# Portfolio Theory without a Risk-Free Asset: Safety as a Fragmented and Endogenous Concept in a Multipolar World

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**Abstract**

Modern portfolio theory and asset pricing rely fundamentally on the existence of a global, universally accepted risk-free asset. However, rising geopolitical fragmentation, fiscal dominance, strategic capital controls, and the emergence of multipolar reserve systems suggest this foundational assumption no longer holds in contemporary financial markets. This paper develops a portfolio theory framework in which no asset is universally risk-free across all investors. Instead, safety is modeled as an investor-specific utility component that varies systematically across geographical, political, and institutional boundaries based on information asymmetries and institutional constraints. We formalize investor-specific safety preferences through a modified utility function and derive optimal portfolios under heterogeneous safety valuations. We develop an equilibrium pricing framework based on marginal investor arbitrage conditions. This theoretical framework provides explanations for cross-sectional patterns in portfolio holdings, cross-border basis spreads, and home bias phenomena. Our results demonstrate that portfolio theory must evolve beyond assumptions of universal safety toward models that explicitly account for the heterogeneous nature of modern risk perception.

**Keywords:** Portfolio Theory, Risk-Free Asset, Safety Premium, Geopolitical Risk, Asset Pricing, Multipolar Finance, Capital Market Segmentation

## 1 Introduction: The Collapse of Universal Safety

### 1.1 The Traditional Paradigm and Its Growing Irrelevance

Portfolio theory, as originally formalized by Markowitz [1952], and the Capital Asset Pricing Model (CAPM) developed by Sharpe [1964], both depend critically on the existence of a universally accessible, truly risk-free asset. This theoretical construct has enabled elegant mathematical formulations of optimal portfolio allocation, risk-return relationships, and equilibrium asset pricing for over half a century. The traditional CAPM framework expresses expected returns as:

$$\mathbb{E}[R_i] = R_f + \beta_i(\mathbb{E}[R_M] - R_f) \tag{1}$$

where $R_f$ represents the universal risk-free rate, accessible to all investors regardless of their location, institutional type, or political affiliation.

However, the assumption of a universal risk-free asset has become increasingly problematic in the contemporary global financial landscape due to information asymmetries, institutional constraints, and regulatory fragmentation that create heterogeneous asset safety perceptions across investor types.

### 1.2 Empirical Motivation

Consider the following cross-sectional empirical patterns that traditional portfolio theory struggles to explain:

- **Cross-Sectional Basis Spreads:** Currency-hedged returns differ systematically across investor nationalities even for similar assets [Du et al., 2018].
- **Portfolio Home Bias Patterns:** The degree of home bias varies systematically across investor types and countries.
- **Cross-Border Investment Patterns:** Investment flows follow predictable patterns based on geographical, political, and regulatory proximity [Arslanalp et al., 2022].
- **Differential Asset Holdings:** Identical assets show different ownership patterns across investor types.

### 1.3 Our Theoretical Contribution

This paper develops a theoretical framework where safety is incorporated directly into investor utility functions based on information asymmetries and institutional constraints. Our key innovations include:

1. **Microfounded Safety Preferences:** We model safety preferences as arising from information asymmetries and institutional monitoring capabilities.
2. **Marginal Investor Pricing:** We develop pricing based on marginal investor arbitrage conditions rather than market averages.
3. **Institutional Constraint Framework:** We analyze how regulatory and institutional barriers create persistent cross-sectional pricing differences.

## 2 Microfoundations of Safety Preferences

### 2.1 Information Asymmetries and Monitoring Capabilities

Safety preferences arise from fundamental information asymmetries and differential monitoring capabilities across investor types. Investors have superior information about and monitoring capabilities for assets that are:

1. **Institutionally Familiar:** Assets issued under legal and regulatory frameworks similar to the investor's home jurisdiction, where the investor has accumulated expertise and monitoring infrastructure.
2. **Informationally Accessible:** Assets for which the investor has better access to information through language, cultural understanding, regulatory reporting requirements, and professional networks.
3. **Enforcement Reliable:** Assets where the investor has confidence in legal enforcement mechanisms based on historical experience and institutional relationships.

## 2.2 Investor-Specific Safety Valuations

**Definition 1** (Safety Valuation Function)**.** *For each investor $i \in \mathcal{I}$, we define a safety valuation function $V_i : \mathcal{A} \to \mathbb{R}^+$ that captures the utility value from reduced monitoring costs and enhanced information quality.*

The safety valuation reflects information quality and monitoring cost differentials:

$$
\begin{aligned}
V_i(A_j) =& \alpha_i \cdot \text{InformationQuality}_i(A_j) + \beta_i \cdot \text{MonitoringEfficiency}_i(A_j) \\
&+ \gamma_i \cdot \text{EnforcementReliability}_i(A_j) + \varepsilon_{ij}
\end{aligned}
\tag{2}
$$

where:

- $\alpha_i, \beta_i, \gamma_i > 0$ are investor-specific parameters reflecting the relative importance of each factor
- $\text{InformationQuality}_i(A_j)$ measures the quality of information investor $i$ has about asset $A_j$
- $\text{MonitoringEfficiency}_i(A_j)$ captures investor $i$'s ability to monitor asset $A_j$ cost-effectively
- $\text{EnforcementReliability}_i(A_j)$ reflects investor $i$'s confidence in legal enforcement for asset $A_j$
- $\varepsilon_{ij}$ captures idiosyncratic factors

This linear specification allows for complementarities and interactions between different information and monitoring advantages while maintaining tractability.

## 2.3 Safety-Augmented Utility Function

Each investor maximizes utility incorporating both financial returns and information/monitoring advantages:

$$
U_i(w) = \mathbb{E}[w^\top R] - \frac{\gamma_i}{2} w^\top \Sigma w + \delta_i \sum_{j=1}^{n} w_j V_i(A_j) \tag{3}
$$

where $\delta_i > 0$ represents the value investor $i$ places on reduced monitoring costs and enhanced information quality.

# 3 Optimal Portfolio Allocation

## 3.1 First-Order Conditions

The first-order conditions yield:

$$\mu - \gamma_i \Sigma w_i^* + \delta_i V_i = \lambda_i \mathbf{1} \tag{4}$$

where $V_i = [V_i(A_1), V_i(A_2), \ldots, V_i(A_n)]^\top$.

## 3.2 Optimal Portfolio Solution

Solving for the optimal portfolio:

$$w_i^* = \frac{1}{\gamma_i} \Sigma^{-1} [\mu + \delta_i V_i - \lambda_i \mathbf{1}] \tag{5}$$

where:

$$\lambda_i = \frac{\mathbf{1}^\top \Sigma^{-1} (\mu + \delta_i V_i) - \gamma_i}{\mathbf{1}^\top \Sigma^{-1} \mathbf{1}} \tag{6}$$

# 4 Marginal Investor Pricing Framework

## 4.1 Asset-Specific Marginal Investors

Rather than using market-wide averages, we identify the marginal investor for each asset based on their relative advantage in information and monitoring.

**Definition 2** (Marginal Investor)**.** *For asset $j$, the marginal investor $i^*(j)$ is the investor with the highest effective valuation:*

$$i^*(j) = \arg\max_{i \in \mathcal{I}} \left\{ \frac{\delta_i V_i(A_j)}{\gamma_i} \right\} \tag{7}$$

This captures the idea that assets are priced by the investors who have the greatest comparative advantage in evaluating and monitoring them.

## 4.2 Marginal Investor Pricing Condition

**Theorem 1** (Marginal Investor Pricing)**.** *In equilibrium, asset $j$'s expected return is determined by its marginal investor's first-order condition:*

$$\mathbb{E}[R_j] = \frac{Cov(R_j, M_{i^*(j)})}{\mathbb{E}[M_{i^*(j)}]} + \frac{\delta_{i^*(j)} V_{i^*(j)}(A_j)}{\gamma_{i^*(j)}} \tag{8}$$

*where $M_{i^*(j)}$ is the marginal investor's pricing kernel.*

*Proof.* The marginal investor $i^*(j)$ has the highest willingness to pay for asset $j$ due to their information and monitoring advantages. In equilibrium, this investor's first-order condition determines the asset's price, as they will bid up the price until their marginal benefit equals the marginal cost. Other investors with lower valuations will not be marginal price-setters for this asset. □

### 4.3 Cross-Asset Pricing Differences

Assets with different marginal investors will exhibit pricing differences reflecting the heterogeneity in information quality and monitoring capabilities:

**Corollary 1** (Cross-Asset Safety Premia). *For two assets $A_i$ and $A_k$ with different marginal investors, the difference in safety premia is:*

$$\frac{\delta_{i^*(i)} V_{i^*(i)}(A_i)}{\gamma_{i^*(i)}} - \frac{\delta_{i^*(k)} V_{i^*(k)}(A_k)}{\gamma_{i^*(k)}} \tag{9}$$

## 5 Institutional Constraints and Limited Arbitrage

### 5.1 Institutional Arbitrage Constraints

Arbitrage is limited by institutional constraints that vary across investor types and asset characteristics.

**Definition 3** (Institutional Constraint Function). *For investor $i$ attempting to arbitrage between assets $j$ and $k$, the constraint is:*

$$IC_{i,j,k} = \max\{0, \mathit{CapitalRequirement}_{i,j,k} - \mathit{AvailableCapital}_i\} \tag{10}$$

*where capital requirements reflect regulatory and risk management constraints.*

### 5.2 Arbitrage Equilibrium Condition

**Theorem 2** (Limited Arbitrage Equilibrium). *For two assets $A_j$ and $A_k$ with similar cash flow characteristics, the return difference in equilibrium satisfies:*

$$|\mathbb{E}[R_j] - \mathbb{E}[R_k]| \leq \min_{i \in \mathcal{I}} \left\{ \frac{IC_{i,j,k}}{\mathit{TradingCapacity}_i} + \left| \frac{\delta_i V_i(A_j)}{\gamma_i} - \frac{\delta_i V_i(A_k)}{\gamma_i} \right| \right\} \tag{11}$$

*Proof.* If the return difference exceeded this bound, the investor with the lowest combined institutional constraints and valuation differences would have profitable arbitrage opportunities. This investor would increase arbitrage activity until the constraint binds or the return difference narrows to eliminate the opportunity. □

This provides a rigorous foundation for understanding why seemingly similar assets can trade at different prices without creating unlimited arbitrage opportunities.

## 6 Cross-Sectional Empirical Implications

### 6.1 Information-Based Home Bias

**Theorem 3** (Information-Driven Home Bias). *Optimal portfolios will exhibit home bias when domestic assets provide superior information quality and monitoring efficiency relative to foreign assets, specifically when:*

$$\delta_i [\alpha_i \Delta \mathit{InfoQuality}_i + \beta_i \Delta \mathit{MonitoringEff}_i] > \gamma_i \frac{\mathit{DiversificationBenefit}_i}{2} \tag{12}$$

*where $\Delta$ terms represent the advantage of domestic over foreign assets.*

*Proof.* The condition ensures that the utility gain from superior information and monitoring capabilities for domestic assets outweighs the risk-reduction benefits of international diversification. The left side represents information advantages, while the right side represents diversification costs. □

This provides a non-circular explanation for home bias based on fundamental information asymmetries rather than assumed geographical preferences.

## 6.2 Cross-Sectional Investment Patterns

**Proposition 1** (Information-Based Investment Patterns)**.** *Across investors, portfolio weights in asset $j$ are positively correlated with:*

1. *Investor's information quality about asset $j$*
2. *Investor's monitoring efficiency for asset $j$*
3. *Investor's confidence in legal enforcement for asset $j$*
4. *Inverse of investor's regulatory constraints on asset $j$*

These predictions can be tested using cross-sectional portfolio holdings data, information quality proxies, and institutional capability measures.

# 7 Empirical Implementation

## 7.1 Measurable Proxies for Information and Monitoring

- **Information Quality:** Language similarity, shared legal systems, accounting standard compatibility, analyst coverage overlap
- **Monitoring Efficiency:** Physical proximity to headquarters, time zone overlap, presence of local offices, cultural similarity indices
- **Enforcement Reliability:** Bilateral investment treaty coverage, historical dispute resolution success rates, judicial system quality indices

## 7.2 Parameter Estimation Strategy

The preference parameters can be estimated using cross-sectional variation in portfolio allocations:

$$(\alpha_i, \beta_i, \gamma_i, \delta_i) = \arg\min \sum_{j=1}^{n} (w_{ij}^{observed} - w_{ij}^{model})^2 \tag{13}$$

subject to the constraint that $w_{ij}^{model}$ solves the optimization problem in equation (5).

# 8 Policy Implications

## 8.1 Financial Market Integration

Our framework provides insights into policies that can improve financial market integration:

1. **Information Standardization:** Harmonizing accounting standards and disclosure requirements reduces information quality differentials

2. **Regulatory Coordination:** Mutual recognition agreements reduce monitoring costs for cross-border investments

3. **Legal Framework Development:** Bilateral investment treaties and dispute resolution mechanisms enhance enforcement reliability

## 8.2 Crisis Resilience

Markets with lower information asymmetries and institutional constraints exhibit greater resilience during stress periods, as investors maintain confidence in their ability to monitor and enforce claims across a broader range of assets.

# 9 Conclusion

This paper has developed a theoretical framework for portfolio theory without universal risk-free assets based on information asymmetries and institutional constraints. By grounding safety preferences in fundamental economic factors rather than arbitrary assumptions, we provide explanations for cross-sectional patterns in portfolio holdings and asset prices.

## 9.1 Key Contributions

1. Microfounded safety preferences based on information asymmetries and monitoring capabilities

2. Marginal investor pricing framework that avoids conceptual problems with market averages

3. Rigorous limited arbitrage conditions based on institutional constraints

4. Non-circular explanations for home bias and international investment patterns

5. Empirically implementable framework with measurable proxies

The framework demonstrates that portfolio theory can be extended beyond traditional assumptions while maintaining theoretical rigor and empirical tractability.

### 9.2 Future Research Directions

1. Empirical calibration using international portfolio data and information quality measures
2. Dynamic extensions incorporating learning about information quality over time
3. Applications to specific policy questions such as financial market integration initiatives
4. Integration with models of information production and dissemination in financial markets

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