Problem Set 8: Simulation/Estimation

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Problem 1. Consider the same problem as we worked on in Problem 2 from PS7. That is, suppose we have an infinitely lived worker with assets a and an income y who is saving for the future. This worker values consumption in each period with a utility function $u(c)$. They can save at a risk-free rate r, but cannot borrow, and they discount the future at a rate β. Their income is a random variable. In particular, income tomorrow follows a log-normal AR(1) process:

$$\log(y') = \rho \log(y) + \epsilon$$

where $\epsilon \sim N(0, \sigma_\epsilon)$. Their problem can be written recursively as:

$$\begin{aligned} v(a,y) = \max_{c,a'} \quad & u(c) + \beta \mathbb{E}\left[v(a',y') \mid y\right] \\ \text{s.t.} \quad & c + a' \leq (1+r)a + y \\ & a' \geq 0 \\ & \log(y') \sim N(\rho \log(y), \sigma_\epsilon) \end{aligned}$$

For this problem, set $\beta = 0.95$, $\rho = 0.8$, $\sigma_\epsilon = 0.1$, and $u(c) = c^{1-\sigma}/(1-\sigma)$ for $\sigma = 2$

Start with an interest rate of $r = 0.03$ and solve this model, like last week. We know that in a model without borrowing constraints, the consumption decisions of workers would satisfy the Euler equation

$$u'(c) = \beta(1+r)\mathbb{E}[u'(c')]$$

where c' is the value of consumption that they would choose tomorrow.

(a) Does the Euler equation hold at all of the points on your asset/income grids? Try plotting the residuals of the Euler equation (LHS minus RHS). You'll want to pick a representative set of income values, and plot the slices along the asset grid. If it fails to hold everywhere, can you say something about where agents are off their Euler equation? What is the intuition here?

(b) Try solving the model, and simulating the distribution of assets for 4 different interest rates: $r = 0.01, 0.02, 0.03, 0.04$. How does the average level of assets vary with the interest rate? What's the intuition here?

(c) Suppose I told you that in the actual economy, we have average asset holdings of 5.1. What is the interest rate (holding all of the other parameters constant) which causes the model to match this moment of the data?[1]

(d) Based on our last lecture, is this really the right way to think about the interest rates? How would you choose parameters of this model to match the asset holdings of the economy if you had an infinite computational budget (i.e, if you weren't concerned about how long it would take)?

[1] Warning: Depending on how fast your solution algorithm is, this problem could take a long time to run.
You will need to simulate a large number of agents (probably a few hundred thousand or a million).
You should set a very loose tolerance for your solution of the interest rate. If you find the interest rate within a factor of 10^{-3} or 10^{-4} that should be good enough. You'll struggle to do better than this because of simulation noise.