PAPER

Use of the Harmonic Mean: On Averaging Count-Rate Data for Variable-Time, Fixed-Count Intervals

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The average of count rates from measurements performed by accumulating a fixed total number of counts over a variable time interval should be obtained with a harmonic mean. Use of an arithmetic mean in these cases will give intuitively incorrect results. Uncertainty estimators for this harmonically-averaged counting rate, such as the standard deviation, must also be suitably transformed to correspond to that for the harmonic mean.

As part of a recent collaboration, a colleague had occasion to present me with a summary of results from some very low-level atmospheric ^{222}Rn concentration measurements. An abbreviated version of the findings is summarized in Table 1.[*1+] The six ^{222}Rn assays given in Table 1 were intended to be "averaged", and were based on count-rate measurements for fixed-count intervals, i.e. by measurement of the variable time intervals t_i required to accumulate a given, predetermined, total number of counts C, which in this case was $C = 5000$. The assays were performed in this way to achieve uniform statistical "counting errors" (and thereby equal weights) on each count-rate determination.[*2]

I should like to use the data and analyses of Table 1 to illustrate the blind misapplication of the arithmetic mean for the counting data obtained from such an experimental design. The example and supporting discourse may be of value to other workers who may use or may have previously used a similar type of variable-time counting situation. Parenthetically, one may note that this experimental condition can be contrasted with the more common procedure used for replicate count-rate measurements in which C is variable and t is fixed. Considering that almost every automated counting instrument used for radionuclidic metrology (e.g., multichannel analyzers, liquid scintillation counters, etc.) has a feature that allows counting for a variable-time, fixed-count interval, I almost hate to speculate how often this misapplication of the arithmetic mean may be occurring in our discipline.

Examination of Table 1 reveals that the count rates R_i of the six processed samples were simply obtained from $R_i = C/t_i$. The count rates were in turn converted to activity concentrations A_i by application of several normalizing factors: a measured sample volume V_i; an independently determined ^{222}Rn calibration factor ϵ_i (in units of detected counts per second per Bq of ^{222}Rn) which includes both detection efficiency and sample handling (processing and transfer) efficiency; and a decay correction to a common reference time (i.e., $D_i = \exp(-\lambda T_i)$ where λ is the ^{222}Rn decay constant and T_i is the time interval between the reference time and the midpoint of the counting time period). Hence, $A_i = R_i/V_i\epsilon_i D_i$. The six A_i values were then averaged in the usual way, yielding an estimated mean activity concentration $A_a = 0.3311$ Bq·m^{-3} and standard deviation of the mean $s_m(A_a) = 0.0151$ Bq·m^{-3}. On first appearances, the treatment appears to be quite reasonable. Nevertheless, it is not. The analysis is incorrect in the sense that the arithmetic mean A_a that was obtained gives an intuitively incorrect estimate of the mean activity A for reasons that will be given below. A "correct" estimate of mean A would be the *harmonic mean* A_h which is defined as the reciprocal of the arithmetic mean of the reciprocals of A_i, i.e. $A_h = n/\Sigma_i(1/A_i)$ for n determinations of A_i (and where each A_i value must be positive).[*3] The reason A_h is considered to be a correct estimate (while A_a is not) arises from the fact that A_i is in an inverse proportional (or reciprocal) relation with the dependent variable t_i. For the example of Table 1, the harmonic mean is $A_h = 0.3275$ Bq·m^{-3} with a corresponding standard deviation of the mean of $s_m(A_h) = 0.0154$ Bq·m^{-3}. The derivation of $s_m(A_h)$ will be shown later. These estimates may be compared to the tabulated arithmetic mean of $A_a = 0.3311$ Bq·m^{-3} and $s_m(A_a) = 0.0151$ Bq·m^{-3} which

+ *All notes, sequentially numbered, are given as supplementary endnotes.

Total accumulated counts C	Measured counting time interval t_i(s)	Count rate R_i (s^{-1})	Sample volume V_i (m^3)	Calibration factor ε_i	Decay correction D_i	Activity concentration A_i ($Bq \cdot m^{-3}$)
5000	20147	0.2482	0.3033	2.541	0.8352	0.3856
5000	36553	0.1368	0.2534	2.434	0.8076	0.2746
5000	26018	0.1922	0.2667	2.591	0.7891	0.3524
5000	25777	0.1940	0.2940	2.588	0.7682	0.3319
5000	29794	0.1678	0.2659	2.590	0.7519	0.3241
5000	30319	0.1649	0.2834	2.497	0.7334	0.3178
					mean (arithmetic) =	0.3311
					standard deviation of the mean =	0.0151

Table 1 Typical results for assays of atmospheric ^{222}Rn concentrations as obtained from count-rate measurements using fixed-count, variable-time intervals and which illustrate the misapplication of the arithmetic mean for this experimental design.

were originally given (Table 1). The small 1% difference between A_h and A_a may appear to be insignificant, particularly in regard to the variability in the A_i values as reflected in the $s_m(A_a)$ and $s_m(A_h)$ estimators. The difference however, irrespective of its magnitude, is just a needless introduction of an error[*4] that arises from use of an inappropriate measurement analysis model. Although its magnitude may appear to be somewhat trivial in this case, it may be quite important for other high-level metrology applications where overall uncertainties are a few tenths of one percent at one standard deviation.

The simplest way to demonstrate that the above arithmetic average leads to an incorrect result is to use a simpler example that does not require any normalizations or corrections for sample sizes, efficiencies, or decay. Imagine the following hypothetical experiment. A sample of a very long-lived radionuclide having a "true" activity of exactly 1 Bq is sequentially counted four times. Each count is performed for a variable time t_i that results in the accumulation of a constant number of counts $C = 100$. Assume, because it is long lived, that there is negligible radioactive decay over the entire period. Suppose further that the detection efficiency is 100%, so that one obtains one detected count for every disintegration. The results for this hypothetical experiment could very well look something like the following:

$C =$	100	$t_i =$	107 s
	100		96 s
	100		102 s
	100		84 s

The four t_i values given here are representative of what one might obtain by random selection from a Poisson process having a population mean activity of $\mu = 1$ Bq (i.e., 100/100 s). The variability in the t_i values is just that from selecting four random values from this Poisson process.

A "correct" measure of central tendency for the estimated activity in this case could be defined as the total number of disintegrations (or counts) divided by the total time: $A = 400/(389\ s) \simeq 1.028\ s^{-1}$. It seems intuitive that this approach, which obviously utilizes all four data sets, should lead to a correct result. On doing the algebra to avoid any possibility of rounding errors, we can see that the arithmetic mean of the estimated activity is

$$A_a = [(100/107) + (100/96) + (100/102) + (100/84)]/4 = 364989600/4(88010496) = 1267325/1222368 \simeq 1.037\ \mathrm{Bq}$$

which differs from the intuitively "correct" A; whereas the harmonic mean

$$A_h = 4/[1/(100/107) + 1/(100/96) + 1/(100/102) + 1/(100/84)] = 4/(389/100) = 400/389 \simeq 1.028\ \mathrm{Bq}$$

leads to an identical result for A.

Some elementary texts on statistics (cf., references 1,4-6) and popularizations of mathematics (e.g., reference 7) give similar examples on the appropriateness of the harmonic mean in specific types of problems.[*5] For illustrative purposes, the most well-known academic exercises, as now invoked, involve the computation of an average velocity for an automobile[5,6] or an airplane.[7] One such example, revised from that given by Freund,[5] involves determining the average velocity of an automobile that has traveled the first 100 km of a 200-km trip at 100 $km \cdot h^{-1}$ and the second 100 km at 50 $km \cdot h^{-1}$. Initially, and unthinkingly, one might suppose that the average velocity is just the simple arithmetic mean of 100 $km \cdot h^{-1}$ and 50 $km \cdot h^{-1}$, i.e. 75 $km \cdot h^{-1}$. Average velocity in this situation, however, would normally (and intuitively) be defined as the total trip distance divided by the total time.[*6] Here the total distance is 200 km, and the total time is 3 h (1 h for the first leg plus 2 h for the second), which yields an average velocity of 66 2/3 $km \cdot h^{-1}$ rather than the 75 $km \cdot h^{-1}$ obtained from the arithmetic mean. This average velocity could have been determined directly obtained (without calculating the total distance and total time) from the harmonic mean of the two velocities. That is, 2 divided by the sum (1/100 + 1/50). Curiously, the wholly analogous type of averaging problem as it applies to count-rate determinations with variable-time, fixed-count intervals is not treated in any of the most common radioactivity texts (cf., references 9-11) or the authoritative references on radioactivity measurements,[12-17] many of which have extensive sections on the statistical treatment of experimental counting data.

Now that I, hopefully, have convinced the reader that the harmonic mean is a more appropriate measure of central tendency for these variable-time, fixed-count counting situations, we must turn to addressing the issue of selecting an appropriate measure of dispersion (i.e., an uncertainty estimator on the precision or variability in the data). A sample variance s^2 (or standard deviation s) as it would apply for the harmonic mean is an obvious choice.[*7]

A convenient way to evaluate s^2 would be to perform a simple reciprocal transformation of the count-rate data, and then to evaluate the sample variance on the transformed data in the usual way. The variance s^2 may then be approximated from the calculated variance on the transformed data using the familiar "Propagation of Uncertainty" law.[18,19] To wit, for a random variable x having data values x_i ($i = 1,2,3\ldots,n$) and a harmonic mean of $H_x = n/[\Sigma_i(1/x_i)]$, it can demonstrated that: (1) with a variable transformation of $y = 1/x$, the arithmetic mean of y (i.e., $A_y = \Sigma_i y_i/n$) becomes equal to the reciprocal of the harmonic mean of x, $H_x = n/\Sigma_i y_i = 1/A_y$; and (2) the calculated sample variance on A_y, given by $s^2[A_y] = \Sigma_i(y_i - A_y)^2/(n-1)$, can be used to approximate $s^2[H_x]$ through use of the "propagation of uncertainty" relation $s^2[H_x] \simeq (\partial x/\partial y)^2 s^2[A_y] + \ldots$ where the partial derivative is evaluated at the central locations of x and y, i.e. $\partial x/\partial y = 1/y^2 = A_y^2$. Therefore, we have for the sample variance and standard deviation $s^2[H_x] \simeq (1/A_y^4)s^2[A_y]$, and $s[H_x] \simeq (1/A_y^2)s[A_y]$. The standard deviation of the mean is then $s_m[H_x] = s[H_x]/\sqrt{n} \simeq (1/A_y^2)s[A_y]/\sqrt{n} = (1/A_y^2)s_m[A_y]$ which on a fractional (coefficient of variation v_m) basis is $v_m[H_x] = s_m[H_x]/H_x \simeq (1/A_y^2)(1/H_x)s_m[A_y] \simeq (1/A_yH_x)v_m[A_y] = v_m[A_y]$. This approach has the advantage of using A_y and $s[A_y]$, which may be directly calculated from the usual arithmetic operations, to derive $H_x = 1/A_y$ and $s[H_x] = (1/A_y^2)s[A_y]$. The expression given above for $s^2[H_x]$ is based on a first-order Taylor series approximation of the functional form,[18] and therefore is a valid approximation only if the higher order terms in the Taylor series are negligible. In general, this requires that the partial derivative when evaluated at the mean values is small; that the relative magnitudes of $s_m[A_y]$ and $s_m[H_x]$ compared to A_y and A_x (i.e., $v_m[A_y]$ and $v_m[H_x]$ are small; and that the second- and higher-order partial derivatives, as evaluated at the means, do not give rise to significantly large values. The approximation for the reciprocal transformation used here should be sufficiently accurate for almost any conceivable precision in $v_m[A_y]$ and $v_m[H_x]$ since powers of the mean appear in the denominators of the partial derivatives.

For the activity concentration data given in Table 1, the calculated mean and standard deviation of the mean for the reciprocal transformation were $A_y = 3.0531\ \mathrm{m^3{\cdot}Bq^{-1}}$ and $s_m[A_y] = 0.1432\ \mathrm{m^3{\cdot}Bq^{-1}}$ which yields a harmonic mean of $H_x = 0.3275\ \mathrm{Bq{\cdot}m^{-3}}$ (for A_h) and standard deviation of $s[H_x] = 0.0154\ \mathrm{Bq{\cdot}m^{-3}}$ (for $s_m[A_h]$). This uncertainty estimator may be directly combined with those from other uncertainty components to obtain a propagated, or overall uncertainty on the measurement. For example, the derived $s_m[A_h]$ from the data of Table 1 would be combined with the "standard uncertainty"[19] estimates for the sample volume $u[V]$, calibration factor $u[\in]$, and decay correction $u[D]$, as well as with other components (such as those due to counting system stability and linearity, dead-time corrections, live-time determination, etc.) to arrive at the combined standard uncertainty on A: e.g., $u[A] \simeq (s_m^2[A_h] + u^2[V] + u^2[\in] + u^2[D] + \ldots)^{1/2}$.

In conclusion, it should be emphasized that the sample harmonic mean H_R is an appropriate measure of central tendency for count rate $R = C/t$ determinations which are based on replicate measurements with variable times t_i to accumulate a fixed total number of counts C. In these situations, its use results in a more intuitively "correct" mean than that obtained by arithmetic averaging. Its use would not be appropriate in the usual cases when the counting time interval t is fixed and the accumulated counts C_i is variable (or perhaps when both C_i and t_i are variable). The harmonic mean may be subsequently treated just like one treats the arithmetic (or any other) mean, such as in combining it with other measured quantities (like a detection efficiency) to derive subsequent quantities of interest (like an activity). Derivation of appropriate, corresponding measures of dispersion about the harmonic mean, such as the derived standard deviation of the mean $s_m[H_R]$ are straightforward. These harmonic mean dispersion estimators may be combined in the usual way with other uncertainty component estimators to obtain a propagated, combined standard uncertainty for the measurement.

One last point may be noted for emphasis. The harmonic mean is always a more intuitively appropriate measure of central tendency when averaging data for which the variable is in an inverse (i.e., reciprocal) relationship with the quantity of interest. This is true whether one is dealing with a radionuclidic counting rate obtained with variable times, or with a "price index" rate for variable money, or any other kind of rate quantity. It is not true however, as has been suggested in a few mistaken texts on statistics, that the harmonic mean is appropriate for any rate average. It is only appropriate when the variable appears in the denominator of the rate.

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Endnotes

***1**

The data and analysis presented here in Table 1 has been simplified for illustrative purposes. For example, the necessary background subtraction corrections, which just makes this illustration more convoluted, are not given in this presentation. Similarly, the details of the exact experimental methodology used for these measurements (e.g., on the sample collection and measurement of sample volume, on the sample processing, on the detectors and their calibration, etc.) are not terribly relevant to this exposition. Rather, I should merely like to draw attention to the data analysis as originally invoked.

***2**

The term "counting error", a misnomer in several ways, arises from the assumption that the radioactive decay process itself (but *not* necessarily the counting process) is Poisson distributed and that its mean and variance are equal, i.e., the variance (C) is equal to the mean number of total observed counts (C).

***3**

The arithmetic and harmonic means are only two of an inexhaustible number of different types of averages. A generalized definition for many, but not all possible ones, can be given by $m_b = (\Sigma_i x_i^b / n)^{1/b}$ for a series of n positive values of x_i. Hence, different means are obtained as b takes on different values. With $b = 1$, $m_{b=1}$ yields the familiar arithmetic mean. The harmonic mean is obtained with $b = -1$. With $b = 2$, $m_{b=2}$ is what is termed the "quadratic" mean. With $b = 0$, the above expression for $m_{b=0}$ yields an indeterminate form, which has been shown[1] to be the "geometric mean" $m_{b=0} = (\Pi_i x_i)^{1/n}$. And so on, through all possible values of b. It is hard to imagine, however, that quantities like the $m_{b=5}$ quintic mean can be of much practical use! It may also be of some historical interest to note that the concept of various means, at least in terms of equivalent geometrical representations, is at least 2500 years old. Fragments of the writings of Eudemus (ca. 335 BC), a pupil of Aristotle, have been preserved in the *Eudemian Summary* of Procolus (410 - 485 AD), and clearly indicate that three means were known in the time of Pythagoras (ca. 540 BC): the "arithmetic", the "geometric", and the "subcontrary". The last was later renamed to "harmonic mean" by Archytas (b. ca. 428 BC) and Hippasus (ca. 470 BC) because of ascribed mystical connotations for various numbers which were important for the numerologically-based philosophic speculations of the day. The Pythagorean definition of this "harmonic mean" devolved from the joint conditional that (for two positive numbers a and b) the harmonic mean H between a and b (with $a > b$) satisfies $a = H + a/m$ and $b = H \div b/m$ if a number m exists [n.b., the second term signs are reversed if $b > a$). The curious relationship between a and b and the two means A and H, $a{:}A = H{:}b$, was known as the "musical" proportion. From geometrical considerations it was known that, for the two positive numbers a and b, the arithmetic mean was $A = (a + b)/2$, the geometric mean was $G = \sqrt{ab}$, and the harmonic mean was $H = 2ab/(a+b)$. It was also known that $A \geq G \geq H$ where the equalities hold if and only if $a = b$. From whence comes our presently held generality for positive-valued discrete data sets:

arithmetic mean > geometric mean > harmonic mean.

For the set of the three means as calculated with the same data, the geometric is also equal to the geometric mean of the other two means. This relation is true, however, only in the case of averaging two values. Both the early *Elements* of Euclid (ca. 320 BC) and the much later, exhaustive *Mathematical Collection* of Pappus (300 AD) contain a number of such interesting problems on the properties of the various means. These historical details on the Greek means are treated briefly by Eves,[2] and more thoroughly by Heath.[3] Returning to our generalized expression for the mean, it is clear that m_b yields the smallest mean when $b = -\infty$, and the largest measure when $b = \infty$. In all cases, m_b increases as b increases from $-\infty$ to ∞, which proves that the harmonic mean ($m_{b=-1}$) is less than the geometric mean ($m_{b=0}$) which is less than the arithmetic mean ($m_{b=1}$) which is less than the quadratic mean ($m_{b=2}$), etc. Examination of the generalized expression also proves that all of the means given by m_b are internal means (i.e., they fall within the range of the x_i values in the series). One should note for completeness at this point that other "measures of central tendency" (i.e., a value used to locate some center of the distribution of x_i values), beyond the means defined by m_b, exist, such as the "median" (i.e., the middle value in the distribution) and the "mode" (the value which occurs most probably or most frequently in the distribution).

***4**

The word error as used here does not refer to an estimated measurement uncertainty, but rather is used in the context of a difference from a "true" or canonically known value.

***5**

Interestingly, of the seemingly innumerable number of elementary, university-level statistical texts, only a small minority make any mention of the harmonic mean at all. Of those that do, they typically merely define it with scant consideration of its use beyond the obvious kind of example (a small percentage of which are clearly wrong). Invariably, the older texts (e.g., references 1,4) provide more thorough treatments. In practicality, the harmonic mean is now most widely employed for the averaging of prices and costs for the construction of index numbers, e.g., the "Price Index"[8] or "Cost of Living Index".[7] It has also been used for other types of averages of the quantities of various commodities that can be obtained for fixed money,[1,4,7] as well as for "work limit" measures (i.e., the time required to perform a given, fixed quantity of work) in psychological tests and in industrial applications.

***6**

It must be emphasized that there is nothing inherently absolute about the choice of any measure of central tendency for any given situation. In any of the examples given previously one could have, for example, arbitrarily chosen to have calculated and reported geometric means or more robust medians. The choice of a particular measure of central tendency (or measure of dispersion) should be based on its reasonableness and usefulness for the situation. This is largely what is meant by intuitive "correctness". The arithmetic mean and sample variance for the arithmetic mean are so widely employed because of their unique properties when applied to normal distributions (e.g., that they are unbiased and efficient estimators of their corresponding population parameters). As such, these estimators are reasonable choices in most (but not all) cases, have great utility, and can be considered to lead to intuitively "correct" results. The use of the harmonic mean for the examples given here are justified on this same basis. Justifications for the use of the harmonic mean as opposed to an arithmetic mean in certain specific situations have been more fully treated by Kelley[1] and Ferger.[8] Ferger also gave some harsh criticisms of the treatment given to the harmonic mean in the extant statistical texts of the day. Despite the passage of 64 years, textbook treatments have not improved.

*7

Guidance in university-level texts on statistics for obtaining a measure of dispersion when using the harmonic mean is virtually nonexistent. Almost unbelievably, of over 30 statistical texts (out of a total of nearly 200) that make some reference to the existence of the harmonic mean and that have been cursorily consulted, only one of them[1] addresses how to place a confidence interval on the harmonic mean. The approach suggested by Kelley[1] is in essence equivalent to that given here. A more rigorous approach that would derive a closed-form sample variance estimator $s^2[H_x]$ for this case would be desirable, but is beyond the scope of this report. It must be emphasized here that one should *not* calculate a kind-of-equivalent sample variance from the harmonic mean residuals [i.e., from $\Sigma_i(x_i - H_x)^2/(n-1)$]. One might mistakenly suppose that this expression would be suitably analogous to that used for the arithmetic mean, and as valid. It is not since the expectation value $E[\Sigma_i(x_i - H_x)^2/(n-1)]$ in this case is not an unbiased estimator of the population variance (second central moment) of x_i about H_x even if one assumes that the expectation value $E[x_i]$ of x_i is equal to the population harmonic mean of the x_i values.

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