Duke University

Department of Economics

ECON702 — Dynamic Discrete Choice

Assignment 1 - September 2022

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Problem 1.

To ease on notation, we omit the i subscript from our variables. Let $X_j \perp \epsilon_j$ and consider utility's CDF:

$$\begin{aligned} F_{\epsilon_j}(t) &= \mathbb{P}(X\beta_j + \epsilon_j \leq t) \\ &= \mathbb{P}(\epsilon_j \leq t - X\beta_j) \\ &= \exp(-\exp(t - X\beta_j)) \end{aligned}$$

So $u_j = X\beta_j + \epsilon_j \sim EV1(X\beta_j, 1)$

Now we evaluate the distribution of the maximum of utilities:

$$\begin{aligned} F_{\max_j u_j}(t) &= \mathbb{P}(\max_j u_j \leq t) \\ &= \prod_{j=1}^{J} \exp(-\exp(-t - X\beta_j)) \\ &= \exp(-\sum_{j=1}^{J} \exp(-t - X\beta_j)) \\ &= \exp\left(-\exp(-t) \cdot \sum_{j=1}^{J} X\beta_j)\right) \\ &= \exp\left(-\exp(-t) \cdot \exp\left(\log \sum_{j=1}^{J} X\beta_j\right)\right) \\ &= \exp\left(-\exp(-(t - \log \sum_{j=1}^{J} X\beta_j)\right) \end{aligned}$$

So $\max_j u_j \sim EV1(\log \sum_{j=1}^{J} X\beta_j, 1)$

As for the expectation, since maximum utility is just a standard EV1 distribution added to a location parameter - by linearity of expectations $\mathbb{E} \max_j u_j = \gamma + \log \sum_{j=1}^{J} X\beta_j$, since γ is the mean of a standard EV1 random variable.

Problem 2

Throughout this problem we used a step and optimality tolerance of 10^-10. The variables in Z are transformed to $\hat{Z}_{ij} = Z_{ij} - Z_{i1}$. This transformation does not affect the differences in utility but allows to set utility from the first option to 0.

a)

For this part, agent $i's$ utility from alternative j is defined as

$$u_i j = \beta_{j1} X_{1,1,i} + \beta_{j2} X_{1,2,i} + \gamma Z_j + \varepsilon_{ij}$$

β_{01} and β_{02} are normalized to 0.

The unconstrained optimization algorithm converges to the values reported in table 1 after 23 iterations. The result is robust to different initial points.

Table 1

Parameter	Estimate
β_{11}	1.567
β_{12}	0.303
β_{21}	0.632
β_{22}	0.805
γ	-0.365

b)

Utility is defined as above, tolerance levels as well. Results are robust to different initial guesses. The results in table 2 consider as an initial 0.2 for all parameters, and the optimization concludes after 73 steps.

Table 2

Parameter	Estimate
β_{11}	1.443
β_{12}	0.353
β_{21}	0.698
β_{22}	0.733
γ	-0.295
ρ	0.748

c)

For this part, agent $i's$ utility from alternative j is defined as

$$u_i j = \beta_{j1} X_{1,1,i} + \beta_{j2} X_{1,2,i} + \beta_{j3} X_{2,i} + \gamma Z_j + \varepsilon_{ij}$$

β_{01}, β_{02} and β_{03} are normalized to 0. .

The unconstrained optimization algorithm converges to the values reported in table 3 after 36 iterations. The result is robust to different initial points.

d)

Utility is defined as above, tolerance levels as well. Results are robust to different initial guesses. The results in table 4 consider as an initial 0.2 for all parameters, and the optimization concludes after 51 steps.

Table 3

Parameter	Estimate
β_{11}	1.954
β_{12}	0.490
β_{13}	1.446
β_{21}	0.956
β_{22}	0.997
β_{23}	1.456
γ	-0.393

Table 4

Parameter	Estimate
β_{11}	1.946
β_{12}	0.492
β_{13}	1.446
β_{21}	0.959
β_{22}	0.993
β_{23}	1.456
γ	-0.389
ρ	0.987

Problem 3

Throughout this problem, we used a step and optimality tolerance of 10^-10. The variables in Z are transformed to $\hat{Z}_{ij} = Z_{ij} - Z_{i1}$. This transformation does not affect the differences in utility but allows to set utility from the first option to 0.

For this part, agent $i's$ utility from alternative j is defined as

$$u_i j = \beta_{j1} X_{1,1,i} + \beta_{j2} X_{1,2,i} + \gamma Z_j + \varepsilon_{ij}$$

β_{01} and β_{02} are normalized to 0.

Starting from the point $.3 * \mathbf{1}$ (where $\mathbf{1}$ is a vector of ones) the unconstrained optimization algorithm converges to the values reported in the first column of table 5. These estimation results are consistent with the static BST model. However, starting from points that are relatively close to $\mathbf{1}$ such as $0.7 * \mathbf{1}$ the algorithm converges to a different solution reported in the second column. These estimates are not consistent with the static BST model.

The difference between estimates is relatively small for all parameters except the ρ's which dictate the degree of substitution within nests.

Table 5

Parameter	Estimate	
β_{11}	1.985	2.056
β_{12}	0.511	0.500
β_{21}	0.973	1.012
β_{22}	1.040	1.055
β_{31}	2.007	2.081
β_{32}	1.003	1.016
β_{41}	0.967	1.004
β_{42}	0.494	0.483
γ	-0.408	-0.422
ρ_1	0.528	0.846
ρ_2	0.731	1.075
BST consistency	Yes	No