Duke University

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ECON — Dynamic Discrete Choice

Assignment 2 - September 2022

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Problem 1.

a)

We solve the dynamic program by backward recursion and estimate parameters by maximum likelihood. Notation-wise, $delta_j$ is the choice-specific coefficient associated with $X1t$, and γ_{jk} is the choice-specific coefficient associated with each of the time-invariant regressors. Note also that coefficients related to choice 0 are normalized to 0.

b)

Even though we do not have an exclusion restriction, β can still be identified since we have terminal choice and, more importantly, we observe characteristics of individuals who are at the terminal state. So, given that flow parameters are identified and that agents have perfect foresight, variation in when agents choose to go to option 0 identify the discount factor

c)

Consider two agents i, i' with the same sequence of observable characteristics, but different initial choices. Let for instance $Y_{i0} = 1$, $Y_{i'0} = 2$. Assume the δs and γs are identified and are such that $\delta_1 > \delta_2$, and $\gamma_{k1} > \gamma_{k2}$ - that is, option 1 is better for both i, i'. Without switching costs, we should expect $Y_{i1} = Y_{i'1} = 1$. The analyst not seeing that in the data very much implies c is high.

Table 1

Parameter	Estimate
δ_0	0.00
δ_1	0.99
δ_2	0.51
δ_3	0.49
δ_4	-0.99
γ_{10}	0.00
γ_{11}	-1.20
γ_{12}	-1.72
γ_{13}	-1.24
γ_{14}	-1.67
γ_{20}	0.00
γ_{21}	0.42
γ_{22}	0.92
γ_{23}	0.45
γ_{24}	0.92
β	0.96
c	0.43

Problem 2

a)

We estimate these by maximum likelihood, though OLS would be equivalent:

Table 2

Parameter	Estimate
α_0	0.00
α_1	0.50
α_2	0.20
α_3	-0.10
α_4	-0.20
σ	1.00

b)

In this problem, agents do not have perfect foresight therefore in every state, for every decision we need to calculate the expected values over future shocks to X_{1t}.

If we use a recursive function and a finite approximation to the distribution of the normal shocks of size s:

- In every period (10), we need to calculate the conditional valuation of every non-terminal option (4) taking the average across all shocks (s). This means that from one period to another the size of the problem increases by a factor of $10 \times 4 \times s$. This is, the problem grows expontentially with every recursion!!

- There is no straightforward way to store the calculations made at time τ and use them in previous periods since at time τ part of the uncertainty faced in previous periods is resolved which changes the calculations.

To solve the problem we tried two approaches:

- A simple function that loops over states, choices and realizations. This is not a problem for the computer's memory but was too slow.
- A vectorized approach. This was faster but required storing a matrix that grew expontetially in every iteration. At some point the computer was not able to store it.

Without using finite dependence or CCP, we could only solve the problem using a substed of the data. Table 2 reports the results for samples that start from different periods and use different approximations to the normal distribution.

We find that the estimates are robust to the sample (starting period) and the precision of the normal approximation. Even when instead of an approximation we just asume that the shocks are equal to their average of zero, the estimates remain stable (although β increases a little bit) suggesting that perhaps a simplified code with simplifying assumptions could have worked.

Table 3

Starting period	$t = 9$			$t = 7$		$t = 6$	
Normal draws	$s = 15$	$s = 2$	$\epsilon = 0$	$s = 2$	$\epsilon = 0$	$s = 2$	$\epsilon = 0$
Parameter							
γ_{11}	-1.073	-1.077	-1.077	-1.076	-1.080	-1.057	-1.066
γ_{12}	-1.504	-1.506	-1.506	-1.524	-1.531	-1.504	-1.519
γ_{13}	-1.064	-1.066	-1.067	-1.012	-1.016	-1.012	-1.019
γ_{14}	-1.643	-1.641	-1.643	-1.570	-1.573	-1.522	-1.528
γ_{21}	0.455	0.458	0.458	0.467	0.466	0.487	0.486
γ_{22}	0.933	0.935	0.935	0.981	0.980	1.005	1.002
γ_{23}	0.481	0.485	0.484	0.456	0.456	0.471	0.470
γ_{24}	1.182	1.181	1.184	1.034	1.037	0.979	0.983
δ_1	0.228	0.221	0.225	0.207	0.220	0.181	0.212
δ_2	0.092	0.091	0.091	0.077	0.087	0.063	0.090
δ_3	0.185	0.180	0.182	0.164	0.176	0.160	0.187
δ_4	-0.236	-0.239	-0.240	-0.240	-0.229	-0.245	-0.222
c	0.371	0.367	0.371	0.385	0.384	0.390	0.393
β	0.911	0.898	0.908	0.924	0.935	0.908	0.925

c)

β is identified by an exclusion restriction argument - variable X_1 is time invariant and does not affect the state's transition, though it does affect utility. So even though people with the same past choice and $X1t$ have the same probability have the same distribution of states, their flow payoffs are different. This variation identifies β.

Problem 3

a)

Using notation from class, λ is the player's move arrival parameter. It is estimated as a relative frequency - times he was allowed to move divided by time frame. As for nature's transition matrix Q, we get estimates also by taking ratios - q_{10} for instance, is the number of times nature moved out of state 1 divided by the time it spent there.

Table 4

Parameter	Estimate
λ	1.00
q_{10}	0.51
q_{11}	0.49
q_{01}	0.49
q_{00}	0.51

b)

The estimation is similar to a nested fixed point algorithm - in the inner nest we take parameters (flow payoffs u_0, u_1 and switching cost c) as given and solve for the value functions V_0, V_1 using value function iteration. On the outer nest we use the value functions to estimate the three parameters by maximum likelihood.

As for the data itself to input in the likelihood, we only need counts for each truple of incumbency status, action, and state, all of those conditional on nature not moving. The associated choice probabilities would have the logit forms (but, as usual, as a function of the value functions) - in case an outside player decides to enter, we discount the switching cost from the corresponding value function term.

Results for the three parameters are on the table below

Table 5

Parameter	Estimate
u_0	-0.91
u_1	-1.24
c	2.06