Signals and Differential Equations

Concept and computations

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Signals play a very important role in modern society even though many would disagree at first thought. The fact is that most things that surround us every day can be regarded or thought of as signals; common things like wireless radio, WiFi, everything inside our computers, music, television, but also things like images, movement, heartbeats, behavioural patterns, brain activity. Seeing that signals are so abundant in our world it seems reasonable to also be able to process and treat them right, be able to use them for what they are worth. In this article we will talk about the connection between signals and concepts in signal processing and the field of differential equations. We will not dwelve into the theory of how to solve the equations but rather give a general overview of the connection between the two fields.

1 How we represent signals

At their essence (continuous) signals are values representing something that varies in time, as such we can represent them as functions:

$$y : \mathbb{R} \to \mathrm{A}, \qquad \mathrm{y} = \mathrm{y(t)}$$

Depending on the situation we're dealing with these signals (functions) might be mathematically continuous, piecewise continuous or not continous at all.

2 Behold the Differential Equation

If we now assume that we are given some system as a black box model; we don't know what's going on on the inside or how it works, but we are given ways to feed it information and and retrieve a response. Let's say we have a description of this system as a differential equation:

$$\sum a_i \frac{d^i y}{dx^i} = x(t)$$

Laplace transforming this equation gives us (setting all initial conditions to zero):

$$\sum a_i s^i Y(s) = X(s) \implies Y(s) = \frac{X(s)}{\sum a_i s^i} = T(s)X(s)$$

The function T above is called the transer function and is at the core of our discussion. This being in the complex s plane These functions will in general be complex. To obtain the frequency response we just have to limit our complex parameter to lie on the unit circle.