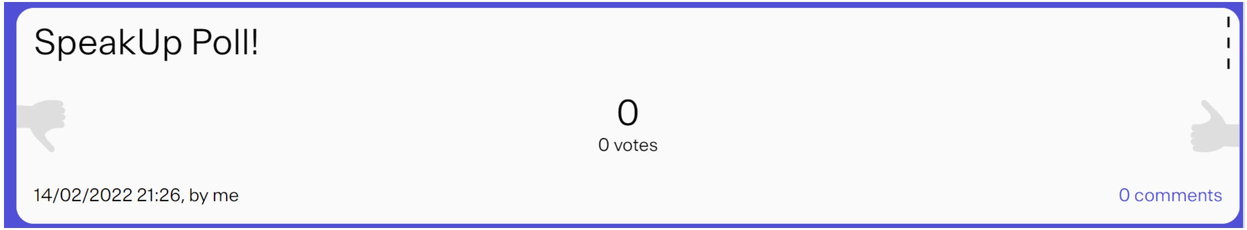
https://go.epfl.ch/mlbd-lecture

Short quiz about the past…

[Model Evaluation] Given a data set {1,2,3,4} , one possible bootstrap set is {1,1,1,1}:

a) True

b) False



Short quiz about the past…

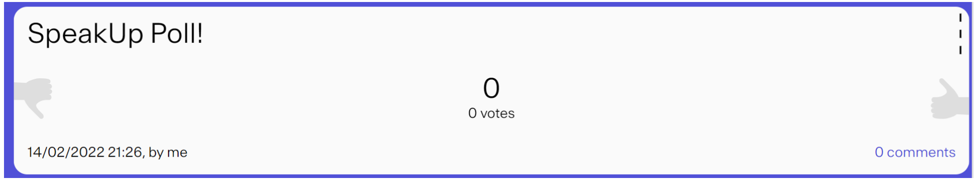
[Model Evaluation] Which of the following statements about k-fold cross validation are wrong? N denotes the number of samples in the data set, k the number of folds:

a) k must always be smaller than N.

b) The smaller k is, the more expensive it is to compute the error.

c) Cross validation can be used to tune model hyperparameters.

d) Cross validation is not a valid method for computing the generalization error of a model.



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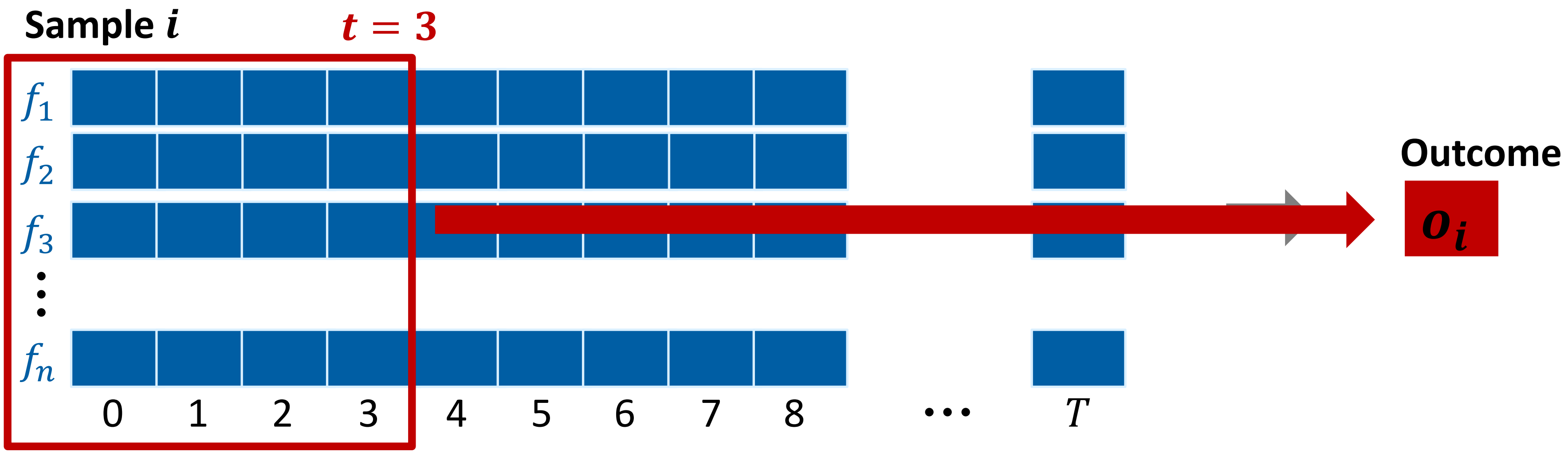
Supervised learning on time series:
- **Probabilistic graphical models**
- **Neural networks: LSTM, GRU, etc.**

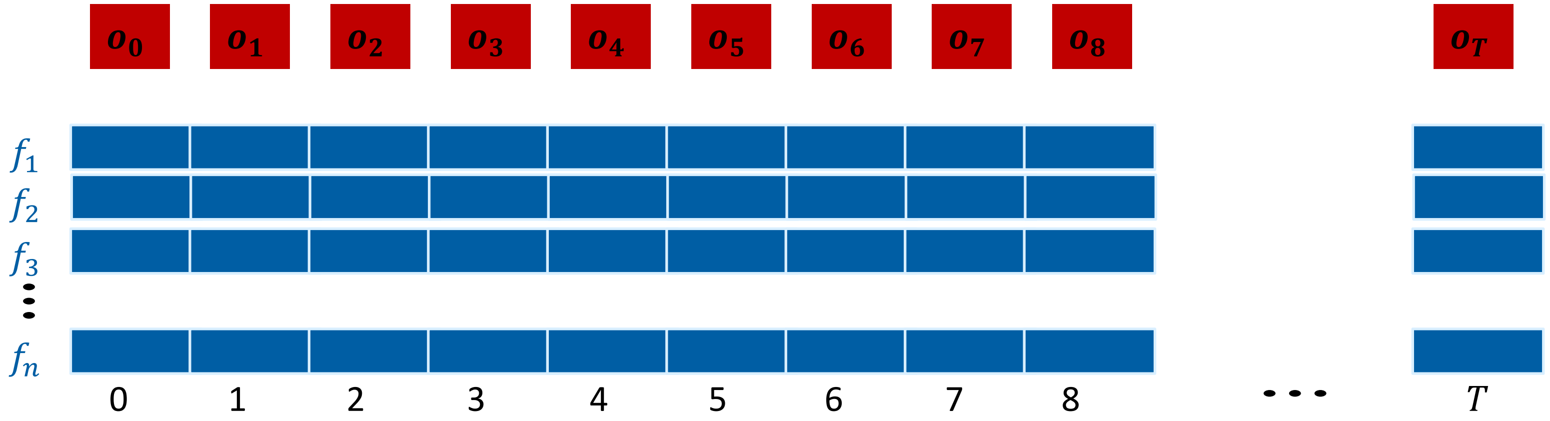
Time Series – Prediction Task

- Prediction of a target variable after $t < T$ time steps, where T is the total number of time steps



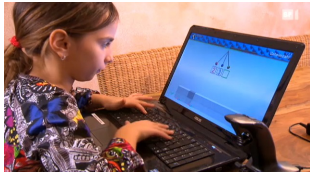
Time Series – Tracing Task

- Prediction of a target variable after $t < T$ time steps, where T is the total number of time steps
- Prediction of a variable in time step $t + 1$, based on time steps $0, \dots, t$



Today: Tracing Student Knowledge

- Is the student learning?
  - Measure what the student *knows* at a specific time *t*
  - More specifically: knowledge of the student about relevant knowledge components (skills)



Task:	$50 - 23 = ?$	$75 - 12 = ?$	$38 - 14 = ?$
Answer:	27	61	24

Tracing Knowledge – why is it useful?

- Is the student learning?
  - Measure what the student *knows* at a specific time *t*
  - More specifically: knowledge of the student about relevant knowledge components (skills)

➡ Choose the next appropriate activity

➡ Know which activities support learning

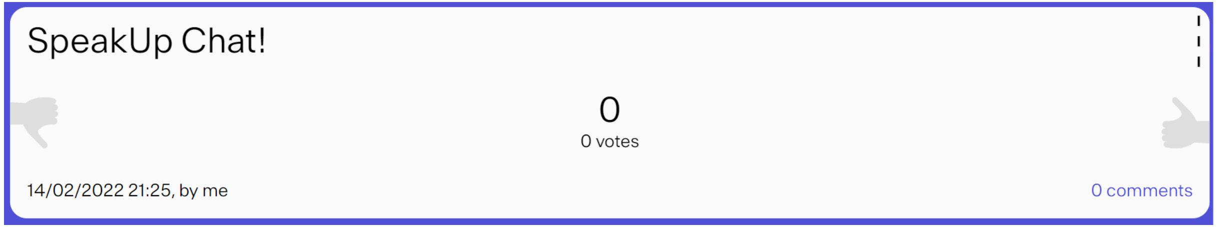
Today's Use Case

- ASSISTments is a free tool for assigning and assessing math problems and homework
- All math problems (tasks/items) are associated to a specific skill/knowledge component
- 4,217 middle-school students
- 525,534 observations

Today: Tracing Student Knowledge

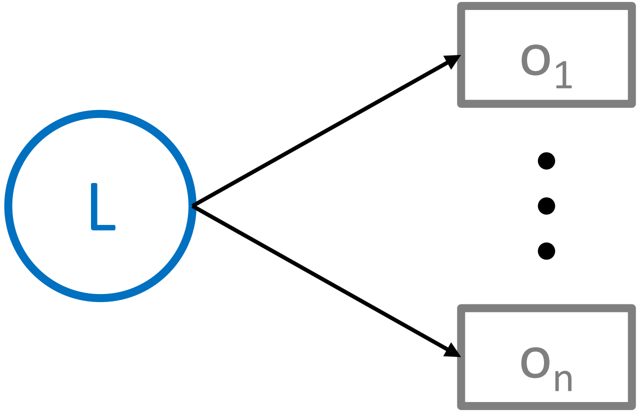
- **Bayesian Knowledge Tracing (BKT)**
- Learning Curves

What is a latent variable?



What is a latent variable?

- A latent variable L is a variable which is not directly observable/cannot be measured
- It is assumed to affect the outcome of other variables $\boldsymbol{o}$, which can be observed (directly measured)

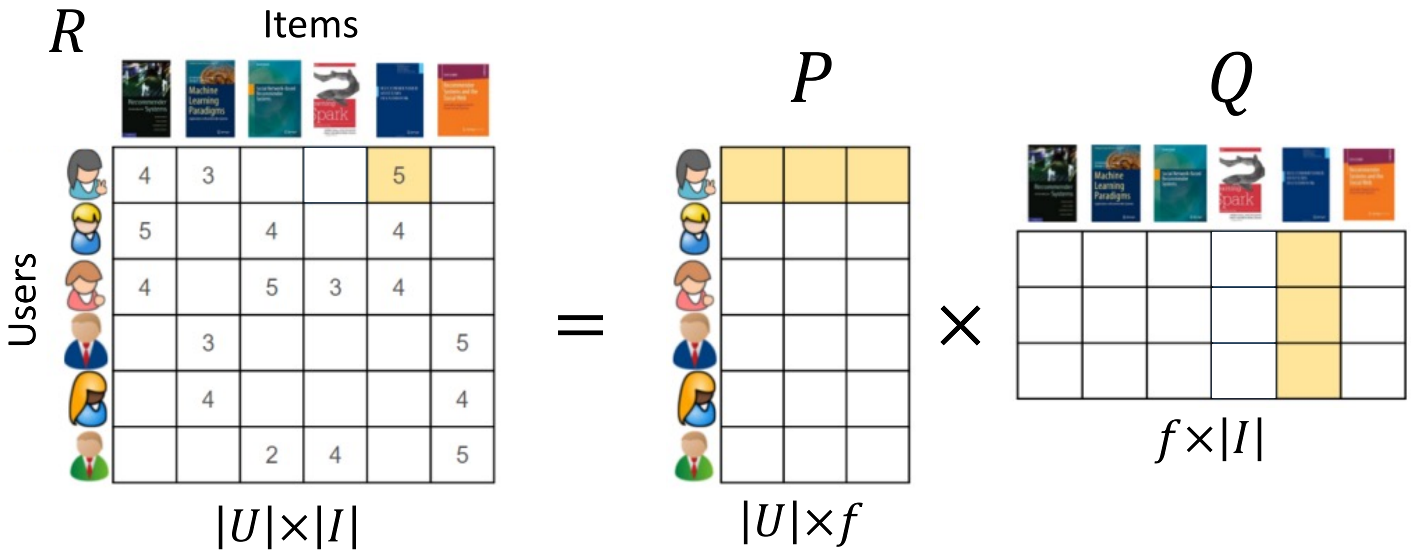


Why should we use latent variables?

- In many scientific fields, we are interested in concepts/factors that cannot directly be measured/observed:
  - Political sciences: leadership, political competence, etc.
  - Psychology: stress, self-worth, personality characteristics, talent, etc.
  - Education: memory, spatial ability, cognitive abilities, etc.
- We represent underlying concepts/factors by latent variables and infer them from the observed variables

Example 1: Recommender Systems

- Given: ratings of users u for items i (e.g., books)



[Image source: https://heartbeat.fritz.ai/recommender-systems-with-python-part-iii-collaborative-filtering-singular-value-decomposition-5b5dcb3f242b]

Example 2: Education

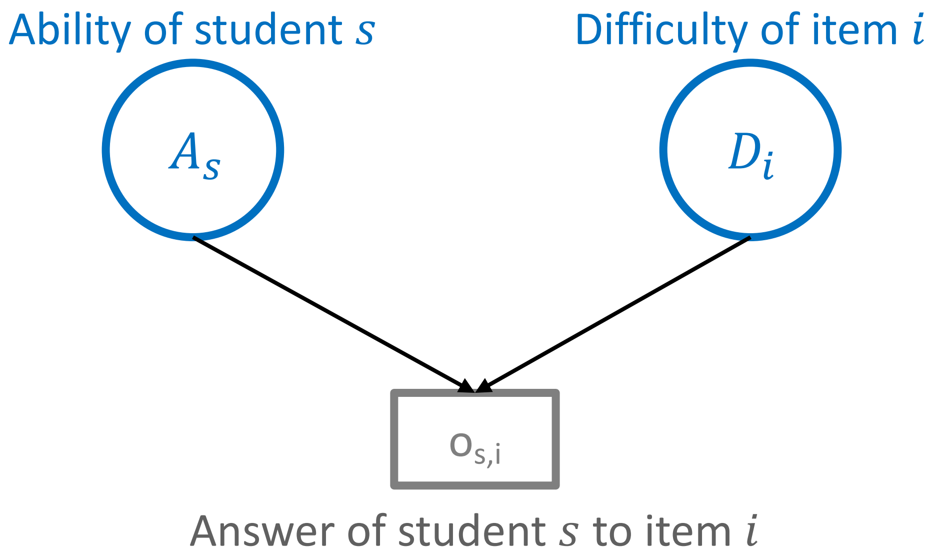
- Observations: binary answers (correct/wrong) of students to items (tasks)

$$\boxed{o_{s,i}}$$

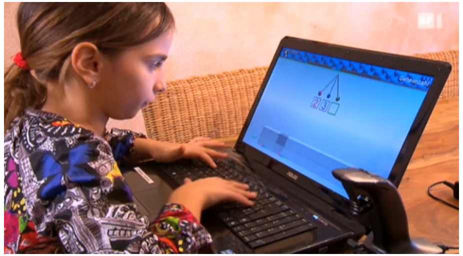
Answer of student s to item i

Example 2: Education

- Observations: binary answers (correct/wrong) of students to items (tasks)

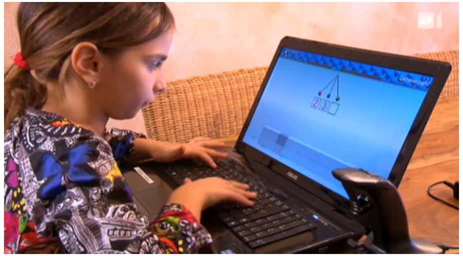


Is the student learning?



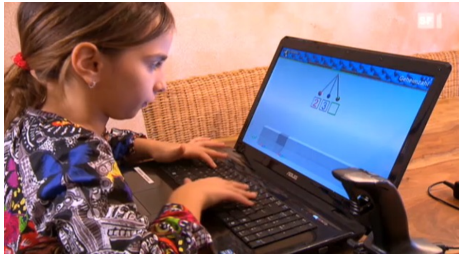
Task:	$50 - 23 = ?$	$75 - 12 = ?$	$38 - 14 = ?$
Answer:	27	61	24

What are we measuring?



Task:	$50 - 23 = ?$	$75 - 12 = ?$	$38 - 14 = ?$
Answer:	27	61	24
	1	0	1

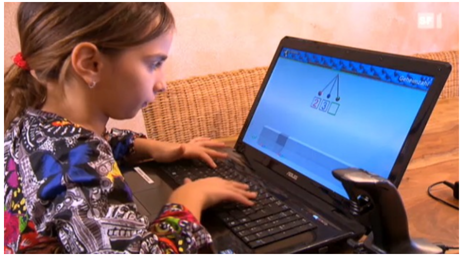
Binary observations of student answers



Subtraction 0-100

1	2		...		n
0	0	1	0	1	1

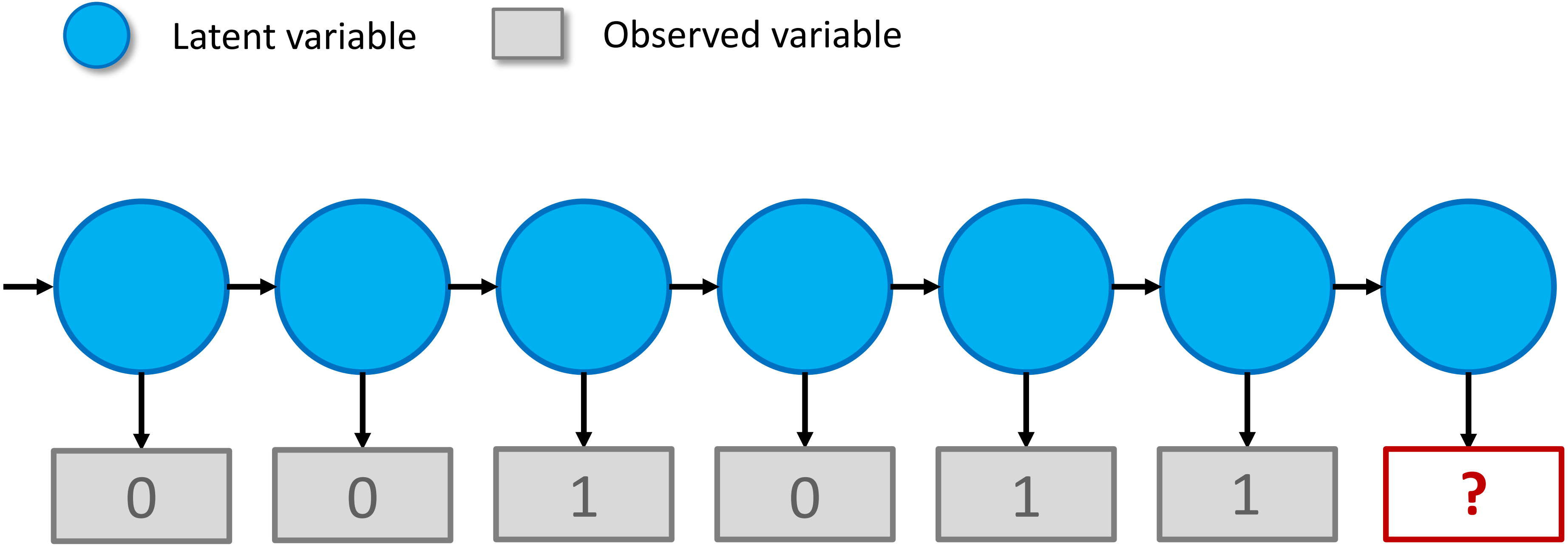
Predicting future performance



Subtraction 0-100

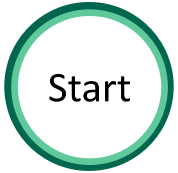
1	2		…		n	n+1
0	0	1	0	1	1	?

Bayesian Knowledge Tracing (BKT)



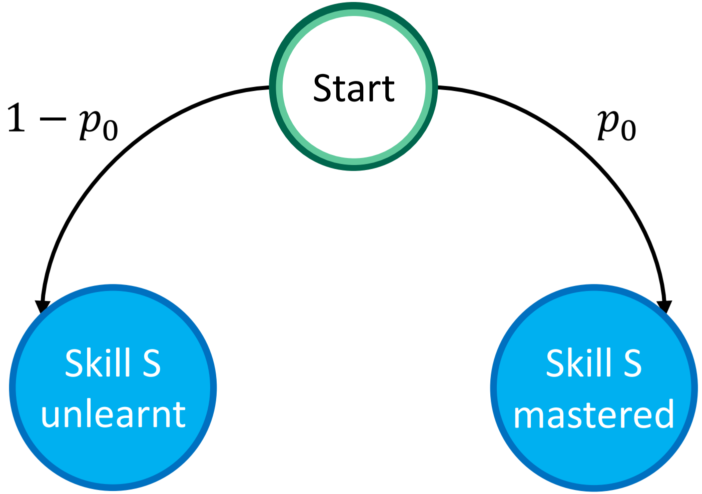
[Corbett & Anderson, UMUAI 1994]

Bayesian Knowledge Tracing (BKT)



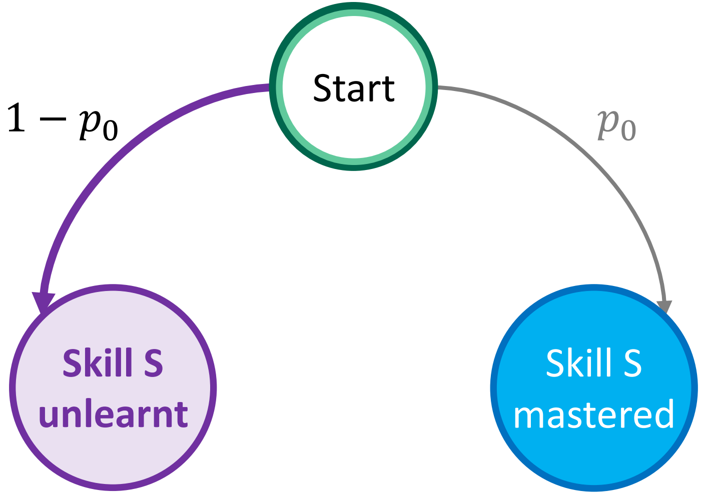
Observations for student s:

Bayesian Knowledge Tracing (BKT)



Observations for student s:

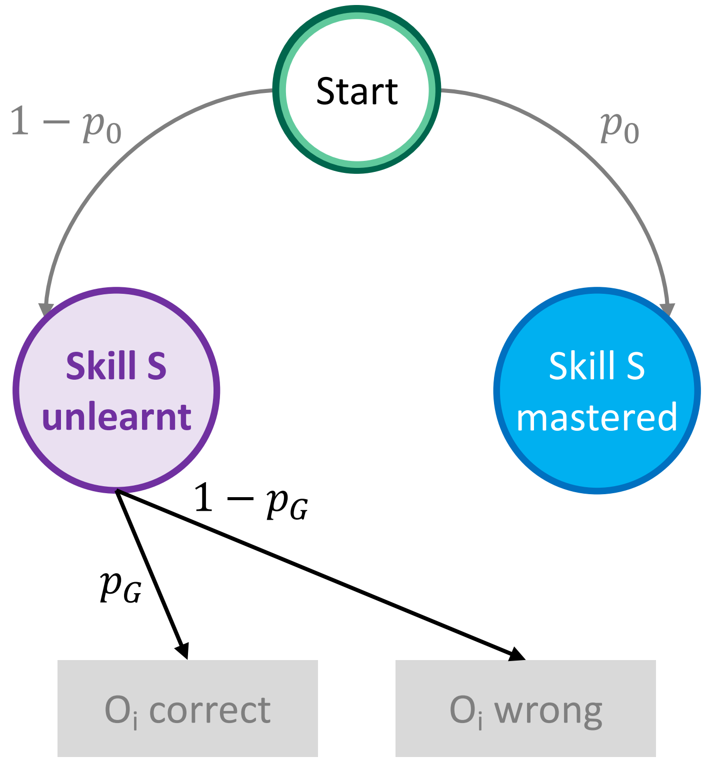
Bayesian Knowledge Tracing (BKT)



Observations for student s:

$t = 0$:

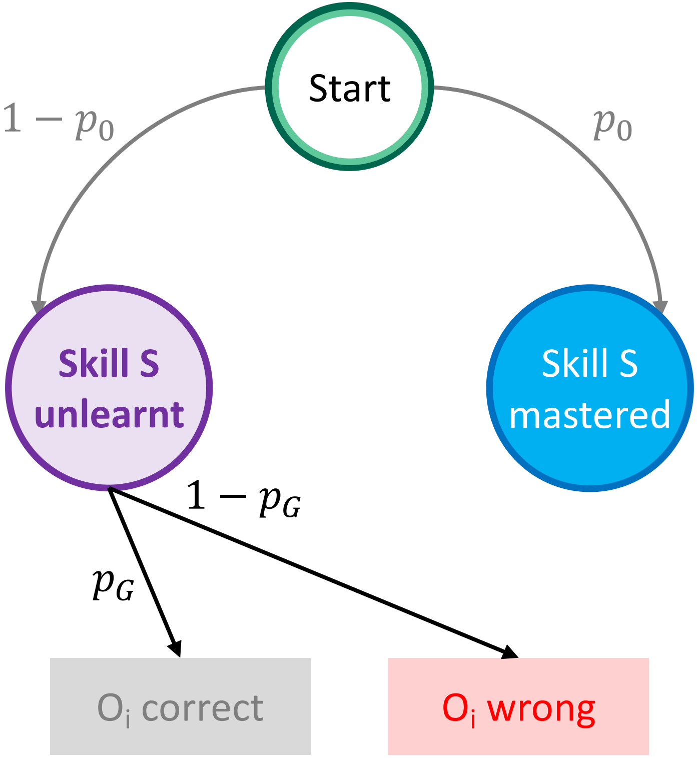
Bayesian Knowledge Tracing (BKT)



Observations for student s:

$t = 0$:

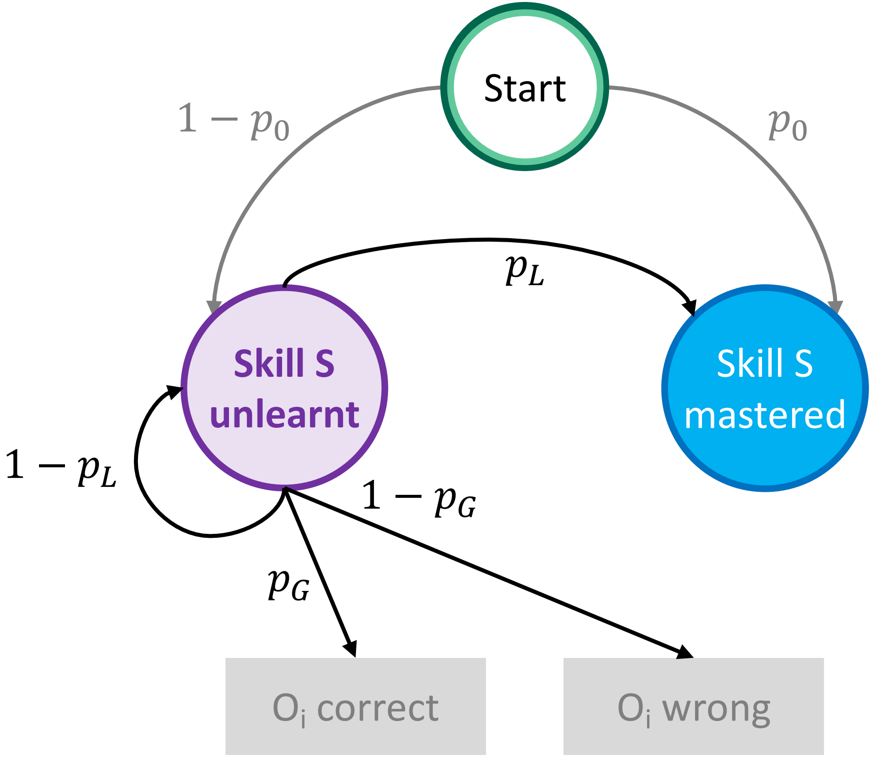
Bayesian Knowledge Tracing (BKT)



Observations for student s:

$t = 0$: 0

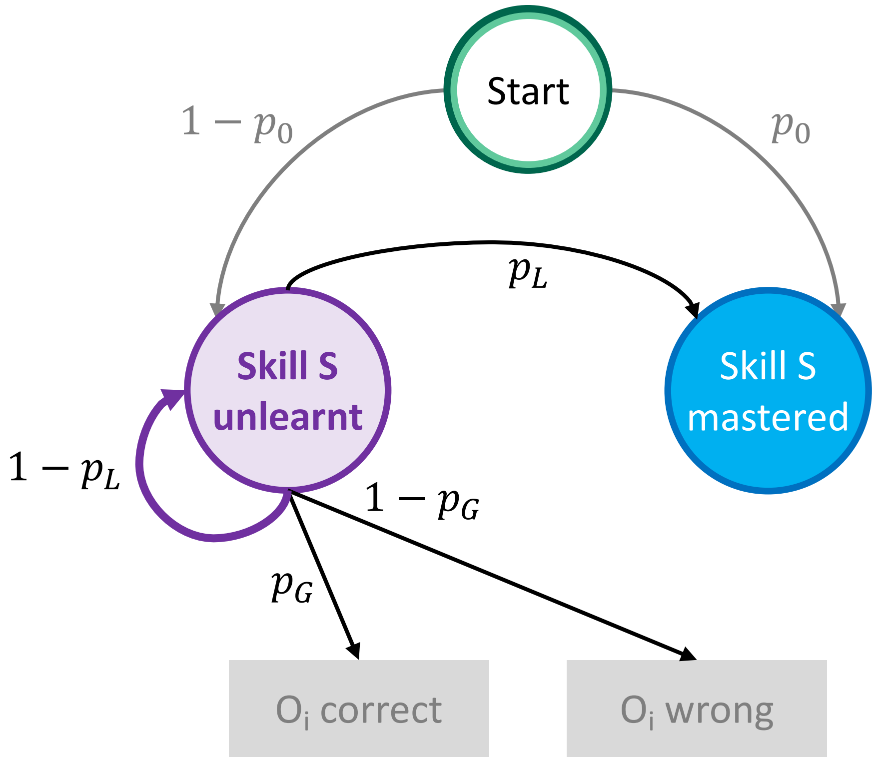
Bayesian Knowledge Tracing (BKT)



Observations for student s:

$t = 0$: 0

Bayesian Knowledge Tracing (BKT)

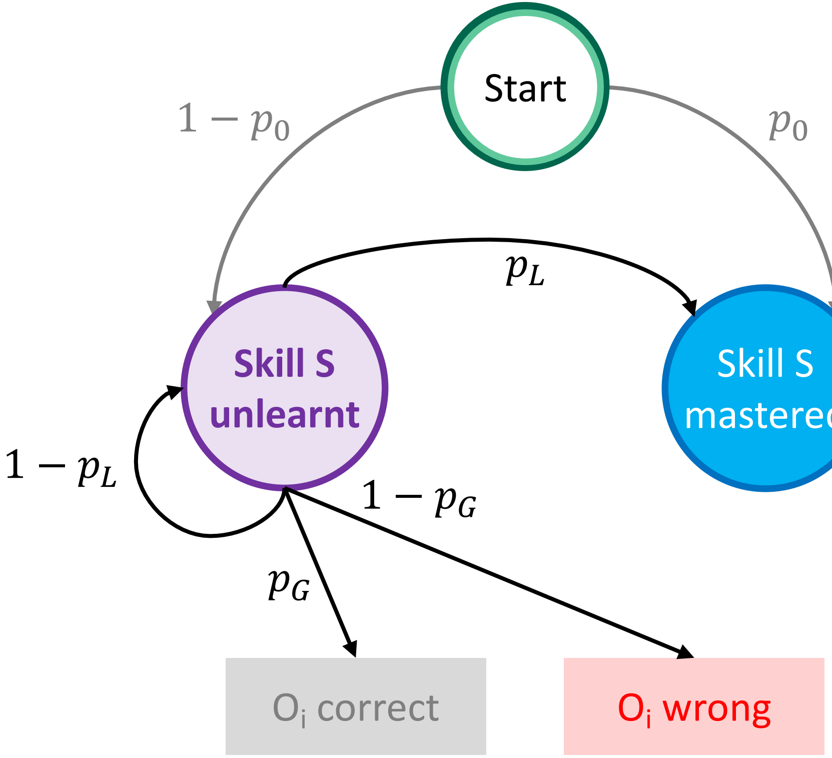


Observations for student s:

$t = 0$: 0

$t = 1$:

Bayesian Knowledge Tracing (BKT)

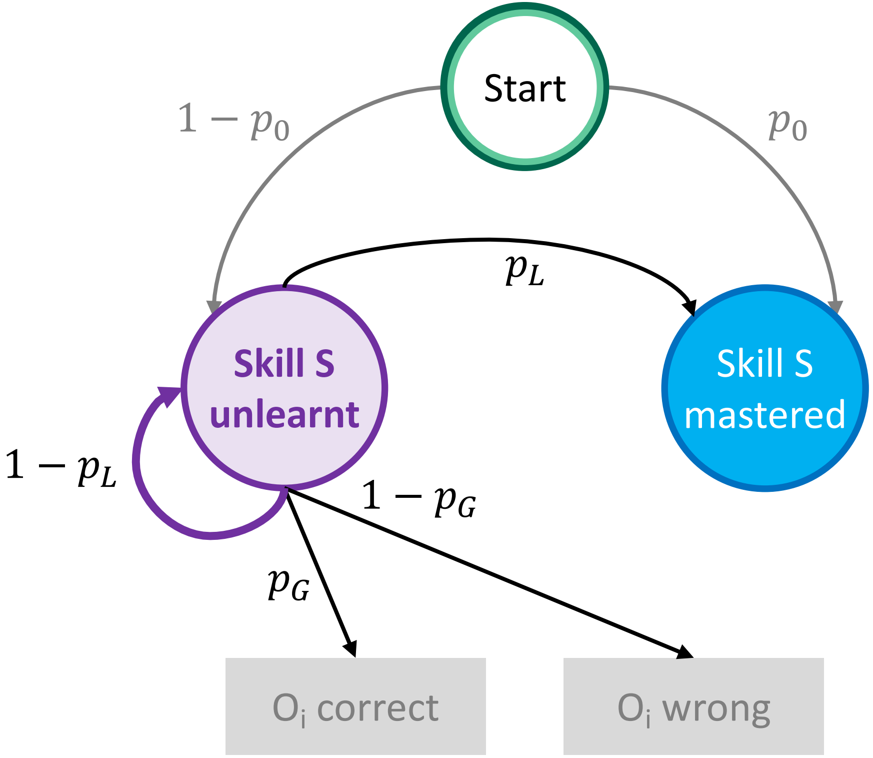


Observations for student s:

$t = 0$: 0

$t = 1$: 0

Bayesian Knowledge Tracing (BKT)



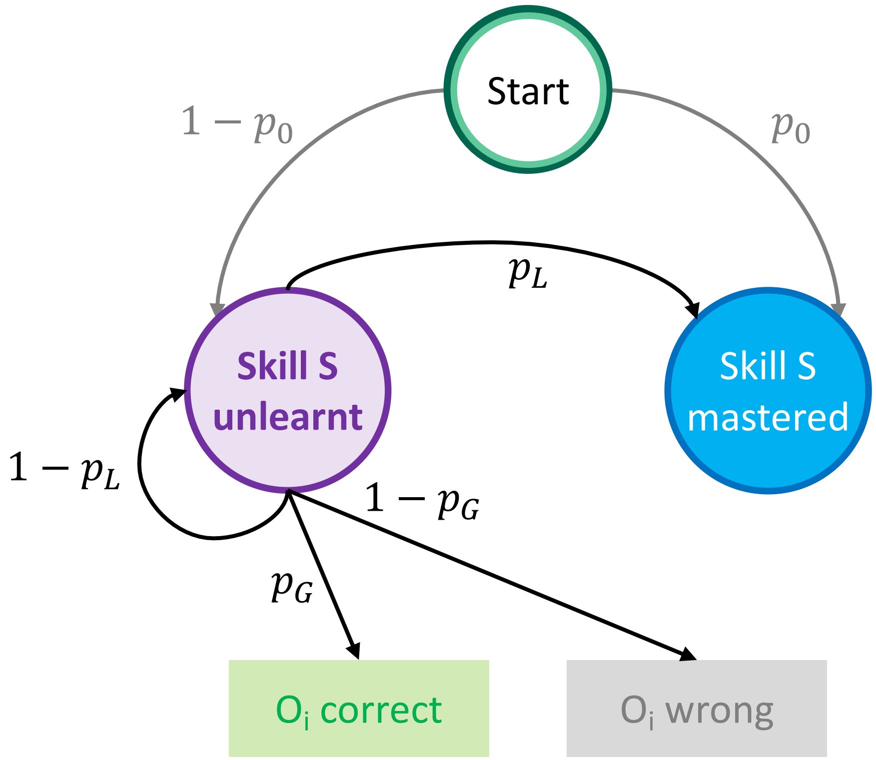
Observations for student s:

$t = 0$: 0

$t = 1$: 0

$t = 2$:

Bayesian Knowledge Tracing (BKT)



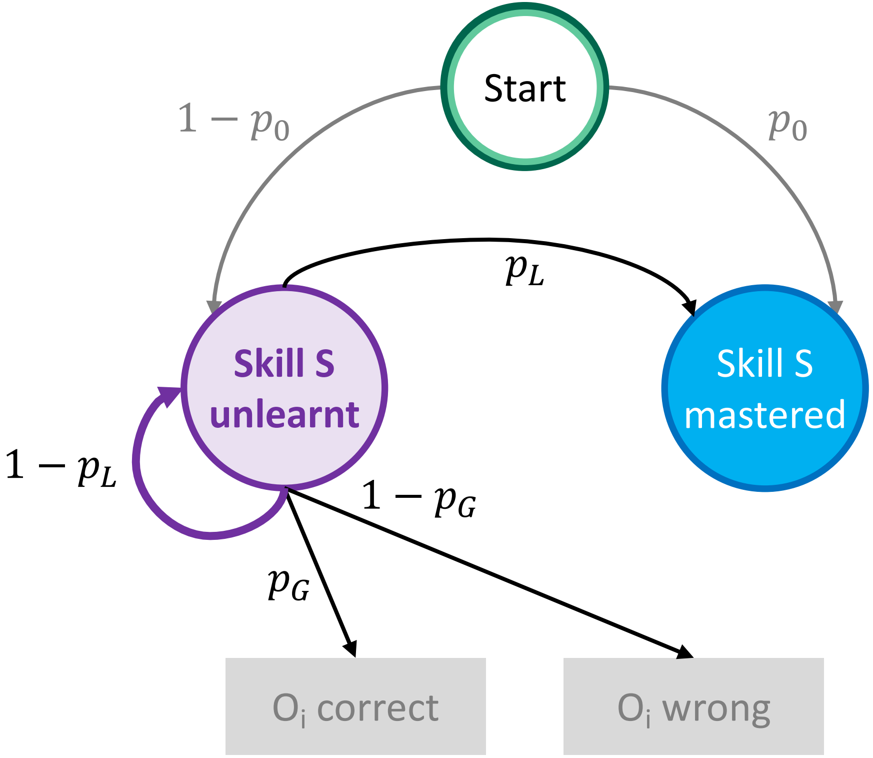
Observations for student s:

$t = 0$: 0

$t = 1$: 0

$t = 2$: 1

Bayesian Knowledge Tracing (BKT)



Observations for student s:

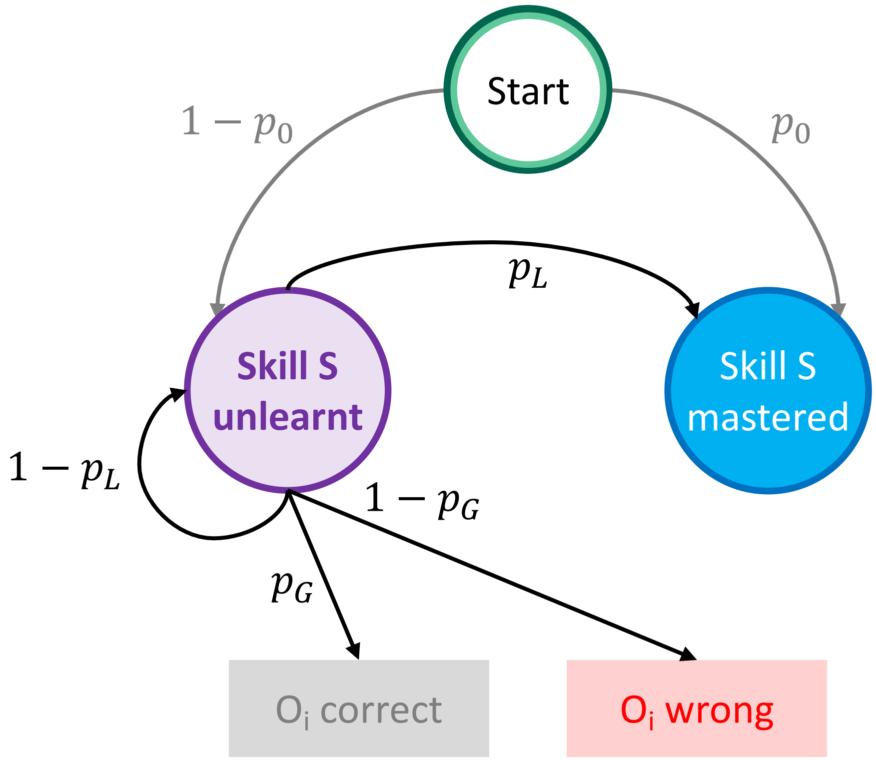
$t = 0$: 0

$t = 1$: 0

$t = 2$: 1

$t = 3$:

Bayesian Knowledge Tracing (BKT)



Observations for student s:

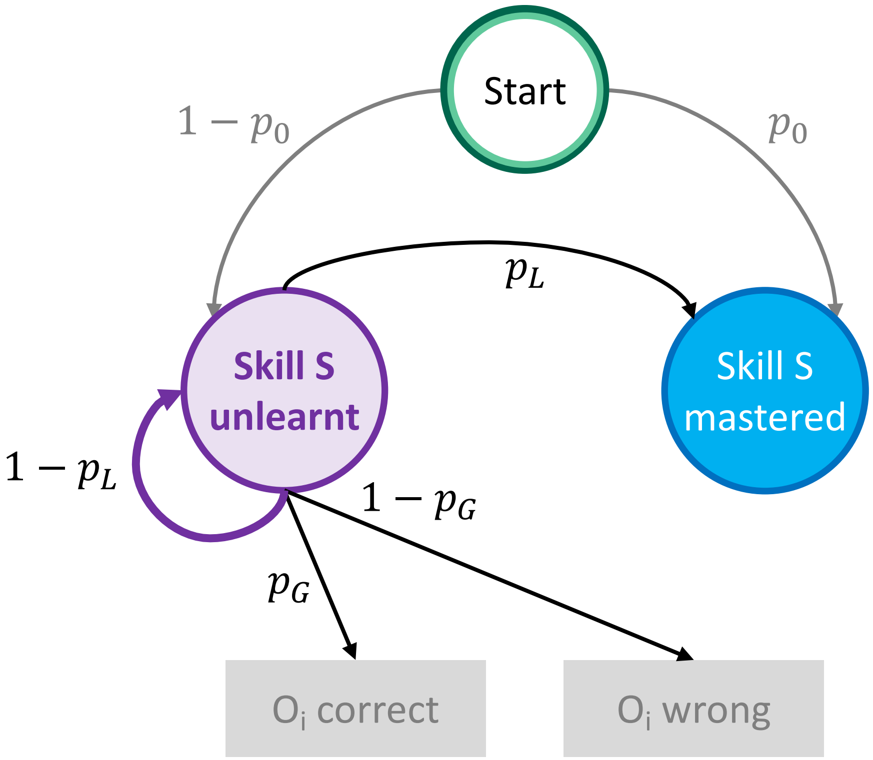
$t = 0$: 0

$t = 1$: 0

$t = 2$: 1

$t = 3$: 0

Bayesian Knowledge Tracing (BKT)



Observations for student s:

$t = 0$: 0

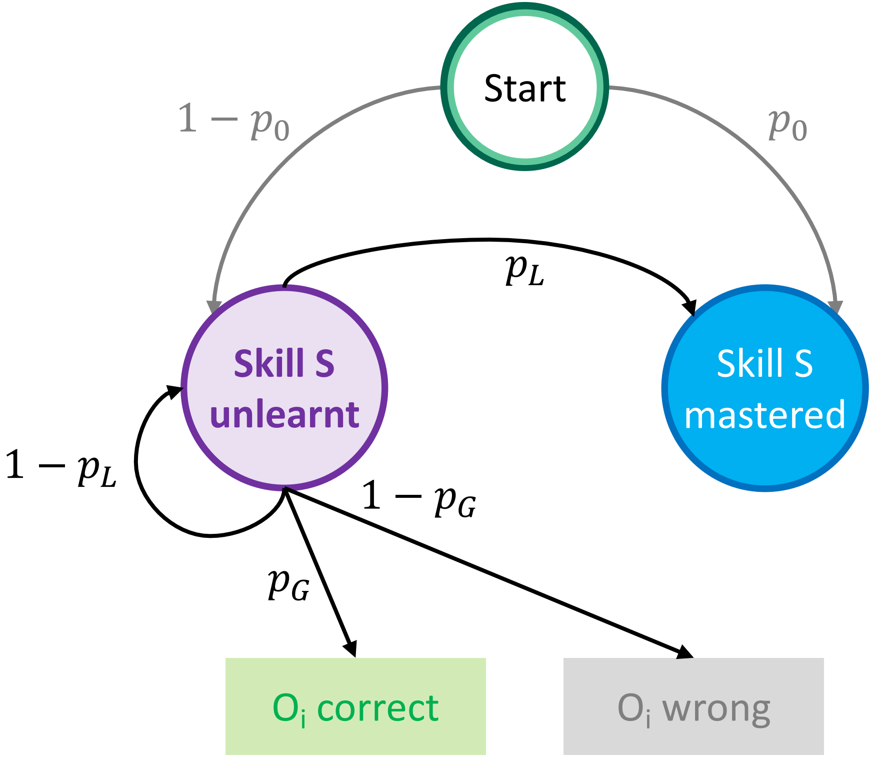
$t = 1$: 0

$t = 2$: 1

$t = 3$: 0

$t = 4$:

Bayesian Knowledge Tracing (BKT)



Observations for student s:

$t = 0$: 0

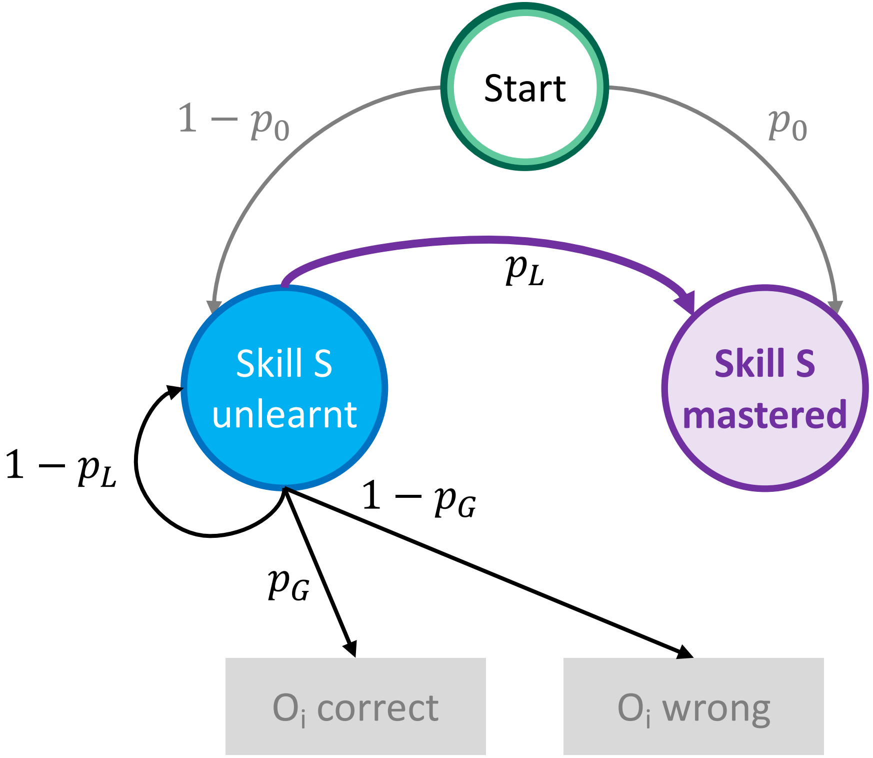
$t = 1$: 0

$t = 2$: 1

$t = 3$: 0

$t = 4$: 1

Bayesian Knowledge Tracing (BKT)



Observations for student s:

$t = 0$: 0

$t = 1$: 0

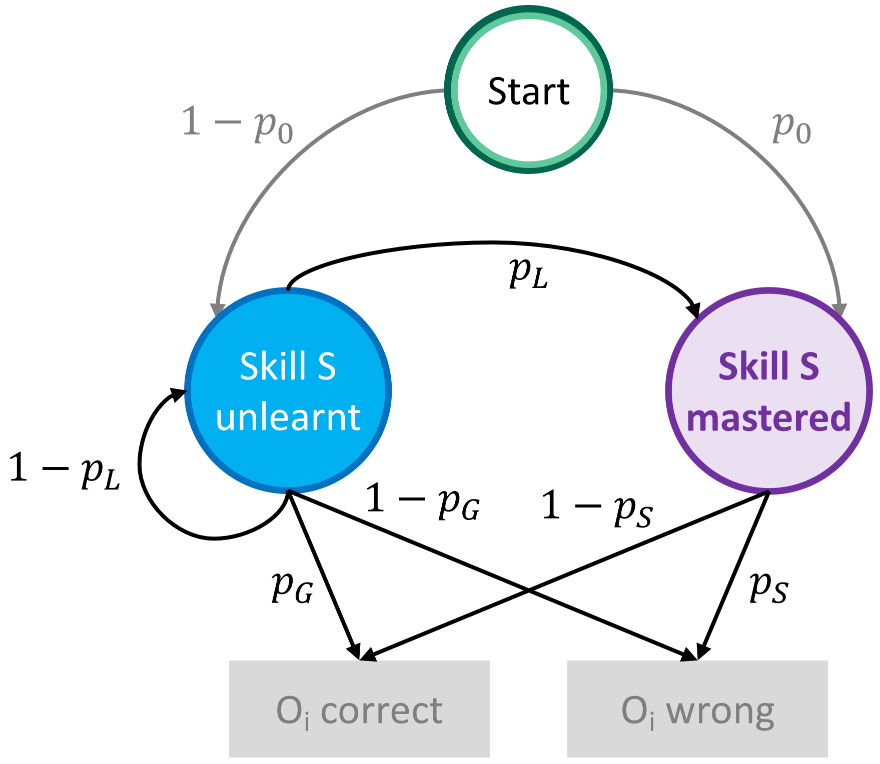
$t = 2$: 1

$t = 3$: 0

$t = 4$: 1

$t = 5$:

Bayesian Knowledge Tracing (BKT)



Observations for student s:

$t = 0$: 0

$t = 1$: 0

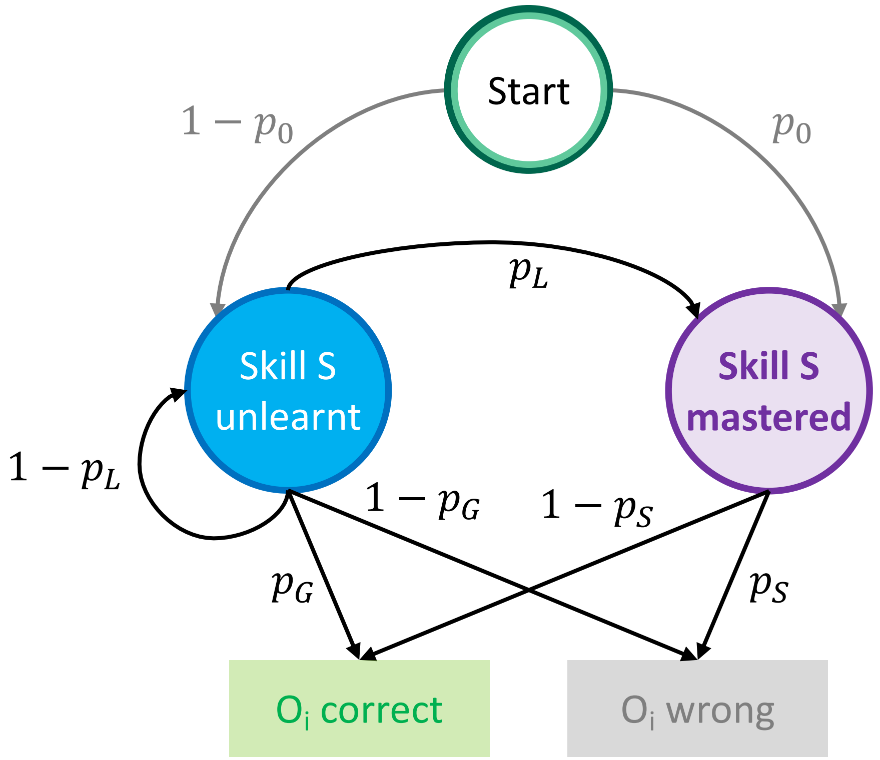
$t = 2$: 1

$t = 3$: 0

$t = 4$: 1

$t = 5$:

Bayesian Knowledge Tracing (BKT)



Observations for student s:

$t = 0$: 0

$t = 1$: 0

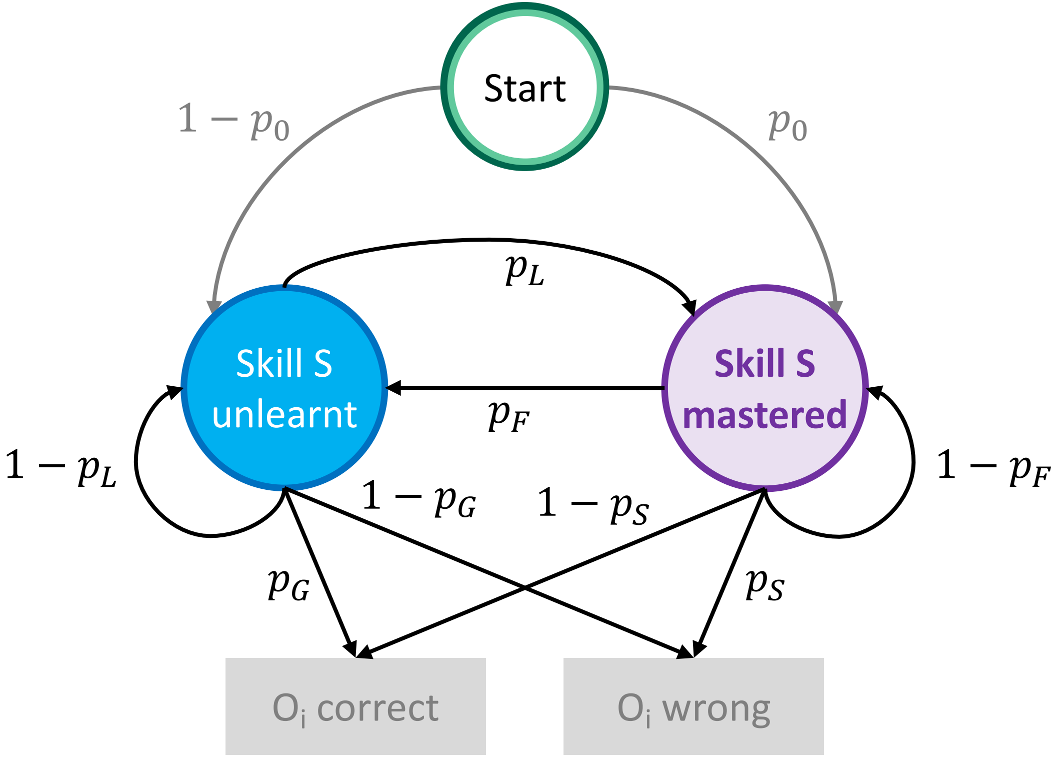
$t = 2$: 1

$t = 3$: 0

$t = 4$: 1

$t = 5$: 1

Bayesian Knowledge Tracing (BKT)



Observations for student s:

$t = 0$: 0

$t = 1$: 0

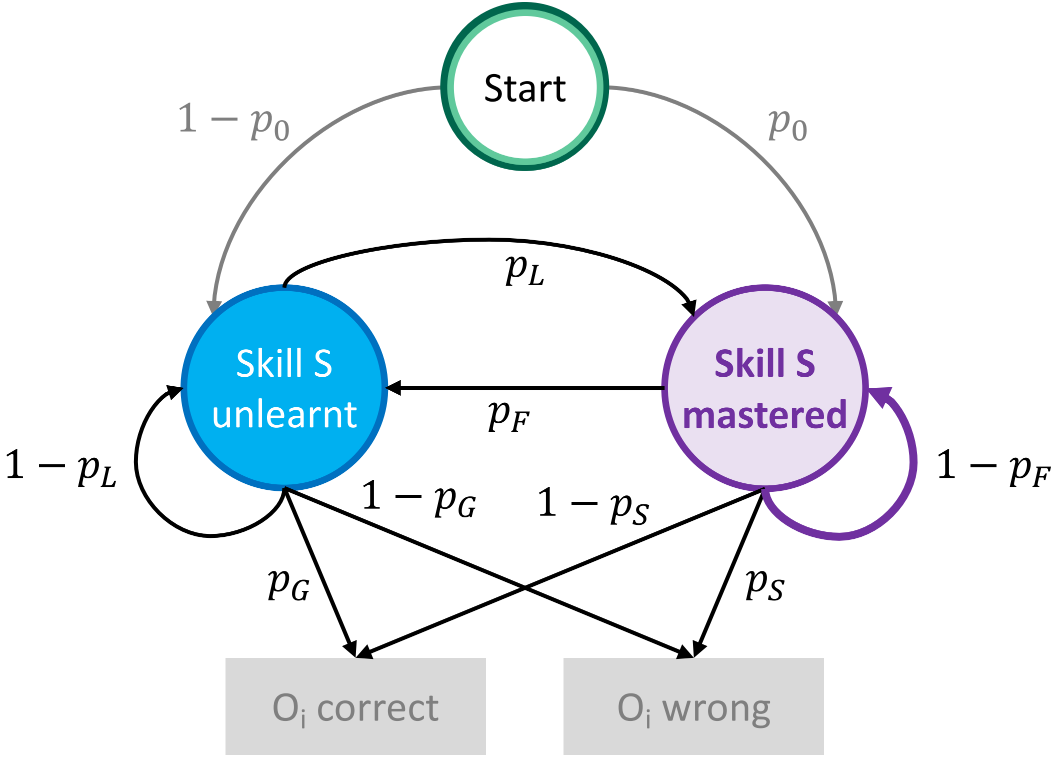
$t = 2$: 1

$t = 3$: 0

$t = 4$: 1

$t = 5$: 1

Bayesian Knowledge Tracing (BKT)



Observations for student s:

$t = 0$: 0

$t = 1$: 0

$t = 2$: 1

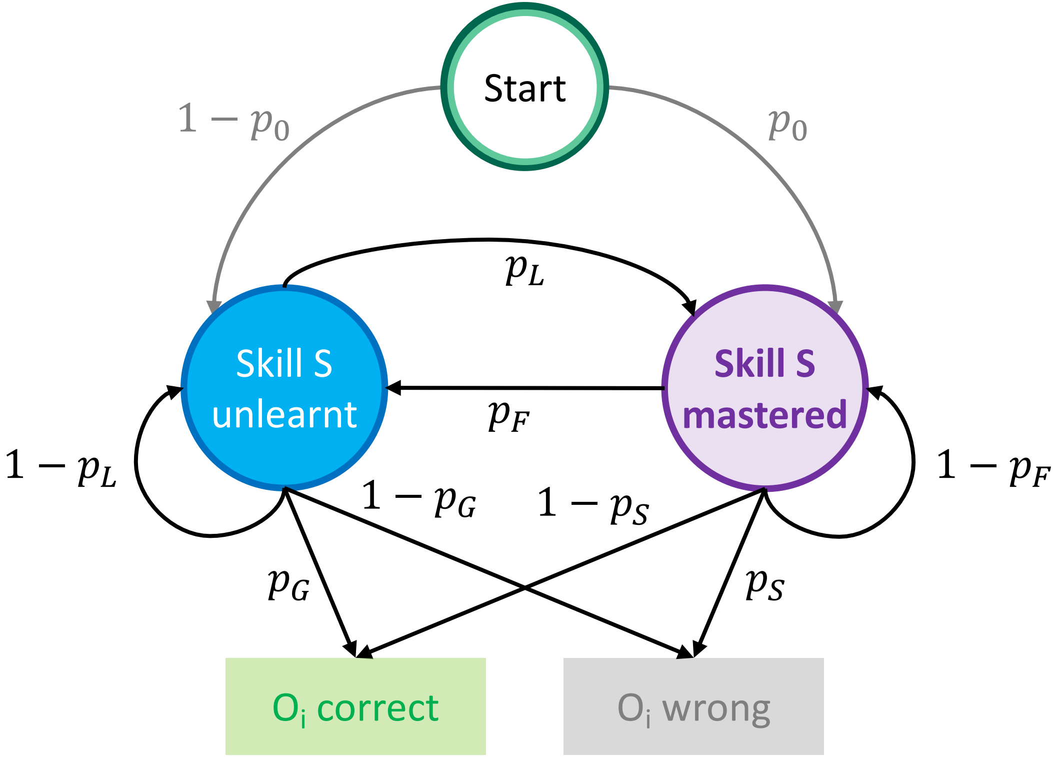
$t = 3$: 0

$t = 4$: 1

$t = 5$: 1

$t = 6$:

Bayesian Knowledge Tracing (BKT)



Observations for student s:

$t = 0$: 0

$t = 1$: 0

$t = 2$: 1

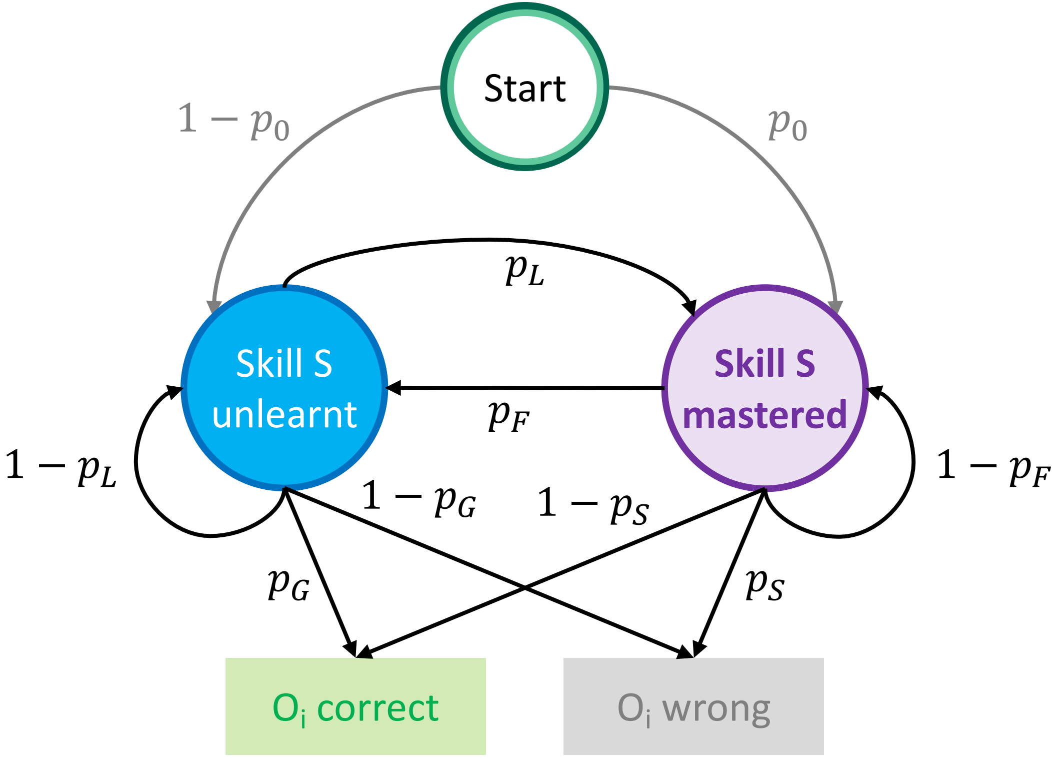
$t = 3$: 0

$t = 4$: 1

$t = 5$: 1

$t = 6$: 1

Bayesian Knowledge Tracing (BKT)



Observations for student s:

$t=0$: 0

$t=1$: 0

$t=2$: 1

$t=3$: 0

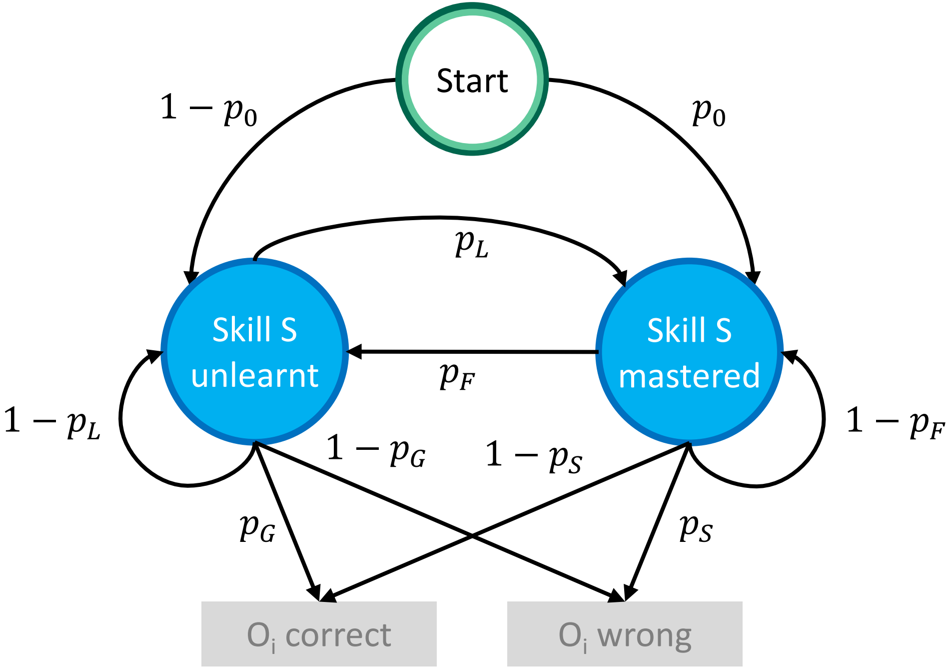
$\boxed{\boldsymbol{o_s} = [0,0,1,0,1,1,1]}$

$t=4$: 1

$t=5$: 1

$t=6$: 1

BKT - Terminology



One latent variable (S) with two possible states

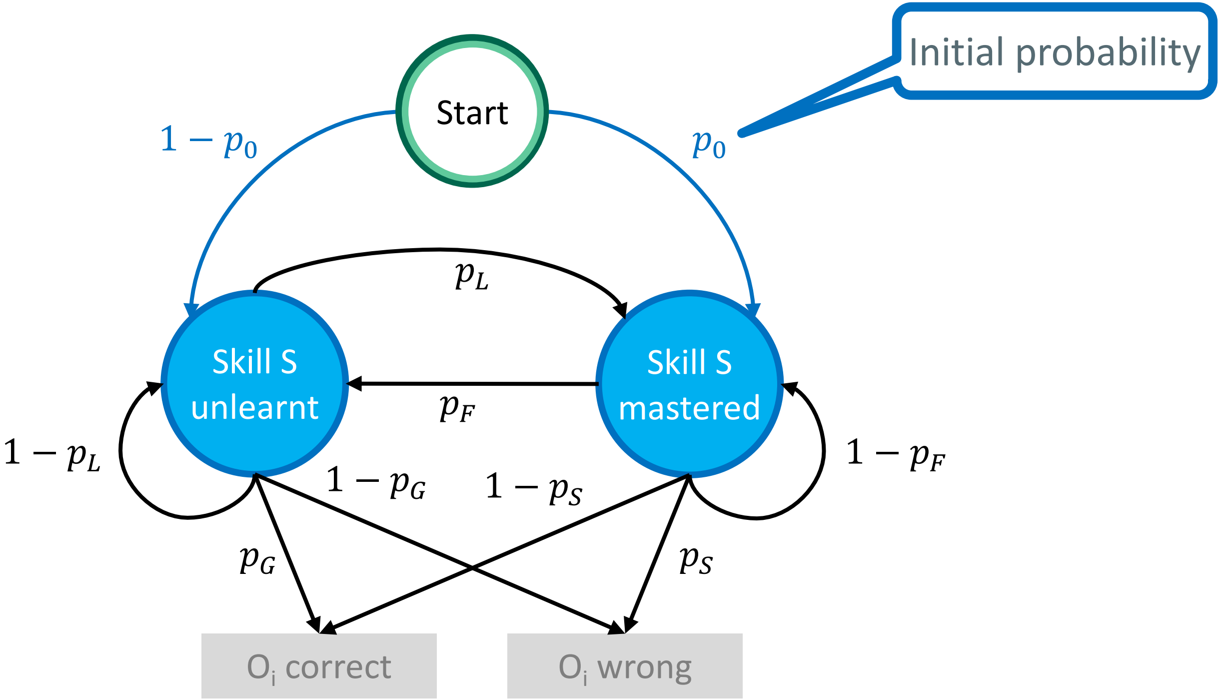
Observations (also binary)

Five parameters:

$$\theta = \{p_0, \underbrace{p_L, p_F}_{\text{Transition probabilities}}, \overbrace{p_S, p_G}^{\text{Emission probabilities}}\}$$

[Corbett & Anderson, UMUAI 1994]

BKT parameters are interpretable



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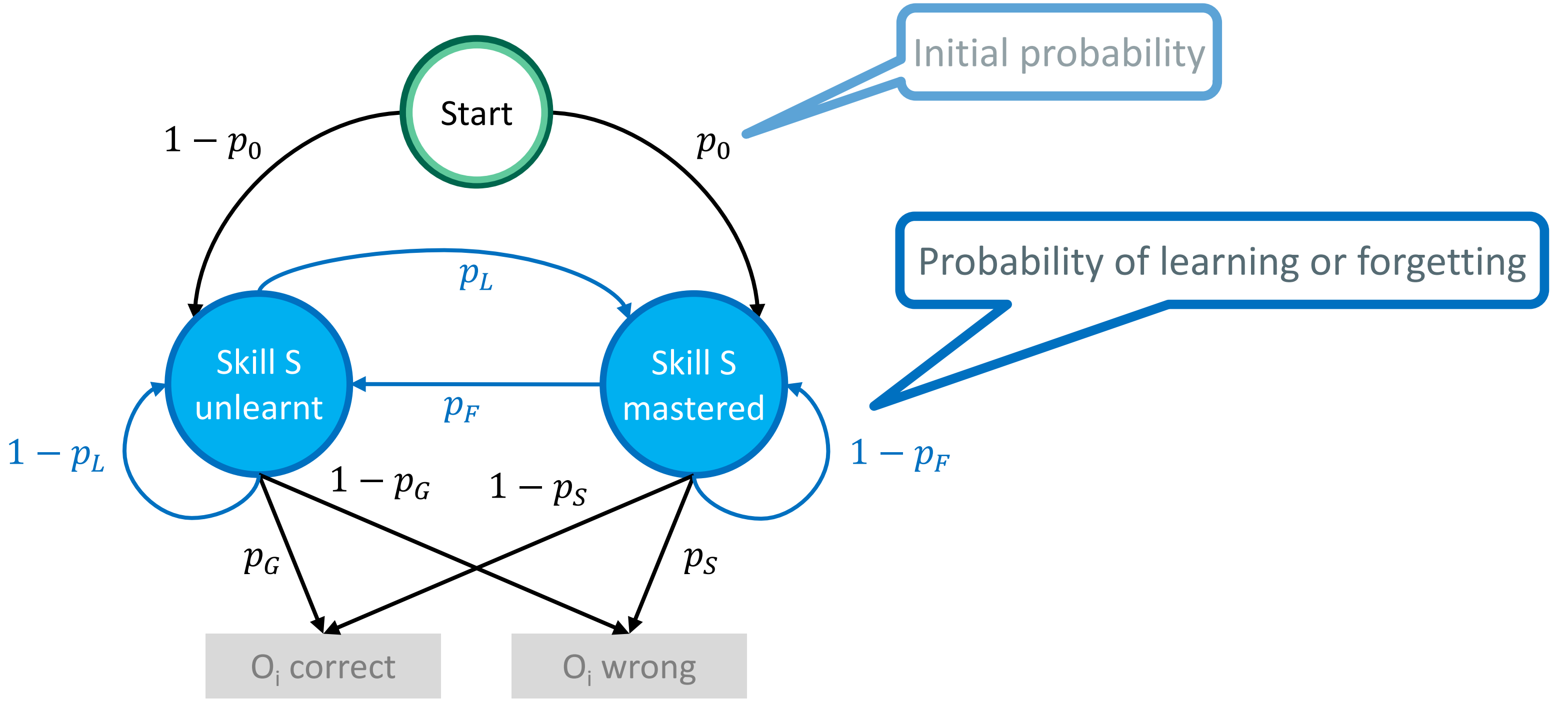
BKT parameters are interpretable



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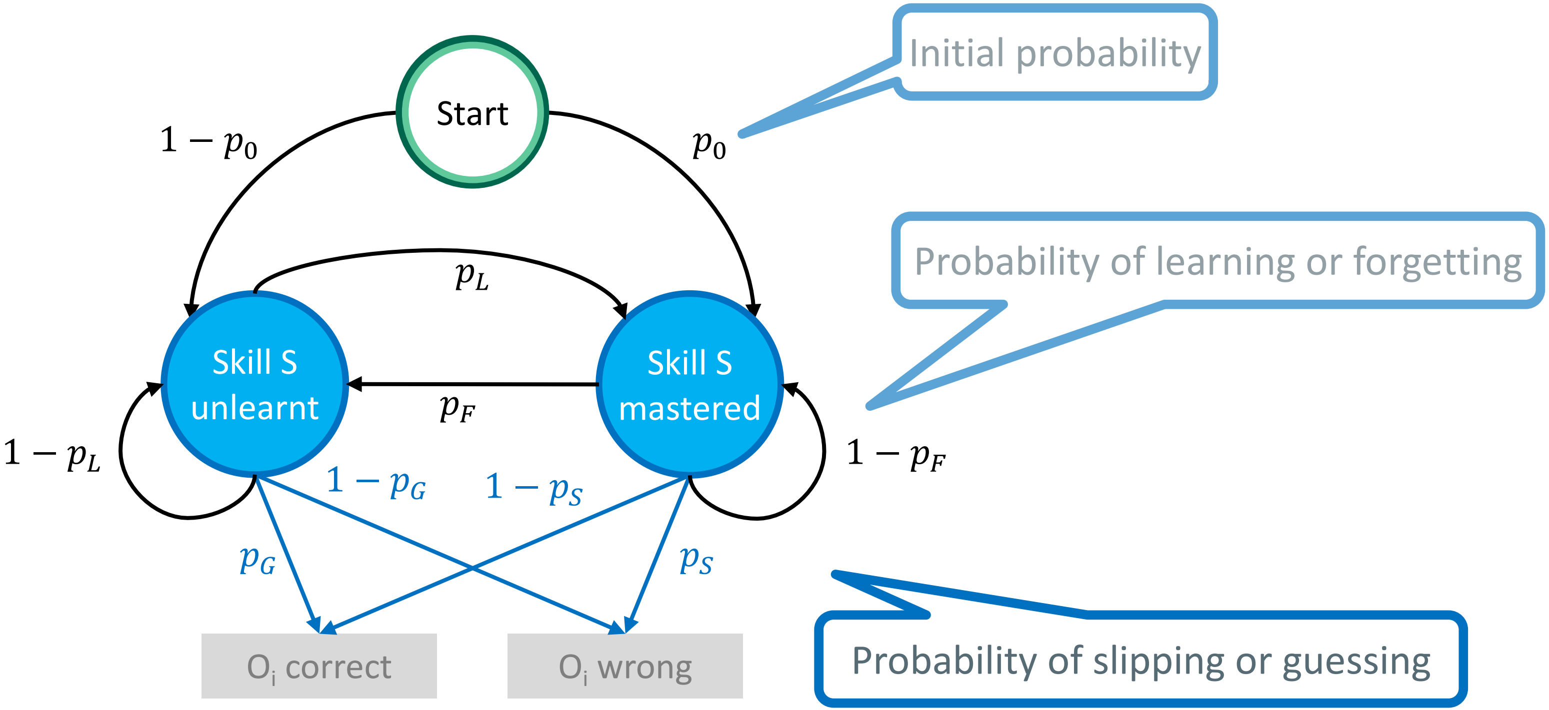
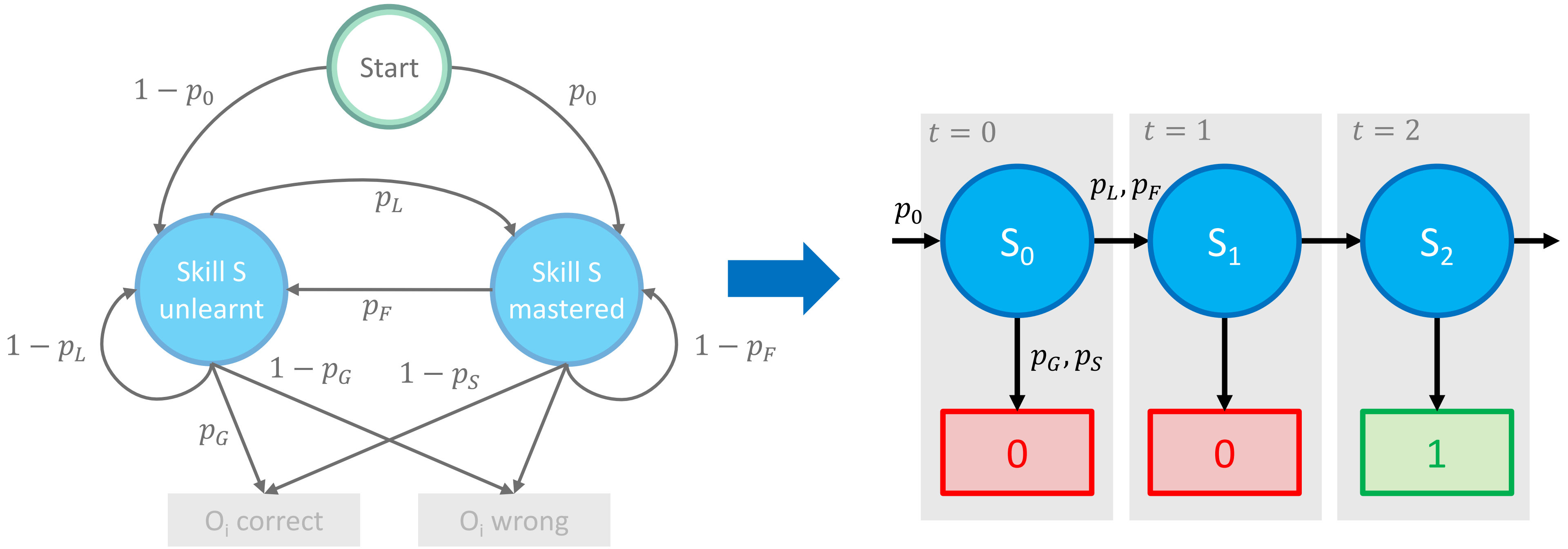
BKT parameters are interpretable



[Corbett & Anderson, UMUAI 1994]

BKT – unrolled over time



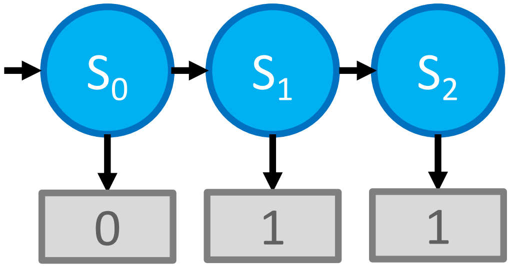
Two tasks need to be solved in practice

- Given a model with parameters $\theta = \{p_0, p_L, p_F, p_S, p_G\}$ and a sequence of observations $\boldsymbol{o} = [o_0, \dots, o_t]$ from a student s, predict o_{t+1}

o_0	o_1	o_2	…		o_t	o_{t+1}
0	0	1	0	1	1	?

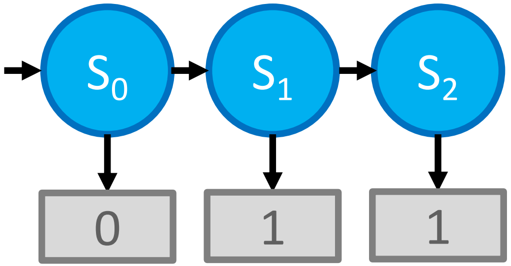
Inference

Inference Example



$$p_0 = 0.5$$
$$p_S = 0.2$$
$$p_G = 0.3$$
$$p_L = 0.4$$
$$p_F = 0.0$$

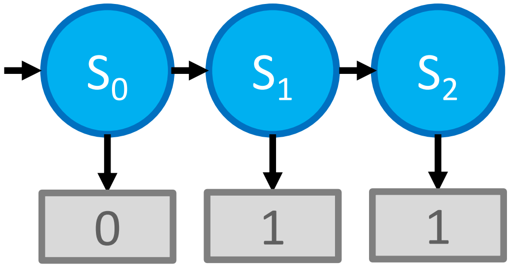
Inference Example – Your Turn



$p_0 = 0.5$
$p_S = 0.2$
$p_G = 0.3$
$p_L = 0.4$
$p_F = 0.0$

- $p(s_0 = 1)$?
- $p(o_0 = 1)$?
- $p(s_1 = 1 | o_0 = 0)$?
- $p(o_1 = 1 | o_0 = 0)$?
- $p(s_2 = 1 | o_0 = 0, o_1 = 1)$?

Inference Example – Your Turn



$p_0 = 0.5$
$p_S = 0.2$
$p_G = 0.3$
$p_L = 0.4$
$p_F = 0.0$

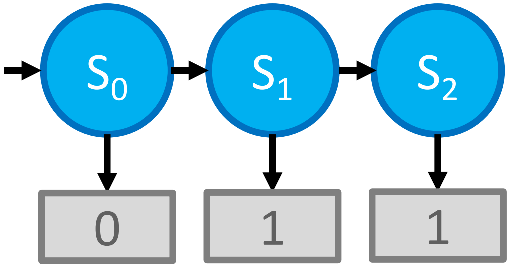
S_0	$p(S_0)$
1	p_0
0	$1\text{-}p_0$

S_t	S_{t+1}	$p(S_{t+1} \mid S_t)$
0	0	$1 - p_L$
0	1	p_L
1	0	p_F
1	1	$1 - p_F$

S_t	O_t	$p(O_t \mid S_t)$
0	0	$1 - p_G$
0	1	p_G
1	0	p_S
1	1	$1 - p_S$

- $p(s_0 = 1)$?
- $p(o_0 = 1)$?
- $p(s_1 = 1 | o_0 = 0)$?
- $p(o_1 = 1 | o_0 = 0)$?
- $p(s_2 = 1 | o_0 = 0, o_1 = 1)$?

Inference Example – Your Turn



$p_0 = 0.5$
$p_S = 0.2$
$p_G = 0.3$
$p_L = 0.4$
$p_F = 0.0$

S_0	$p(S_0)$
1	p_0
0	1-p_0

S_t	S_{t+1}	$p(S_{t+1}\|S_t)$
0	0	$1 - p_L$
0	1	p_L
1	0	p_F
1	1	$1 - p_F$

S_t	O_t	$p(O_t\|S_t)$
0	0	$1 - p_G$
0	1	p_G
1	0	p_S
1	1	$1 - p_S$

Some useful rules:

$$p(A, B) = p(A|B) \cdot p(B)$$

$$p(A|B) = \frac{p(B|A) \cdot p(A)}{p(B)}$$

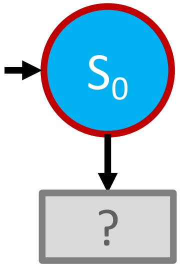
$$p(A = 1) = p(A = 1, B = 1) + p(A = 1, B = 0)$$

- $p(s_0 = 1)$?
- $p(o_0 = 1)$?
- $p(s_1 = 1|o_0 = 0)$?
- $p(o_1 = 1|o_0 = 0)$?
- $p(s_2 = 1|o_0 = 0, o_1 = 1)$?

Inference in BKT models

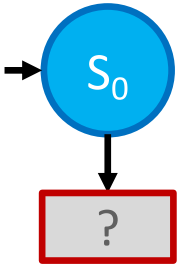
Equations for time step 0:

$$p(s_0 = 1) = p_0$$
$$p(s_0 = 0) = 1 - p_0$$



Inference in BKT models

Equations for time step 0:



$$p(s_0 = 1) = p_0$$
$$p(s_0 = 0) = 1 - p_0$$

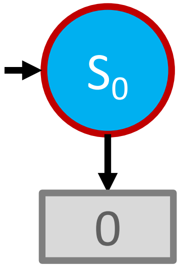
$$p(o_0 = 1) = p(o_0 = 1, s_0 = 1) + p(o_0 = 1, s_0 = 0)$$
$$= (1 - p_s) \cdot p_0 + p_G \cdot (1 - p_0)$$

$$p(o_0 = 0) = p_S \cdot p_0 + (1 - p_G) \cdot (1 - p_0)$$

Inference in BKT models

$$\underbrace{p(s_0 = 1|o_0 = 0)}_{p_{s_0|0}} = \frac{p(o_0 = 0|s_0 = 1) \cdot p(s_0 = 1)}{p(o_0 = 0)}$$

$$= \frac{p_S \cdot p_0}{p_s \cdot p_0 + (1 - p_G) \cdot (1 - p_0)}$$

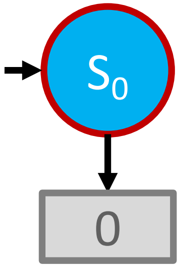


$$p(s_0 = 0|o_0 = 0) = 1 - p_{s_0|0}$$

Inference in BKT models

$$\underbrace{p(s_0 = 1|o_0 = 1)}_{p_{s_0|1}} = \frac{p(o_0 = 1|s_0 = 1) \cdot p(s_0 = 1)}{p(o_0 = 1)}$$

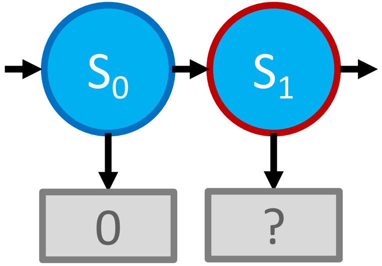
$$= \frac{(1 - p_S) \cdot p_0}{(1 - p_s) \cdot p_0 + p_G \cdot (1 - p_0)}$$



$$p(s_0 = 0|o_0 = 1) = 1 - p_{s_0|1}$$

Inference in BKT models

Equations for time step 1:



$$p(s_1 = 1 | o_0 = 0) = \frac{p(s_1 = 1, o_0 = 0)}{p(o_0 = 0)}$$

$$= \frac{p(s_1 = 1, s_0 = 1, o_0 = 0)}{p(o_0 = 0)} + \frac{p(s_1 = 1, s_0 = 0, o_0 = 0)}{p(o_0 = 0)}$$

$$= \frac{p(s_1 = 1 | s_0 = 1) \cdot p(o_0 = 0 | s_0 = 1) \cdot p(s_0 = 1)}{p(o_0 = 0)}$$

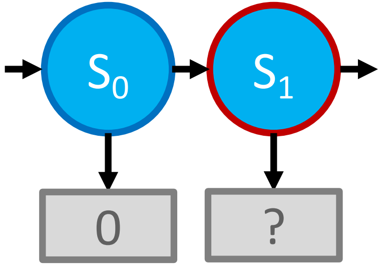
$$+ \frac{p(s_1 = 1 | s_0 = 0) \cdot p(o_0 = 0 | s_0 = 0) \cdot p(s_0 = 0)}{p(o_0 = 0)}$$

$$p(s_1 = 1 | o_0 = 0) = (1 - p_F) \cdot p_{s_0|0} + p_L \cdot (1 - p_{s_0|0})$$

Inference in BKT models

$$p(s_1 = 1|o_0 = 1) = (1 - p_F) \cdot p_{s_0|1} + \ p_L \cdot (1 - p_{s_0|1})$$

$$p(s_1 = 1|o_0 = 0) = (1 - p_F) \cdot p_{s_0|0} + \ p_L \cdot (1 - p_{s_0|0})$$

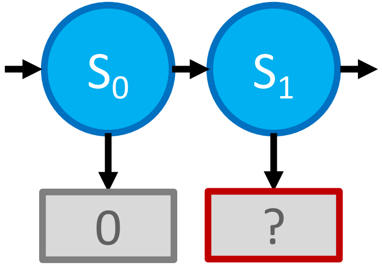


$$\Downarrow$$

$$p_{s_1|o_0} = (1 - p_F) \cdot p_{s_0|o_0} + \ p_L \cdot (1 - p_{s_0|o_0})$$

Inference in BKT models

$$p(o_1 = 1 | o_0 = 0) = \frac{p(o_1 = 1, o_0 = 0)}{p(o_0 = 0)}$$



$$+ \frac{p(o_1 = 1, s_1 = 1, s_0 = 1, o_0 = 0)}{p(o_0 = 0)}$$
$$+ \frac{p(o_1 = 1, s_1 = 1, s_0 = 0, o_0 = 0)}{p(o_0 = 0)}$$
$$+ \frac{p(o_1 = 1, s_1 = 0, s_0 = 1, o_0 = 0)}{p(o_0 = 0)} + \frac{p(o_1 = 1, s_1 = 0, s_0 = 0, o_0 = 0)}{p(o_0 = 0)}$$

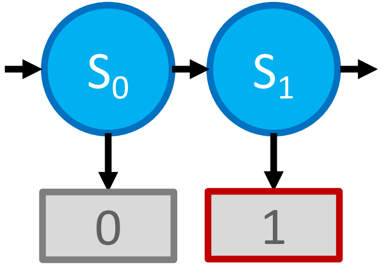
$$= p(o_1 = 1 | s_1 = 1) \cdot (p(s_1 = 1 | s_0 = 1) \cdot p(o_0 = 0 | s_0 = 1) \cdot p(s_0 = 1)/p(o_0 = 0))$$
$$+ p(o_1 = 1 | s_1 = 1) \cdot (p(s_1 = 1 | s_0 = 0) \cdot p(o_0 = 0 | s_0 = 0) \cdot p(s_0 = 0)/p(o_0 = 0))$$
$$+ p(o_1 = 1 | s_1 = 0) \cdot (p(s_1 = 0 | s_0 = 1) \cdot p(o_0 = 0 | s_0 = 1) \cdot p(s_0 = 1)/p(o_0 = 0))$$
$$+ p(o_1 = 1 | s_1 = 0) \cdot (p(s_1 = 0 | s_0 = 0) \cdot p(o_0 = 0 | s_0 = 0) \cdot p(s_0 = 0)/p(o_0 = 0))$$

$$p(o_1 = 1 | o_0 = 0) = (1 - p_S) \cdot p_{s_1 | o_0 = 0} + p_G \cdot (1 - p_{s_1 | o_0 = 0})$$

Inference in BKT models

$$p(o_1 = 1|o_0 = 1) = (1 - p_S) \cdot p_{s_1|o_0=1} + p_G \cdot (1 - p_{s_1|o_0=1})$$

$$p(o_1 = 0|o_0 = 1) = p_S \cdot p_{s_1|o_0=1} + (1 - p_G) \cdot (1 - p_{s_1|o_0=1})$$

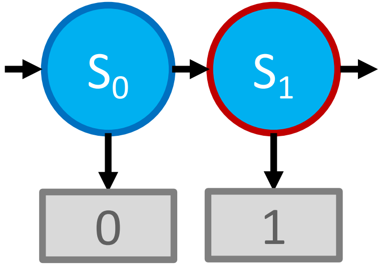


$$\Downarrow$$

$$p(o_1 = 1|o_0) = (1 - p_S) \cdot p_{s_1|o_0} + p_G \cdot (1 - p_{s_1|o_0})$$

$$p(o_1 = 0|o_0) = p_S \cdot p_{s_1|o_0} + (1 - p_G) \cdot (1 - p_{s_1|o_0})$$

Inference in BKT models



$$p(s_1 = 1 | o_1 = 1, o_0) = \frac{p(s_1 = 1, o_1 = 1, o_0)}{p(o_1 = 1, o_0)}$$

$$\begin{aligned} p(o_1 = 1, o_0) &= p(o_1 = 1 | o_0) \cdot p(o_0) \\ &= ((1 - p_S) \cdot p_{s_1|o_0} + p_G \cdot (1 - p_{s_1|o_0})) \cdot p(o_0) \end{aligned}$$

$$\begin{aligned} p(s_1 = 1, o_1 = 1, o_0) &= p(o_1 = 1 | s_1 = 1\,) \cdot p(s_1 = 1 | s_0 = 1\,) \cdot p(o_0 | s_0 = 1) \cdot p(s_0 = 1) \\ &+ p(o_1 = 1 | s_1 = 1\,) \cdot p(s_1 = 1 | s_0 = 0\,) \cdot p(o_0 | s_0 = 0) \cdot p(s_0 = 0) \\ &= (1 - p_s) \cdot p_{s_1|o_0} \cdot p(o_0) \end{aligned}$$

$$p(s_1 = 1 | o_1 = 1, o_0) = \frac{(1 - p_s) \cdot p_{s_1|o_0}}{((1 - p_S) \cdot p_{s_1|o_0} + p_G \cdot (1 - p_{s_1|o_0}))}$$

Inference in BKT models

$\boldsymbol{o_{t-1}} = [o_0, \dots, o_{t-1}]$

Equations for time $t = 0$:

Equations for time steps $t = 1, \dots, T$:

Belief about latent state before observation

$$p(s_0 = 1) = p_0$$

$$p_{s_t|\boldsymbol{o_{t-1}}} = (1 - p_F) \cdot p_{s_{t-1}|\boldsymbol{o_{t-1}}} + p_L \cdot (1 - p_{s_{t-1}|\boldsymbol{o_{t-1}}})$$

Predicted observation at time *t*

$$p(o_0 = 1) = (1 - p_S) \cdot p_0 + p_G \cdot (1 - p_0)$$
$$p(o_0 = 0) = p_S \cdot p_0 + (1 - p_G) \cdot (1 - p_0)$$

$$p(o_t = 1|\boldsymbol{o_{t-1}}) = (1 - p_S) \cdot p_{s_t|\boldsymbol{o_{t-1}}} + p_G \cdot (1 - p_{s_t|\boldsymbol{o_{t-1}}})$$
$$p(o_t = 0|\boldsymbol{o_{t-1}}) = p_S \cdot p_{s_t|\boldsymbol{o_{t-1}}} + (1 - p_G) \cdot (1 - p_{s_t|\boldsymbol{o_{t-1}}})$$

Posterior: belief about latent state after observation

$$\left.\begin{aligned} p_{s_0|1} &= \frac{(1 - p_S) \cdot p_0}{(1 - p_S) \cdot p_0 + p_G \cdot (1 - p_0)} \\ p_{s_0|0} &= \frac{p_S \cdot p_0}{p_S \cdot p_0 + (1 - p_G) \cdot (1 - p_0)} \end{aligned}\right\} p_{s_0|o_0}$$

$$\left.\begin{aligned} p_{s_t|1,\boldsymbol{o_{t-1}}} &= \frac{(1 - p_S) \cdot p_{s_t|\boldsymbol{o_{t-1}}}}{(1 - p_S) \cdot p_{s_t|\boldsymbol{o_{t-1}}} + p_G \cdot (1 - p_{s_t|\boldsymbol{o_{t-1}}})} \\ p_{s_t|0,\boldsymbol{o_{t-1}}} &= \frac{p_S \cdot p_{s_t|\boldsymbol{o_{t-1}}}}{p_S \cdot p_{s_t|\boldsymbol{o_{t-1}}} + (1 - p_G) \cdot (1 - p_{s_t|\boldsymbol{o_{t-1}}})} \end{aligned}\right\} p_{s_t|\boldsymbol{o_t}}$$

[Corbett & Anderson, UMUAI 1994]

Making predictions using a BKT model

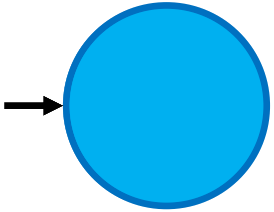
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



Making predictions using a BKT model

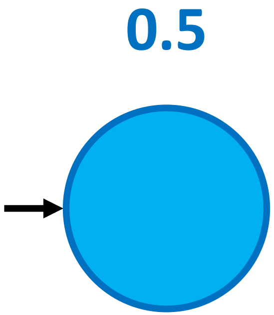
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



$p(s_0 = 1) = p_0$

Making predictions using a BKT model

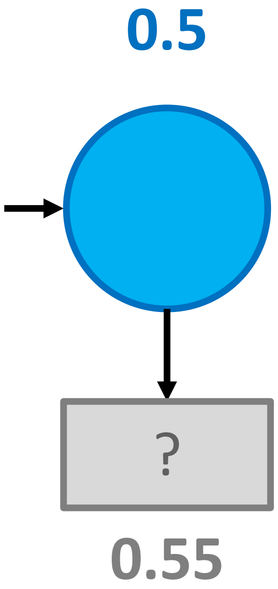
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



$$p(s_0 = 1) = p_0$$

$$p(o_0 = 1) = (1 - p_s) \cdot p_0 + p_G \cdot (1 - p_0)$$

Making predictions using a BKT model

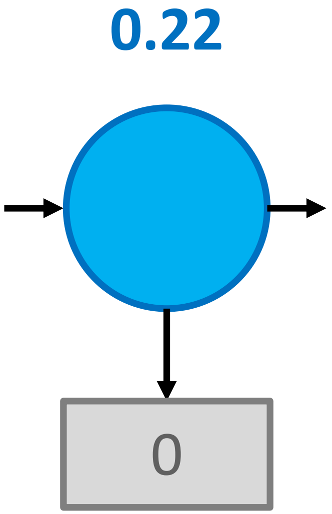
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



$$p_{s_0|0} = \frac{p_S \cdot p_0}{1 - p(o_0 = 1)} = \frac{0.2 \cdot 0.5}{0.45}$$

Making predictions using a BKT model

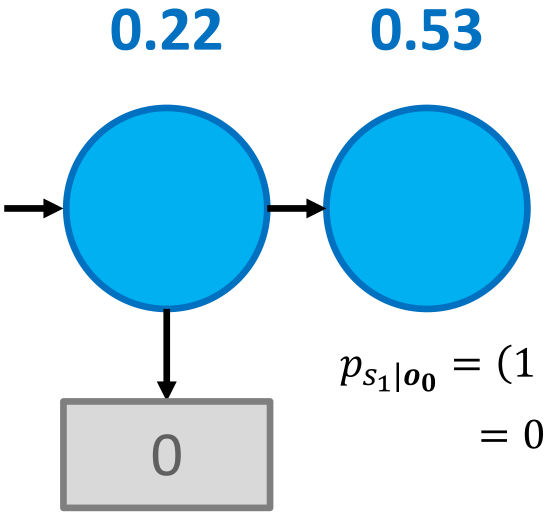
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



$$p_{s_1|\boldsymbol{o_0}} = (1 - p_F) \cdot p_{s_1|\boldsymbol{o_0}} + \ p_L \cdot (1 - p_{s_1|\boldsymbol{o_0}})$$
$$= 0.22 + 0.4 \cdot 0.78$$

Making predictions using a BKT model

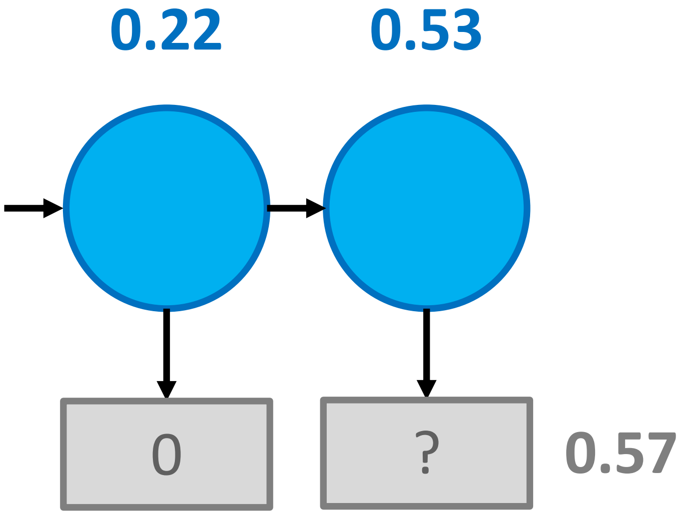
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



$$
\begin{aligned}
p(o_1 = 1|\boldsymbol{o_0}) &= (1 - p_S) \cdot p_{s_1|\boldsymbol{o_0}} + p_G \cdot (1 - p_{s_1|\boldsymbol{o_0}}) \\
&= 0.8 \cdot 0.53 + 0.3 \cdot 0.47
\end{aligned}
$$

Making predictions using a BKT model

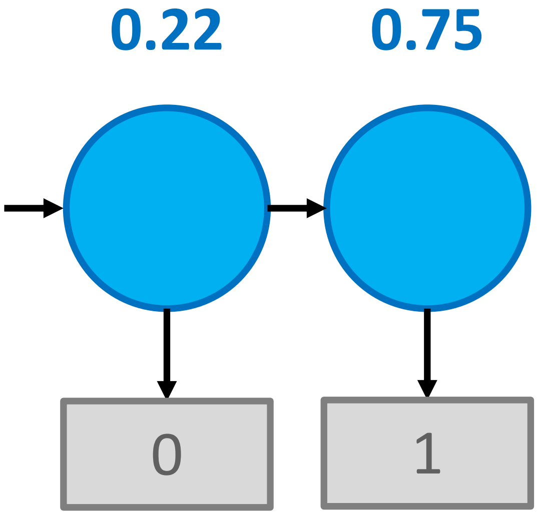
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



$$p_{s_1|1,\boldsymbol{o_0}} = \frac{(1-p_S)\cdot p_{s_1|\boldsymbol{o_0}}}{(1-p_S)\cdot p_{s_1|\boldsymbol{o_0}} + p_G\cdot(1-p_{s_1|\boldsymbol{o_0}})}$$

$$= \frac{0.8\cdot 53}{0.57}$$

Making predictions using a BKT model

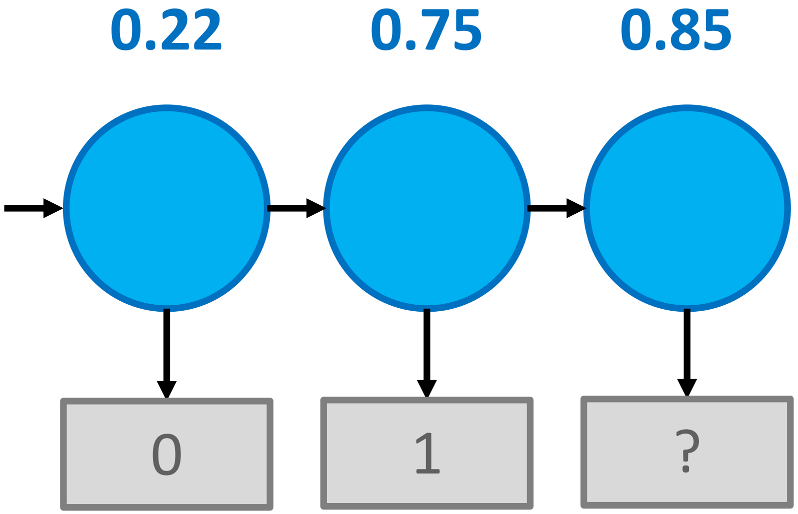
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



Making predictions using a BKT model

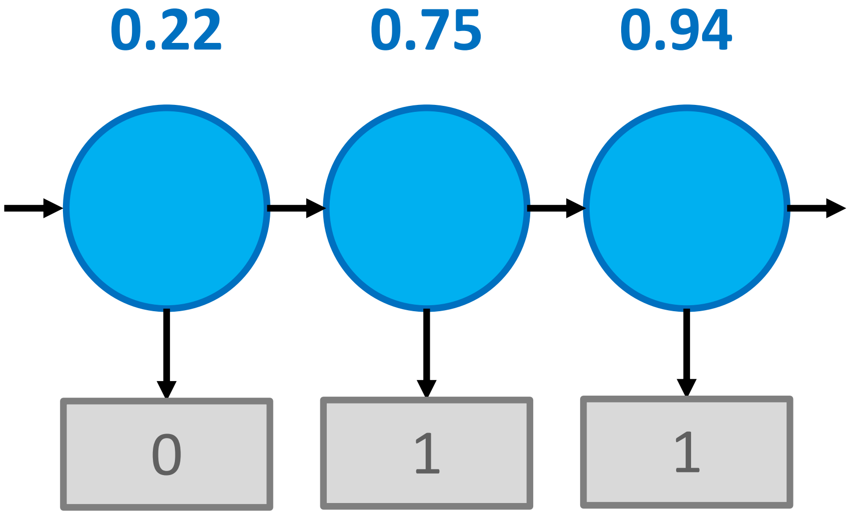
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



Making predictions using a BKT model

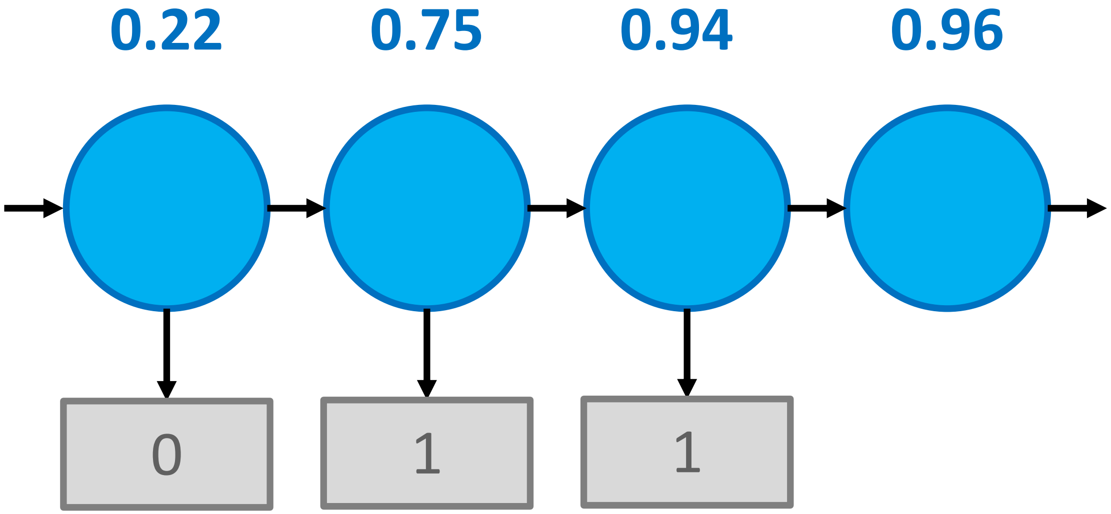
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



Making predictions using a BKT model

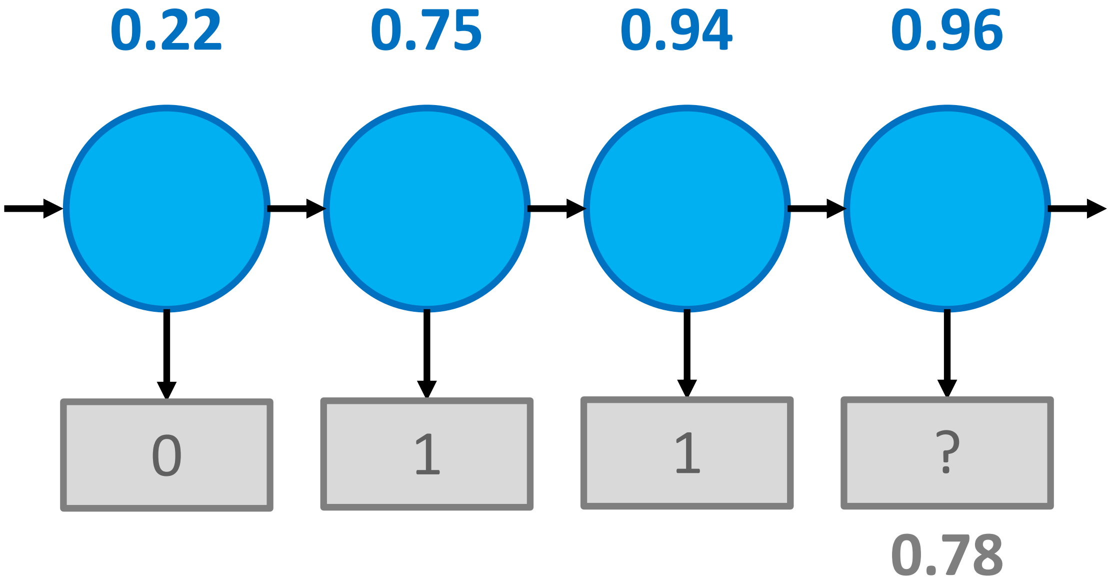
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



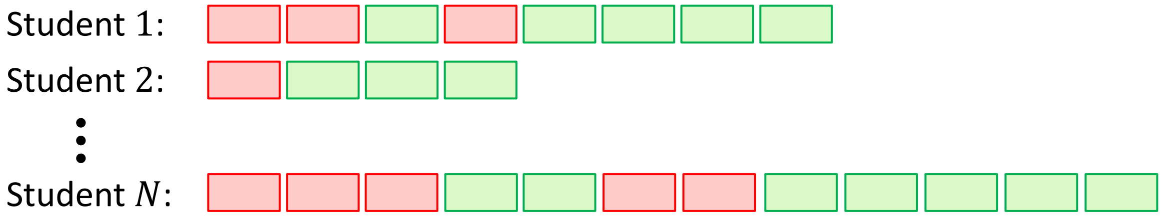
Two tasks need to be solved in practice

- Given a model with parameters $\theta = \{p_0, p_L, p_F, p_S, p_G\}$ and a sequence of observations $\boldsymbol{o} = [o_0, \dots, o_t]$ from a student s, predict o_{t+1}

o_0	o_1	o_2	•••		o_t	o_{t+1}
0	0	1	0	1	1	?

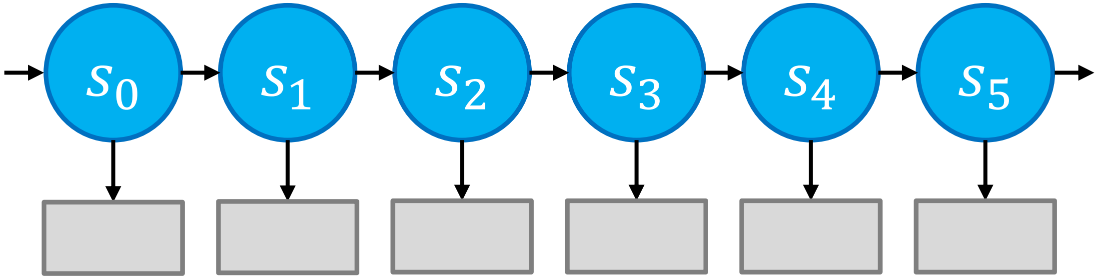
Inference

- Given sequences of observations $\boldsymbol{o} = [o_0, \dots, o_T]$ of N students, learn the parameters $\theta = \{p_0, p_L, p_F, p_S, p_G\}$ that maximize the likelihood of the observed data



Parameter Learning

Training a BKT model



$$\theta = \{p_0, p_L, p_F, p_S, p_G\}$$

Student l_0: $\mathbf{o}_{l_0} = [0,1,1]$

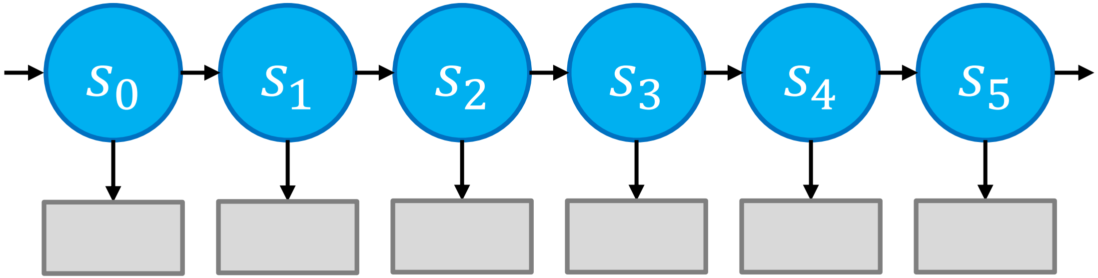
⋮

$$\max_{\theta} p\left(\mathbf{o}_{l_0}, \dots, \mathbf{o}_{l_{N-1}}, \mathbf{o}_{l_N}\right)$$

Student l_{N-1}: $\mathbf{o}_{l_{N-1}} = [1,0,1,1,1,0,0,1,1,1]$

Student l_N: $\mathbf{o}_{l_N} = [0,1,0,1]$

Training a BKT model



$$\theta = \{p_0, p_L, p_F, p_S, p_G\}$$

Student l_0: $\mathbf{o}_{l_0} = [0,1,1]$

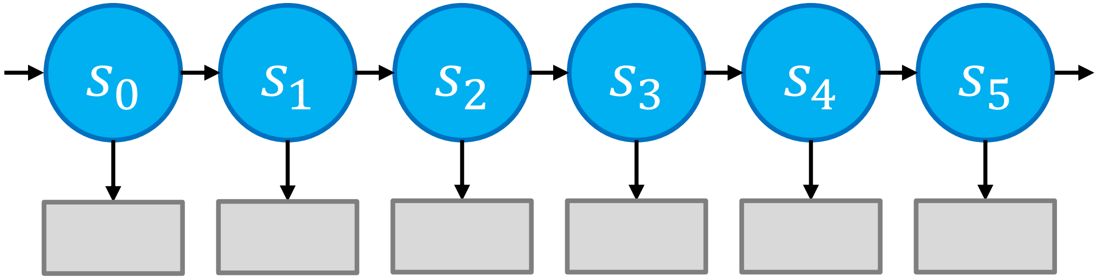
⋮

Student l_{N-1}: $\mathbf{o}_{l_{N-1}} = [1,0,1,1,1,0,0,1,1,1]$

Student l_N: $\mathbf{o}_{l_N} = [0,1,0,1]$

$$\max_{\theta} \prod_{i=1}^{N} p(\boldsymbol{o}_{l_i})$$

Training a BKT model



$$\theta = \{p_0, p_L, p_F, p_S, p_G\}$$

Student l_0: $p(\mathbf{o}_{l_0}) = \sum_{\boldsymbol{s}} p(\mathbf{o}_{l_0}, \boldsymbol{s})$

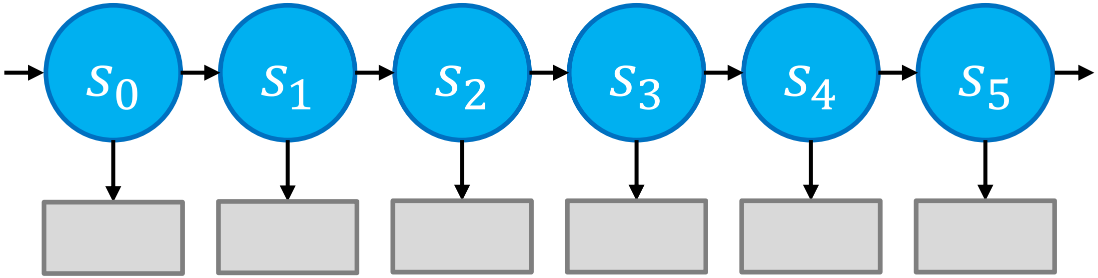
⋮

Student l_{N-1}: $p(\mathbf{o}_{l_0}) = \sum_{\boldsymbol{s}} p(\mathbf{o}_{l_0}, \boldsymbol{s})$

Student l_N: $p(\mathbf{o}_{l_0}) = \sum_{\boldsymbol{s}} p(\mathbf{o}_{l_0}, \boldsymbol{s})$

$$\max_{\theta} \prod_{i=1}^{N} \sum_{\boldsymbol{s}_{l_i}} p(\mathbf{o}_{l_i}, \boldsymbol{s}_{l_i})$$

Training a BKT model



$$\theta = \{p_0, p_L, p_F, p_S, p_G\}$$

$$\max_{\theta} \prod_{i=1}^{N} \sum_{\boldsymbol{s}_{l_i}} p(\mathbf{o}_{l_i}, \boldsymbol{s}_{l_i}) \Rightarrow \min_{\theta} - \sum_{i=1}^{N} \log\left(\sum_{\boldsymbol{s}_{l_i}} p(\mathbf{o}_{l_i}, \boldsymbol{s}_{l_i}) \right)$$

- Brute-Force Grid Search
- Expectation Maximization
- Gradient Descent
- Nelder-Mead Optimization

Your Turn – Evaluating a BKT model

- In the student notebook, you have:
  - A trained BKT model for six selected skills
  - A data frame containing the predictions of the BKT model for each observation in the test set
- Your task:
  - Compute the RMSE or AUC separately for each skill
  - Provide a visualization of the mean RMSE (or AUC) + standard deviation over all skills as well as the per skill RMSE (or AUC)

Assumptions behind BKT

- Knowledge can be divided into different skills
- Definition of skills is accurate/detailed enough
- Each task corresponds to a single skill (original)
- There is **no** connection between the skills
- Mastery can be achieved through practice
- There is no forgetting: $p_F = 0$ (original)

Today: Tracing Student Knowledge

- Bayesian Knowledge Tracing (BKT)
- **Learning Curves**

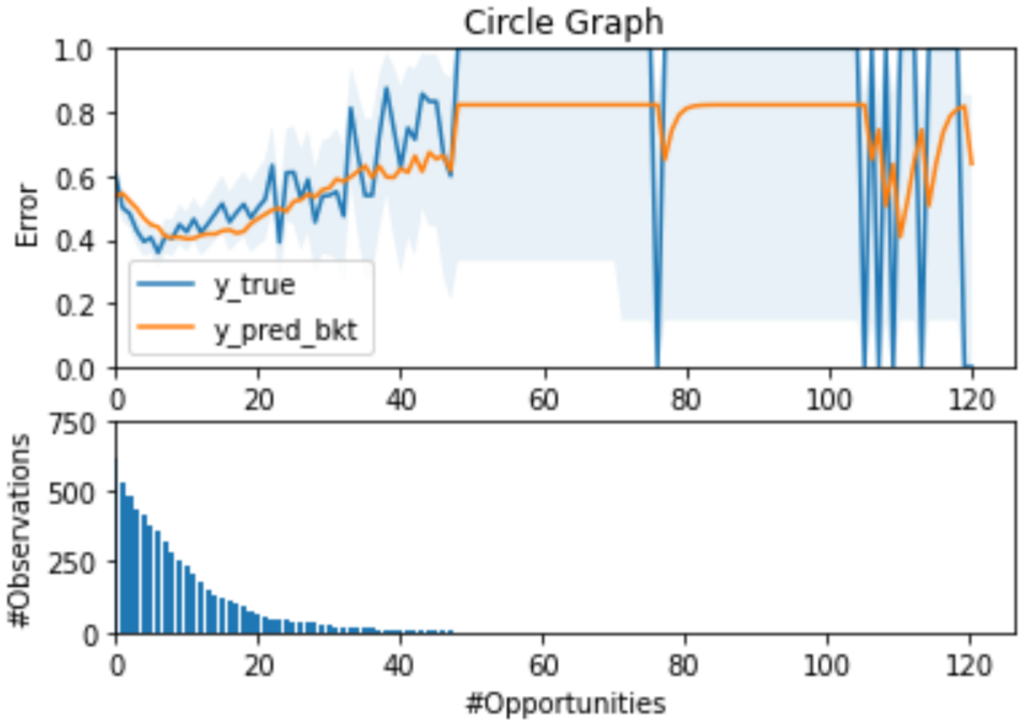
Tracing Knowledge – why is it useful?

- Is the student learning?
  - Measure what the student *knows* at a specific time *t*
  - More specifically: knowledge of the student about relevant knowledge components (skills)

➡ Choose the next appropriate activity

➡ Know which activities support learning

What could this curve indicate?



Your Turn – Learning Curves

- In the student notebook, you have:
  - BKT model trained on all skills and students
  - List of available skills
  - Function for plotting learning curves and student numbers for a specific skill
- Your task:
  - Pick 1-2 skills, generate the learning curves for them, and interpret them
  - Send us your plots and interpretations

Tracing Knowledge – why is it useful?

- Is the student learning?
  - Measure what the student *knows* at a specific time *t*
  - More specifically: knowledge of the student about relevant knowledge components (skills)

➡ Choose the next appropriate activity

➡ Know which activities support learning

If you want additional practice…

- You can solve tasks from last year's homework
  - independently
  - during the tutorial sessions on Wednesday morning, the TAs will be happy to help and answer questions
- For this lecture: Knowledge Tracing Exercise
- We are happy to provide feedback on your solution:

Upload your Jupyter Notebook here:

https://go.epfl.ch/mlbd-activities