

#1

$$m \left( \frac{x_{i+1} - 2x_i + x_{i-1}}{\Delta t^2} \right) + d \left( \frac{x_{i+1} - x_{i-1}}{2\Delta t} \right) + Kx_i + mg + O(\Delta t^3) = 0$$

$$\Rightarrow \frac{m}{\Delta t^2} x_{i+1} - \frac{2m}{\Delta t^2} x_i + \frac{m}{\Delta t^2} x_{i-1} + \frac{d}{2\Delta t} x_{i+1} - \frac{d}{2\Delta t} x_{i-1} + Kx_i + mg + O(\Delta t^3) = 0$$

$$\Rightarrow \left( \frac{m}{\Delta t^2} + \frac{d}{2\Delta t} \right) x_{i+1} + \left( K - \frac{2m}{\Delta t^2} \right) x_i + \left( \frac{m}{\Delta t^2} - \frac{d}{2\Delta t} \right) x_{i-1} + mg + O(\Delta t^3) = 0$$

$$\Rightarrow x_i = \frac{\left( \frac{2m}{\Delta t^2} - K \right) x_{i-1} + \left( \frac{d}{2\Delta t} - \frac{m}{\Delta t^2} \right) x_{i-2} - mg + O(\Delta t^3)}{\left( \frac{d}{2\Delta t} + \frac{m}{\Delta t^2} \right)}$$

$$\Rightarrow x_i = \left[ \left( \frac{2m}{\Delta t^2} - K \right) x_{i-1} + \left( \frac{d}{2\Delta t} - \frac{m}{\Delta t^2} \right) x_{i-2} - mg \right] \frac{\Delta t}{\left( \frac{d}{2} + \frac{m}{\Delta t} \right)} + \frac{O(\Delta t^3)}{\frac{d}{2} + \frac{m}{\Delta t}}$$

$$x_i = [c_1 \cdot x_{i-1} + c_2 \cdot x_{i-2} + c_3] \cdot c_4 + O(\Delta t^3)$$

#2

$$Y_1 = Y_0 + v_0 dt + \frac{dt^2}{2m} F_0 + O(dt^3), \text{ où } F = -KY(t) - mg - 2v(t)$$

$$= Y_0 + v_0 dt + \frac{dt^2}{2m} (-KY_0 - mg - 2v_0) + O(dt^3)$$

$$= Y_0 + v_0 dt - \frac{K dt^2}{2m} Y_0 - \frac{mg dt^2}{2m} - \frac{2 dt^2}{2m} v_0 + O(dt^3)$$

$$\Rightarrow Y_1 = \left(1 - \frac{K dt^2}{2m}\right) Y_0 + \left(dt - \frac{2 dt^2}{2m}\right) v_0 - \frac{mg dt^2}{2m} + O(dt^3)$$

$$\Rightarrow Y_1 = \left(1 - \frac{K dt^2}{2m}\right) Y_0 + \left[\left(1 - \frac{2 dt}{2m}\right) dt\right] v_0 - \frac{g dt^2}{2} + O(dt^3)$$