Debt Sustainability Under Endogenous Yields

A Theoretical Model of Safe Asset Demand

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Outline

Traditional Debt Dynamics: The Foundation

The Standard Framework

Traditional debt accumulation equation:

$$\Delta d_t = \left(\frac{R_t - G_t}{1 + G_t} \right) d_{t-1} + p_t \tag{1}$$

Where:

- R_t = nominal interest rate on debt
- G_t = nominal GDP growth
- p_t = primary deficit as share of GDP
- d_t = debt-to-GDP ratio

The Critical Limitation

Exogenous Interest Rate Assumption

Traditional models treat R_t as exogenous

- Interest rates determined independently of debt dynamics
- Overlooks feedback mechanisms in global financial markets
- Incomplete for major economies with globally significant debt

Classical stability condition:

$$G > R \tag{2}$$

Extended Stability Framework

Incorporating Market Realities

More advanced perspective:

$$G > R \iff (\pi^* - \pi^e) + (\hat{r} + tp) < 0 \tag{3}$$

Where:

- $\pi^* - \pi^e$ = unexpected inflation
- $\hat{r}$ = liquidity/safety premium
- tp = term premium

Key insights:

- Inflation surprises help sustainability
- High bond premia worsen sustainability
- Need negative sum for debt stability under deficits

Research Motivation

The U.S. Puzzle

- U.S. debt-to-GDP: 60% (2008) $\rightarrow$ 100%+ (2024)
- Treasury yields remain historically low
- Traditional metrics suggest unsustainability
- Markets continue absorbing increasing issuances

Challenge

This requires a new theoretical framework

Core Innovation

Making Interest Rates Endogenous

Key insight: For major economies, especially reserve currency issuers, interest rates are dynamically influenced by:

- Global capital flows
- Investor demand patterns
- Unique position in global financial system

Solution

Endogenize R_t through global safe asset demand

Model Components

Essential Variables

Global Safe Asset Demand:

- A_t = total global safe asset demand (scaled to world GDP)
- Captures international appetite for secure investments

Allocation Function:

- $\phi(R_t)$ = proportion allocated to U.S. Treasuries
- $\frac{\partial \phi(R_t)}{\partial R_t} > 0$ (higher yields attract more demand)

Total Demand:

$$\text{Demand}_t^{\text{Treas}} = \phi(R_t) \cdot A_t \tag{4}$$

Market Clearing Mechanism

From Demand to Yields

Market equilibrium condition:

$$d_t = \phi(R_t) \cdot a_t^* \tag{5}$$

Where $a_t^* = A_t / Y_t$ (scaled to U.S. GDP)

Inverse demand function:

$$R_t = \phi^{-1}\left(\frac{d_t}{a_t^*}\right) \tag{6}$$

Critical insight: R_t now depends on:

- Debt level (d_t) $\uparrow \rightarrow$ yields $\uparrow$
- Global safe asset demand (a_t^*) $\uparrow \rightarrow$ yields $\downarrow$

The Complete System

Integrated Debt Dynamics

Final system of equations:

$$\Delta d_t = \left(\frac{R_t - G_t}{1 + G_t} \right) d_{t-1} + p_t \tag{7}$$

$$R_t = \phi^{-1} \left(\frac{d_{t-1}}{a_t^*} \right) \tag{8}$$

Timing structure resolves simultaneity:

- R_t determined by previous period's debt d_{t-1}
- Current debt accumulation uses current rates R_t
- Creates dynamic feedback without bias

Functional Form Example

Practical Implementation

Isoelastic specification:

$$\phi(R_t) = \left(\frac{R_t}{\bar{R}}\right)^{\epsilon} \tag{9}$$

Yields endogenous rate:

$$R_t = \bar{R} \cdot \left(\frac{d_t}{a_t^*}\right)^{1/\epsilon} \tag{10}$$

Where:

- $\epsilon > 0$ = elasticity of global allocation
- $\bar{R}$ = benchmark yield on other safe assets

The Privilege Explained

Three Pillars of Advantage

1. **Reserve Currency Status**
   - Dollar's global dominance
   - Central bank reserve holdings
   - Trade settlement currency
2. **Market Depth & Liquidity**
   - Unparalleled Treasury market infrastructure
   - High effective elasticity ϵ
   - Lower price impact of debt issuance
3. **Safe Haven Demand**
   - Flight-to-quality flows during crises
   - Persistent high a_t^*
   - Global financial stability anchor

Mathematical Formalization

How Privilege Works

High and stable a_t^* directly suppresses R_t:
For given debt level d_t:

- Higher $a_t^* \Rightarrow$ lower R_t
- U.S. can tolerate higher d_t before unsustainable dynamics
- Feedback loop mitigation

"Service flow" benefit:

- Economic value from global financial role
- Ability to run larger deficits with less consequence
- Externalization of fiscal costs through systemic importance

Comparative Advantage

Traditional vs. Endogenous Models

Debt increase impact:

- Traditional: $\frac{\partial R}{\partial d} = 0$
- This model: $\frac{\partial R_t}{\partial d_{t-1}} = \frac{1}{a_t^* \phi'(R_t)} > 0$

Global demand shock:

- Traditional: No direct sustainability impact
- This model: Lower a_t^* directly increases required yields

Implication

Traditional models underestimate U.S. unique position

Regime change possibility:

$$\phi(R_t) = \begin{cases} \phi_0(R_t) & \text{if } d_t/a_t^* < \tau \\ \phi_0(R_t) \cdot \delta(d_t/a_t^*) & \text{if } d_t/a_t^* \geq \tau \end{cases} \tag{11}$$

Where:

- τ = confidence threshold
- $\delta(d_t/a_t^*) < 1$ = confidence penalty

Crisis Risk Factors

Potential Threats to Privilege

1. **Geopolitical Shifts**
   - Challenges to dollar dominance
   - Reduced a_t^*
2. **Fiscal Sustainability Concerns**
   - Persistent high deficits
   - Confidence effects
3. **Alternative Safe Assets**
   - Competing safe asset markets
   - Reduced $\phi(R_t)$
4. **Market Structure Changes**
   - Central bank balance sheet normalization
   - Liquidity effects

Early Warning System

Risk Monitoring

Proposed risk index:

$$\text{Risk Index}_t = \omega_1 \frac{d_t}{a_t^*} + \omega_2 \text{Spread}_t + \omega_3 \text{FX Vol}_t \tag{12}$$

Key constraints for model validity:

- $0 \leq \phi(R_t) \leq \phi_{\max} \leq 1$
- $d_t \leq \phi_{\max} \cdot a_t^*$ (absorptive capacity)
- $0 < \epsilon < \epsilon_{\max}$ (elasticity bounds)

Strategic Insights

For U.S. Policymakers

Key policy insights:

1. **Maintain Global Confidence**
   - Debt sustainability depends on reserve currency status
   - Monitor global safe asset demand as much as domestic metrics
2. **Traditional Rules Inadequate**
   - Fiscal rules for reserve currency issuers differ
   - Unique position requires unique framework
3. **Non-Linear Risk Management**
   - Gradual privilege loss could create sudden deterioration
   - Need early warning systems

Required Primary Balance

Fiscal Sustainability Metric

Required primary surplus for stability:

$$s^* = \frac{(R - G) \cdot d^*}{1 + G} \tag{13}$$

Implications:

- If $G > R$: Can run primary deficit ($s^* < 0$)
- If $R > G$: Primary surplus required ($s^* > 0$)

U.S. Advantage

Lower R through privilege allows higher sustainable deficits

Comparative Analysis

Other Developed Economies

Different constraints:

- Limited reserve currency status
- Lower market liquidity
- Reduced safe-haven demand

Result: More stringent market constraints despite similar:

- Demographic pressures
- Fiscal challenges
- Economic fundamentals

Key Contributions

Model Advances

1. **Endogenized Interest Rates**
   - Move beyond exogenous R_t assumption
   - Capture global macro-financial factors
2. **Formalized Exorbitant Privilege**
   - Mathematical framework for U.S. advantage
   - Quantitative mechanism through R_t equation
3. **Global Financial Integration**
   - Links debt sustainability to global demand structure
   - Emphasizes systemic role in financial architecture

Research Agenda

Future Directions

Empirical Validation:

- Estimate elasticity parameters ϵ
- Calibrate threshold effects τ
- Validate absorptive capacity constraints

Model Extensions:

- Time-varying elasticity: $\epsilon_t = f(\text{VIX}_t, \text{Crisis}_t)$
- Political economy considerations
- Endogenous growth and inflation feedbacks

Policy Applications:

- Regime transition dynamics
- Alternative safe asset development
- Crisis scenario modeling

Model Limitations

Areas for Improvement

1. **Behavioral Factors**
   - Assumes rational investor behavior
   - May underestimate sentiment effects
2. **Political Economy**
   - Not explicitly modeled
   - Important for sustainability assessment
3. **Transition Dynamics**
   - Regime shifts need further development
   - Empirical validation limited
4. **Feedback Effects**
   - GDP growth and inflation treated as exogenous
   - High debt could affect these variables

Final Thoughts

Strategic Implications

For the United States:

- Unique fiscal flexibility from global financial role
- Need to maintain structural conditions generating safe asset demand
- Any threat to dollar dominance affects fiscal sustainability

For Global Economy:

- U.S. debt dynamics fundamentally different from other nations
- Global safe asset demand critical for financial stability
- Understanding privilege essential for policy coordination

Key Takeaway

The model shows that U.S. debt sustainability is less about traditional fiscal prudence and more about maintaining its unique position in the global financial architecture.

Thank You

Questions & Discussion

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Key Takeaway

The U.S. "exorbitant privilege" is not just qualitative advantage—it's a quantitative mechanism that fundamentally alters debt dynamics through endogenous yield determination.