Re: Finding general solutions using reduced matrices

Taylor Y. Ka <taylorka@uw.edu>
Mon 3/18/2019 10:17 PM
To: Arthur Liu <artliu@cs.washington.edu>

What a god

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From: Arthur Liu <artliu@cs.washington.edu>
Sent: Monday, March 18, 2019 10:10:20 PM
To: Taylor Y. Ka
Subject: Fw: Finding general solutions using reduced matrices

The B in Bennet stands for the Best

From: Bennet Goeckner <goeckner@uw.edu>
Sent: Monday, March 18, 2019 9:58 PM
To: Arthur Liu
Subject: Re: Finding general solutions using reduced matrices

Hi Arthur,

From your description, I think you are doing this problem in a correct way. You should be considering the augmented system

[a_1 a_2 a_3 a_4 | 3a_4 - a_5]

Now we use the echelon form B given above -- say that B = [b_1 b_2 b_3 b_4 b_5]. The above augmented system is then equivalent to

[b_1 b_2 b_3 b_4 | 3b_4 - b_5]

Solving this, we get the given answer. From what I can tell, this sounds like what you did.

The key reason why this works is a theorem that says that if A and B are equivalent (i.e. you can get from one to the other using row operations), then the dependencies between the columns of A and exactly the same as the dependencies of the columns of B. (On a related note, this is why "Option 2" from class works.)

Let me know if I can explain this more clearly, or if you have any other questions.

-Bennet

On Mon, Mar 18, 2019 at 9:30 PM Arthur Liu <artliu@cs.washington.edu> wrote:

> Hi Bennet!

I'm struggling to understand how to solve one of the problems on the practice finals. I got the right answer, but I'm worried that it was just a coincidence because I'm not sure I fully understand the process. (This is question 3c on Kevin Lui's 308L Autumn 2017 final, and I included a picture of it at the bottom of this email).

The process I used was sort of an extrapolation from question 3b where I could pretend the reduced echelon they gave us was an augmented matrix that solved for linear combinations of the 5th column in terms of the first 4. This gives x1 = -2 + t1 - 2t2... etc.
To then solve for 3a4 - a5, I just subtracted the general solution of a5 from the general solution for 3a3 ie: (0, 0, 0, 3).

I'm not very sure if this is correct since is there the potential that 3a3 could be the linear combination of other vectors besides just being (0, 0, 0, 3)?

I don't know if my approach makes sense, but I'm wondering if the real way to solve this question involves doing something else or if there's something that I might have overlooked.

Thanks so much!
Arthur Liu

3. Let A and B be equivalent matrices defined by

$$A = \begin{bmatrix} -3 & 3 & -1 & -9 & 3 \\ 2 & -2 & 1 & 7 & -1 \\ 4 & -4 & 5 & 23 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 2 & -2 \\ 0 & 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = B.$$

(a) (4 points) Find a basis for the solution space of $Ax = 0$.

Solution: $\{(1,1,0,0,0), (-2,0,-3,1,0), (2,0,-3,0,1)\}$

(b) (4 points) Let a_1, a_2, a_3, a_4, a_5 be the columns of A. Define $C = [a_1\ a_2\ a_3\ a_4]$. What is a particular solution to $Cx = a_5$?

Solution: $(-2, 0, 3, 0)$.

(c) (4 points) Using the same variables as (b), what is the general solution to $Cx = 3a_4 - a_5$?

Solution: $(8,0,6,0) + s_1(1,1,0,0) + s_2(-2,0,-3,1)$.