Duke University

Department of Economics

ECON — Industrial Organization

Assignment 2 - November 2022

Bomi Kang

Ricardo Miranda

Arthur Pereira de Souza

Problem 1.

1.1

Fixed price case: We remove i and t subscripts for convenience.

The firm solves the following problem, in which we factor out the expectation of the shock using the LIE:

$$\max_{M,L} \quad p\mathbb{E}[e^u] \cdot \min\{a \cdot e^\omega L^{\alpha_L} K^{\alpha_K}, b \cdot M\} - p^M \cdot M - L$$

It is optimal to set $a \cdot e^\omega L^{\alpha_L} K^{\alpha_K} = b \cdot M$, so that we substitute out M and solve for the optimal choice of L:

$$\max_{L} \quad \left(p\mathbb{E}[e^u] - \frac{p^M}{b}\right) \cdot a \cdot e^\omega L^{\alpha_L} K^{\alpha_K} - L$$

Note that for $p\mathbb{E}[e^u] < \frac{p^M}{b}$ profits are decreasing in L, implying $L = 0$ in such a case. We compute the FOC assuming this is not true:

$$\left(p\mathbb{E}[e^u] - \frac{p^M}{b}\right) \cdot a \cdot \alpha_L \cdot e^\omega \cdot L^{\alpha_L - 1} K^{\alpha_K} = 1$$

$$\begin{aligned}
L^* &= \left(\left(p\mathbb{E}[e^u] - \frac{p^M}{b}\right) \cdot a \cdot \alpha_L \cdot e^\omega \cdot K^{\alpha_K}\right)^{\frac{1}{1-\alpha_L}} \\
M^* &= \frac{1}{b}\left(p\mathbb{E}[e^u] - \frac{p^M}{b}\right)^{\frac{\alpha_L}{1-\alpha_L}} \alpha_L^{\frac{\alpha_L}{1-\alpha_L}} \left(a \cdot e^\omega \cdot K^{\alpha_K}\right)^{\frac{1}{1-\alpha_L}} \\
l(K, \omega) &= \max\{L^*, 0\} \\
m(K, \omega) &= \max\{M^*, 0\}
\end{aligned}$$

It is clear that $m(K,\omega)$ is monotonically increasing in ω assuming we are not at the corner solution: $\frac{\partial \log m(K,\omega)}{\partial \omega} = \frac{1}{1-\alpha_L} > 0$

Now we take the difference $\log m - \log l$ (assuming not in the corner solution) and check that it is a constant:

$$\log l(K,\omega) - \log m(K,\omega) = \log b + \log\left(p\mathbb{E}[e^u] - \frac{p^M}{b}\right) + \log \alpha_L$$

Monopoly case Define $\theta \equiv \frac{1}{\eta} + 1$. Without loss of generality, assume $\mathbb{E}[e^u\theta] = 1$.

The monopolist now solves the following:

$$\begin{aligned} \max_{L,M} \quad & A^\theta L^{\alpha_L \theta} - \frac{p^M}{b} A L^{\alpha_L} - L \\ s.t \quad & M = \frac{1}{b} A L^{\alpha_L} \\ & A \equiv a e^\omega K^{\alpha_K} \end{aligned}$$

The FOC writes as

$$\alpha_L \theta A L^{\alpha_L - 1} \cdot p(L,K,\omega) - \frac{\alpha_L p^M}{b} A \alpha_L^{\alpha_L - 1} = 1$$

In which we write $p(L)$ to emphasize that the output's price is now an endogenous object. Labor and material demand is now quite similar to before, only now that they are implicitly determined by

$$\begin{aligned} &\left(\theta p(L,K,\omega) - \frac{p^M}{b}\right) \cdot a \cdot \alpha_L \cdot e^\omega \cdot L^{\alpha_L - 1} K^{\alpha_K} = 1 \\ & L^* = \left(\left(\theta p(L,K,\omega) - \frac{p^M}{b}\right) \cdot a \cdot \alpha_L \cdot e^\omega \cdot K^{\alpha_K}\right)^{\frac{1}{1-\alpha_L}} \\ & M^* = \frac{1}{b}\left(\theta p(L,K,\omega) - \frac{p^M}{b}\right)^{\frac{\alpha_L}{1-\alpha_L}} \alpha_L^{\frac{\alpha_L}{1-\alpha_L}} \left(a \cdot e^\omega \cdot K^{\alpha_K}\right)^{\frac{1}{1-\alpha_L}} \\ & l(K,\omega) = \max\{L^*, 0\} \\ & m(K,\omega) = \max\{M^*, 0\} \end{aligned}$$

Proving the monotonicity of the intermediate input's demand is much trickier now since we don't have an implicit solution. One could brute force the problem and use the implicit function theorem or monotone comparative statics, but we take other approach.

First note that labor and materials are complements for fixed capital levels, so showing that either increases in ω is enough. For a given output price, we've already shown that labor increases in ω, and so do materials. Also, given that this is true, obviously output Q also increases in ω (even more than proportionately).

Now it remains to be shown that this still holds even when prices also vary in response to ω (through both the direct effect on quantity, and the indirect one on inputs). The monopolist will always operate on the elastic portion of the demand curve (which in our case is constant, so $|\eta| > 1$). This means that total revenue always moves in the same direction as output. Keep this in mind.

Now use that $R(\omega) = Q((L(\omega), M(\omega)) \cdot p(Q(M(\omega), L(\omega)))$, and since labor and materials are in fixed proportions given capital, we can WLOG write $R(\omega) = Q(M(\omega)) \cdot p(Q(M(\omega)))$

$$
\begin{aligned}
\frac{\partial \log R(\omega)}{\partial \omega} &= \frac{\partial \log Q(M(\omega))}{\partial \omega} + \frac{\partial \log p(Q(M(\omega)))}{\partial \omega} \\
&= \frac{1}{Q} Q'(M(\omega)) \cdot M'(\omega) + \frac{1}{p} p'(Q) \cdot Q'(M(\omega)) \cdot M'(\omega) \\
&= Q'(\omega) M'(\omega) \left[\frac{1}{Q} + \frac{Q}{Q} \frac{p'(Q)}{p} \right] \\
&= \frac{Q'(\omega) M'(\omega)}{Q} \left[1 + \frac{Q p'(Q)}{p} \right] \\
&= \frac{Q'(\omega) M'(\omega)}{Q} \left[1 + \frac{1}{\eta} \right] \\
&= \frac{Q'(\omega) M'(\omega)}{Q} \cdot \theta
\end{aligned}
$$

Since $\eta < -1$, $\theta > 0$. Therefore:

$$\begin{aligned}\text{sign}(R'(\omega)) = \text{sign}\left(\frac{\partial \log R(\omega)}{\partial \omega}\right) &= \text{sign}(\theta \cdot Q'(\omega)M'(\omega)) \\ &= \text{sign}(Q'(\omega)M'(\omega))\end{aligned}$$

As said before, output and revenue move together, and $Q'(\omega) > 0$, also said before. The only way for the previous equality to hold is then with $M'(\omega) > 0$.

Problem 2

Observation - we don't worry about corner solutions here because we assume we don't observe the zeros in the data (those firms are out of the market). This could potentially yield additional selection issues, which would not change our (non) identification results, only worsen them.

Fixed price case: we show that α_L is unidentified in a Levinsohn-Petrin (LP) 2003 style first stage due to the functional dependence problems highlighted in ACF 2015.

In more abstract terms, we show that labor is completely pinned down if we condition on material and capital choice.

To see this, note from the previous derivations that we can write $\log L_{it} = \log M_{it} + \log b + \log \alpha_L + \log \left(p - \frac{p^M}{b}\right)$. That is, conditional on material choices, labor is fixed, which is especially true given the assumption that prices are all fixed.

So if we were to use LP's two-stage procedure, the 1st stage would entail estimating nonparametrically an equation of the form:

$$\log Y_{it} = \alpha_L \log L_{it} + \phi_{it}(K_{it}, M_{it}) + u_{it}$$
$$\phi_{it}(K_{it}, M_{it}) = \log a + \alpha_K \log K_{it} + m_{it}^{-1}(K_{it}, M_{it})$$

Where, as we have shown, $m(.)$ is invertible due to material inputs demand's monotonicity with respect to ω. But given that labor is pinned down by K_{it}, M_{it}, we can't separate the identification of α_L and that of ϕ_{it}

Monopoly: note that now $\log M - \log L$ is *not* a constant anymore. In an analogous fashion to before, we have that

$$\log L_{it} = \log M_{it} + \log b + \log \alpha_L + \log\left(\theta p(L_{it}, M_{it}, \omega_{it}) - \frac{p^M}{b}\right)$$

Using the inversion (which can be done as we've proved monotonicity before), this writes as

$$\log L_{it} = \log M_{it} + \log b + \log \alpha_L + \log\left(\theta p(L_{it}, M_{it}, m_{it}^{-1}(K_{it}, M_{it})) - \frac{p^M}{b}\right)$$

Even if we didn't write it as an explicit function, once again labor is pinned down once we condition on K_{it}, M_{it}. For this reason, α_L is still unidentified in LP 2003's 1st stage.

Problem 3

3.1

Once again removing subscripts for conciseness, conditional input demand for materials is given as follows, in which firms observe the shock to labor ϵ, persistent TFP ω, but not u. We use the LIE to factor out the expectation of $\exp(u)$:

$$
\begin{aligned}
\max_{M} \quad & \mathbb{E}(e^{u\theta)})(e^{\omega+\alpha_L\epsilon}L^{\alpha_L}K^{\alpha_K})^{\theta}\cdot M^{\alpha_M\theta} - p^M M \\
\iff \max_{M} \quad & \mathbb{E}(e^{u\theta)})A^{\theta}\cdot M^{\alpha_M\theta} - p^M M \\
\theta &\equiv \frac{1}{\eta}+1 \\
A &\equiv e^{\omega+\alpha_L\epsilon}L^{\alpha_L}K^{\alpha_K}
\end{aligned}
$$

The corresponding FOC is

$$
\begin{aligned}
\implies \frac{\alpha_M\theta}{p^M}\mathbb{E}(e^{u\theta)})A^{\theta}M^{\alpha_M\theta-1} = 1 \\
\log M = \frac{1}{1-\alpha_M\theta}\left(\log\frac{\alpha_M\theta}{p^M} + \log\mathbb{E}(e^{u\theta)}) + \theta\log A\right)
\end{aligned}
$$

So the conditional material demand function is increasing in ω and given by

$$
M(L,K,\omega) = \left(\frac{\alpha_M}{p^M}\theta\right)^{\frac{1}{1-\alpha_M\theta}}\cdot\mathbb{E}(e^{u\theta})^{\frac{1}{1-\alpha_M\theta}}\cdot\left(e^{\omega+\alpha_L\epsilon}L^{\alpha_L}K^{\alpha_K}\right)^{\frac{\theta}{1-\alpha_M\theta}}
$$

Putting this back in the objective function:

$$
\begin{aligned}
\theta\log Q &= \log\mathbb{E}(e^{u\theta}) + \theta(\log A + \alpha_M\log M) \\
&= \log\mathbb{E}(e^{u\theta}) + \theta\left(\log A + \frac{\alpha_M}{1-\alpha_M\theta}\left[\log\frac{\alpha_M\theta}{p^M} + \theta\log A + \log\mathbb{E}(e^{u\theta})\right]\right) \\
&= \frac{\alpha_M\theta}{1-\alpha_M\theta}\log\frac{\alpha_M\theta}{p^M} + \frac{1}{1-\alpha_M\theta}\log\mathbb{E}(e^{u\theta}) + \frac{\theta}{1-\alpha_M\theta}\log A \\
\implies Q^{\theta} &= \left(\frac{\alpha_M}{p^M}\theta\right)^{\frac{\alpha_M\theta}{1-\alpha_M\theta}}\cdot\mathbb{E}(e^{u\theta})^{\frac{1}{1-\alpha_M\theta}}\cdot A^{\frac{\theta}{1-\alpha_M\theta}} \\
\implies \pi(L,K,\omega) &= \left(\frac{\alpha_M}{p^M}\theta\right)^{\frac{\alpha_M\theta}{1-\alpha_M\theta}}\cdot\mathbb{E}(e^{u\theta})^{\frac{1}{1-\alpha_M\theta}}\cdot\left(e^{\omega+\alpha_L\epsilon}L^{\alpha_L}K^{\alpha_K}\right)^{\frac{\theta}{1-\alpha_M\theta}} \\
&\quad - p^M\left(\frac{\alpha_M}{p^M}\theta\right)^{\frac{1}{1-\alpha_M\theta}}\cdot\mathbb{E}(e^{u\theta})^{\frac{1}{1-\alpha_M\theta}}\cdot\left(e^{\omega+\alpha_L\epsilon}L^{\alpha_L}K^{\alpha_K}\right)^{\frac{\theta}{1-\alpha_M\theta}} \\
\implies \pi(L,K,\omega) &= \mathbb{E}(e^{u\theta})^{\frac{1}{1-\alpha_M\theta}}\cdot\left(e^{\omega+\alpha_L\epsilon}L^{\alpha_L}K^{\alpha_K}\right)^{\frac{\theta}{1-\alpha_M\theta}}\cdot\left(\left(\frac{\alpha_M}{p^M}\theta\right)^{\frac{\alpha_M\theta}{1-\alpha_M\theta}} - p^M\cdot\left(\frac{\alpha_M}{p^M}\theta\right)^{\frac{1}{1-\alpha_M\theta}}\right)
\end{aligned}
$$

3.2

We assume we are in a stationary environment, in that prices are fixed. The FOCs and envelope equations associated with the Bellman equation are

$$\begin{aligned}
\beta\mathbb{E}[V_L(L', K', \omega')] &= w \\
\beta\mathbb{E}[V_K(L', K', \omega')] &= p^K \\
V_L(L, K, \omega) &= \pi_L(L, K, \omega) \\
V_K(L, K, \omega) &= \pi_K(L, K, \omega) + p^K(1-\delta)
\end{aligned}$$

The Euler equations are, therefore:

$$\begin{aligned}
\beta\mathbb{E}_t[\pi_L(L_{it+1}, K_{it+1}, \omega_{it+1})] &= w \\
\beta\mathbb{E}[\pi_K(L_{it+1}, K_{it+1}, \omega_{it+1}) + p^K(1-\delta)] &= p^K
\end{aligned}$$

Where $\pi(L, K, \omega)$ is defined as in the previous question.

3.3

We start by evaluating the profit function at parameter values. So we evaluate the following objects:

$$\begin{aligned}
\theta &= 0.8 \\
\frac{\theta}{1-\alpha_M\theta} &= \frac{4}{3} \\
\frac{\alpha_M}{p^M}\theta &= \frac{2}{5} \\
\frac{\alpha_M\theta}{1-\alpha_M\theta} &= \frac{2}{3} \\
\frac{1}{1-\alpha_M\theta} &= \frac{5}{3} \\
\mathbb{E}[e^{\theta u}] &= e^{0.064}
\end{aligned}$$

The last expression comes from $e^{\theta u}$ being a lognormal random variable. For conciseness, define $z_t \equiv \frac{5}{3}(0.064 + 0.24\epsilon_t + 0.8\omega_t)$. And now omit the i subscript:

$$\begin{aligned}
\pi(L_t, K_t, \omega_t) &= e^{\frac{5}{3}\cdot 0.064}(e^{\omega+0.3\epsilon})^{0.8\cdot\frac{5}{3}} \cdot (L^{0.3}K^{0.2})^{\frac{4}{3}} \cdot \underbrace{\left(\left(\frac{2}{5}\right)^{\frac{2}{3}} - \left(\frac{2}{5}\right)^{\frac{5}{3}}\right)}_{\approx 0.3} \\
&\approx \frac{3}{10}e^{z_t} \cdot (L_t^{0.3}K_t^{0.2})^{\frac{4}{3}}
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}_t[\pi_L(L_{t+1}, K_{t+1}, \omega_{t+1})] &= \frac{12}{100} \cdot K_{t+1}^{\frac{4}{15}} \cdot L_{t+1}^{-\frac{3}{5}} \cdot \mathbb{E}_t[e^{z_{t+1}}] \\
\mathbb{E}_t[\pi_K(L_{t+1}, K_{t+1}, \omega_{t+1})] &= \frac{8}{100} \cdot K_{t+1}^{-\frac{11}{15}} \cdot L_{t+1}^{\frac{2}{5}} \cdot \mathbb{E}_t[e^{z_{t+1}}]
\end{aligned}$$

Note now that $z_{t+1}|\mathcal{F}_t \sim \mathcal{N}\left(\frac{8}{75} + 1.2\omega_t, 2.25\right)$ since it is a sum of independent normal random variables. Therefore $\mathbb{E}_t[e^{z_{t+1}}] = \exp(1.23 + \frac{6}{5}\omega_t)$

So the Euler equations simplify to:

$$\frac{12}{100} \cdot K_{t+1}^{\frac{4}{15}} \cdot L_{t+1}^{-\frac{3}{5}} \cdot e^{1.23+1.2\omega_t} = \frac{10}{9}$$
$$\frac{8}{100} \cdot K_{t+1}^{-\frac{11}{15}} \cdot L_{t+1}^{\frac{2}{5}} \cdot e^{1.23+1.2\omega_t} = \frac{91}{90}$$
$$\implies K_t \approx 0.73 \cdot L_t$$
$$\implies L_{t+1} = (0.108)^3 \cdot (0.73)^{0.8} \cdot e^{3.69+3.6\omega_t}$$
$$K_{t+1} = (0.108)^3 \cdot (0.73)^{1.8} \cdot e^{3.69+3.6\omega_t}$$
$$M_t = \left(\frac{2}{5}\right)^{\frac{5}{3}} \cdot e^{\frac{8}{75}} \cdot (e^{0.3\epsilon_t+\omega_t} L_t^{0.3} K_t^{0.2})^{\frac{4}{3}}$$

3.4

We start with the ACF approach. Note that in our setting there are a few differences from the standard ACF framework

- We have a gross production function, not value-added - Materials are also Cobb-Douglas, not in fixed proportions.
- We assume to have *revenue* data, not output data - this isn't much of a problem under the assumption of constant demand elasticity (and therefore constant markups)
- Not only does the econometrician not observe persistent productivity ω_{it}, it doesn't observe the temporary shock ϵ_{it} to labor, which is observed by firms when choosing materials M_{it} - if we aren't careful, this violates the scalar unobservable assumption

What we do then is write out a log-linear equation for revenue :

$$r_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \beta_m m_{it} + \nu_{it} + u_{it}$$
$$\nu_{it} = \gamma\omega_{it} + \lambda\epsilon_{it}$$

In which the β's, γ, and λ are functions of model parameters - $(\rho, \alpha_L, \alpha_K, \eta)$. Simply running this regression of course won't work, since materials are chosen after observing ν_{it}.

That is why we approximate material input's demand in a filtering stage, just as ACF. Note that writing ν_{it} as a composite of the two unobservables preserves the scale unobservable assumption - The Cobb-Douglas assumption is doing a lot of work here.

So our first stage is

$$r_{it} = \phi(k_{it}, m_{it}, l_{it}) + u_{it}$$

Due to collinearities in capital and labor demands, we consider ϕ as a linear function.

In the second stage we follow ACF and form moment conditions as follows:

$$\mathbb{E}[(\hat{\phi}_{it} - \beta_0 - \beta_l l_{it} - \beta_k k_{it} - \rho(\hat{\phi}_{it-1} - \beta_0 - \beta_l l_{it-1} - \beta_k k_{it-1})) \otimes Z_{it}] = 0$$

$$Z_{it} = \begin{bmatrix} 1 \\ k_{it} \\ l_{it} \\ \hat{\phi}_{t-1} \end{bmatrix}$$

These moment conditions, using the identity as the weighting matrix, should yield consistent estimates of β_0, β_k, β_l. We need then to use these estimates to solve for the structural parameters $(\alpha_k, \alpha_l, \theta)$, where $\theta = 1 + \frac{1}{\eta}$.

So we need to partial out materials from our revenue function since the ACF approach does *not* include materials in the second stage (note that our moment conditions also do not have them). Using then the analytical expression for revenue $R(L, K, \omega)$:

$$r_{it} = \frac{1}{1 - \alpha_M\theta} \cdot \left(\alpha_M\theta \log \alpha_M\theta + \log \mathbb{E}[e^{\theta u}]\right) + \frac{\theta\alpha_L}{1 - \alpha_M\theta} l_{it} + \frac{\theta\alpha_K}{1 - \alpha_M\theta} k_{it}$$

$$r_{it} = \frac{1}{1 - \alpha_M\theta} \cdot \left(\alpha_M\theta \log \alpha_M\theta + \frac{\theta^2}{10}\right) + \frac{\theta\alpha_L}{1 - \alpha_M\theta} l_{it} + \frac{\theta\alpha_K}{1 - \alpha_M\theta} k_{it} + \nu_{it} + u_{it}$$

Matching coefficients, our estimates are the solution to a nonlinear system of equations:

$$
\begin{aligned}
\beta_0 &= \frac{1}{1-\alpha_M\theta}\cdot\left(\alpha_M\theta\log\alpha_M\theta+\frac{\theta^2}{10}\right)\\
\beta_l &= \frac{\theta\alpha_L}{1-\alpha_M\theta}\\
\beta_k &= \frac{\theta\alpha_k}{1-\alpha_M\theta}\\
\alpha_M &= 1-\alpha_L-\alpha_K\\
\theta &= \frac{1}{\eta}+1
\end{aligned}
$$

Now we go to the dynamic panel approach. We follow the slides really closely:

$$
\begin{aligned}
r_{it} &= \beta_0+\beta_l l_{it}+\beta_k k_{it}+\alpha_L\epsilon_{it}+\omega_{it}+u_{it}\\
r_{it}-\rho r_{it-1} &= \beta_0(1-\rho)+\beta_l(l_{it}-\rho l_{it-1})+\beta_k(k_{it}-\rho k_{it-1})+\alpha_L(\epsilon_{it}-\rho\epsilon_{it-1})+\xi_{it}+(u_{it}-\rho u_{it-1})
\end{aligned}
$$

Unlike on the slides, labor and capital are chosen one period in advance, so that we can form moment conditions based on the equation above. If they weren't, we would still need to take first differences. Our moment conditions are the standard ones:

$$
\begin{aligned}
&\mathbb{E}[(r_{it}-\rho r_{it-1}-\beta_0(1-\rho)-\beta_l(l_{it}-\rho l_{it-1})-\beta_k(k_{it}-\rho k_{it-1}))\otimes W_{it}]=0\\
&W_{it}=\begin{bmatrix}1\\ k_{it}\\ l_{it}\\ k_{it-1}\\ l_{it-1}\end{bmatrix}
\end{aligned}
$$

Having estimated ρ, β_l, and β_k, we back out structural parameters solving the same nonlinear system as before.

Results

Using Stata and Mata (see code attached) we simulated the data generating process. We started by simulating 100 periods of the serially correlated shocks ω and then kept the last 10 to make sure that our data is in stationary state.

Simulating the independent shocks and using the analytical formulas derived in the previous section we obtained the realizations of revenue, labor, capital and intermediate inputs for 1000 firms over 10 periods.

Because production technology is Cobb-Doublas, the log of revenue is a linear combination of inputs. This allows us to estimate the first stage model using OLS of R on Y, M and L.

For the second stage we obtained the first-stage predictions and calculated lags of capital, labor and predicted revenue to be used as regressors and instruments.

Results for a single simulation are shown in the table below. We managed to accurately estimate the second stage regression coefficients on labor and capital as well as the serial correlation parameter ρ. As shown in the table and in the montecarlo excercise, they are unbiased around the true population value. However, we were not able to estimate β_0. This prevented us from using the structural equations to accurately recover the model parameters α_l, and α_k. Results are shown in table 1.

The code we used to invert the regression coefficients is shown in the appendix.

Table 1: Model parameters estimates using the approach of Ackerberg et al

	Regression coefficients			
	β_l	β_k	β_0	ρ
Estimate	0.398	0.266	-1.001	0.907
True value	0.4	0.266	-1.13	0.9

	Model parameters			
	α_l	α_k	η	ρ
Estimate	0.217	0.145	0.845	0.907
True value	0.3	0.2	0.8	0.9

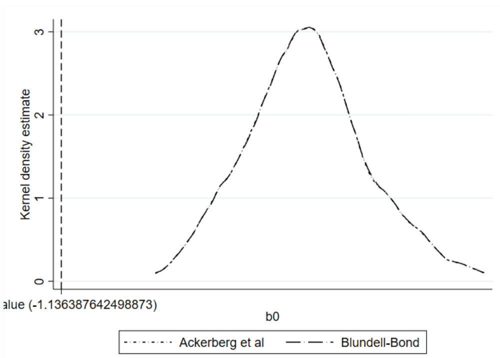
For the Blundell-Bond approach, we use the same variables, this time we include contemporaneous and lagged labor and capital as instruments. As in the previous case, we were able to accurately estimate all the regression coefficients except the constant. This yields erroneous estimates of the model parameters. Results are shown in table 2.

Table 2: Model parameters estimates using the Blundell-Bond approach

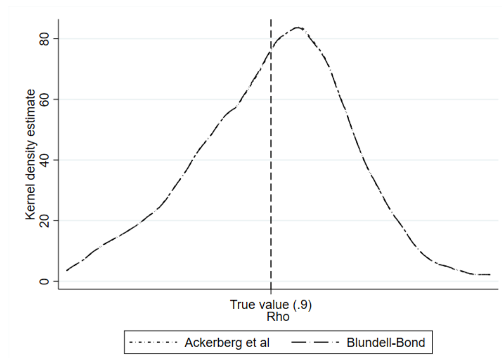
	Regression coefficients			
	β_l	β_k	β_0	ρ
Estimate	0.394	0.270	-1.002	0.907
True value	0.4	0.266	-1.13	0.9
	Model parameters			
	α_l	α_k	η	ρ
Estimate	0.217	0.145	0.845	0.907
True value	0.3	0.2	0.8	0.9

4

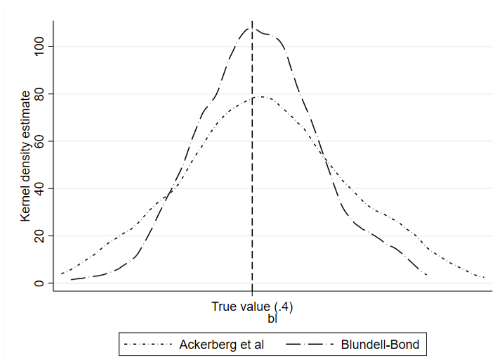
Since we were not able to estimate the parameter β_0, we cannot compare performance of the methods using our estimates for α_k and α_l. Instead, we will compare how accurately the linear parameters are estimated as well as the serial correlation parameter ρ.



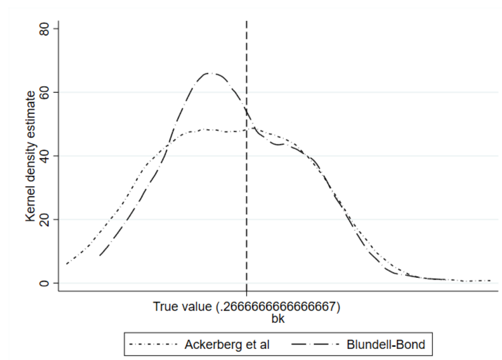
(a) Distribution of $\hat{\beta}_0$



(b) Distribution of $\hat{\rho}_0$



(c) Distribution of $\hat{\beta}_l$

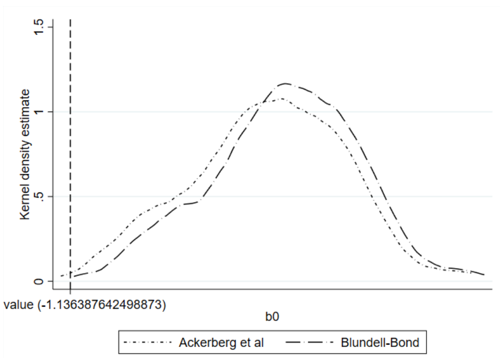


(d) Distribution of $\hat{\beta}_k$

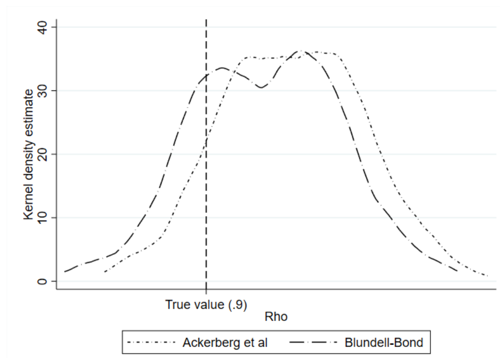
Bonus- Measurement error in capital

Now we repeat the same excercise but allowing for measurement error in capital. This means that the firm's decisions will be exactly the same as before, however, instead of observing K_{it} we observe (and use for estimation) $\tilde{K_{it}} = K_{it} exp(\varepsilon)$.

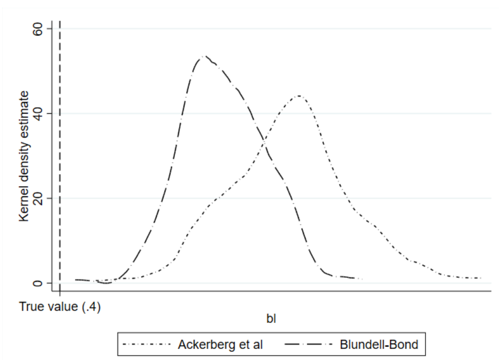
As in the previous section, we compare estimations of the linear parameters (because those are the ones we were able to accuratelye estimate)



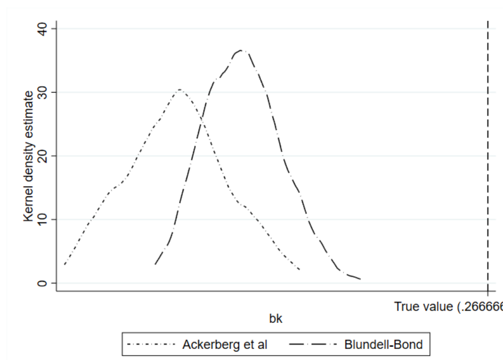
(a) Distribution of $\hat{\beta}_0$



(b) Distribution of $\hat{\rho}_0$



(c) Distribution of $\hat{\beta}_l$



(d) Distribution of $\hat{\beta}_k$

Consistent with attenuation bias, we find that our estimates for the coefficient in capital are downward biased while our estimates for the coefficient on labor are uppward biased.

Appendix 1: Code for Montecarlo simulations

```
*Initialize simulation
set seed 2022
matrix BlundellBond=J(200,4,0)
matrix AckerbergEtAl=J(200,4,0)

forvalues i=1/200{

*Create data
clear
set obs 1000
gen Firm=_n

global Rho=0.9

*Simulate 100 periods and keep the last 10 to make sure that the series is close to a st
gen Omega_0=rnormal(0,1)
forvalues t=1/100{
local tt=`t'-1
gen Omega_`t'=$Rho * Omega_`tt'+rnormal(0,1)
}

forvalues t=0/100{
```

```
if ‘t’<90{
drop Omega_‘t’
}
else{
local tt=‘t’-90

rename Omega_‘t’ Omega_‘tt’
}
}

*Use analytical relations to construct inputs and revenue (in the process we also create
gen Epsilon_0=rnormal(0,3)
forvalues t=0/9{

local tt=‘t’+1
gen Epsilon_‘tt’=rnormal(0,3)
gen U_‘tt’=rnormal(0,.2)
gen L_‘tt’=(0.108)^3*(0.73)^0.8*exp(3.69+3.6*Omega_‘t’)
gen K_‘tt’=0.73*L_‘tt’
replace L_‘tt’=L_‘tt’*exp(Epsilon_‘tt’)
gen M_‘tt’=(2/5)^(5/3)*exp(8/75)*(L_‘tt’^(0.3)*K_‘tt’^(0.2)*exp(Omega_‘tt’))^(4/3)

gen Y_‘tt’=(exp(U_‘tt’+Omega_‘tt’)*L_‘tt’^(0.3)*K_‘tt’^(0.2)*M_‘tt’^(0.5))^(0.8)

}

*Reshape data and get log-variables
reshape long Y_ L_ K_ M_ Omega_ U_ Epsilon_, i(Firm) j(t)
```

```
drop if t==0

foreach var in Y K L M {
gen log_`var'_=log(`var'_)
}

*First stage predictions
reg log_Y_ log_K_ log_L_ log_M_
predict log_Y_Hat

*Get lags
sort Firm t
bysort Firm: gen log_Y_Hat_lag=log_Y_Hat[_n-1]
bysort Firm: gen log_K_lag=log_K_[_n-1]
bysort Firm: gen log_L_lag=log_L_[_n-1]
bysort Firm: gen log_M_lag=log_M_[_n-1]
bysort Firm: gen log_Y_lag=log_Y_[_n-1]

***********************************************************
*GMM- Ackerberg et Al
***********************************************************
gmm (log_Y_-({b0=-.1}+{bk}*log_K_+{bl}*log_L_)-{RhoHat=.9}*(log_Y_Hat_lag-({b0}+{bk}*log

global b0=_b[/b0]
global bl=_b[/bl]
global bk=_b[/bk]
global RhoHat=_b[/RhoHat]
```

```
matrix AckerbergEtAl[`i',1]=$b0
matrix AckerbergEtAl[`i',2]=$bl
matrix AckerbergEtAl[`i',3]=$bk
matrix AckerbergEtAl[`i',4]=$RhoHat

*di .8*((1/.6)*ln(.4)+(1/.6)*(.8^2/10))

************************************************************
*GMM- Blundell-Bond
************************************************************
gmm (log_Y_-{RhoHat=.8}*log_Y_Hat_lag-{b0}*(1-{RhoHat})-{bk}*(log_K_-{RhoHat}*log_K_lag)

global b0=_b[/b0]
global bl=_b[/bl]
global bk=_b[/bk]
global RhoHat=_b[/RhoHat]

matrix BlundellBond[`i',1]=$b0
matrix BlundellBond[`i',2]=$bl
matrix BlundellBond[`i',3]=$bk
matrix BlundellBond[`i',4]=$RhoHat

}

*******************************************************************************
*Comparisons
*******************************************************************************
```

```
local b0_true=.8*((1/.6)*ln(.4)+(1/.6)*(.8^2/10))

local bk_true=.8*.2/.6

local bl_true=.8*.3/.6

local Rho_true=.9


clear

matrix Results=[AckerbergEtAl, BlundellBond]

matrix colnames Results = "b0_Ack" "bl_Ack" "bk_Ack" "Rho_Ack" "b0_BB" "bl_BB" "bk_BB" "


svmat Results, names(col)


foreach param in b0 bk bl Rho{

twoway (kdensity ‘param’_Ack, lcolor(black) lpattern(shortdash_dot)) (kdensity ‘param’_B


graph export "D:\DDC\PhD_IO\pset2\MS_‘param’.png", as(png) name("Graph") replace


}
```

Appendix 2: Code for parameter inversion

```
clear mata


mata
```

```
void AckerbergSolve(todo, p, lnf,  S, H){

b0=$b0
bl=($bl)
bk=($bk)

al=p[1]
ak=p[2]
theta=p[3]

lnf = (al*theta)/(1-theta*(1-al-ak))-bl \
(ak*theta)/(1-theta*(1-al-ak))-bk \
theta*((1/(1-theta*(1-al-ak))) *(ln(theta*(1-al-ak))+theta^2/10))-b0

lnf=lnf'*lnf

}

S = optimize_init()

optimize_init_evaluator(S, &AckerbergSolve())

optimize_init_evaluatortype(S, "v0")

optimize_init_params(S, (.4,.4,.1))

optimize_init_which(S,  "min" )
```

```
optimize_init_tracelevel(S,"none")

optimize_init_conv_ptol(S, 1e-16)

optimize_init_conv_vtol(S, 1e-16)

p = optimize(S)

p

st_matrix("x",p)

end
```