Dot or Scalar Product

20.7. $\mathbf{A} \cdot \mathbf{B} = AB \cos \theta \qquad 0 \leqq \theta \leqq \pi$

where θ is the angle between $\mathbf{A}$ and $\mathbf{B}$.

Fundamental results follow:

20.8. $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$ Commutative law

20.9. $\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$ Distributive law

20.10. $\mathbf{A} \cdot \mathbf{B} = A_1B_1 + A_2B_2 + A_3B_3$

where $\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}$, $\mathbf{B} = B_1\mathbf{i} + B_2\mathbf{j} + B_3\mathbf{k}$.

Cross or Vector Product

20.11. $\mathbf{A} \times \mathbf{B} = AB \sin \theta\, \mathbf{u} \qquad 0 \leqq \theta \leqq \pi$

where θ is the angle between $\mathbf{A}$ and $\mathbf{B}$ and $\mathbf{u}$ is a unit vector perpendicular to the plane of $\mathbf{A}$ and $\mathbf{B}$ such that $\mathbf{A}$, $\mathbf{B}$, $\mathbf{u}$ form a *right-handed system* (i.e., a right-threaded screw rotated through an angle less than 180° from $\mathbf{A}$ to $\mathbf{B}$ will advance in the direction of $\mathbf{u}$ as in Fig. 20-5).

Fundamental results follow:

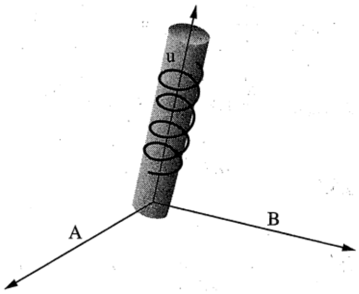

Fig. 20-5

20.12.
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} = (A_2B_3 - A_3B_2)\mathbf{i} + (A_3B_1 - A_1B_3)\mathbf{j} + (A_1B_2 - A_2B_1)\mathbf{k}$$

20.13. $\mathbf{A} \times \mathbf{B} = -(\mathbf{B} \times \mathbf{A})$

20.14. $\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$

20.15. $|\mathbf{A} \times \mathbf{B}|$ = area of parallelogram having sides $\mathbf{A}$ and $\mathbf{B}$

Miscellaneous Formulas Involving Dot and Cross Products

20.16.
$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = A_1B_2C_3 + A_2B_3C_1 + A_3B_1C_2 - A_3B_2C_1 - A_2B_1C_3 - A_1B_3C_2$$

20.17. $|\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})|$ = volume of parallelepiped with sides $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$

20.18. $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

20.19. $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{A}(\mathbf{B} \cdot \mathbf{C})$

20.20. $(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})$

20.21.
$$\begin{aligned}(\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D}) &= \mathbf{C}\{\mathbf{A} \cdot (\mathbf{B} \times \mathbf{D})\} - \mathbf{D}\{\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})\} \\ &= \mathbf{B}\{\mathbf{A} \cdot (\mathbf{C} \times \mathbf{D})\} - \mathbf{A}\{\mathbf{B} \cdot (\mathbf{C} \times \mathbf{D})\}\end{aligned}$$