Minimum Criteria for Signal Processing

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Abstract

Analyzing light from distant astrophysical objects is the main way astronomers are able to uncover properties pertaining to celestial objects. Signal processing is at the forefront of recreating an accurate picture of the object we are trying to study. That is why having a good understanding of signal processing is of great importance. We need to be able to recover information about a signal and be able to do science with the information gathered. There are three main points that people working with signals should have a basic understanding of. Those concepts are the Nyquist criteria, implementing Fourier transforms and mixers.

1 Introduction

Signal processing is essential in analyzing light from distant objects. In this paper I will outline the essential information one must know in order to acquire accurate data about an object one might be interested in studying. The reason for going over this is that anyone who is studying a celestial objects needs to know the limitations that one has on processing signal and have confidence in their data acquisition. The ultimate goal is to acquire data correctly and provide these data set to the community where people may do science with the data one has collected. This data may be used to test our understanding of theoretical models and to improve the current literature. In section 2 we will go over the Nyquist Criteria, which is a threshold on the sampling rate one must sample to recreate information about a signal accurately. Section 3 will go over a vital tool in signal processing called the Fourier Transform. With the Fourier transform at our disposal, one can determine at what frequency the most amount of power is coming from which can be used to deduce and interpret information about the object emitting the signal we receive. Then we conclude this paper by going over mixers and their application in section 4. Mixers are used to change the frequency of an incoming signal while retaining the original information of the signal.

2 Nyquist Criteria

The Nyquist Criteria is a fundamental concept in signal processing. What the Nyquist Criteria allows one to do is to uniquely characterize an incoming wave. You need to be able to sample your signal enough times to gather information regarding its period, amplitude, relative phase, etc. Albeit one could sample at a really high frequency but the data files that we would generate would be too big for us to use. The minimum sampling rate is given by the Nyquist criteria. This criteria is the minimum sampling frequency one must sample a signal and be able to uniquely characterize it. The Nyquist criteria is given by the mathematical formula:

$$\nu_{sampling} \geq 2\nu_{max} \tag{1}$$

Here $\nu_{sampling}$ is the sampling frequency of the detector and ν_{max} is the maximum frequency in the signal. If you fall below this sampling limit the signal that you will measure will be aliased. Which can give inaccurate information regarding the incoming signal.

2.1 Aliasing

Aliasing is a consequence of not taking enough samples of the signal you want to measure. If you sample a fast signal too slowly what will happen is that you will mask the true frequency of the fast signal you want to measure to one of a lower frequency. Below are some examples of aliasing using a sampling rate of 12.5 Mhz and changing the signal frequency to intervals of the sampling frequency.

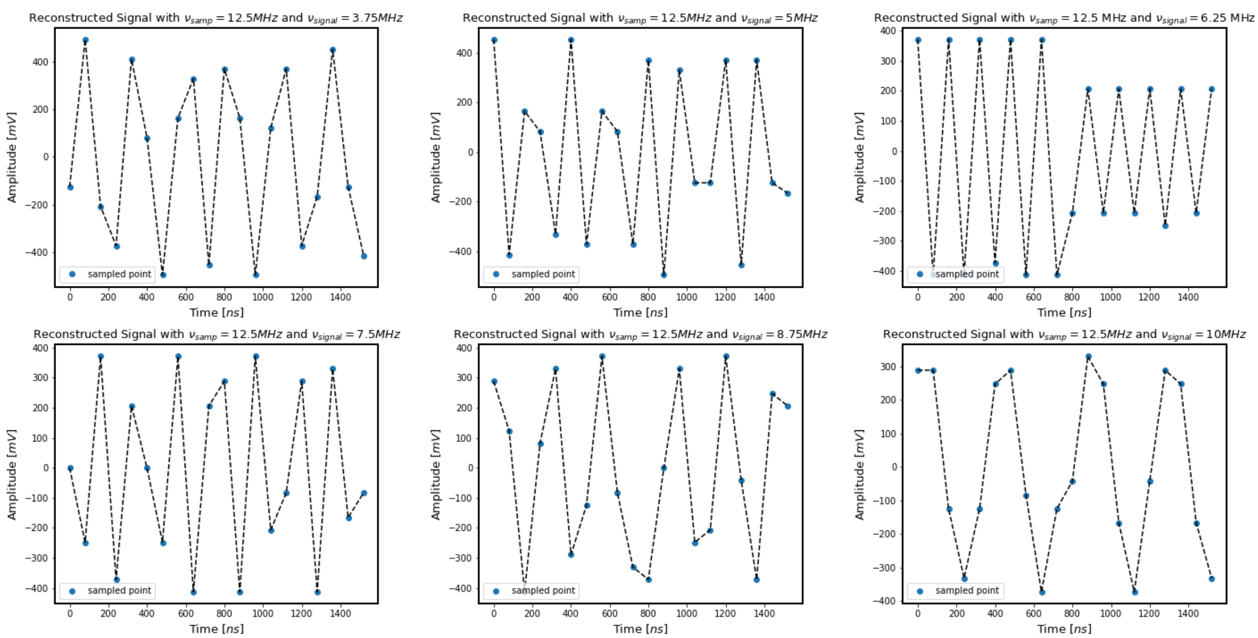


Figure 1: Plot showing the progression of our signal reconstruction as we increase the frequency of our signal and fall below the Nyquist criteria. Top three figure have signals that are below or equal to the Nyquist criteria and the bottom three are reconstructed signals that are above the Nyquist criteria. Here you can see the aliasing occur for the faster frequencies as they are masked onto lower frequency waves.

Due to our sampling rate of 12.5 MHz we can accurately and uniquely characterize signals that have a frequency of 6.25 MHz. We can see from the plots that signals above that frequency seem to revert back to a signal with a slower frequency. Looking at the 5 MHz and 7.5 MHz plots we sort of see that it is hard to distinguish these two waves. They seem to both have similar period and amplitude. This goes to show that sampling a signal at or above the Nyquist limit is crucial in accurately representing the signal.

3 Fourier Transforms

Fourier transforms are the basis for signal processing. With Fourier transforms we are able to determine unique features of a signal that otherwise would have been missed. In the lab one usually detects a signal in a time domain as one samples the signal in discrete time intervals. While these measurements allow us to get an idea about the shape of the incoming signal, there is not much useful information that we can gather. What a Fourier transform allows one to do is to change the measurements from the time domain into the frequency domain while retaining the original information of the signal. The main reason for doing this is that any signal can be decomposed as a sum of sine and cosine waves. The Fourier transform essentially tells someone that for a given frequency range, which frequency is contributing the most to the signal.

So how does one use Fourier transforms and what exactly is it? Suppose you have a function of time f(t), then the Fourier transform of this function f(t) into the frequency domain is given by:

$$F(\nu) = \int_{-\infty}^{\infty} f(t) e^{2\pi\nu i t} dt \tag{2}$$

However, in the real world this mathematical expression does not apply since we can not take infinite time series or can make our time increments infinitesimally small. Thus, we modify the above equation to envelope a period. Moreover, since we can only take measurements in discrete samples and not a continuum we need to replace the integral with a summation. The proper Fourier equation becomes:

$$F(\nu) = \frac{1}{T} \sum_{t=-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{2\pi\nu i t} \Delta t \tag{3}$$

The amazing thing about the Fourier transform is that it is reversible. In the sense that if you go into the frequency domain you can go back to the time domain. This reversibility is given by the inverse Fourier transform which has the form:

$$f(t) = \frac{1}{T} \sum_{t=-\frac{\nu}{2}}^{\frac{\nu}{2}} F(\nu) e^{-2\pi\nu i t} \Delta\nu \tag{4}$$

Once we uncover useful information about the signal in the frequency domain we have the option to go back to the time domain and reconstruct a wave at a certain frequency for example.

Some applications of Fourier transforms are presented in the upcoming section.

3.1 Power Spectrum

A big part of signal processing is being able to take a signal and determine where most of the power is coming from. This tells one where the most dominant part of the wave we are measuring is at. To answer this question we need to apply a Fourier transform on the signal and calculate the power spectrum.

The power spectrum is defined as:

$$P(\nu) = F(\nu)F(\nu)* \tag{5}$$

This is a measure of the power at a given frequency. The most prominent frequency in the signal is contributing the most to the signal and one expects that the power spectrum will reflect that. By taking the Fourier transform and multiplying by its complex conjugate we make sure to eliminate the imaginary component and keep only real values. Some examples of power spectra are shown below.

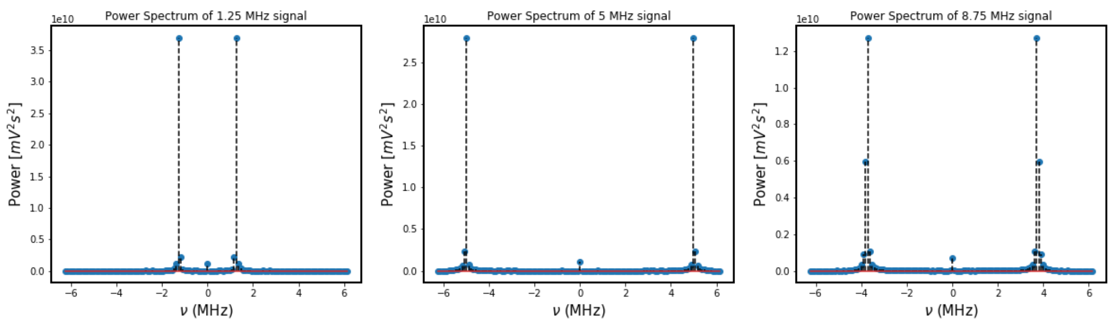


Figure 2: Power spectra of three different data sets. We can see from the plot that as we approach the Nyquist limit, which in our case is 6.25 MHz, the power spectrum goes outward to the Nyquist Frequency. However as we go beyond this limit the power spectrum goes back to the lower frequency range. This is due to aliasing of the signal. This makes the 8.75 MHz signal peak at about 3.75MHz.

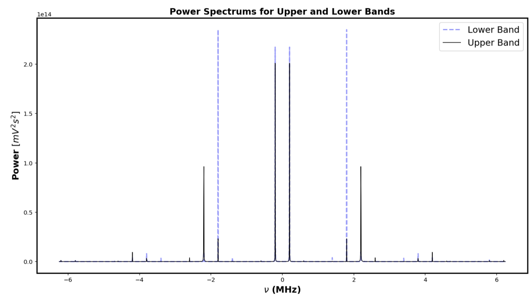
We can see by the plots that we have 2 spikes. These two peaks correspond to the frequency of the signal in both the positive and negative frequency range. Take note that the maximum power frequency matches up well with the signal frequency which is to be expected. If we had a more complicated wave say one that is a sum of two or more sine waves then we expect the power spectrum to reflect this as we would have a peak from the two frequencies of the wave.

4 Mixers

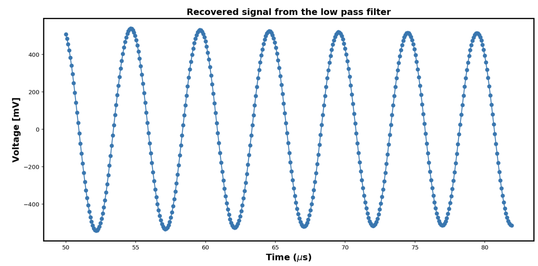
Due to the Nyquist criteria we know that we need to sample our signal at a frequency equal to twice the maximum frequency in our signal. However, in real world applications, we are limited in the sampling frequency by our detectors. In our specific case we were limited to sampling at a maximum sampling rate of 62.5MHz. Which means that we could sample a maximum signal frequency of 31.25 MHz. If we were to have signals with frequencies above 31.25 MHz we would not be able to characterize the incoming signal uniquely. Since signal can come to us from a whole range of frequency we need a way to make sure that the signal we receive fall below the

31.25MHz limit. To answer this problem we use something called a mixer. Mixer is a fancy word for multiplying since what we are doing is multiplying the incoming signal by a Local Oscillation so that the frequency of the wave can fall within our detection limit.

In the lab experiment we had our local oscillator (LO) set to 1 MHz and had an upper frequency of 1.2MHz and lower frequency of .8 MHz. This gave us a $\Delta\nu$ = .2 MHz. What we set out to do was to see if we can distinguish between negative frequencies and positive frequencies in our sample.



[a]



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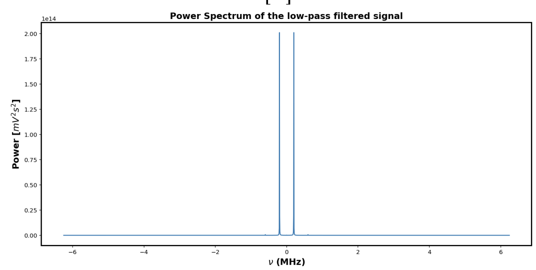

Figure 3: (a): Power spectra of the Lower and Upper bandwidth and we can see that both have different upper frequency limits but the $\Delta\nu$ that we are interested in fall in the range of .2MHz. (b): Filtered out power spectrum to only look at the low frequency signal. (c): Recovered signal from the central Power Spectrum

The main take away from the power spectrum is that the upper and lower bands can be distinguished. This is particularly useful when we have a signal as it allows us to look at the upper band and we can say that a particular signal is above or below the LO frequency. Then we can reduce the signal even more by taking only the frequency that falls within our detection limit. Once we filtered out the higher frequency signal we were able to recover information of the input signal using the lower frequency signal. This is not the same for the double sideband mixer.

To test the double sideband lab we had to offset the LO by 90 degrees so that we can make one output real and the other imaginary and we can actually measure these in our lab through the ports of our PICO-sampler. In our reconstruction of the signal we need to make these into complex values. Calculate power spectrum and see what we observed. However, we fell short in our delay between the LO because when we took the power spectrum we were still able to recover peaks that we could distinguish from positive and negative frequencies. When we should be getting is just one peak.

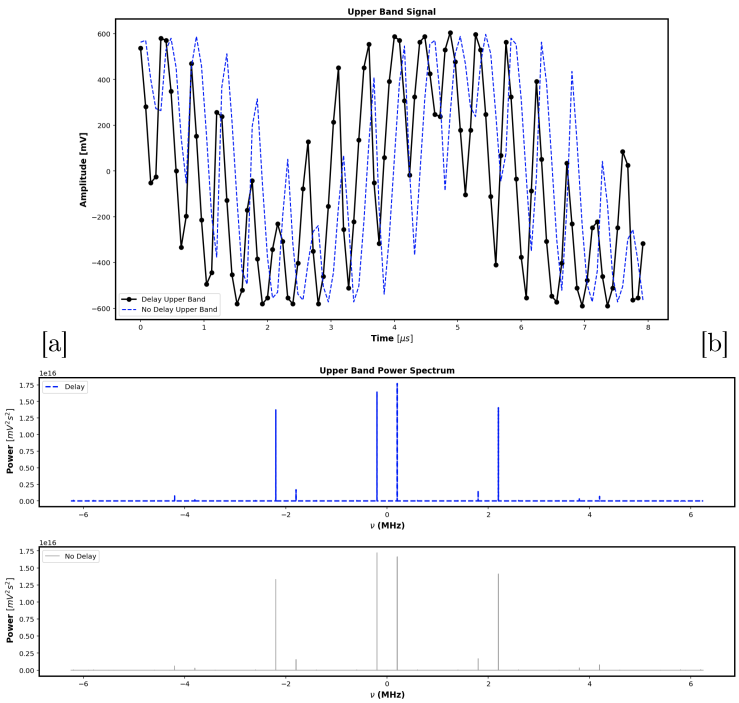

Figure 4: (a): Plot of the signal and we can see that there is a delay but it also shows that we did not get the 90 degree shift we were after. (b): the Power spectra for the Upper Band signal with delay and no delay

4.1 Conclusion

In the double sideband mixer we had to utilize everything covered in this paper. We made sure we utilized a sampling rate that satisfied the Nyquist criteria. We used Fourier transforms to derive a power spectrum. However, in the end we were not able to get the single spike in our power spectrum that we were expecting in this lab. This is due to the relative phase between the two signals not being in a perfect 90 degree offset. This offset could have been noticed if we had hooked up the cables into an oscilloscope. But we failed to do that and as a result we took a power spectra that were not quite out of phase. For future works we would need to make sure that the signals are indeed offset by 90 degrees. We would check this by looking at the oscilloscope and properly taking the data.

5 Contributions and Acknowledgments

In our lab group we each sectioned off different portions of the lab to different people. Arthur Martirosyan handled the lab equipment. Getting everything set up so that we may take our measurements of the data. Arthur also worked out the signal plots and defining function in our data analysis. He also worked on making the plots more interactive for demonstration purposes. Matthew Cardozo was in charge of handling the mixer section and he wrote code that plotted the signal and took its power spectrum. He also worked on the data analysis of power spectra and noise. I wrote code that worked primarily with the Nyquist section. Making plots for the the

Nyquist windows, frequency resolution and leakage power. I also write the code for the noise section of the lab, making a function to calculate the average power given a number of data sets. I also worked on the code to plot the signals, wrote functions to filter out upper band for the mixer data. We wrote the code in a Jupyter notebook and it can be found in the repository on Github following: https://github.com/arthurm101/radiolab and selecting Lab1.