## Problem Set 3

## Due October 11th by 3pm

For this assignment use dataassign3.mat. The variables in this data set are:

- Firm1 an  $N \times T$  matrix of whether a firm is in the market is in each of the N markets in each of the T time periods.
- State an  $N \times T$  matrix denoting the (transitory) state of each of the markets in each of the time periods.
- PState an  $N \times 1$  vector denoting the permanent state of the market.
- Y an  $N \times T$  matrix denoting the price in each of the markets in each of the time periods.

Firms are solving an infinite horizon problem and the environment is stationary. Firms maximize lifetime profits and the discount factor is 0.9. Flow profits for entering (or staying in) market n at time t are given by:

$$\Pi_{nt} = \alpha_0 + \alpha_1 State_{nt} + \alpha_2 PState_{nt} + \alpha_3 (1 - Firm1_{nt-1}) + \epsilon_{1nt}$$

where  $Firm1_{nt-1}$  is an indicator for whether the firm was an incumbent (at t = 1 this variable should be set to zero for all markets) and  $\epsilon_{1nt}$  is distributed Type 1 extreme value. Profits for staying out a market are given by  $\epsilon_{0nt}$  which is also distributed Type 1 extreme value. Once a firm chooses to stay out the market, there are no future profits and a new potential entrant arrives at the market in the next period. Firms only have expectations regarding future values of State. The probabilities of transitioning to different value of  $State_t$  only depend on the values of  $State_{t-1}$ . Since the environment is stationary, these transition probabilities do not vary with time.

The price process, which is not relevant for firms' decisions, is given by:

$$Y_{nt} = \gamma_0 + \gamma_1 State_{nt} + \gamma_2 PState_{nt} + \gamma_3 Firm1_{nt} + \zeta_{nt}$$

where  $\zeta_{nt}$  is distributed  $N(0, \sigma)$ .

For all the estimation problems below, use CCP's in formulating the differenced conditional value functions.

- 1. Estimate the dynamic discrete choice process using CCP's for the future value terms. These CCP's should be taken directly from the data. (Note that you will need to estimate the transition process on *State* and that this should be done outside of the estimation of the dynamic discrete choice model)
- 2. Estimate the full model (dynamic discrete choice, prices, and transitions on *State*) where *PState* is now a missing variable. Use both ways of updating the CCP's and compare your results from the two methods.
- 3. Estimate the full model where *State* is now a missing variable (*PState* is known). This will involve also estimating the transitions on the unobserved variable.
- 4. Estimate the full model with *State* as a missing variable in two stages. First, estimate the distribution of the unobserved heterogeneity and the parameters of the pricing process using the empirical likelihood of the choices in the EM algorithm. Next, with the conditional probabilities of being a particular type in hand, obtain estimates of the profit parameters.