Problem Set 1: Fundamentals of Programming

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Exercise 1. Arrays and matrix operations (20 points). Given the list from the previous exercise

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l1 = [2, 5, 6, 4, 5 ,9, 3, 2, 2]
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a. Convert the list into a 3×3 matrix (2-d array). The outcome should be the matrix
   $A = \begin{bmatrix} 2 & 5 & 6 \\ 4 & 5 & 9 \\ 3 & 2 & 2 \end{bmatrix}$
b. Find the maximum of the matrix A. Find the index of the maximum.
c. Transpose the matrix$A$.
d. Squared the matrix (i.e. AA'). Also, raise to the power of 2 all the elements in matrix A.
e. Compute the eigenvalues of matrix A.
f. Multiply matrix A by a matrix of zeros (f1), by the identity matrix (f2),and by a matrix of ones (f3).
g. Create an even grid from 1 to 9 with 9 elements. Convert the grid into a 3x3 matrix called B. Multiply matrix A by matrix B.

Exercise 2. Conditional statements and operations (20 points). Given the list from exercise 1

```
l1 = [2, 5, 6, 4, 5 ,9, 3, 2, 2]
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a. Create a new list that contains only the elements in list l1 that are smaller than 5.
b. Create a new list that contains only the elements in list l1 bigger or equal than 3 and smaller than 7.
c. Given matrix A from exercise 2, write a code that checks whether 5 belongs to A.
d. Create a new matrix B that is equal to matrix A but where numbers below 4 are replaced by zeroz.
e. Write a code that counts the number of zeros in matrix B.

Exercise 3. CES production function and comparative statistics (30 points). The Constant Elasticity of Substitution production function is a commonly used production function that takes the following form

$$Y = F(K, L) = A\left(\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

Where K and L are the capital and labor factors. A represents the total factor productivity, α is the capital's share, and σ is the constant elasticity of substitution between the two production factors. A, α, σ are strictly positive. It can be shown (using L'Hôpital's rule) that when $\sigma \to 1$ the CES production function boils down to a Cobb-Douglas production function

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha}$$

a. Create a function that given the arguments K, L, A, α, σ, returns output Y. You can have inside the function an if statement for when $\sigma = 1$ the output Y comes from Cobb-Douglass production function, else from the CES function.

From now on work with the following parameterization: $A = 1.5, \alpha = 0.33$.

b. **Cobb-Douglass production function**. First consider the Cobb-Douglass case with $\sigma = 1$. Compute output Y for an even-spaced grid of K, $G_k = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and a fixed $L = 3$. Plot the resuls—make an x-y plot with the grid of K in the x axis and output Y in the axis y.

c. From b, recompute output Y for the 3 cases $\alpha = 0.25, \alpha = 0.5, \alpha = 0.75$. Make an $x - y$ plot with the 3 production functions in the same graph.

d. **CES production function.** Redo exercise b but for $\sigma = 0.33$.

e. Keeping $\alpha = 0.33$, plot output Y vs the grid of capital for the cases of $\sigma = 0.25, \sigma = 0.5, \sigma = 1, \sigma = 2, \sigma = 4$.

f. How does output Y changes along K for the different σ specifications? Can you provide the economic interpretation? **Hint:** σ captures the relative degree of substitutability/complementarity between the two inputs K, L.

Exercise 4. Transitions in the Solow model of growth (30 points) Consider a basic Solow model with a Cobb-Douglass production function $Y_t(K_t, L_t) = AK_t^{\alpha} L_t^{1-\alpha}$, no population growth, and depreciation rate δ. The capital per worker law of motion is

$$k_{t+1} = sAk_t^{\alpha} + (1 - \delta)k_t \tag{1}$$

The parameter values are $\alpha = 0.3$, $\delta = 0.7$, $A = 4$, $s = 0.4$.

a. What is the steady state capital per worker and output per worker in the economy?

b. Suppose the economy in the first three periods, $t = 0$ to $t = 2$, is in steady state. Then, at period $t = 3$ the economy experiences a permanent negative shock such that A decreases by 25 percent. Compute and plot the levels of capital per worker and output per worker from period $t = 0$ till the economy reaches the new steady state. Use a tolerance level of $\varepsilon = 0.5$ to find convergence to the new steady state.

c. Consider the economy is in the original steady state with $A = 4$ from period $t = 0$ to period $t = 2$. Then, from period $t = 3$ to $t = 10$ the economy experiences a temporary shock that leads the saving rates up to $s = 0.6$ and after period $t = 10$ the saving rate goes back to $s = 0.4$. Compute and plot the output per worker level from period $t = 0$ to period $t = 100$. Explain how a temporary change in the savings rate affects output.