Duke University

Department of Economics

ECON 881 — Dynamic Discrete Choice

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Problem 1.

We solve the dynamic program using CCPs. Let's set up the firm's problem to show which CCPs are relevant in our case.

When we see all the data, relevant state variables are the transitory state s, the permanent state p, and finally the firm's lag position on the market l.

Let π be the transition matrix of the transitory state

$$\pi = \begin{pmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{pmatrix}$$

We can then recursively define a firm's conditional valuation function of choosing to stay on the market, letting u_1 be is associated flow payoff

$$v_1(s,p,l) = u_1(s,p,l) + \beta[\pi_{ss'}\mathbb{E}_t[\max\{v_1(s',p,1) + \epsilon_{1t+1}, \epsilon_{0t+1}\}] + \pi_{ss}\mathbb{E}_t[\max\{v_1(s,p,1) + \epsilon_{1t+1}, \epsilon_{0t+1}\}]]$$

With the usual logit error term, this simplifies to:

$$v_1(s,p,l) = u_1(s,p,l) + \beta[\pi_{ss'}\log(\exp(v_1(s',p,1)) + 1) + \pi_{ss}\log(\exp(v_1(s,p,1)) + 1)]$$

We can express this in terms of choice probabilities. As an example, let $p_0(s,p,1)$ be the conditional probability of choosing option 0 given states $(s,p,1)$. It has the usual logit form:

$$\begin{aligned} p_0(s,p,1) &= \frac{\exp(v_0(s,p,1))}{\exp(v_0(s,p,1)) + \exp(v_1(s,p,1))} \\ &= \frac{1}{1 + \exp(v_1(s,p,1))} \\ \implies \log(1 + \exp(v_1(s,p,1))) &= -\log p_0(s,p,1) \end{aligned}$$

We recast the conditional valuation function then as

$$v_1(s,p,l) = u_1(s,p,l) + \beta[-\pi_{ss'}\log(p_0(s',p,1)) - \pi_{ss}\log(p_0(s,p,1)]$$

One can then see that we only need 4 CCPs to estimate the dynamic program - $p_0(0,0,1)$, $p_0(0,1,1)$, $p_0(1,0,1)$, and $p_0(1,1,1)$.

The likelihood has the usual logit form, with the loading on the term associated with individual n, at time t, given state $k = (s,p,l)$ given by

$$L_{nkt} = \mathcal{L}_{nkt}^{\mathbf{1}\{d_{nt}=1\}} \cdot (1-\mathcal{L}_{nkt})^{\mathbf{1}\{d_{nt}=0\}}$$
$$\mathcal{L}_{nkt} = \frac{\exp(v_1(s,p,l))}{1+\exp(v_1(s,p,l)}$$

After estimating CCPs and state transitions in a first stage using bin estimators, results for the parameters on flow payoffs are as follows

Table 1: Question 1 estimates

Parameter	Estimate
α_0	-0.49
α_1	-1.00
α_2	0.31
α_3	-1.58

Problem 2.

Now we need to use the EM algorithm to jointly estimate the pricing process and firm's dynamic program assuming the Permanent State variable is missing. This involves getting expressions for the likelihood assuming the Permanent State is observed at either value (0 or 1) and weighting these by conditional type probabilities.

Note that the pricing process is exogenous to the firm's decision making and that $Y|(s,p,d) \sim \mathcal{N}(\gamma_0 + \gamma_1 \cdot s + \gamma_2 \cdot \cdot p + \gamma_3 \cdot d, \sigma)$, where d is firm's current choice.

This leads us to the expression for our likelihood given state $k = (s,p,l)$:

$$\mathcal{L}_{nkt} = L_{nkt} \cdot \phi \left(\frac{\gamma_0 + \gamma_1 \cdot State_{nt} + \gamma_2 \cdot p + \gamma_3 \cdot d_{nt}}{\sigma} \right)$$

Where ϕ is the Standard Normal's pdf, and L_{nkt} is the likelihood for firm's decision making, as defined before.

The formula's for updating the distribution of $PState$ and conditional type probabilities $q_n s$ are the usual, taken from the slides. We report results considering both methods of updating CCPs - with the data (using a bin estimator but replacing $PState$ with weights $q_n s$) or with the likelihood.

To initialize the algorithm, we need to guess values for α's, γ's, conditional type probabilities q_{ns}, CCPs (same ones in problem 1), and the unobserved heterogeneity's distribution. We are clever and guess values close to the ones taken/estimated using the full data, so the algorigthm should converge using not-clever values.

Results are as follows for both methods:

It appears that the data method of updating CCPs leads to estimates that are closer to the ones obtained under full data, so we shall use this method for the subsequent problems.

Problem 3

Now the transitory state is missing - we once again use the EM algorithm, which requires now the estimation of time-varying conditional type probabilities q_{nst}, as well as estimation of transition probabilities of the state, and its initial distribution, which we assume to not depend on observables in the model.

Table 2: Question 2 estimates

	Estimate	
Parameter	CCP - Data	CCP - Model
α_0	-0.50	0.43
α_1	-1.00	-1.49
α_2	0.30	0.94
α_3	-1.58	-1.57
γ_0	6.97	6.99
γ_1	1.01	1.01
γ_2	-0.25	-0.25
γ_3	-0.71	-0.71
σ	0.99	0.99

Results are as follows:

Table 3: Question 3 estimates

Parameter	Estimate
α_0	-0.48
α_1	-1.01
α_2	0.30
α_3	-1.59
γ_0	6.98
γ_1	1.03
γ_2	-0.29
γ_3	-0.70
σ	0.99

Problem 4

Now we do the estimation in two stages. In the first we use the EM algorithm to esitmate parameters of the pricing process - note that this there are no dynamics here, we do not need to continuously update CCPs, we only compute them using the as a function of the estimated conditional type probabilities.

We use then use those two to maximize the likelihood of associated to the firm's problem, treating the transitory state as observed each period and using q_{nst}'s as weights to obtain estimates for the α's.

Results are below:

Table 4: Question 4 estimates

Parameter	Estimate
α_0	-0.78
α_1	-0.09
α_2	0.28
α_3	-1.51
γ_0	7.15
γ_1	0.98
γ_2	-0.21
γ_3	-0.10
σ	0.99

Note that results are further away from the full data benchmark. This makes sense, since estimation of the conditional type probabilities does not make use the full variation provided by the data, in that the EM algorithm only uses the pricing process to infer conditional type probabilities - this of course contaminates the estimation of firms' dynamic program.