COLLÉ
20 JUNE 2018

PUTTING A COVARIANCE INTO AN UNCERTAINTY BUDGET

Assume you have two variables x & y both with determined uncertainties s_x and s_y (i.e., standard deviations by type-A assessments – of necessity)

And further assume that there is an established covariance correlation between x and y [as given by Winkler in his eq(9) as CORR(x,y)]

Then the propagation of uncertainties considering the correlation between x & y is

$$\left[\left(\frac{s_x}{x}\right)^2 + \left(\frac{s_y}{y}\right)^2 - 2\,\mathrm{CORR}(x,y)\left(\frac{s_x}{y}\right)\left(\frac{s_y}{y}\right) \right]^{1/2}$$

N.B. CORR(x,y) CAN BE POSITIVE OR NEGATIVE

IF THE UNCERTAINTY BUDGET WILL BE TREATED BY COMBINING ALL COMPONENTS AS SUM OF SQUARES & THEN MULTIPLYING BY K=2, IT IS ADVISABLE TO USE $\left(\frac{t_{0.95,\nu}}{2}\right)$ AS COEFFICIENT ON ALL $\left(\frac{s_x}{x}\right)$ & $\left(\frac{s_y}{y}\right)$ VALUES FOR LESS THAN 12 DEGREES FREEDOM.

BUDGET TABLE

component	RELATIVE STANDARD Uncertainty
X	S_x/x
Y	S_y/y
X,Y correlation	$\sqrt{2\ \text{CORR}(X,Y)\left(\frac{S_x}{x}\right)\left(\frac{S_y}{y}\right)}$
RELATIVE COMBINED UNCERTAINTY	$\left[\sum u_i^2\right]^{1/2}$
expanded Unc.	$2\left[\sum u_i^2\right]^{1/2}$

OR USE FOR $\nu < 12$

$$\frac{t}{2}\left(\frac{S_x}{x}\right)$$

$$\frac{t}{2}\left(\frac{S_y}{y}\right)$$

$$\frac{t}{2}\sqrt{2\ \text{CORR}(x,Y)\left(\frac{S_x}{x}\right)\left(\frac{S_y}{y}\right)}$$