PROJECT OUTLINE

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1. Introduction

We study how noise propagates in the *Golub-Kahan iterative bidiagonalization (GKIB)* in the problem

$$Ax \approx b, \quad b = b^{exact} + b^{noise} \in \mathbb{R}^n. \tag{1}$$

In the above equation the noise level of b^{noise} is unknown, but through this bidiagonalization process, it is possible to determine it. The first part of this study will be to replicate results from the numerical experiments done in [**?**]. More specifically, we will find at what iteration step the noise is revealed, and how we can find the noise level from this, as done in [**?**].

The chosen test problem will be **shaw**, so in (**??**), A is the blurring matrix and b^{exact} is the blurred image that is returned from the `shaw` function. We will generate b^{noise} and add it to b^{exact} to create b. Using this information, we apply the Golub-Kahan iterative bidiagonalization using MATLAB code that we found on-line `http://www.cs.cas.cz/krylov` to perform our numerical experiments. From this code we will obtain a sequence of real numbers α_j, β_j and vectors s_j, w_j for $k = 1, 2, 3, \ldots$ that are going to be use to determine when the noise is revealed and how to get the noise level.

2. Replicate results from main paper

- Brendan's part goes here

3. Use NCP approach to obtain results

We will attempt to find k_{noise} using the *Normalized Cumulative Periodogram (NCP)* approach. This involves computing the cumulative periodogram of the basis vectors s_k and seeing if as k increases the noise becomes more visible in the basis vector s_k, that is, the cumulative periodogram of those vectors will begin to approach the diagonal that connects the points with 0 and 1 ordinates.

In order to do this, we will need a way to measure how noise-like the cumulative periodogram is and at what point we consider it to be noise. Two methods that we will consider for this measurement are the deviations of the basis vectors from a straight diagonal "white noise" line and the other one will be to asses the portion of the basis vectors that lies outside the *Kolmogorov-Smirnov test.* The latter approach requires us to provide

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a confidence level for the white noise line and that is something that we will experiment with. From this approach, we will investigate if we can find the bidiagonalization iteration that is noise revealing. For more information on this topic we will check [**?**].

4. Compare the two methods

The two methods described above to find the moment where the noise is revealed will be compared. We will look for differences and advantages of those methods, in special, how the predicted noise level changes with the k_{noise} acquired by the Normalized Cumulative Periodogram and the method described in [**?**]. To get the noise level δ_{noise} we are going to use

$$\delta_{noise} = \frac{\|b^{noise}\|}{\|b^{exact}\|} \approx \frac{1}{2}\rho_{k_{noise}}, \quad \text{where} \quad \rho_k = \left(\prod_{j=1}^{k} \frac{\alpha_j}{\beta_{j+1}}\right)^{-1},$$

as described on section 4.2 of [**?**]. Moreover, plots of the basis vectors s_k will be provided to be clear at which iteration the noise is revealed.

5. More information

To find more information and source code that is current in development you can access the Github repository `https://github.com/arthurmitrano/noisePropagationBidiagonalization`. You can also find versions of the report file there. No unauthorized code will be uploaded, since the project is open currently public.

References

[1] Hnetynkova, I. and Plesinger, M.. *The regularizing effect of the Golub-Kahan iterative bidiagonalization and revealing the noise level in the data.* BIT Numer Math (2009) 49: 669696.

[2] Rust, B. W. and O'Leary, D. P.. *Residual periodograms for choosing regularization parameters for ill-posed problems.* Inverse Problems 24 (2008) 034005 (30pp).