Reprinted from RADIOELEMENT ANALYSIS, Proceedings of the 23rd Conference on Analytical Chemistry in Energy Technology, W.S. Lyon (ed.), Ann Arbor Science (Ann Arbor, Michigan: 1980).

TREATMENT AND REPORTING OF UNCERTAINTIES FOR ENVIRONMENTAL RADIATION MEASUREMENTS

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ABSTRACT

Recommendations for a practical and uniform method for treating and reporting uncertainties in environmental radiation measurements data are presented. The method requires that each reported measurement result include the value, a total propagated random uncertainty expressed as the standard deviation, and a combined overall uncertainty. The uncertainty assessment should be based on as nearly a complete assessment as possible and should include every conceivable or likely source of inaccuracy in the result. Guidelines are given for estimating random and systematic uncertainty components, and for propagating and combining them to form an overall uncertainty.

INTRODUCTION

During the past few years, the need for improvements in the quality and usability of environmental radiation data has been emphasized many times. One of the problem areas frequently identified concerned the need for greater uniformity in the methods used to report data, particularly in the treatment and reporting of uncertainties associated with measurement results. Unfortunately, almost all environmental radiation data are presently reported either without any statements of uncertainties, or with unspecified uncertainties, or with uncertainties which are based on incomplete assessments. Further, federal practices pertaining to the reporting of uncertainties not only contribute to these abuses, but are also as diverse as the number of agencies and requirements specified within an agency's regulations.

To address these problems, the National Bureau of Standards (NBS) coordinated an interagency and multi-organizational project with broad national participation. The project was initiated in 1977 at the request of the Task Force on Radiation Measurements of the Conference of Radiation Control Program Directors (CRCPD). At approximately the same time, a Health Physics Society (HPS) ad hoc committee was organized for the purpose of upgrading environmental radiation data. One of the nine project objectives identified by that HPS committee had a scope nearly equivalent to the CRCPD Task Force recommendation. As a result, NBS agreed to coordinate and implement the project under the auspices of the HPS committee and an appropriate subcommittee was formed to serve as a working group.[1] The subcommittee prepared a detailed *Guide and Recommendations* for reporting of environmental radiation measurements data which will be published as an NBS Special Publication.[2] A condensed version of the *Guide* will be included, along with eight other reports, in a publication of the HPS that will present the combined output of the ad hoc committee.[3] Some highlights from that part of the *Guide* concerned with the assessment, propagation and reporting of uncertainties are presented here. Of necessity, this summary must be very brief, and the reader should consult the complete *Guide*[2] for additional detail.

REQUIREMENTS FOR UNCERTAINTY STATEMENTS AND THE COMPLETE UNCERTAINTY ASSESSMENT PROCESS

There are three necessary conditions for the reporting of uncertainties. They are:

i) Every reported value must include an assessment of its uncertainty.
ii) The reported uncertainty must be clearly understood and must convey sufficient information so that its meaning, using correct terminology, is unambiguous.
iii) The reported uncertainty must be based on as nearly complete an assessment as possible.

These requirements should be self evident. A reported value without an accompanying uncertainty statement is for nearly all purposes worthless. Similarly, an unspecified uncertainty statement forces the user to speculate as to its meaning. Lastly, if the uncertainty is meant to be an estimate of the likely accuracy or the likely limits to the absolute error in the measurement result, then the uncertainty must be based on as nearly complete an assessment as possible. The uncertainty assessment must consider every conceivable or likely source of inaccuracy in the result.

A large part of the *Guide* is devoted to this process of making a complete uncertainty assessment of all of the conceivable sources of inaccuracy, and the mechanics of treating the individual components from their initial assessment through their propagation. The philosophical basis for this approach has been described[4] and will not be considered here. Figure 1 attempts to illustrate this complete assessment process, and the division of the sources of inaccuracy between random and systematic components. Every conceivable source of inaccuracy should be classified into one of the two categories depending on how the uncertainty is estimated. Until combined for the measurement result, the individual uncertainty components in both categories should be retailed separately.

The random uncertainty components are statements of precision (or more correctly, of *im*precision) and can be considered to be those sources of inaccuracy which *can be* and *are* assessed and propagated by statistical methods. Sample statistics, such as the standard deviation s, which are computed entirely from the measurement data, are used to estimate the population parameters such as σ. For completeness, the systematic uncertainty components (δ) can be considered to be the residual set of conceivable sources of inaccuracy which are biased and not subject to random fluctuations, and those which may be due to random causes or stochastic process but *cannot be* or *are not* assessed by statistical methods. The systematic uncertainty includes sources of inaccuracy in addition to biases in order to obtain a complete accounting of all sources of inaccuracy in the overall uncertainty. Finally, the overall uncertainty (u) used as an estimate of the inaccuracy is then just some combination of the random and systematic uncertainties.

A third category of conceivable sources of inaccuracy is that termed "blunders" (Figure 1). These are the outright mistakes in the measurement process. If one or more blunders are known to exist, then the value should be discarded, or a correction should be applied to the measurement. One must recognize that blunders do and will occur and an internal system of independent checks should be established for each stage of the measurement and data reduction system to detect and avoid them.

STATISTICAL TREATMENT OF RANDOM UNCERTAINTIES

Each random uncertainty component should be estimated in terms of its standard deviation, or the standard error

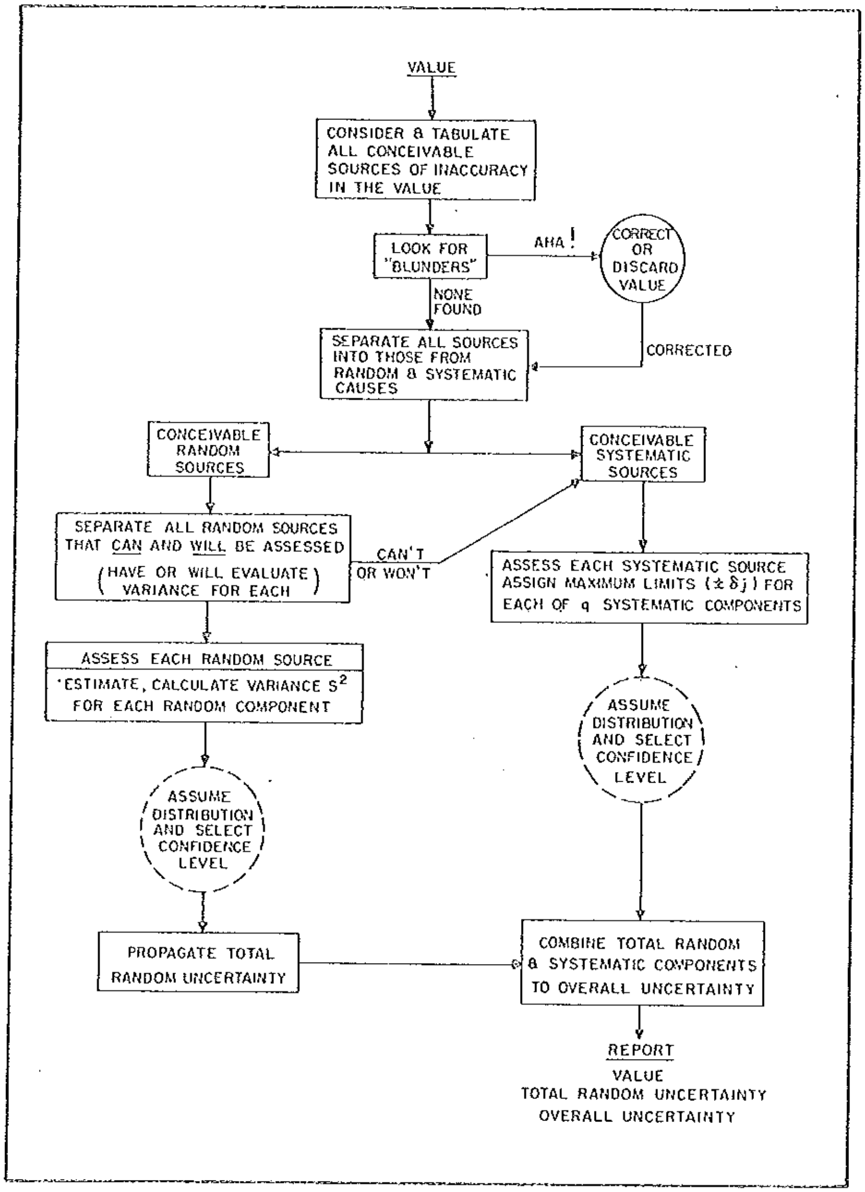

Figure 1. Complete Uncertainty Assessment Process.

of the mean for independent multiple measurements. These components are estimated by a statistical analysis of replicate measurements. The random "counting error" contribution may be estimated from single measurements by making the familiar Poisson assumption. Although the use of the term "counting error" will probably continue to prevail, the term is a gross misnomer. It takes into consideration only the random scatter about the mean from the radioactive decay process itself. To presume that this is the only source of random fluctuation in the overall measurement process is fatuous. Unfortunately, federal guidance, as it pertains to reporting environmental radiation data, does not suggest otherwise. Other sources may be random timing uncertainties, random variations in the sample preparation, positioning of the sample at the detector, etc. The list is nearly endless. Many of these are addressed in a detailed appendix to the *Guide*.

The only way to realistically assess the overall random uncertainty is to make replicate determinations and calculate the standard deviation of the mean by the usual statistical methods. For reported values derived from several component quantities, the individual uncertainty estimates (obtained from replicate measurements for each independent variable must be propagated to obtain the overall standard deviation of the final value. It is recognized that replicate measurements of environmental samples are uncommon and may not be feasible because of time and cost constraints. Yet, an uncertainty assessment based on some procedure beyond calculating the square root of the total number of counts is clearly needed. The *Guide* recommends that the total random uncertainty be obtained by propagating the individual variances for each random uncertainty component. This would include not only the Poisson counting error and random uncertainty derived from the particular measurement under conconcern, but also the random components from corrections, constants, calibration factors and any other measurements that make up the final result. These contributions may be combined by summing in quadrature or by the use of other propagation of error formulae given in the *Guide*. It should be emphasized that one benefit of making a complete assessment is that the process will require the evaluation and statistical control of many previously unevaluated measurement parameters. This will ultimately improve the quality of measurements and aid quality control within laboratories.

ASSESSMENT OF SYSTEMATIC UNCERTAINTIES

A clear distinction between random and systematic uncertainties is often difficult and troublesome. In part, this is because many experimental processes embody both systematic and stochastic (random) elements. The *Guide* contains a number of examples which attempt to clarify the distinction. Although a general guideline for the approach to the assessment can be formulated, there are, unfortunately, no rules to objectively assign a magnitude to the systematic uncertainties. This is a subjective process based mainly on the judgement and experience of the person making the measurement.

In estimating the magnitude of the systematic uncertainties, the *Guide* recommends that they be considered as upper bounds, overall or maximum limits. By what method then should the magnitude of these maximum limits to the systematic uncertainties be assigned? It may be based on a comparison to a standard or verification with two or more independent and reliable measurement methods. Additionally, it may be based on judgement; based on experience; based on intuition; based on other measurements and data; or just a pure guess (which may be better than nothing). Or it may include combinations of, or all of the above factors. The *Guide* also attempts to make this subjective process clearer through the use of examples of typical measurement situations. The fact that a conceivable source of inaccuracy is ascribable to random causes is not a sufficient condition for treating it as a random uncertainty in error reporting. It must be assessable by statistical methods derived from an analysis of repeated measurements. In the absence of these data, the effect of this random cause must be treated as a systematic uncertainty.

APPROACH TO AN OVERALL UNCERTAINTY

As mentioned, one can obtain a combined total random uncertainty stated in terms of some dispersion statistic such as the standard deviation (s). Conceptually, one can propagate and express the total random uncertainty in terms of some confidence limits at any chosen confidence level. This requires a number of assumptions, such as to the underlying population distributions, and can very rapidly become exceedingly complicated requiring computations based on the number of measurements involved in each random uncertainty component, the number of degrees of freedom, the magnitude of the counts in the Poisson counting error contribution, etc. One is therefore confronted with the choice between making simplifying assumptions (which ultimately may provide results that are misleading), or performing complex calculations (whose efficacy, in terms of the time and cost required, can be seriously questioned).

Additionally, one can obtain a list of each conceivable source of systematic uncertainty expressed as a confidence limit ($\pm\delta_j$) which can be at some high (say 99% or greater) confidence level.

Actually, without making assumptions about the distributions of the systematic uncertainties it is impossible to proceed. One of the simplest set of assumptions that can be made is that 1) the component systematic uncertainties are all independent, and 2) they are distributed such that all values within the estimated limits are equally likely (rectangular distribution). This approach, often termed the PTB approach[5], is gaining in popularity. With the above two assumptions, the rectangular systematic uncertainty distributions can be folded together to obtain a combined probability distribution for which the variance may be computed. This may then be combined in quadrature with that for the random uncertainty. In effect, the assumed Normal distribution of the random uncertainty is convoluted with the combined systematic uncertainty distribution to obtain an overall distribution. With this, the overall uncertainty limits at any given confidence level can be evaluated.

RECOMMENDED STATEMENT OF OVERALL UNCERTAINTY AND FINAL RESULT

There is no one clearly superior method out of the range of approaches to reporting an overall uncertainty. Yet, the diversity of current practices and methods either recommended or required by governmental bodies, or used by those who report data make it difficult if not impossible to combine or compare these data in a manner that would be useful for determining trends or possible necessary actions. There is, therefore, a great need for uniformity in the method used to report the uncertainties of environmental radiation data. In hope of achieving greater uniformity, the PTB method is recommended. In its simplest form, the random and systematic uncertainties should be combined to form an overall uncertainty in the result x by

$$u = \sqrt{s^2 + 1/3 \sum_{j=1}^{q} \delta_j^2}$$

where s is the standard deviation corresponding to the total random uncertainty and δ_j is the magnitude of the estimated systematic uncertainty for each of the q systematic uncertainty components. Additional details and other special cases are given in the *Guide*.

The recommended method requires that each reported measurement result include the value, the total random uncertainty expressed as the standard deviation, and a combined overall uncertainty. Examples illustrating the method are given in a detailed appendix to the *Guide*. This recommended approach may not be the best for all purposes. It is intended to be useful and practical without imposing unnecessary burdens on the time, money and personnel resources of laboratories. Adoption of this method may eventually demonstrate its most serious shortcomings and ultimately lead to better methods.

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