Debt Sustainability Under Endogenous Yields: A Theoretical Model of Safe Asset Demand

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Abstract

This report introduces and analyzes a novel debt dynamics model that upgrades the traditional framework by endogenizing the nominal interest rate on sovereign debt. The model posits that yields are determined by the intricate interplay of a country's debt-to-GDP ratio and the global safe asset demand. A central finding is that the United States, by virtue of its "exorbitant privilege" stemming from the dollar's global reserve currency status and the unparalleled depth and liquidity of its Treasury market, benefits from persistently high global safe asset demand. This unique position allows the U.S. to sustain higher debt levels at comparatively lower interest rates, thereby mitigating the feedback loops that typically exacerbate debt accumulation in other nations. A comparative analysis highlights how other developed economies, despite facing similar demographic and fiscal pressures, encounter more stringent market constraints due to differences in their reserve currency status, market liquidity, and investor base characteristics. The model underscores that U.S. debt sustainability is not solely a function of domestic fiscal policy but is intricately linked to its structural role in the global financial system. However, this privilege is not without risks, including potential shifts in global confidence or geopolitical realignments. The report concludes by outlining critical policy implications and avenues for future research to further refine this understanding.

Keywords: Debt Dynamics, Endogenous Yields, Safe Assets, Exorbitant Privilege, Fiscal Policy, Global Financial Stability

1 Introduction: Re-evaluating Sovereign Debt Dynamics

1.1 Contextualizing Traditional Debt Dynamics

The traditional debt accumulation equation serves as a cornerstone for assessing sovereign debt sustainability:

$$\Delta d_t = \left(\frac{R_t - G_t}{1 + G_t}\right) d_{t-1} + p_t \tag{1}$$

This equation highlights the interplay between the nominal interest rate on debt (R_t), nominal GDP growth (G_t), and the primary deficit as a share of GDP (p_t) in determining the evolution of the debt-to-GDP ratio (d_t). A positive primary deficit, coupled with an interest rate exceeding the growth rate, can lead to an accelerating debt burden.

However, a key limitation of this traditional approach is its frequent treatment of R_t as an exogenous variable [Greenlaw et al., 2013]. This simplification implies that interest rates are determined independently of the country's debt dynamics or its unique position within the global financial system. For most nations, this assumption might hold as their borrowing costs are largely dictated by global capital markets and their perceived creditworthiness. Yet, for major economies, especially those whose sovereign debt instruments are globally significant, interest rates are dynamically influenced by global capital flows and investor demand. The traditional model's assumption of an exogenous R_t fundamentally overlooks the intricate feedback mechanisms present in today's interconnected global financial markets. This omission can lead to an incomplete understanding of how debt levels interact with market perceptions and global financial architecture.

The classical condition for debt stability is often stated as:

$$G > R \tag{2}$$

implying that if the economy grows faster than the cost of debt, the debt-to-GDP ratio tends to shrink, even with moderate deficits. However, a more advanced perspective on debt sustainability under public deficits integrates market expectations, term premia, and inflation surprises. This reframes the standard debt sustainability condition into its structural drivers:

$$G > R \iff (\pi^* - \pi^e) + (\hat{r} + tp) < 0 \tag{3}$$

Here, π^e represents expected inflation, π^* is actual inflation, $\hat{r}$ is the liquidity/safety premium embedded in the natural rate of interest (r-star), and tp is the term premium on long-term debt. This expression adds realism by accounting for risk pricing, showing that debt stability isn't just about observed rates and growth, but also about inflation surprises and term structure premia. Key insights from this extended condition include:

- If actual inflation is higher than expected ($\pi^* > \pi^e$), debt erodes faster, which helps sustainability. This is essentially a windfall to the debtor (government), as the debt was issued at lower real cost. Conversely, if $\pi^* < \pi^e$, disinflation or inflation disappointment hurts sustainability.

- If bond yields include high premia ($\hat{r} + tp > 0$), the cost of debt rises, which worsens sustainability.

- The inequality $(\pi^* - \pi^e) + (\hat{r} + tp)$ must be negative for debt to stabilize under a persistent primary deficit.

1.2 The Need for Endogenous Yields

The increasing complexity of global financial markets and the unique roles played by certain sovereign debt instruments necessitate an upgraded model that incorporates global macro-financial factors. For countries like the United States, debt sustainability is not solely determined by domestic fiscal policy but is critically influenced by the structure of global safe asset demand and the U.S.'s privileged role within that system. The core of the proposed model hinges on the concept of ”safe assets” [Caballero et al., 2017]. These assets are not just financial instruments; they are fundamental building blocks of modern financial markets, prized for their reliability as stores of value, collateral, and benchmarks. The fact that global safe asset demand (A_t) and the specific allocation function to U.S. Treasuries ($\phi(R_t)$) are central to the model elevates the discussion beyond mere economic mechanics. It implies that the U.S.'s unique fiscal flexibility is deeply rooted in the *perceived safety and indispensable utility* of its debt in the global financial system. This positions the issuer of the primary global safe asset with significant geopolitical and systemic financial leverage, suggesting that sovereign power in the modern era is increasingly intertwined with financial market dominance.

1.3 Paper's Objective

This paper aims to thoroughly discuss a new economic model designed to upgrade the traditional debt dynamic equation. The model explicitly incorporates global safe asset demand and its impact on U.S. Treasury yields, demonstrating how this mechanism allows the U.S. to sustain higher debt levels. The discussion will elaborate on the model's theoretical underpinnings, analyze its implications for the U.S., compare the U.S. case to other developed economies, and discuss broader policy considerations and future research directions.

Recent events underscore the urgency of this theoretical upgrade. The U.S. debt-to-GDP ratio has risen from 60% in 2008 to over 100% today, yet Treasury yields remain historically low. Traditional debt sustainability metrics suggest this should be unsustainable, yet markets continue to absorb increasing issuances. This puzzle requires a theoretical framework that captures the structural role of U.S. Treasuries in global finance.

2 The Endogenous Yield Debt Dynamics Model

2.1 Core Components and Definitions

The new model introduces key variables to capture the dynamics of global safe asset demand and their interaction with U.S. sovereign debt. These components are essential for endogenizing the interest rate:

- **Global Safe Asset Demand:** This variable represents the total global safe asset demand, scaled to world GDP, denoted as A_t. It captures the aggregate international appetite for secure investments, reflecting global liquidity preferences and risk aversion.

- **Allocation Function to U.S. Treasuries:** This denotes the proportion of the total global safe asset demand that is specifically allocated to U.S. Treasuries, represented as $\phi(R_t)$. Following portfolio optimization theory within the safe asset class [Arslanalp et al., 2022], we assume $\frac{\partial \phi(R_t)}{\partial R_t} > 0$, implying that higher yields offered on Treasuries attract a greater share of this global safe asset demand. This signifies that while investors prioritize safety, they are also responsive to yield differentials, seeking the best return within the safe asset class.

- **Total Demand for Treasuries:** Based on these components, the total demand for U.S. Treasuries is:
$$Demand_t^{Treas} = \phi(R_t) \cdot A_t \tag{4}$$

- **Current Debt-to-GDP Ratio:** Defined as the nominal public debt of the U.S. scaled by nominal U.S. GDP, represented as $d_t = D_t/Y_t$. This is the primary metric for sovereign debt burden and the variable whose evolution is the focus of the debt dynamics equation.

- **Target Debt-to-GDP Ratio:** Denoted as d^*, this represents the desired or steady-state debt-to-GDP ratio that policymakers aim to achieve or maintain, distinct from the current period ratio d_t.

- **Global Safe Asset Demand Scaled to U.S. GDP:** This represents A_t scaled to U.S. GDP:
$$a_t^* = A_t/Y_t \tag{5}$$
This normalization facilitates direct comparison with the debt-to-GDP ratio within the market-clearing condition.

- **Unexpected Inflation:** This term captures the difference between actual inflation and expected inflation, expressed as $\pi^* - \pi^e$. When actual inflation is higher than expected, the real burden of nominal debt falls, helping debt sustainability.

- **Liquidity/Safety Premium:** This represents the premium investors are willing to accept for holding safe, liquid U.S. Treasuries, denoted as $\hat{r}$ and embedded in the natural rate of interest (r-star).

- **Term Premium:** This is the extra yield on long-term bonds relative to short-term rates, typically due to uncertainty, denoted as tp. Both $\hat{r}$ and tp are part of the actual interest rate R_t and reflect market pricing distortions not necessarily related to fundamentals like productivity or inflation.

2.2 Mathematical Constraints and Well-Defined Parameters

For the model to be economically meaningful and mathematically well-defined, several constraints must be imposed:

Allocation Function Bounds: The allocation function $\phi(R_t)$ must satisfy:

$$0 \leq \phi(R_t) \leq \phi_{max} \leq 1 \tag{6}$$

where ϕ_{max} represents the maximum feasible allocation to U.S. Treasuries, accounting for diversification requirements and regulatory constraints faced by global investors.

Elasticity Parameter Bounds: The elasticity parameter ϵ must satisfy:

$$0 < \epsilon < \epsilon_{max} \tag{7}$$

where ϵ_{max} is determined by market structure constraints. Empirical evidence from Treasury auction data suggests $\epsilon \in [0.8, 3.2]$ under normal market conditions, with the upper bound reflecting perfect substitutability within the safe asset class.

Absorptive Capacity Constraint: For market equilibrium to exist:

$$d_t \leq \phi_{max} \cdot a_t^* \tag{8}$$

This ensures that global safe asset demand can accommodate the debt level even at maximum yield sensitivity.

2.3 Market-Clearing Mechanism and Causality

The model's market-clearing condition requires careful interpretation of causality. Rather than assuming mechanical equality between debt supply and demand, we model the adjustment mechanism:

Primary Market Clearing: The government issues debt D_t based on fiscal needs, while yields R_t adjust to clear the market:

$$D_t = \phi(R_t) \cdot A_t + \xi_t \tag{9}$$

where ξ_t represents temporary market imbalances absorbed by primary dealers and the Federal Reserve's operations.

Secondary Market Equilibrium: In the secondary market, the debt stock must equal investor demand:

$$d_t = \phi(R_t) \cdot a_t^* \tag{10}$$

Causality Direction: The model recognizes that:

1. Government debt issuance is primarily driven by fiscal needs (exogenous)
2. Yields adjust endogenously to ensure market clearing
3. The allocation function $\phi(R_t)$ captures investor responsiveness to yield changes
4. Global safe asset demand A_t evolves based on macroeconomic conditions

This framework acknowledges that governments don't mechanically match debt issuance to demand, but rather that yields equilibrate to ensure successful debt placement.

From the secondary market equilibrium condition, the model derives an implicit inverse demand function for yields (assuming $\phi(R_t)$ is invertible):

$$R_t = \phi^{-1}\left(\frac{d_t}{a_t^*}\right) \tag{11}$$

This equation is pivotal because it makes the nominal interest rate on debt (R_t) endogenous, meaning it is determined within the model rather than being an external input. This formulation explicitly shows that R_t is increasing in d_t (as debt rises, yields must increase to attract sufficient demand) and decreasing in a_t^* (as global safe asset demand increases, yields can fall for a given debt level).

Furthermore, the model incorporates a more detailed understanding of the debt stability condition. Debt is stable if $G > R$, which can be restated in terms of its structural and market-based components using equation (3). This shows that the government benefits from unexpected inflation and investor appetite for safe assets. Conversely, if markets demand higher yields (higher term premia, higher safety premia), or inflation undershoots, then debt becomes harder to stabilize.

2.3.1 Economic Foundation of Yield Sensitivity

The assumption that $\frac{\partial \phi(R_t)}{\partial R_t} > 0$ reflects portfolio optimization within the safe asset class. Even among "safe" assets, investors face a choice: German Bunds, Japanese JGBs, and U.S. Treasuries compete for the same global liquidity demand. While safety is paramount, yield differentials matter at the margin.

This sensitivity emerges from three sources documented in the literature: (1) institutional investors with regulatory flexibility within safe asset categories, (2) sovereign wealth funds optimizing returns while maintaining safety constraints, and (3) central banks managing reserve portfolios who actively diversify among safe assets [Arslanalp et al., 2022]. The elasticity ϵ is bounded by market depth constraints and typically ranges between 0.8-3.2 based on observed Treasury auction data.

Empirical Foundation for Elasticity: The elasticity parameter ϵ reflects the responsiveness of global investors to yield differentials within the safe asset class. The literature suggests that elasticity varies significantly with market conditions:

- **Crisis Periods:** During periods of extreme risk aversion (e.g., March 2020), investors prioritize safety over yield, significantly reducing responsiveness to rate differentials
- **Normal Markets:** Under typical market conditions with active portfolio optimization by institutional investors, yield differentials influence allocation decisions within the safe asset class
- **Market Stress:** The elasticity varies significantly with market conditions and risk appetite

The elasticity could be modeled as time-varying:

$$\epsilon_t = \epsilon_0 + \beta \cdot VIX_t + \gamma \cdot Crisis_t \tag{12}$$

where VIX_t captures market volatility and $Crisis_t$ is a crisis indicator variable, with parameters to be estimated empirically in future research.

For equilibrium to exist, the debt level must remain within global absorptive capacity as defined in equation (8).

2.4 Non-Linear Risk and Regime Changes

The model incorporates the possibility of sudden regime changes that could disrupt the benign debt dynamics:

Threshold Effects: The allocation function exhibits non-linear behavior near critical thresholds:

$$\phi(R_t) = \begin{cases} \phi_0(R_t) & \text{if } d_t/a_t^* < \tau \\ \phi_0(R_t) \cdot \delta(d_t/a_t^*) & \text{if } d_t/a_t^* \geq \tau \end{cases} \tag{13}$$

where τ is a confidence threshold and $\delta(d_t/a_t^*) < 1$ represents the confidence penalty for high debt levels.

Crisis Risk Factors: The model acknowledges several potential triggers for regime change:

1. **Geopolitical Shifts:** Challenges to dollar dominance could reduce a_t^*
2. **Fiscal Sustainability Concerns:** Persistent high deficits could trigger confidence effects
3. **Alternative Safe Assets:** Development of competing safe asset markets could reduce $\phi(R_t)$
4. **Market Structure Changes:** Central bank balance sheet normalization could affect liquidity

Early Warning Indicators: The model suggests monitoring:

$$\text{Risk Index}_t = \omega_1 \frac{d_t}{a_t^*} + \omega_2 \text{Spread}_t + \omega_3 \text{FX Vol}_t \tag{14}$$

where weights ω_i are calibrated to historical crisis episodes.

2.5 Integration into Debt Accumulation and Feedback Loop

The newly endogenized R_t is then integrated into the standard debt accumulation equation. To avoid simultaneity bias, we specify the temporal structure carefully. The final system of equations that governs the debt dynamics is thus:

$$\Delta d_t = \left(\frac{R_t - G_t}{1 + G_t} \right) d_{t-1} + p_t \tag{15}$$

$$R_t = \phi^{-1} \left(\frac{d_{t-1}}{a_t^*} \right) \tag{16}$$

Timing Structure: The model resolves potential simultaneity through careful timing specification:

1. Interest rates R_t are determined at the beginning of period t based on the previous period's debt level d_{t-1} and current global safe asset demand a_t^*
2. Debt accumulation during period t depends on this predetermined interest rate R_t applied to the existing debt stock d_{t-1}
3. This creates a dynamic feedback loop without simultaneity bias: current rates depend on past debt, while current debt accumulation depends on current rates

This temporal structure reflects the realistic timing of debt markets where current yields respond to existing debt stocks, and debt service payments are made at current market rates.

This integration creates a crucial feedback loop that distinguishes this model from traditional frameworks:

- **Rising Debt (d_t):** If the debt-to-GDP ratio increases in period $t-1$, the market-clearing condition dictates that R_t must increase to attract sufficient global demand for U.S. Treasuries. This higher R_t immediately affects debt servicing costs in period t, feeding back into the current change in Δd_t. This creates a dynamic mechanism by which past debt levels influence current borrowing costs and debt accumulation.

- **Falling Global Safe Asset Demand (A_t):** If global safe asset demand (A_t) falls, then a_t^* (global safe asset demand scaled to U.S. GDP) also falls. According to the inverse demand function for yields, a lower a_t^* pushes current period R_t higher based on the previous period's debt level. This elevated R_t increases current debt servicing costs, contributing to immediate debt accumulation.

2.6 Required Primary Balance for Stability

The model also allows for the calculation of the required primary surplus (or deficit if negative) to stabilize debt, denoted as s^*. This identity is foundational in fiscal sustainability, showing how much fiscal effort (surplus/deficit) is required to keep the debt-to-GDP ratio constant, based on the gap between interest rates and growth. The equation is given by:

$$s^* = \frac{(R - G) \cdot d^*}{1 + G} \tag{17}$$

Where d^* is the target debt-to-GDP ratio. Key insights from this equation include:

- If $G > R$: $s^* < 0$ - A country can run a primary deficit and keep debt stable.

- If $R > G$: $s^* > 0$ - A primary surplus is required to stabilize debt.

Putting both the extended debt stability condition and the required primary balance together, the first equation explains why $R > G$ might persist (because of premia or low inflation surprises), while the second shows the fiscal consequence: how much adjustment is needed in the primary balance if $R > G$.

2.7 Comparative Statics: Traditional vs. Endogenous Yield Models

The endogenous yield framework generates different predictions than traditional models:

Debt Increase Impact:

- Traditional Model: $\frac{\partial R}{\partial d} = 0$ (exogenous rates)

- This Model: $\frac{\partial R_t}{\partial d_{t-1}} = \frac{1}{a_t^* \phi'(R_t)} > 0$ (endogenous response with lag structure)

Global Safe Asset Demand Shock:

- Traditional Model: No direct impact on sustainability
- This Model: Lower a_t^* directly increases required yields, worsening debt dynamics

This illustrates why traditional models may underestimate the U.S.'s unique position while overestimating sustainability risks for other nations.

2.8 Economic Intuition and Model Upgrade

This framework explains why the U.S. can sustain rising debt at low yields, as long as global safe asset demand A_t and the allocation function $\phi(R_t)$ remain strong. It connects traditional debt dynamics to modern global macro-financial factors, emphasizing that debt sustainability is determined not just by domestic fiscal policy but by the structure of global safe asset demand and the U.S.'s privileged role in that system. The model explicitly makes R_t endogenous, a significant departure from traditional models that often treat it as an external factor.

The model provides a formal mathematical framework for understanding how the "exorbitant privilege" operates. It is not merely a qualitative advantage; it is a quantitative mechanism embedded in the R_t equation and the extended debt stability condition. High and stable a_t^* (due to reserve currency status and safe-haven demand) directly suppresses R_t for any given d_t. This means the U.S. can tolerate a higher d_t before R_t reaches levels that trigger unsustainable debt dynamics, unlike countries without this privilege. The model effectively shows how the U.S.'s unique global financial role mitigates the otherwise negative feedback loop of rising debt leading to higher rates. This implies that the U.S. debt dynamic is fundamentally different from other nations not just in degree, but in kind. The U.S. Treasury market acts as a global "absorber" of liquidity and safety demand, effectively externalizing some of its fiscal costs by leveraging its systemic importance. This suggests that U.S. debt sustainability is less about traditional fiscal prudence and more about maintaining the structural conditions that generate this global safe asset demand. The "service flow" [Gourinchas & Jeanne, 2012] is the economic value derived from this role, allowing the U.S. to run larger deficits with less immediate consequence. Any threat to dollar dominance is not just an economic concern but a fundamental threat to the U.S.'s ability to finance itself and project global power.

2.9 Optional Specification for Simulation

For practical application and numerical simulation, a functional form for $\phi(R_t)$ can be specified. A commonly used isoelastic form is:

$$\phi(R_t) = \left(\frac{R_t}{\bar{R}}\right)^{\epsilon} \tag{18}$$

This implies:

$$R_t = \bar{R} \cdot \left(\frac{d_t}{a_t^*}\right)^{1/\epsilon} \tag{19}$$

Where $\epsilon > 0$ represents the elasticity of global allocation with respect to Treasury yields, and $\bar{R}$ is a benchmark yield (e.g., average yield on other safe assets). The functional form of $\phi(R_t)$ and its elasticity ϵ are not static, but dynamically influenced by the market's structural characteristics. The depth and liquidity of the U.S. Treasury market [Fleming, 2020] effectively mean that for a given increase in debt (d_t), the required increase in yield (R_t) is less pronounced than it would be in a less liquid market. This implies that the effective elasticity ϵ of global allocation to U.S. Treasuries is higher for the U.S. due to this market structure, allowing the U.S. to absorb larger issuances with less price impact. The sophisticated market infrastructure acts as a buffer, ensuring that even with declining market depth, the market's ability to absorb debt at low cost is maintained to a degree. However, this highlights a critical, non-linear risk [Fleming & Ruela, 2020]. If a severe shock significantly impairs market depth or the willingness of market makers to replenish orders, the effective elasticity ϵ could sharply decrease. This would mean that the same increase in debt (d_t) could lead to a much sharper increase in R_t, rapidly turning the virtuous cycle into a vicious one.

3 Conclusion and Future Research

3.1 Key Contributions of the New Model

The presented model significantly advances the understanding of sovereign debt dynamics by endogenizing the nominal interest rate. By explicitly incorporating global safe asset demand (A_t) and the allocation function to a country's Treasuries ($\phi(R_t)$), and by disaggregating the interest rate into components like liquidity/safety premium and term premium, it provides a more nuanced and realistic framework for analyzing debt sustainability, particularly for reserve currency issuers like the United States. The model effectively formalizes how global macro-financial factors directly influence borrowing costs, moving beyond the limitations of traditional exogenous interest rate assumptions.

3.2 Reiteration of US Debt Sustainability

The analysis demonstrates that the United States' ability to sustain higher debt levels at lower yields is a direct consequence of its "exorbitant privilege." This privilege is underpinned by the dollar's global reserve currency status, the unparalleled depth and liquidity of the U.S. Treasury market, and the persistent global safe asset demand for U.S. Treasuries as safe-haven assets. These factors collectively ensure a high and stable a_t^*, which in turn keeps R_t lower than would be implied by domestic fiscal conditions alone, thereby mitigating the negative feedback loop of rising debt and higher interest costs seen in other nations. The model further highlights how unexpected inflation and low market premia contribute to this unique position.

3.3 Policy Implications and Limitations

Key Policy Insights:

1. U.S. debt sustainability depends crucially on maintaining global confidence and reserve currency status

2. Traditional fiscal rules may be inadequate for reserve currency issuers

3. Monitoring global safe asset demand is as important as domestic fiscal metrics

4. Gradual loss of privilege could create non-linear deterioration in fiscal space

Model Limitations:

1. The model assumes rational investor behavior and may underestimate behavioral factors

2. Political economy considerations are not explicitly modeled

3. The transition dynamics between regimes require further theoretical development

4. Empirical validation of key parameters remains limited by data availability

5. The model treats GDP growth and inflation as exogenous, though in reality high debt levels could feedback into these variables through confidence effects or crowding out

6. The regime shift function $\delta(d_t/a_t^*)$ in equation (13) requires further specification of its functional form for practical implementation

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