1 An online expectation minimization algorithm

Define $\mu = \alpha \exp(f)$, $\nu = \beta \exp(g)$, $x = (\mu, \nu)$. We will change variables without warning in the following. Define the Bregman divergence

$$\begin{aligned} D_\alpha(\mu, \mu_0) &= \langle \alpha, \, \exp(f_0 - f) - 1 - (f_0 - f) \rangle \\ D_\beta(\nu, \nu_0) &= \langle \beta, \, \exp(g_0 - g) - 1 - (g_0 - g) \rangle \\ D_{\alpha,\beta}(x, x_0) &= D_\alpha(\mu, \mu_0) + D_\beta(\nu, \nu_0) \end{aligned}$$

We want to solve the objective

$$\min_x \mathcal{F}(x) \triangleq \mathrm{KL}(\alpha, \mu) + \mathrm{KL}(\beta, \nu) + \langle \mu \otimes \nu, \, \exp(-C) \rangle$$

Define the prox objective

$$\begin{aligned} \mathcal{L}(x, x_t) &= 2F(x_t) + \langle \nabla \mathcal{F}(x_t), \, x - x_t \rangle + D_{\alpha,\beta}(x, x_t) \\ &= \mathbb{E}_{\hat{\alpha} \sim \alpha, \hat{\beta} \sim \alpha} \Big[2F(x_t) + \langle \nabla \mathcal{F}(x_t), \, x - x_t \rangle + D_{\hat{\alpha}, \hat{\beta}}(x, x_t) \Big] \end{aligned}$$

The Sinkhorn iterations then rewrites as

$$x_t = \operatorname*{argmin}_x \mathbb{E}_{\hat{\alpha}, \hat{\beta}} \mathcal{L}_{\hat{\alpha}, \hat{\beta}}(x, x_t)$$

Probably useless ?