Solving Optimization Problems with QAOA and Amazon Braket Hybrid Jobs

Optimization Problems are Ubiquitous

Manufacturing (e.g. supply chain optimization)

Logistics (e.g. route and fleet planning)

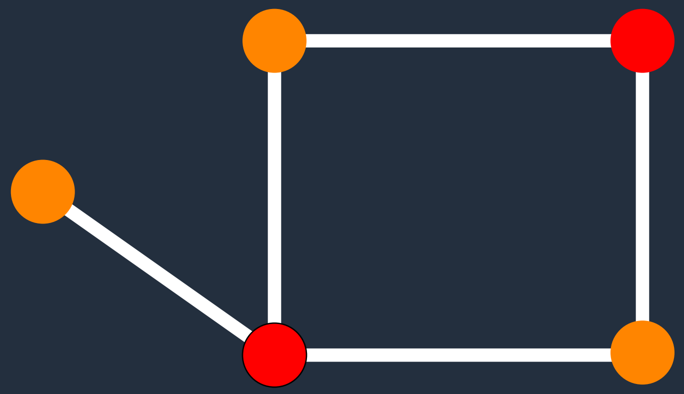
Finance (e.g. portfolio optimization)

Telecom (e.g. wireless sensor network design)

Oil & Gas (e.g. reservoir estimations)

Pharmaceuticals (e.g. candidate configuration selection)

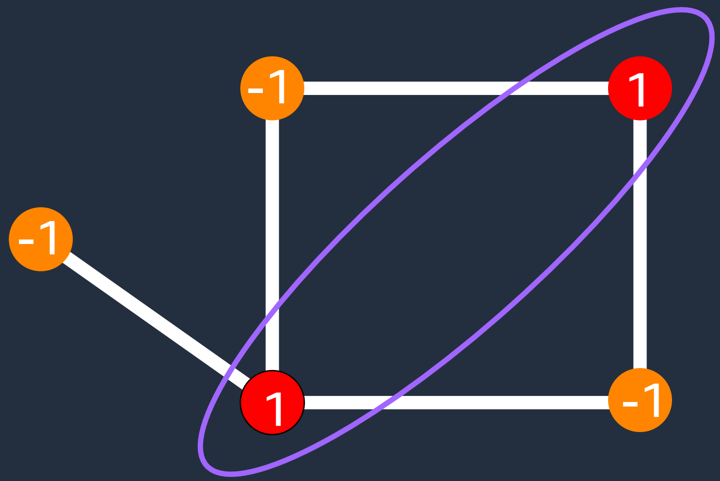
The Maximum Cut Problem



Partition the vertices into two sets such that the number of edges between these two sets is maximized

The Maximum Cut Problem

Maximum Cut



Formulation as an optimization problem:

$$\max_{\mathbf{s}} \frac{1}{2} \sum_{ij \in E} (1 - s_i s_j) \qquad s_i \in \{-1, +1\}$$

Same sign: no edge is cut and no contribution to the objective:

$$\frac{1}{2}(1 - s_i s_j) = 0$$

Mapping the Problem to a Quantum Computer

Classical formulation

$$\max_{\mathbf{s}} C(\mathbf{s}) = \frac{1}{2} \sum_{ij \in E} (1 - s_i s_j)$$

Quantum formulation

Characterize a Hamiltonian by mapping the binary variables s_i onto the eigenvalues of σ_i^z:

$$H_C = \frac{1}{2} \sum_{ij \in E} \left(I - \sigma_i^z \sigma_j^z \right)$$

Finding the ground state of H_C is generically hard.

Quantum Approximate Optimization Algorithm (QAOA)

QAOA approximates ground state by preparing the parametrized ansatz

$$|\beta,\gamma\rangle = U_x(\beta_p)\, U_{zz}(\gamma_p)\, U_x(\beta_{p-1})\, U_{zz}(\gamma_{p-1}) \ldots U_x(\beta_1)\, U_{zz}(\gamma_1)\, |s\rangle$$

consisting of p layers of

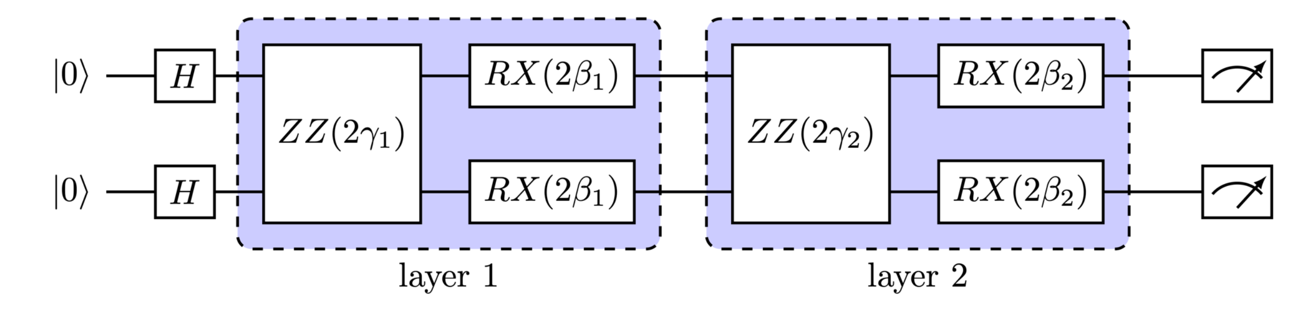
the Mixer Hamiltonian $U_x(\beta) = \exp[-i\beta \sum_i \sigma_i^x]$ (single-qubit rotations)

and the Problem Hamiltonian $U_{zz}(\gamma) = \exp[-i\gamma\, H_C]$ (spin-spin interactions)

starting from uniform superposition basis states $|s\rangle = |+,+,\ldots,+\rangle$

Quantum Approximate Optimization Algorithm (QAOA)

Example: QAOA circuit with 2 qubits and 2 layers



$2p$ classical parameters for p layers of QAOA blocks.

Measurement in computation basis with several shots allows to evaluate $E(\beta,\gamma) = \langle H_C \rangle$.

Classical optimizer varies $2p$ parameters to minimize $E(\beta,\gamma)$

Hands-on Lab