Aplicando a Teoria Mie-Debye para Caracterização de Parâmetros Físicos em Pinças Óticas

Defesa de Dissertação de Mestrado

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Conteúdo

Introdução

Goal

Describe the process of scattering of EM waves by a spherically symmetric object.
Discuss the application of this model to experiments that try to measure Casimir interactions.

Introduction

Path:

Find a solution for a scalar wave equation in spherical coordinates

$\downarrow$

Adapt the result to a vector field $\rightarrow$ Electromagnetic field description

$\downarrow$

Scattering description with developed tools

$\downarrow$

Aplications

Spherical Wave Solutions of the Scalar Wave Equation

Spherical Wave Solutions of the Scalar Wave Equation

Consider the scalar function $\psi(\vec{r}, \omega)$, which satisfies the Helmholtz wave equation

$$\left(\nabla^2 + k^2\right) \psi(\vec{r}, \omega) = 0.$$

Note that $k^2 = \omega^2/c^2$ and $\psi(\vec{r}, \omega) = FT_t \psi(\vec{r}, t)$.

Spherical Wave Solutions of the Scalar Wave Equation

The laplacian ∇ is then writen in spherical coordinates and, after separating the variables and solving the angular part, we find:

$$\psi(\vec{r}, \omega) = \sum_{l,m} F_{l,m}(r) Y_{l,m}(\theta, \phi)$$

and $F_{l,m}(r)$ is the solution for the radial part of the equation (don't depend on m).

With the transformation:

$$F_l(r) = \frac{1}{\sqrt{r}} u_l(r)$$

the radial part of the equation becomes a Bessel equation, which solutions are:

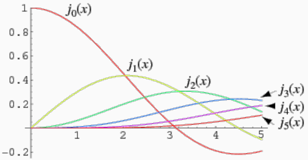
$$F_l(r) = \frac{1}{\sqrt{r}} \left(A_{l,m} J_{l+1/2}(r) + B_{l,m} N_{l+1/2}(r) \right)$$

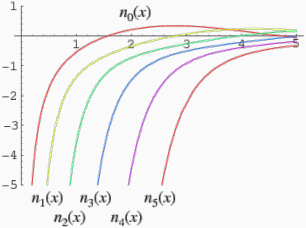
Spherical Wave Solutions of the Scalar Wave Equation

Now, we define the *Spherical Bessel and Henkel Functions*:

$$j_l(kr) = \left(\frac{\pi}{2kr}\right)^{1/2} J_{l+1/2}(kr)$$
$$n_l(kr) = \left(\frac{\pi}{2kr}\right)^{1/2} N_{l+1/2}(kr)$$
$$h_l^{\pm}(kr) = j_l(kr) \pm i n_l(kr)$$

which look like:





Spherical Wave Solutions of the Scalar Wave Equation

When $kr >> l$, those functions will behave as follows:

$$j_l(kr) \to \frac{1}{kr} sin\left(kr - \frac{l\pi}{2}\right)$$

$$n_l(kr) \to -\frac{1}{kr} cos\left(kr - \frac{l\pi}{2}\right)$$

$$h_l^+(kr) \to (-i)^{l+1} \frac{e^{ikr}}{kr}$$

Now, we have a complete set of functions to describe ψ:

$$\psi(\mathbf{r}, \omega) = \sum_{l,m} [A_{lm}^{(+)} h_l^{(+)}(kr) + A_{lm}^{(-)} h_l^{(-)}(kr)] Y_{lm}(\theta, \phi)$$

Electromagnetic Fields description

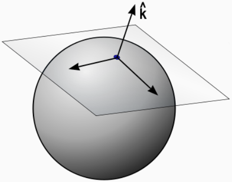
Electromagnetic Fields description

Motivation:

Right now we have the wave equation solution for a scalar field.

Transverse field can be obtained by multiplying the solution by a transverse vector.

But which transverse vector?

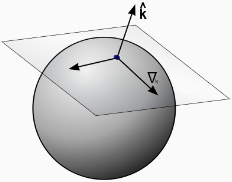


Electromagnetic Fields description

The vector operator ∇_k (in reciprocal space) is a good way to start. Assuming the field has no longitudinal component ($\vec{k} \cdot \vec{A} = 0$), we can write the operator as:

$$\nabla_k = \frac{1}{k}\left(\hat{\theta}\frac{\partial}{\partial\theta} + \hat{\phi}\frac{1}{sen\theta}\frac{\partial}{\partial\phi}\right)$$

where is clear that it is defined in the tangencial plane.

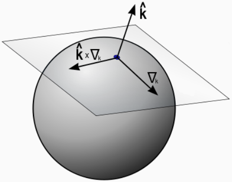


Electromagnetic Fields description

Now, the last component should be perpendicular to, not just $\hat{k}$, but also ∇_k. The obvious choice is to take the vector product:

$$\hat{k} \times \nabla_k = \alpha \vec{L}$$

and, now, we finally have a set of vetors that describes our vector field:



Electromagnetic Fields description

Since the spherical harmonics are eigenfunctions of the operator $\vec{L}$, we can build our fields with the normalized vector

$$\vec{X}_{lm}(\theta,\phi) = \frac{1}{\sqrt{l(l+1)}}\vec{L}Y_{lm}(\theta,\phi).$$

First, we build the magnetic multipole field, using the scalar field expansion found earlier:

$$\vec{E}^{(M)}_{lm}(\vec{r}) = c^2 g_l(kr)\vec{X}_{lm}(\theta,\phi)$$
$$\vec{B}^{(M)}_{lm}(\vec{r}) = \frac{-i}{kc^2}\nabla\times\vec{E}^{(M)}_{lm}(\vec{r})$$

where

$$g_l(kr) = B^{(+)}_l h^{(+)}_l(kr) + B^{(-)}_l h^{(-)}_l(kr).$$

Electromagnetic Fields description

The same ideia is used to construct the eletric multipole field, which leads us to the final result:

$$\vec{E}(\vec{r}) = c^2 \sum_{l,m} \left[\frac{i}{k} a_E(l,m) \nabla \times f_l(kr) \vec{X}_{lm}(\theta,\phi) + a_M(l,m) g_l(kr) \vec{X}_{lm}(\theta,\phi) \right],$$

$$\vec{B}(\vec{r}) = \sum_{l,m} \left[a_E(l,m) f_l(kr) \vec{X}_{lm}(\theta,\phi) - \frac{-i}{k} a_M(l,m) \nabla \times g_l(kr) \vec{X}_{lm}(\theta,\phi)) \right].$$

Electromagnetic Fields description

Plane waves also have a spherical wave expansion. As an example, we have the case of $\hat{k} = \hat{z}$:

$$e^{ikz} = \sum_{l=0}^{\infty} i^l (2l+1) j_l(kr) P_l(cos\theta))$$

where

$$P_l(cos\theta) = Y_{l0}(\theta)$$

Electromagnetic Fields description

This leads to the field:

$$\vec{E}(\vec{r}) = \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left[j_l(kr)\vec{X}_{l,\pm 1}(\theta,\phi) \pm \frac{1}{k} \nabla \times j_l(kr)\vec{X}_{l,\pm 1}(\theta,\phi) \right],$$

$$\vec{B}(\vec{r}) = \frac{1}{c} \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left[\frac{-i}{k} \nabla \times j_l(kr)\vec{X}_{l,\pm 1}(\theta,\phi) \mp i j_l(kr)\vec{X}_{l,\pm 1}(\theta,\phi) \right].$$

Those will be a sum of *ingoing* spherical waves of various l's.

Scattering of Electromagnetic Waves by a sphere (Mie)

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We begin to discuss our problem by stating that we can divide our fields in 2 parts:

$$\vec{E}(\vec{r}) = \vec{E}_{inc}(\vec{r}) + \vec{E}_{sc}(\vec{r})$$
$$\vec{B}(\vec{r}) = \vec{B}_{inc}(\vec{r}) + \vec{B}_{sc}(\vec{r})$$

We use the multipole basis to describe them all! But with caution...

Scattering of Electromagnetic Waves by a sphere (Mie)

$\vec{A}_{inc}(\vec{r})$ is an incoming wave, described by the plane wave expansion.

$\vec{A}_{sc}(\vec{r})$ is the scattered wave. Only one Henkel function describe *outgoing* spherical wave: $H_l^+(kr)$.

$$\vec{E}_{sc}(\vec{r}) = \frac{1}{2}\sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)}\left[\alpha_\pm h_l^+ \vec{X}_{l,\pm 1} \pm \beta_\pm \frac{1}{k}\nabla \times h_l^+ \vec{X}_{l,\pm 1}\right],$$

$$\vec{B}_{sc}(\vec{r}) = \frac{1}{2c}\sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)}\left[\alpha_\pm \frac{-i}{k}\nabla \times h_l^+ \vec{X}_{l,\pm 1} \mp i\beta_\pm h_l^+ \vec{X}_{l,\pm 1}\right].$$

Scattering of Electromagnetic Waves by a sphere (Mie)

$\alpha_{\pm}$ and $\beta_{\pm}$ are both determined by the boundary conditions. Since the scatterer is spherically symmetric, no direction will be treated differently; and assuming there is no absorption, the scattering effect only changes the

Scattering of Electromagnetic Waves by a sphere (Mie)

Conclusion