Evaluating High-dimensional Surrogate Markers of Vaccine Response through Causal Mediation Analysis

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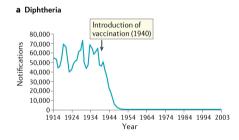






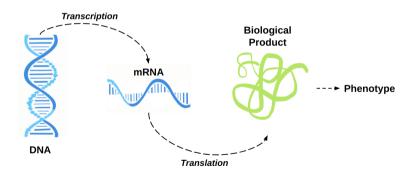
Vaccination

- Exploit immune system
- Most cost-effective measure in public health
 - 154 million lives saved in last 50 years
 - 4-5 million lives saved per year
- Historically developed empirically

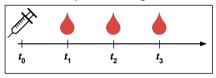


Transcriptomics

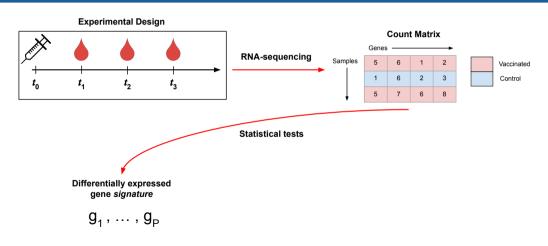
• **Gene expression**: Genes → Product

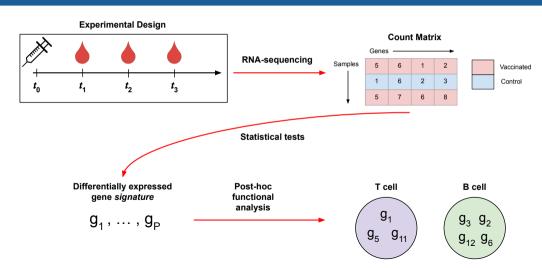


Experimental Design









Huge potential:

Huge potential:



Holistic view of system

Huge potential:



Holistic view of system



Reveal vaccine mechanisms

Huge potential:



Holistic view of system



Reveal vaccine mechanisms



Observed early

Huge potential:



Holistic view of system



Reveal vaccine mechanisms



Observed early

But challenges with high-dimensionality...

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Interpretability

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Sensitivity to investigator choices

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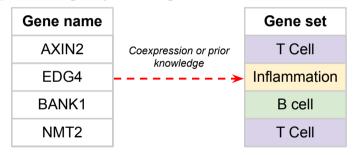


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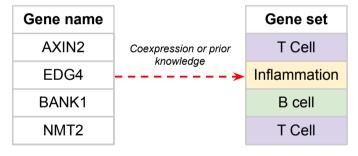


Low signal-noise ratio

Investigate groups of biologically related genes



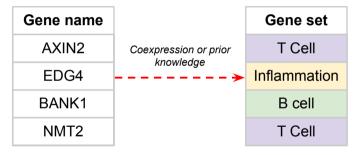
Investigate groups of biologically related genes





Reduced dimensionality

Investigate groups of biologically related genes



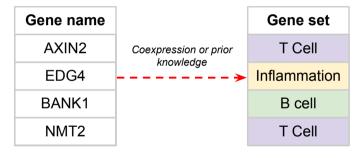


Reduced dimensionality



Biological interpretability

Investigate groups of biologically related genes





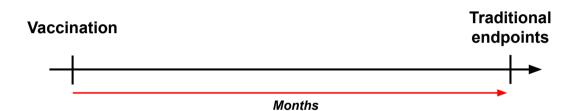
Reduced dimensionality



Biological interpretability



Boost signal







Accelerating validation of candidate vaccines



Accelerating validation of candidate vaccines



Protection of at-risk groups



Accelerating validation of candidate vaccines



Protection of at-risk groups



Mechanistic inference



Accelerating validation of candidate vaccines



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Difficulties in vaccine RCT context:



High-dimensional



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Small sample size



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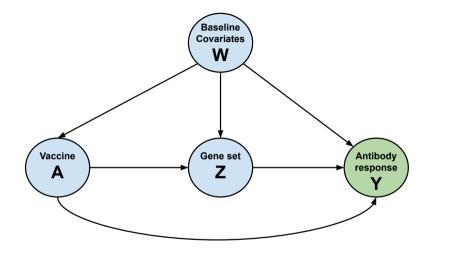
Complex data structures

Evaluating Gene Set Signals as Surrogates

Goal: Ranked list of gene sets by proportion of treatment effect explained

Gene Set	PTE
Z ₁	R ₁
Z ₂	R ₂
Z_3	R_3

where $R_1 > R_2 > R_3 > ...$



- Observed data consists of n i.i.d copies of O = (W, A, Z, Y) where
 - W baseline covariates (e.g. age, sex...)
 - $A \in \{0,1\}$ vaccine indicator
 - Z m genes in the same biological pathway (m > n)
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Define potential outcomes

• Y(a) : Response had treatment been A=a

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- ullet $Y(a,Z(a^\prime))$ response had treatment been A=a and mediators under $A=a^\prime$

Definition of effects

• Total Effect = $\Delta := \mathbb{E}\{Y(1) - Y(0)\}$

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Decompose total effect :

$$\underbrace{\mathbb{E}(Y(1)-Y(0))}_{\text{Total Effect}} = \underbrace{\mathbb{E}(Y(1,Z(0))-Y(0,Z(0))}_{\text{Natural direct effect}} \ + \ \underbrace{\mathbb{E}(Y(1,Z(1))-Y(1,Z(0)))}_{\text{Natural indirect effect}}$$

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Define proportion of treatment effect explained as

$$R_S := \frac{NIE}{\text{Total Effect}} = 1 - \frac{NDE}{\text{Total Effect}}$$

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Optimises bias-variance tradeoff for $\mathbb{E}\{Y|W,A\}$ - not the causal parameter of interest

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Method 2 : Inverse probability weighting



Create comparable treatment groups w.r.t ${\it W}$

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Not robust to sparsity





Target causal parameter of interest to reduce bias

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Optimises bias-variance tradeoff for causal parameter of interest



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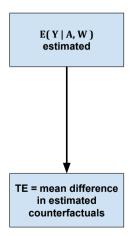
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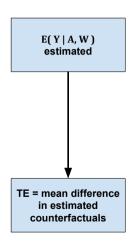
Doubly robust - consistent if $\widehat{\mathbb{E}}\{Y|W,A\}$ or $\widehat{\mathbb{P}}(A=1|W)$ consistently estimated

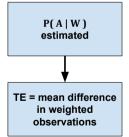
G-Computation



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Inverse Probability Weighting

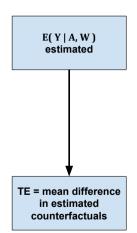


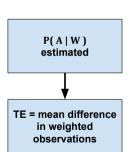


G-Computation

Inverse Probability Weighting

Targeted MLE





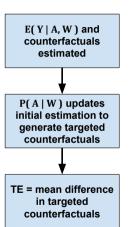


Figure adapted from Schuler and Rose, 2016



 $\mathbb{E}\{Y|W,A\}$ and $\mathbb{P}(A=1|W)$ are **complex**



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Estimate with ensemble machine learning



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Super-Learning

1. Choose m algorithms a priori



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Estimate with ensemble machine learning

- 1. Choose m algorithms a priori
- 2. Use cross-validation to estimate each algorithm performance



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- Initial estimates $\mathbb{E}\{\widehat{Y}|\widehat{W},A\}$ and $\widehat{q}_a=\mathbb{P}(\widehat{A=a}|W)$

TMLE Step 2 : Targeting step

1. Calculate Auxiliary covariate $H_a(A,W)=rac{\mathbb{1}(A=1)}{\widehat{g_1}}-rac{\mathbb{1}(A=0)}{\widehat{g_0}}$

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$$Y = \widehat{Y} + \delta H_a$$

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3. Targeted potential outcomes ⇒

$$\widehat{Y^*(1)} = \widehat{Y(1)} + \widehat{\delta} H_1$$
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Estimating the Natural Direct Effect

Goal: estimate the natural direct effect from n copies of O = (W, A, Z, Y). Let

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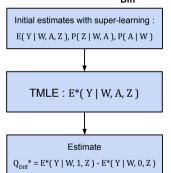
•
$$Q_Y(W, A, Z) = \mathbb{E}(Y|W, A, Z)$$

$$\text{Define NDE} = \mathbb{E}_W \{ \mathbb{E}_Z [\underbrace{Q_Y(W,1,Z) - Q_Y(W,0,Z)}_{Q_{\text{diff}}} | A = 0, W] \}$$
 Residual treatment effect = Δ_S

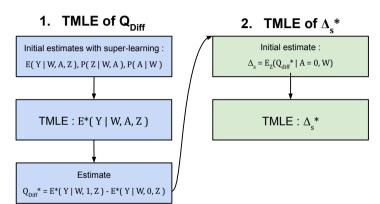
 \implies two quantities to target : Q_{diff} and Δ_S

$NDE = \mathbb{E}_{W} \{ \mathbb{E}_{Z}[Q_{Y}(W, 1, Z) - Q_{Y}(W, 0, Z) | A = 0, W] \}$

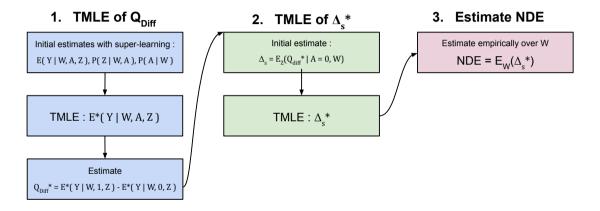
1. TMLE of Q_{Diff}



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Identifiability conditions

Consistency

- Y(a, m) = Y if A = a, Z = m
- Z(a') = Z if A = a'
- Y(a, Z(a')) = Y(a, m) if Z = m

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Conditional independence

- $A \perp \!\!\!\perp Z(a')|W$
- $A \perp Y(a,m)|W$
- $Z \perp Y(a,m)|W,A=a$



$$\triangle$$
 $Z(a') \perp Y(a,m)|W$

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Positivity

- $\mathbb{P}(A=1|W) \in (0,1)$
- $\mathbb{P}(Z = m | W, A = a) \in (0, 1)$

• Potential violation of cross-world independence assumption

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Take-home messages

• Potential and challenges evaluating an early gene expression surrogate of vaccine response

• Targeted learning to reduce bias in causal estimands

Methodological workflow for evaluating PTE by gene sets on vaccine response

Thank you for listening

References

- Pollard, Andrew J. and Else M. Bijker (Dec. 2020). "A guide to vaccinology: from basic principles to new developments". In: *Nature Reviews Immunology* 21.2, pp. 83–100. ISSN: 1474-1741.
- Schuler, Megan S. and Sherri Rose (Dec. 2016). "Targeted Maximum Likelihood Estimation for Causal Inference in Observational Studies". In: *American Journal of Epidemiology* 185.1, pp. 65–73. ISSN: 1476-6256.

Define auxiliary covariate

$$H_Y(Q_Z, g) = \frac{\mathbb{1}(A=1)}{g(1|W)} \cdot \frac{Q_Z(W, 0)}{Q_Z(W, 1)} - \frac{\mathbb{1}(A=0)}{g(0|W)}$$

and working parametric submodel

$$Q_Y(\epsilon_1) = Q_Y + \epsilon_1 H_Y(Q_Z, g)$$

(1)

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TMLE of Q_{diff}

1. Obtain initial estimates $\widehat{Q_Y}$, $\widehat{Q_Z}$, \widehat{g} with super-learner

Step $1: \mathsf{TMLE} \ \mathsf{of} \ Q_{\mathsf{diff}}$

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TMLE of Q_{diff}

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- 2. Find $\epsilon_1^* = \arg\min_{\epsilon} L_Y(Q_Y(\epsilon))$ w.r.t some loss function L_Y

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- 2. Find $\epsilon_1^* = \arg\min_{\epsilon} L_Y(Q_Y(\epsilon))$ w.r.t some loss function L_Y
- 3. Plug in $\widehat{\epsilon_1^*}$ for targeted estimate $\widehat{Q_Y^*}=\widehat{Q_Y}+\widehat{\epsilon_1^*}H_Y(\widehat{Q_Z},\widehat{g})$

Define auxiliary covariate

$$H_Y(Q_Z, g) = \frac{\mathbb{1}(A=1)}{g(1|W)} \cdot \frac{Q_Z(W, 0)}{Q_Z(W, 1)} - \frac{\mathbb{1}(A=0)}{g(0|W)}$$

and working parametric submodel $Q_V(\epsilon_1) = Q_V + \epsilon_1 H_V(Q_Z, q)$

TMLE of Q_{diff}

- 1. Obtain initial estimates $\widehat{Q_Y}$, $\widehat{Q_Z}$, \widehat{g} with super-learner
- 2. Find $\epsilon_1^* = \arg\min_{\epsilon} L_Y(\widehat{Q_Y(\epsilon)})$ w.r.t some loss function L_Y
- 3. Plug in $\widehat{\epsilon_1^*}$ for targeted estimate $\widehat{Q_Y^*} = \widehat{Q_Y} + \widehat{\epsilon_1^*} H_Y(\widehat{Q_Z}, \widehat{g})$
- 4. Estimate $\widehat{Q^*_{\mathrm{diff}}} = \widehat{Q^*_Y}(W, A=1, Z) \widehat{Q^*_Y}(W, A=0, Z)$

Define auxiliary covariate

$$H_Z(g) = \frac{1}{g(0|W)}$$

and working parametric submodel

$$\Delta_S(\epsilon_2) = \psi_Z + \epsilon_2 H_Z(g)$$

(2)

Define auxiliary covariate

$$H_Z(g) = \frac{1}{g(0|W)}$$

and working parametric submodel

$$\Delta_S(\epsilon_2) = \psi_Z + \epsilon_2 H_Z(g)$$

(2)

TMLE of Δ_S

1. Obtain initial estimate of $\Delta_S=\mathbb{E}_{Q_Z}[Q_Y(W,1,Z)-Q_Y(W,0,Z)|A=0,W]$ by regressing $\widehat{Q_{diff}^*}$ on W among controls

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- 3. Plug in $\widehat{\epsilon_2^*}$ for targeted estimate $\widehat{\psi_Z^*(Q_V^*)} = \psi_Z(\widehat{Q_V^*}) + \widehat{\epsilon_2}^* H_Z(\widehat{g})$

Step 3: Empirical estimate of Direct effect

$$\psi_{NDE} = \mathbb{E}_{Q_W}(\psi_Z(Q_Y))$$

⇒ Empirically average over covariates for NDE

$$\widehat{\psi_{NDE}} = \frac{1}{n} \sum_{i=1}^{n} \widehat{\psi_{Z}^{*}(Q_{Y}^{*})}(W_{i})$$

