



Aplicando a Teoria Mie-Debye para Caracterização de Parâmetros Físicos em Pinças Óticas

Defesa de Dissertação de Mestrado

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Introdução

Objetivo

Caracterização de parâmetros na pinça ótica:

→ Teoria MDSA+ para pinças óticas:

→ Correções de aberrações óticas na teoria MDSA → Astigmatismo.

→ Espalhamento Mie → Parâmetro de absorção da microesfera.

→ Cálculo numérico usando a teoria MDSA+:

→ Simulação do experimento de transferência de momento angular na pinça ótica.

→ Cálculos da posição de equilíbrio da esfera variando o parâmetro de absorção.

Breve introdução à teoria de pinças óticas:

Modelagens da pinça ótica: interação de um campo (feixe) com o objeto espalhador (esfera pinçada).

Campo: feixe fortemente focalizado.

Objeto espalhador: simetria esférica \rightarrow espalhamento Mie.

Regimes de tamanhos do objeto espalhador.

Dois regimes distintos:

Regime Rayleigh: raio da esfera muito menor que o comprimento de onda do feixe ($a \ll \lambda$).

Aproximamos a esfera por um dipolo induzido:

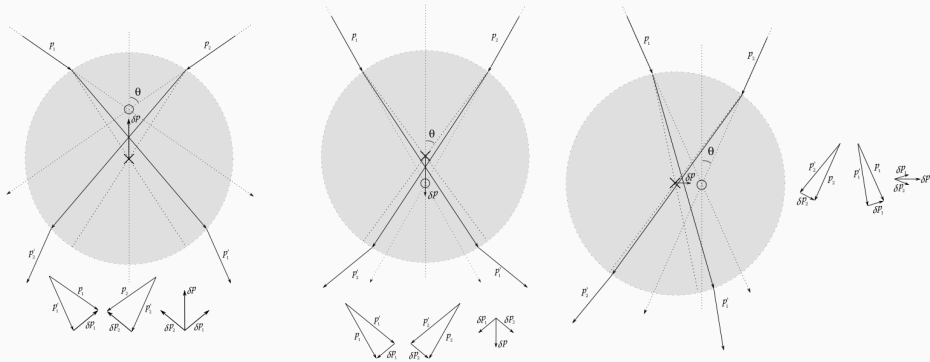
$$\mathbf{F} = \frac{1}{2} \nabla(\mathbf{p} \cdot \mathbf{E}) = \frac{1}{2} \nabla(\alpha \mathbf{E}^2). \quad (1)$$

Força aponta para região de maior intensidade do campo. Para o feixe fortemente focalizado, a região de maior intensidade é o foco.

Regimes de tamanhos do objeto espalhador.

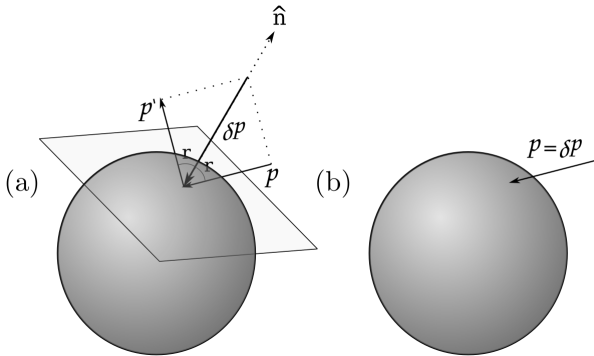
Regime de ótica geométrica: raio da esfera muito maior que o comprimento de onda do feixe ($a \gg \lambda$).

Ignorando efeitos de reflexão e de absorção, raios diametralmente opostos transferem momento para esfera na direção do foco.



Regimes de tamanhos do objeto espalhador.

A posição de equilíbrio não necessariamente é no foco, pois a reflexão (a) transfere momento na direção normal ao plano tangente ao ponto de reflexão na esfera, enquanto a absorção (b) transfere na direção da propagação do raio:



Introdução ao modelo MDSA.

O modelo Mie-Debye para a pinça ótica é um modelo exato para o espalhamento por uma esfera de um campo fortemente focalizado (por uma objetiva).

O modelo para o campo foi desenvolvido por Richards e Wolf, baseado em uma proposição de Debye para um campo escalar produzido por uma objetiva de alta abertura numérica.

O campo é expresso como uma superposição de ondas planas onde os vetores $\mathbf{k}(\theta_k, \phi_k)$ formam um cone de meia abertura θ_0 no espaço de momento:

$$\mathbf{E}_{IN} = \int_0^{\theta_0} d\theta_k \sin \theta_k \int_0^{2\pi} d\phi_k \sqrt{\sin \theta_k} e^{i\mathbf{k} \cdot \mathbf{r}} \hat{\mathbf{e}}_k, \quad (2)$$

Spherical Wave Solutions of the Scalar Wave Equation

Spherical Wave Solutions of the Scalar Wave Equation

Consider the scalar function $\psi(\vec{r}, \omega)$, which satisfies the Helmholtz wave equation

$$(\nabla^2 + k^2) \psi(\vec{r}, \omega) = 0.$$

Note that $k^2 = \omega^2/c^2$ and $\psi(\vec{r}, \omega) = FT_t \psi(\vec{r}, t)$.

Spherical Wave Solutions of the Scalar Wave Equation

The laplacian ∇ is then written in spherical coordinates and, after separating the variables and solving the angular part, we find:

$$\psi(\vec{r}, \omega) = \sum_{l,m} F_{l,m}(r) Y_{l,m}(\theta, \phi)$$

and $F_{l,m}(r)$ is the solution for the radial part of the equation (don't depend on m).

With the transformation:

$$F_l(r) = \frac{1}{\sqrt{r}} u_l(r)$$

the radial part of the equation becomes a Bessel equation, which solutions are:

$$F_l(r) = \frac{1}{\sqrt{r}} (A_{l,m} J_{l+1/2}(r) + B_{l,m} N_{l+1/2}(r))$$

Spherical Wave Solutions of the Scalar Wave Equation

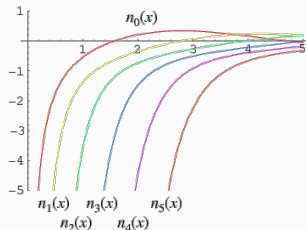
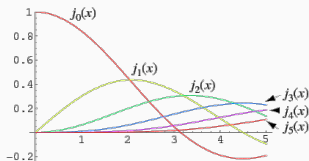
Now, we define the *Spherical Bessel and Henkel Functions*:

$$j_l(kr) = \left(\frac{\pi}{2kr}\right)^{1/2} J_{l+1/2}(kr)$$

$$n_l(kr) = \left(\frac{\pi}{2kr}\right)^{1/2} N_{l+1/2}(kr)$$

$$h_l^{\pm}(kr) = j_l(kr) \pm i n_l(kr)$$

which look like:



Spherical Wave Solutions of the Scalar Wave Equation

When $kr \gg l$, those functions will behave as follows:

$$\begin{aligned}j_l(kr) &\rightarrow \frac{1}{kr} \sin\left(kr - \frac{l\pi}{2}\right) \\n_l(kr) &\rightarrow -\frac{1}{kr} \cos\left(kr - \frac{l\pi}{2}\right) \\h_l^+(kr) &\rightarrow (-i)^{l+1} \frac{e^{ikr}}{kr}\end{aligned}$$

Now, we have a complete set of functions to describe ψ :

$$\psi(\mathbf{r}, \omega) = \sum_{l,m} [A_{lm}^{(+)} h_l^{(+)}(kr) + A_{lm}^{(-)} h_l^{(-)}(kr)] Y_{lm}(\theta, \phi)$$

Electromagnetic Fields description

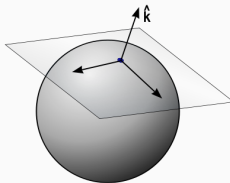
Electromagnetic Fields description

Motivation:

Right now we have the wave equation solution for a scalar field.

Transverse field can be obtained by multiplying the solution by a transverse vector.

But which transverse vector?

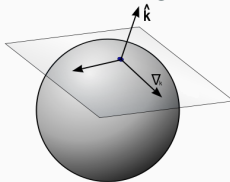


Electromagnetic Fields description

The vector operator ∇_k (in reciprocal space) is a good way to start. Assuming the field has no longitudinal component ($\vec{k} \cdot \vec{A} = 0$), we can write the operator as:

$$\nabla_k = \frac{1}{k} \left(\hat{\theta} \frac{\partial}{\partial \theta} + \hat{\phi} \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} \right)$$

where is clear that it is defined in the tangential plane.

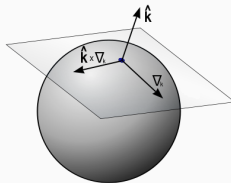


Electromagnetic Fields description

Now, the last component should be perpendicular to, not just \hat{k} , but also ∇_k . The obvious choice is to take the vector product:

$$\hat{k} \times \nabla_k = \alpha \vec{L}$$

and, now, we finally have a set of vectors that describes our vector field:



Electromagnetic Fields description

Since the spherical harmonics are eigenfunctions of the operator \vec{L} , we can build our fields with the normalized vector

$$\vec{X}_{lm}(\theta, \phi) = \frac{1}{\sqrt{l(l+1)}} \vec{L} Y_{lm}(\theta, \phi).$$

First, we build the magnetic multipole field, using the scalar field expansion found earlier:

$$\vec{E}_{lm}^{(M)}(\vec{r}) = c^2 g_l(kr) \vec{X}_{lm}(\theta, \phi)$$

$$\vec{B}_{lm}^{(M)}(\vec{r}) = \frac{-i}{kc^2} \nabla \times \vec{E}_{lm}^{(M)}(\vec{r})$$

where

$$g_l(kr) = B_l^{(+)} h_l^{(+)}(kr) + B_l^{(-)} h_l^{(-)}(kr).$$

Electromagnetic Fields description

The same idea is used to construct the electric multipole field, which leads us to the final result:

$$\vec{E}(\vec{r}) = c^2 \sum_{l,m} \left[\frac{i}{k} a_E(l, m) \nabla \times f_l(kr) \vec{X}_{lm}(\theta, \phi) + a_M(l, m) g_l(kr) \vec{X}_{lm}(\theta, \phi) \right],$$
$$\vec{B}(\vec{r}) = \sum_{l,m} \left[a_E(l, m) f_l(kr) \vec{X}_{lm}(\theta, \phi) - \frac{-i}{k} a_M(l, m) \nabla \times g_l(kr) \vec{X}_{lm}(\theta, \phi) \right].$$

Plane waves also have a spherical wave expansion. As an example, we have the case of $\hat{k} = \hat{z}$:

$$e^{ikz} = \sum_{l=0}^{\infty} i^l (2l+1) j_l(kr) P_l(\cos\theta)$$

where

$$P_l(\cos\theta) = Y_{l0}(\theta)$$

Electromagnetic Fields description

This leads to the field:

$$\vec{E}(\vec{r}) = \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left[j_l(kr) \vec{X}_{l,\pm 1}(\theta, \phi) \pm \frac{1}{k} \nabla \times j_l(kr) \vec{X}_{l,\pm 1}(\theta, \phi) \right],$$
$$\vec{B}(\vec{r}) = \frac{1}{c} \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left[\frac{-i}{k} \nabla \times j_l(kr) \vec{X}_{l,\pm 1}(\theta, \phi) \mp ij_l(kr) \vec{X}_{l,\pm 1}(\theta, \phi) \right].$$

Those will be a sum of *ingoing* spherical waves of various l 's.

Scattering of Electromagnetic Waves by a sphere (Mie)

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We begin to discuss our problem by stating that we can divide our fields in 2 parts:

$$\begin{aligned}\vec{E}(\vec{r}) &= \vec{E}_{inc}(\vec{r}) + \vec{E}_{sc}(\vec{r}) \\ \vec{B}(\vec{r}) &= \vec{B}_{inc}(\vec{r}) + \vec{B}_{sc}(\vec{r})\end{aligned}$$

We use the multipole basis to describe them all! But with caution...

Scattering of Electromagnetic Waves by a sphere (Mie)

$\vec{A}_{inc}(\vec{r})$ is an incoming wave, described by the plane wave expansion.

$\vec{A}_{sc}(\vec{r})$ is the scattered wave. Only one Henkel function describe *outgoing* spherical wave: $H_l^+(kr)$.

$$\vec{E}_{sc}(\vec{r}) = \frac{1}{2} \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left[\alpha_{\pm} h_l^+ \vec{X}_{l,\pm 1} \pm \beta_{\pm} \frac{1}{k} \nabla \times h_l^+ \vec{X}_{l,\pm 1} \right],$$

$$\vec{B}_{sc}(\vec{r}) = \frac{1}{2c} \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left[\alpha_{\pm} \frac{-i}{k} \nabla \times h_l^+ \vec{X}_{l,\pm 1} \mp i\beta_{\pm} h_l^+ \vec{X}_{l,\pm 1} \right].$$

Scattering of Electromagnetic Waves by a sphere (Mie)

α_{\pm} and β_{\pm} are both determined by the boundary conditions. Since the scatterer is spherically symmetric, no direction will be treated differently; and assuming there is no absorption, the scattering effect only changes the

Scattering of Electromagnetic Waves by a sphere (Mie)

Conclusion
