Let's suppose an evol^{n} eq. such as adhesion or cristallis^{n} in steady state criterion:

$$\vec{V} \cdot \nabla \alpha = \frac{D\alpha}{Dt} = f(\alpha, T)$$

$$\Big| \ \alpha = \alpha_0 \text{ on } \Gamma_{in}$$

variational form:

$$\iiint \vec{v}\, \vec{\nabla} \alpha\, \alpha^* - \iiint f\, \alpha^* = R(\alpha) = 0 \quad \forall \alpha^*$$

let us test $\alpha^* = N + \tau\, \vec{v} \cdot \vec{\nabla} N$

$$\iiint v \nabla \alpha\, N + \tau \iiint \nabla^T \alpha\; v \otimes v\, \nabla N$$

$$- \iiint f N - \tau \iiint f\, v \cdot \nabla N = R(\alpha) = 0$$

Non linear problem: zero $R(\alpha)$

~~start~~ $\Rightarrow$ NR step $\alpha^{k+1} = \alpha^k + \delta\alpha$

where $\underbrace{\left.\frac{dR}{d\alpha}\right|_{\alpha_k}}_{J} \times \delta\alpha = R(\alpha_k)$ $(*)$

$$J = \iiint v \nabla \mathcal{N} \, \mathcal{N} + z \iiint \nabla \mathcal{N} \, v \otimes v \nabla \mathcal{N}$$

$$- \iiint \frac{df}{d\alpha} \mathcal{N} \mathcal{N} - z \iiint \mathcal{N} \frac{df}{d\alpha} \vec{v} \cdot \nabla \vec{\mathcal{N}}$$

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implementation in freefem:

α_x is called αold and is a fixed field
f and sensib $= \frac{df}{d\alpha}$ as well.
$\delta\alpha$ is the unknown (trial field)
$\alpha\alpha$ is the test field.

eq ($*$) can be <u>directly</u> obtained by s<u>olv</u>ing
the:

problem ($\delta\alpha$, $\alpha\alpha$) =
int2D [v_x dx($\delta\alpha$) $\alpha\alpha$ + z dx($\delta\alpha$) v_x v_x dx($\alpha\alpha$)
- sensib $\delta\alpha$ x $\alpha\alpha$ - z $\delta\alpha$ sensib v_x dx($\alpha\alpha$)]

- int2D [v_x dx (α) $\alpha\alpha$ + z dx(α) v_x v_x dx($\alpha\alpha$)
- f $\alpha\alpha$ - z f v_x dx ($\alpha\alpha$)

+ on (1, $\delta\alpha$ = 0)

assuming that first NR inialisa[o]
includes the correct dirichlet BC.

(or maybe on(1, $\delta\alpha = \alpha - \alpha_0$) .