Duke University

Department of Economics

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Part 1. Logit

1.1-1.5

The utility for consumer i from product j at market t is as follows.

$$u_{ijt} = X_{jt}\beta + \alpha p_{jt} + \xi_{jt} + \epsilon_{ijt} \tag{1}$$

Here, X_{jt} denotes product characteristics, such as a promotion indicator and brand dummies. P_{jt} is the price and ξ_{jt} is unobserved product characteristics. If we assume that ϵ_{ijt} follows an extreme value type 1 error and normalize the utility of the outside option to 0, the market share has a closed for as below.

$$s_{jt} = \frac{\exp\left(X_{jt}\beta + \alpha p_{jt}\right)}{1 + \sum_k \exp(X_{kt}\beta + \alpha p_{kt})} \tag{2}$$

Taking log yields the equation as follows that can utilize OLS estimation.

$$log(s_{jt}) - log(s_{0t}) = X_{jt}\beta + \alpha p_{jt} \tag{3}$$

We compute market shares as $s_{jt} = \frac{sales_{jt}}{count_t \cdot 0.02}$ - so market shares aren't too low under the rationale that 2% of drug store customers come in looking to buy headache medication. Any of those $count_t \cdot 0.02$ not buying one of the 11 products in the sample is considered to be taking the outside option.

The results are in Table 1. From column (1) to (3), we use price and promotion as product characteristics. Specifically, we added brand dummies in column (2) and store-brand dummies in column (3). There are endogeneity issues with respect to price, since ξ_{it} and p_{jt} can be correlated. Thus, we use two different instrumental variables: wholesale cost and Hausman IV. From columns (4) to (6), we use the same models with (1), (2), and (3) with wholesale cost as an instrument. From columns (7) to (9), we estimate the same models with (1), (2), and (3) using a Hausman instrument.

Table 1: OLS results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Price	-0.0492***	-0.185***	-0.185***	-0.0106***	-0.142***	-0.143***	-0.0493***	-0.188***	-0.189***
	(0.0027)	(0.0024)	(0.0022)	(0.0029)	(0.0024)	(0.0023)	(0.0027)	(0.0024)	(0.0022)
Promotion	0.209***	0.442***	0.438***	0.231***	0.447***	0.443***	0.209***	0.442***	0.438***
	(0.0160)	(0.0143)	(0.0136)	(0.0159)	(0.0142)	(0.0134)	(0.0160)	(0.0143)	(0.0136)
Constant	-7.771***	-7.200***	-7.202***	-7.943***			-7.770***		
	(0.0128)	(0.0108)	(0.0101)	(0.0139)			(0.0129)		
Observations	36476	36476	36476	36476	36476	36476	36476	36476	36476
Brand FE		√			√			√	
Brand × Store FE			√			√			√
Cost IV				√	√	√			
Hausman IV							√	√	√

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

1.6

We can compute the mean own-price elasticities for all brands in the market using the estimates in 1, 2, and 3. Since it is a logit model, own and cross-price elasticities of product j with respect to the price of product k have a closed form as below.

$$\eta_{jkt} = \frac{\partial s_{jt} p_{kt}}{\partial p_{kt} s_{jt}} = \begin{cases} -\alpha p_{jt}\left(1 - s_{jt}\right) & \text{if } j = k \\ \alpha p_{kt} s_{kt} & \text{if } j \neq k \end{cases} \tag{4}$$

The results are in Table 2. Note how OLS does a poor job in estimating elasticities - all of them are less than 1 in absolute value, indicating that firms are operating on the inelastic part of the demand curve, which is not consistent with optimal behavior. The IV estimates do a bit better on that regard, but results still are not satisfactory - we need better instruments or a better model, which we shall use on Part 2.

Table 2: Mean own price elasticties

	OLS	Wholesale IV	Hausman IV
Tylenol(25)	-0.168	-0.633	-0.631
Tylenol(50)	-0.240	-0.906	-0.903
Tylenol(100)	-0.342	-1.289	-1.284
Advil(25)	-0.144	-0.545	-0.543
Advil(50)	-0.247	-0.930	-0.928
Advil(100)	-0.397	-1.496	-1.491
Bayer(25)	-0.131	-0.495	-0.493
Bayer(50)	-0.178	-0.670	-0.668
Bayer(100)	-0.194	-0.730	-0.728
Store(50)	-0.092	-0.347	-0.345
Store(100)	-0.211	-0.794	-0.792

Part 2.

2.1

We start by writing out the expression for indirect utility derived by consumer i when buying product j in market t:

$$\begin{aligned} u_{ijt} &= X_{jt}\beta + \beta_{ib}B_{jt} + \alpha_i p_{jt} + \xi_{jt} + \epsilon_{ijt} \\ &= \beta X_{jt} + \alpha p_{jt} + +\xi_{jt} + \sigma_B v_i B_{jt} + \sigma_I I_i p_{jt} + \epsilon_{ijt} \\ &= \delta_{jt}(\alpha, \beta, \xi_{jt}) + \mu_{ijt}(\sigma_I, \sigma_B) + \epsilon_{ijt} \end{aligned}$$

So, as expected, and contrary to the logit case, preferences are heterogeneous across consumers not only due to the logit error but because there is variation in consumers' tastes for branded products and in their (dis)taste for higher prices.

The distinction between the δ components and μ components is that the latter involves "nonlinear terms", as the jargon says - meaning it involves idiosyncratic terms.

We estimate the model in the standard way - first, for given values of δ_{jt}, σ_I, σ_B, simulate v_i from the normal distribution and income from the data itself to compute average market shares taking advantage of the logit formula:

$$\hat{s}_{jt} = \frac{1}{R}\sum_{i=1}^{R}\left(\frac{\exp\left(\delta_{jt} + \sigma_B v_i B_{jt} + \sigma_I I_i p_{jt}\right)}{1 + \sum_{m=1}^{J}\exp\left(\delta_{mt} + \sigma_B v_i B_{mt} + \sigma_I I_i p_{mt}\right)}\right)$$

With these in hand, since we cannot do a Berry 1994 inversion, we iterate on a contraction to update deltas after comparing model-implied market shares and observed market shares, of course having fixed the values for σ_I, σ_B

$$\delta^{h+1} = \delta^h + \ln(S_{jt}) - \ln(\hat{s}_{jt})$$

After the iteration and having obtained expressions for the linear component δ_{jt}, we extract ξ_{jt} as the difference between the observed components of the product specific term - $X_{jt}\beta + \alpha p_{jt}$ and our δ_{jt}. With ξ_{jt} in hand, we form our moment conditions with our instruments.

Following Nevo's guide, we can express linear (and hence the expression) parameters as a linear function of $\delta(\sigma_I, \sigma_B)$, which helps us give the GMM criterion only maximizes over the σ's. Such a formula is given by

$$[\beta', \alpha] = \left[X_1' Z(Z'Z)^{-1} Z' X_1\right]^{-1} X_1' Z(Z'Z)^{-1} Z' \hat{\delta}\,(\sigma_I, \sigma_B)$$

Table 3 reports parameter estimates for the BLP setting considering two different starting points for the optimization. For the linear term, we include a set of product dummies, price, and promotion. Results are thankfully stable across models.

2.2

For the logit case, the Slutsky substitution matrix has a well-known closed-form expression. Meanwhile, to compute BLP elasticities we need once again to simulate from the normal distribution and from income data - these let us calculate the price coefficient $\alpha + \sigma_I I_i$ for each draw, as well as associated market shares, we average their product across all draws, and then multiply by the ratio between observed prices and market shares:

Table 3: BLP parameter estimates

	Model	
	1	2
Price	-0.624	-0.624
Promotion	0.345	0.345
Tylenol 25	-1.797	-1.797
Tylenol 50	-1.100	-1.100
Tylenol 100	-0.815	-0.815
Advil 25	-2.257	-2.257
Advil 50	-1.955	-1.955
Advil 100	-1.687	-1.687
Bayer 25	-3.509	-3.509
Bayer 50	-3.410	-3.410
Bayer 100	-2.357	-2.357
Store Brand 50	-3.027	-3.027
Store Brand 100	-2.515	-2.515
σ_I	0.024	0.024
σ_B	-0.373	-0.373
GMM criterion	251.48	251.48
Starting point	(0,0)	(1, 1)

$$\eta_{jkt} = \frac{\partial s_{jt} p_{kt}}{\partial p_{kt} s_{jt}} = \frac{p_{kt}}{s_{jt}} \frac{1}{R} \sum_{i=1}^{R} (\alpha + \sigma_I I_i)\, s_{ijt}\, (s_{ikt})^{-1}\, s_{ikt}^{\mathbf{1}[j=k]}$$

Fixing observed prices and market shares to those on store 9 in week 10, tables 4 and 5 display results for the logit and BLP settings, respectively. Thankfully, in either case, own-price elasticities are negative, and cross-price elasticities are positive.

As expected, logit cross-price elasticities η_{jk} are constant across k for fixed j. This is, as we know, due to the restrictive IIA assumption. However, even after including vertical differentiation with BLP, cross-price elasticities within each column are still really close. This is probably due to low estimated values for σ_I (meaning income heterogeneity doesn't matter that much). Still, controlling for taste heterogeneity does lead to lower elasticities overall, even if remarkably the setting seems close to IIA.

Table 4: Substitution matrix for store 9, week 10 - Logit model

	Tylenol 25	Tylenol 50	Tylenol 100	Advil 25	Advil 50	Advil 100	Bayer 25	Bayer 50	Bayer 100	Store Brand 50	Store Brand 100
Tylenol 25	-2.381	0.183	0.156	0.043	0.07	0.016	0.031	0.031	0.022	0.035	0.008
Tylenol 50	0.062	-3.433	0.156	0.043	0.07	0.016	0.031	0.031	0.022	0.035	0.008
Tylenol 100	0.062	0.183	-4.581	0.043	0.07	0.016	0.031	0.031	0.022	0.035	0.008
Advil 25	0.062	0.183	0.156	-2.059	0.07	0.016	0.031	0.031	0.022	0.035	0.008
Advil 50	0.062	0.183	0.156	0.043	-3.858	0.016	0.031	0.031	0.022	0.035	0.008
Advil 100	0.062	0.183	0.156	0.043	0.07	-6.214	0.031	0.031	0.022	0.035	0.008
Bayer 25	0.062	0.183	0.156	0.043	0.07	0.016	-1.982	0.031	0.022	0.035	0.008
Bayer 50	0.062	0.183	0.156	0.043	0.07	0.016	0.031	-2.449	0.022	0.035	0.008
Bayer 100	0.062	0.183	0.156	0.043	0.07	0.016	0.031	0.031	-2.925	0.035	0.008
Store Brand 50	0.062	0.183	0.156	0.043	0.07	0.016	0.031	0.031	0.022	-1.22	0.008
Store Brand 100	0.062	0.183	0.156	0.043	0.07	0.016	0.031	0.031	0.022	0.035	-3.326

Table 5: Substitution matrix for store 9, week 10 - BLP model

	Tylenol 25	Tylenol 50	Tylenol 100	Advil 25	Advil 50	Advil 100	Bayer 25	Bayer 50	Bayer 100	Store Brand 50	Store Brand 100
Tylenol 25	-1.313	0.123	0.105	0.029	0.047	0.011	0.021	0.021	0.015	0.018	0.004
Tylenol 50	0.042	-1.884	0.105	0.029	0.047	0.011	0.021	0.021	0.015	0.018	0.004
Tylenol 100	0.042	0.123	-2.526	0.029	0.047	0.011	0.021	0.021	0.015	0.018	0.004
Advil 25	0.042	0.123	0.105	-1.137	0.047	0.011	0.021	0.021	0.015	0.018	0.004
Advil 50	0.042	0.123	0.105	0.029	-2.133	0.011	0.021	0.021	0.015	0.018	0.004
Advil 100	0.042	0.123	0.105	0.029	0.047	-3.452	0.021	0.021	0.015	0.018	0.004
Bayer 25	0.041	0.123	0.105	0.029	0.047	0.011	-1.095	0.021	0.015	0.018	0.004
Bayer 50	0.042	0.123	0.105	0.029	0.047	0.011	0.021	-1.354	0.015	0.018	0.004
Bayer 100	0.042	0.123	0.105	0.029	0.047	0.011	0.021	0.021	-1.62	0.018	0.004
Store Brand 50	0.033	0.097	0.082	0.022	0.037	0.008	0.016	0.017	0.012	-0.587	0.004
Store Brand 100	0.033	0.097	0.082	0.022	0.037	0.008	0.016	0.017	0.012	0.016	-1.597

2.3

We use firms' first-order conditions and demand estimates to back out marginal costs, which are assumed to be constant. To fix ideas, consider the case of Tylenol, which owns three products and therefore chooses three prices $p = (p_1, p_2, p_3)$ taking as given other products' prices p_{-j}

$$
\begin{aligned}
\max_{p_1,p_2,p_3} M \cdot s_1(p, p_{-j})(p_1 - mc_1) + M \cdot s_2(p, p_{-j})(p_2 - mc_2) + M \cdot s_3(p, p_{-j})(p_3 - mc_3) \\
\implies s_1 + \frac{\partial s_1}{\partial p_1}(p_1 - mc_1) + \frac{\partial s_2}{\partial p_1}(p_2 - mc_2) + \frac{\partial s_3}{\partial p_1}(p_3 - mc_3) = 0 \\
\implies s_2 + \frac{\partial s_1}{\partial p_2}(p_1 - mc_1) + \frac{\partial s_2}{\partial p_2}(p_2 - mc_2) + \frac{\partial s_3}{\partial p_2}(p_3 - mc_3) = 0 \\
\implies s_3 + \frac{\partial s_1}{\partial p_3}(p_1 - mc_1) + \frac{\partial s_2}{\partial p_3}(p_2 - mc_2) + \frac{\partial s_3}{\partial p_3}(p_3 - mc_3) = 0
\end{aligned}
$$

So clearly ownership matters - if a firm owns products j, it takes into account that its choice for p_j changes its market share for product j'. We summarize this relation by stacking FOCs of all brands/products using an ownership matrix Ω, in which $\Omega_{jk} = 1$ is products j, k are produced by the same firm, and 0 otherwise.

$$s + \Omega . * \frac{\partial s}{\partial p}(p - mc) = 0$$

$$mc = p + \left(\Omega . * \frac{\partial s}{\partial p}\right)^{-1} \cdot s$$

$$mc = p + \left(\Omega . * \eta \cdot \frac{s}{p}\right)^{-1} \cdot s$$

Where in the last line we expressed marginal costs as a function of estimated objects or those taken directly from the data, and $\frac{s}{p}$ is element-wise division.

In this context, the ownership matrix is:

$$\Omega = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

And estimated marginal costs, in either case, are:

As expected, marginal cost estimates in table 6 are lower than wholesale costs - the estimates reflect manufacturing marginal costs, which are naturally lower than wholesale costs, reflecting advertising costs, for instance. Note also that BLP estimates for costs are much lower than logit, which is explained by lower demand elasticities.

Table 6: Marginal cost estimates for store 9, week 10

	Estimates		
	Logit	BLP	Observed wholesale cost
Tylenol 25	1.695	0.306	2.1
Tylenol 50	3.356	2.068	3.29
Tylenol 100	4.905	3.668	5.66
Advil 25	1.398	0.213	2.1
Advil 50	3.898	2.763	3.46
Advil 100	7.014	5.903	5.76
Bayer 25	1.305	0.152	1.79
Bayer 50	1.946	0.807	2.08
Bayer 100	2.583	1.454	3.71
Store Brand 50	0.295	-1.208	0.94
Store Brand 100	3.125	1.649	1.92

Part 3.

3.1

Now we take as given the estimated marginal costs from the previous question and firms' FOCs considering a new ownership matrix Ω_2, given below:

$$\Omega_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

However, we need to consider that not only prices change, but demand elasticities too. So taking as given our estimated α coefficient, we solve for prices in the nonlinear system below:

$$s(\alpha, p) + \left(\Omega_2 . * \eta(\alpha, p) . * \frac{s(\alpha, p)}{p}\right) \cdot (p - mc) = 0$$

Where we make explicit that market shares and demand elasticities are endogenous. Note also that in the logit case this isn't so hard, because market shares and the substitution matrix can be expressed in closed form as a function of prices and of the price coefficient.

Table 7: Merger analysis for store 9, week 10 - logit assumption

	Pre-merger	Post merger
Tylenol 25	3.29	3.41
Tylenol 50	4.87	4.96
Tylenol 100	6.38	6.45
Advil 25	2.83	3.15
Advil 50	5.29	5.48
Advil 100	8.39	8.51
Bayer 25	2.71	3.06
Bayer 50	3.34	3.64
Bayer 100	3.97	4.23
Store Brand 50	1.69	1.69
Store Brand 100	4.49	4.49

Table 7 compares pre and post-merger prices. Note that Tylenol, Advil and Bayer products are all a bit more expensive, while Store Brand products have roughly the same prices. It makes sense that the former 3 should be more expensive - the firm that owns them isn't as worried about raising prices of say, Tylenol size 50 because the loss of market share is partially compensated with buyers substituting to products the firm also owns - this effect is more pronounced the stronger the cross-price elasticities within the Tylenol, Advil, Bayer group.

Note too that Store Brand products don't decrease their price either - this is perhaps because under Bertrand competition actions are strategic complements.

3.2

In the BLP case, we would still need to solve the same nonlinear system of equations. However, there is an added difficulty that market shares and demand elasticities cannot be expressed in closed form as a function of prices and price coefficients. We would then proceed with the same approach used in part 2.2 - fixing prices, we draw income and v, compute average market shares and elasticities and then pick prices that solve the system given such averages.