Joining Methods (25 points)

In [ ]:
```
#import packages here
import numpy as np
from matplotlib import pyplot as plt
from scipy.integrate import quad
```

As was explained during the lectures, engineering design of joints should take into consideration pros and cons of each joining method, applied to the specific case we are trying to explore.

Many factors are monitored as design criteria, such as: needed shear strength, necessity for removal to perform inspection and maintenance, lightweight design requirements, possible stress concentrations, desired failure modes, etc.

Overall, understanding the joining of reinforced polymers is essential to obtain proper structural integrity and sufficient service life. Therefore, several aspects of joining methods will be considered and analysed in this notebook, specifically regarding bonded joints and thermoplastic composites (TPC) joining.

NOTE

For the following questions it is essential for you to keep in mind the two main models you will be considering to evaluate peak shear stresses in bonded joints (Klein, B., Gänsicke, T. (2019). Fügetechniken. In: Leichtbau-Konstruktion. Springer Vieweg, Wiesbaden. https://doi.org/10.1007/978-3-658-26846-6_22):

Klein model:

$$\tau(x) = \frac{\omega \cdot \tau_m}{(\beta+2)\cdot(cosh\omega - 1)} \cdot [sinh(\omega(1-\frac{x}{l_u})) + (\beta+1)\cdot sinh(\omega\cdot\frac{x}{l_u})]$$

$$\omega = \sqrt{\frac{G_{Kl}\cdot(E1\cdot t1 + E2\cdot t2)\cdot l_u^2}{E1\cdot t1\cdot E2\cdot t2\cdot d}}$$

$$\beta = \sqrt{\frac{G_{Kl}\cdot(E2\cdot t2 - E1\cdot t1)\cdot l_u^2}{E1\cdot t1\cdot(G_{Kl}\cdot l_u^2 + E2\cdot t2\cdot d)}}$$

Where: τ_m = average shear stess $E1$, $E2$ = adherends' Young's moduli $t1$, $t2$ = adherends' thicknesses l_u = overlap length G_{Kl} = adhesive shear modulus d = adhesive thickness

Volkersen model for the adhesive peak shear:

$$\tau_{Kmax} = \frac{F_{10}}{b}\cdot\sqrt{\frac{1}{2}\cdot\frac{1}{E\cdot t}\cdot\frac{G_K}{t_K}}$$

Where: F_{10} = applied tensile load b = overlap length E = adherends' Young's modulus t = adherends' thickness G_K = adhesive shear modulus t_K = adhesive thickness

In [ ]:
```
### Klein model ###
def tau_klein(x, tau_m, lu, GKl, E1, t1, E2, t2, d):
    omega = np.sqrt(GKl*(E1*t1+E2*t2)*lu**2 / (E1*t1*E2*t2*d))
    beta = (GKl*(E2*t2-E1*t1)*lu**2) / (E1*t1*(GKl*lu**2+E2*t2*d))
    return omega * tau_m / ((beta+2)*(np.cosh(omega)-1)) * (np.sinh(omega
```

In [ ]:
```
### Volkersen model ###
def tau_volkersen(F10, b, E,t,GK,tK):
    return F10/b * np.sqrt(GK / (2*E*t*tK))
```

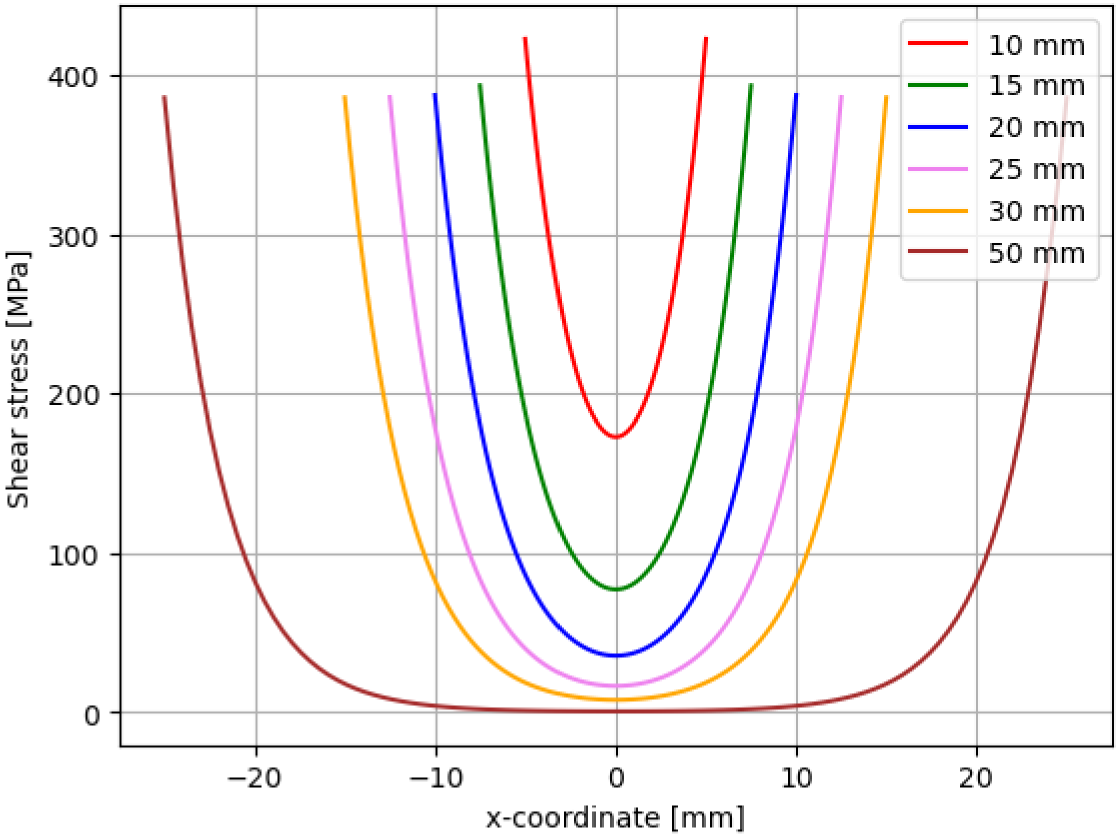
Question 1 (5 points)

Question 1.1: Two carbon fibre reinforced composite parts are to be adhesively bonded, as shown in the figure below. The following information is provided: Adherends' Young's modulus = 70 GPa, adherends' thickness = 2 mm, adherends' width = 20 mm, adhesive shear modulus = 2.0 GPa, adhesive thickness = 0.3 mm, overlap length = 30 mm. The bonded part will be subjected to a tensile load of 50 kN.

- In the lecture you learned about the shear-stress distribution of such a joint. Plot the shear stress distribution over the lap joint for several overlap lengths (including at least: 10, 15, 20, 30, and 50 mm) according to the Klein model. Comment on the results.

- Then, repeat the exercise using the Volkersen model instead. What are the most notable differences?

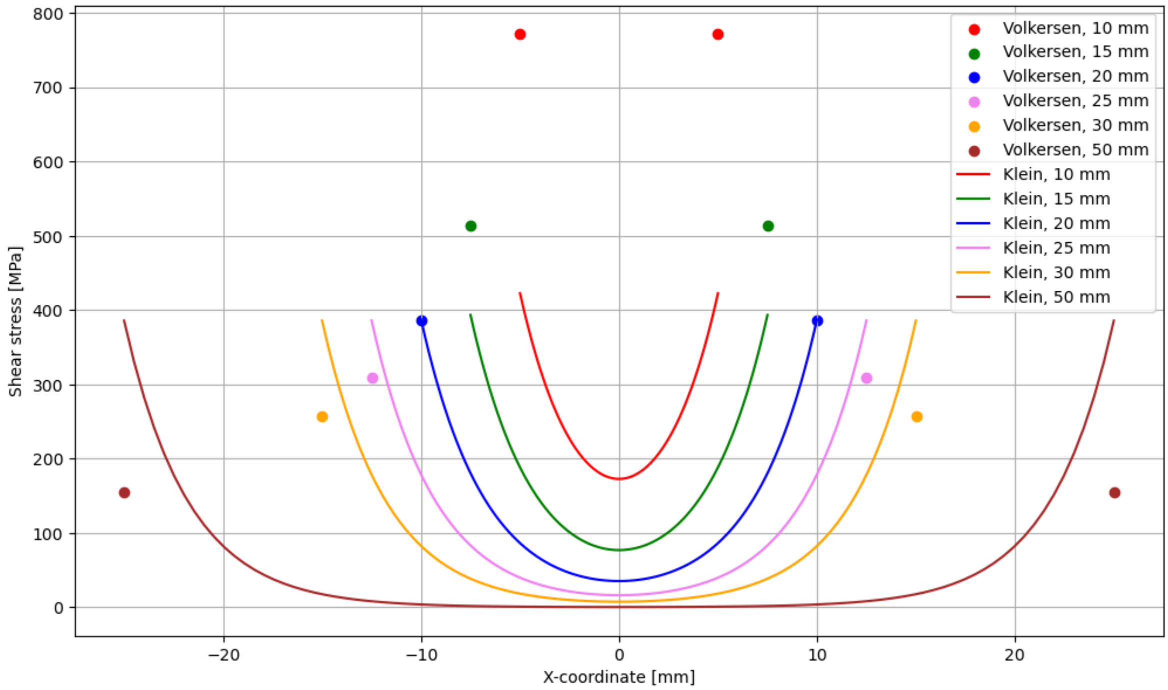
In [ ]:
```
E1, E2, t1, t2, lu, GKl, d, F, w = 70e9, 70e9, 2e-3, 2e-3, 30e-3, 2e9, 0.
overlaps = [10e-3, 15e-3, 20e-3, 25e-3, 30e-3, 50e-3]
colors = ['red', 'green', 'blue', 'violet', 'orange', 'brown']
color_start = 0
for i in overlaps:
    tau_m = F / (w*i)
    x = np.linspace(0,i,100)
    plt.plot((x/1e-3)-i/2*1e3, tau_klein(x, tau_m, i, GKl, E1, t1, E2, t2
    color_start +=1
color_start = 0

plt.legend()
plt.grid()
plt.xlabel('x-coordinate [mm]')
plt.ylabel('Shear stress [MPa]')
plt.show()
```



Comments: The Klein model is plotted above. It shows that the shear stress is symmetrical about the x=0 axis, the middle of the joint, for all overlap lengths. The peak shear stress is achieved at the edges of the joint. As the overlap length increases, the peak shear stress at the edges decreases, but not by much, from just over 420 MPa to 385MPa. However, the minimum shear stress decreases from 172 MPa to almost 0 MPa for the largest overlap, this means that almost no loads are carried by the middle of the joint.

In [ ]:

```
F10, b, E, t, GK, tK = 50e3, 30e-3, 70e9, 2e-3, 2e9, 0.3e-3

plt.figure(figsize=(12,7))
color_start=0
for i in overlaps:
    plt.scatter(i/1e-3/2, tau_volkersen(F10, i, E, t, GK, tK)/1e6, label=
    plt.scatter(-i/1e-3/2, tau_volkersen(F10, i, E, t, GK, tK)/1e6, color
    color_start +=1

color_start=0
for i in overlaps:
    tau_m = F / (w*i)
    x = np.linspace(0,i,100)
    plt.plot((x/1e-3)-i/2*1e3, tau_klein(x, tau_m, i, GKl, E1, t1, E2, t2
    color_start +=1
color_start=0
plt.legend()
plt.grid()

plt.xlabel('X-coordinate [mm]')
plt.ylabel('Shear stress [MPa]')
plt.show()
```

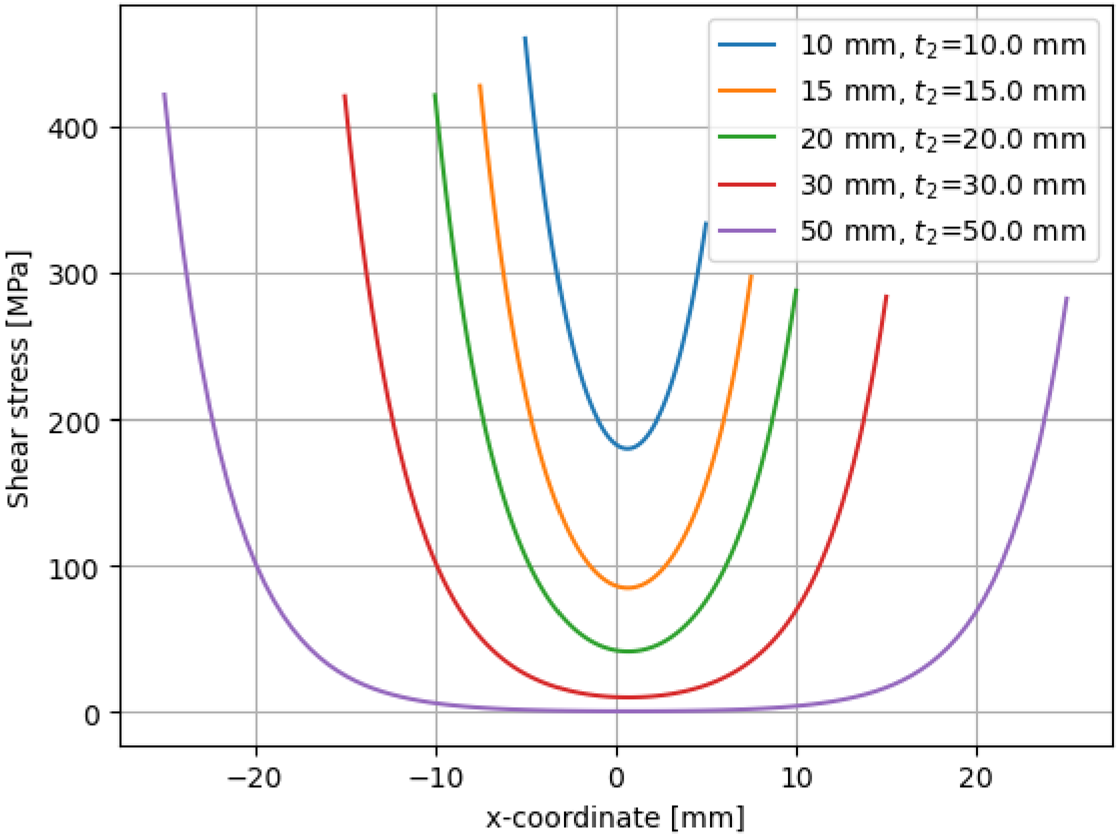


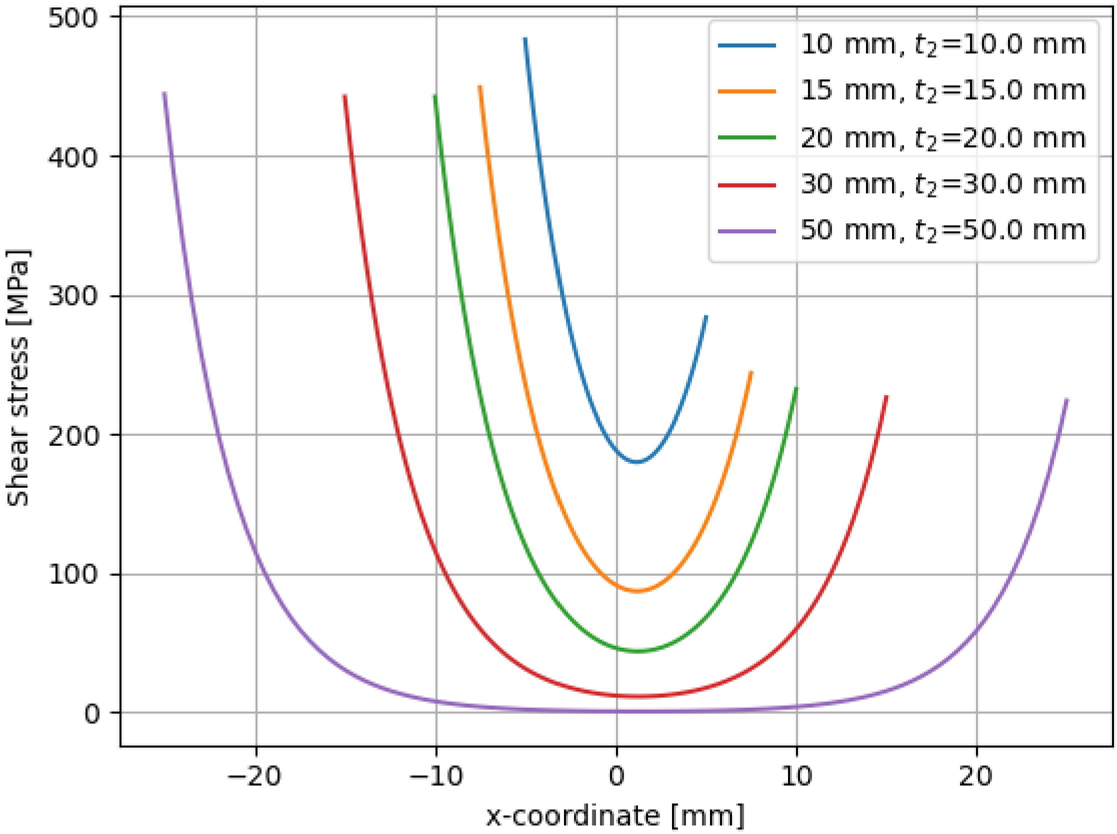
Comments: Plotting the Volkersen and Klein models together, a few notable observations can be made. Firstly, the Volkersen model only provides the peak shear stresses while the Klein model gives the shear stresses throughtout the length of overlap. Second, only for the overlap of 20 mm do the two models agree. For overlaps lower than 20 mm, the Volkersen model overestimates the peak shear stress, while for overlaps higher than 20 mm it underestimates the peak shear stress compared to the other model.

Question 1.2: Show how the stress distribution change when the thickness of the bottom adherend increases to 3mm and 4mm. Create a plot. Why can you expect these shapes of the curves? Answer in the textbox below your plot. Use the Klein model.

In [ ]:
```
E1, E2, t1, t2, lu, GKl, d, F, w = 70e9, 70e9, 2e-3, 2e-3, 30e-3, 2e9, 0.
overlaps = [10e-3, 15e-3, 20e-3, 30e-3, 50e-3]
#overlaps = [10e-3]

for i in overlaps:
    tau_m = F / (w*i)
    x = np.linspace(0,i,100)
    plt.plot((x/1e-3)-i/2*1e3, tau_klein(x, tau_m, i, GKl, E1, 3e-3, E2,

plt.legend()
plt.grid()
plt.xlabel('x-coordinate [mm]')
plt.ylabel('Shear stress [MPa]')
plt.show()

for i in overlaps:
    tau_m = F / (w*i)
    x = np.linspace(0,i,100)
    plt.plot((x/1e-3)-i/2*1e3, tau_klein(x, tau_m, i, GKl, E1, 4e-3, E2,

plt.legend()
```

```
plt.grid()
plt.xlabel('x-coordinate [mm]')
plt.ylabel('Shear stress [MPa]')
plt.show()
```





Answer: A notable change in stress distribution when the thickness of the bottom adherend is increased is that the curves shift rightward, no longer being symmetrical about the y-axis. This not only means that the lowest shear stress in the overlap region is no longer in the center but also that a higher shear stress will be measured at the left most end of the overlap. This is expected since increasing the thickness of one of the adherends induces asymmetry in the joint. Assuming a single lap joint that also undergoes secondary bending due to the applied shear force, a thicker adherend would resist this bending more than the thinner adherend. This difference also results in higher peak shear stresses at one end of the overlap region due to the now offcentered shear stress distribution.

Question 2 (2 points)

In reality, shear stresses are not the only stresses acting on the lap joint. Due to eccentricity, another type of stress occurs which will lower the maximum carrying capacity. Elaborate on this type of stress and why it affects the maximum load that can be carried.

Answer: Due to the eccentricity, secondary bending is also induced in a lap joint. As load is carried by the joint, 'stress lines' bend as they travel from one adherend to another. Naturally, this induces secondary bending forces to straighten these stress lines. Secondary bending depends on the thickness of the adherends, overlap length, as well as the load transfered between them. As the adherends bend, normal stresses in the joint are also introduced as it is pulled apart. The addition of normal stresses with shear stresses increases the peak stresses that the joint must carry and thereby reduces its maximum carrying capacity.

Question 3 (2 points)

Explain how step joints and scarf joints help in increasing the maximum carrying load, compared to single lap joints. Do this by globally analyzing the differences between the 3 joint types, and how these differences result in different stresses.

Answer: Step joints and scarf joints help in increasing the maximum carrying load by reducing secondary bending of the joint. As discussed earlier, a single lap joint undergoes shear stress as well as normal stress which is induced by secondary bending. Increasing the number of steps reduces the impact that secondary bending has on the joint by reducing the normal stress. A scarf joint can be seen as a stepped lap joint with a very high number of steps. By having a scarf angle of 45°, the joint undergoes shear stress and normal stress equally. For angles smaller than 45°, normal stress has a much larger contribution to the peak stress than shear and for angles larger than 45°, the load carrying capacity decreases due the joint's inability to withstand stress.

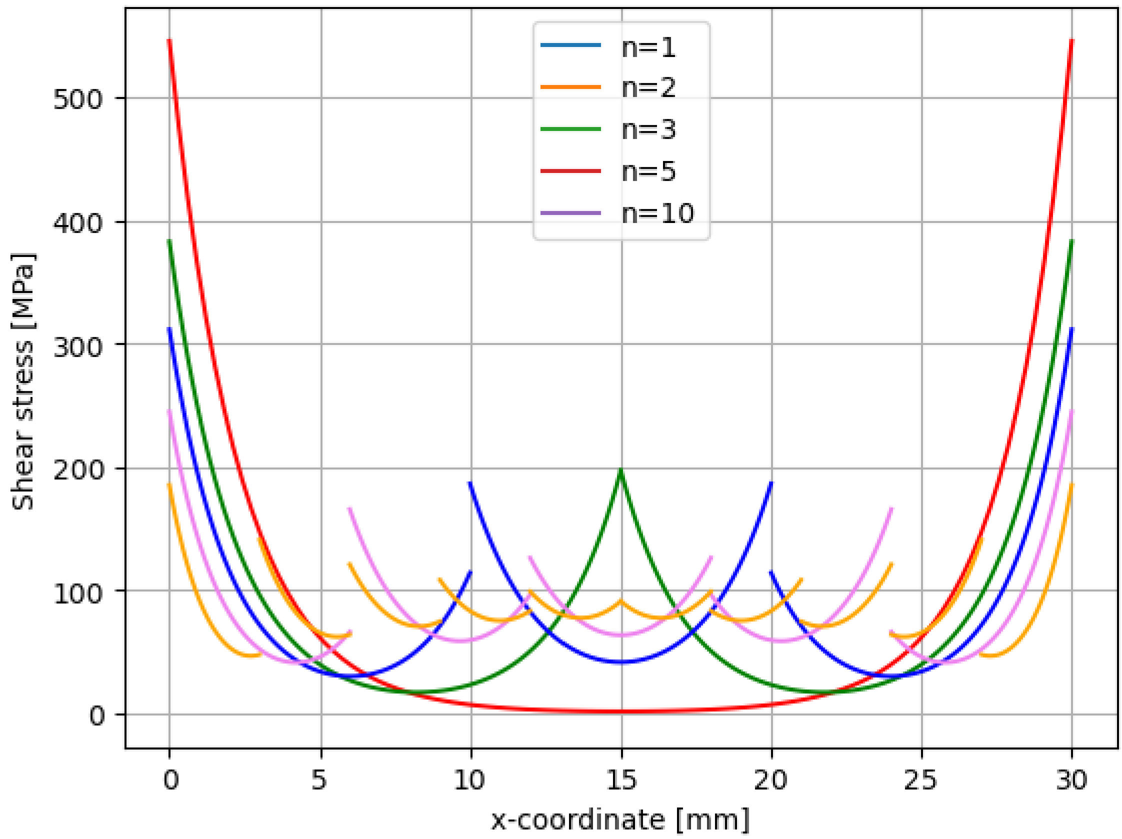
Question 4 (5 points)

Question 4.1: The number of steps n in a step joint has an effect on the shear-stress distribution in the bonded part. Write a function that takes the number of steps of the step joint as input, and returns the stress

distribution over the entire bond length. Assume the steps are always equally spaced.

Show your results by creating a plot below, which shows τ over the given length of 30mm for different values of n. Assume the same material properties as in Question 1 and apply the Klein model (or other analytical models).

In [ ]:
```
E1, E2, t1, t2, lu, GKl, d, F, w = 70e9, 70e9, 2e-3, 2e-3, 30e-3, 2e9, 0.
ns = [1,2,3,5,10]
colors = ['red', 'green', 'blue', 'violet', 'orange', 'brown']
color_start = 0
for n in ns:
    tau_m = F / (w*lu)
    x0 = np.linspace(0,lu/n,100)
    x_vals, y_vals = [], []
    plt.plot(0,0,label=f'n={n}')
    i = 0
    while i < n:
        x = np.linspace(i*lu/n,(i+1)*lu/n,100)
        t = (n-i)*t1/(n+1)
        for j in x:
            x_vals.append(j/1e-3)
        for k in x0:
            y_vals.append(tau_klein(k, tau_m, lu/n, GKl, E1, t, E2, t2-t,
        plt.plot(x/1e-3, tau_klein(x0, tau_m, lu/n, GKl, E1, t, E2, t2-t,
        i +=1
    #plt.plot(x_vals, y_vals, label=f'n={n}')

    color_start +=1
color_start = 0

plt.legend()
plt.grid()
plt.xlabel('x-coordinate [mm]')
plt.ylabel('Shear stress [MPa]')
plt.show()
```

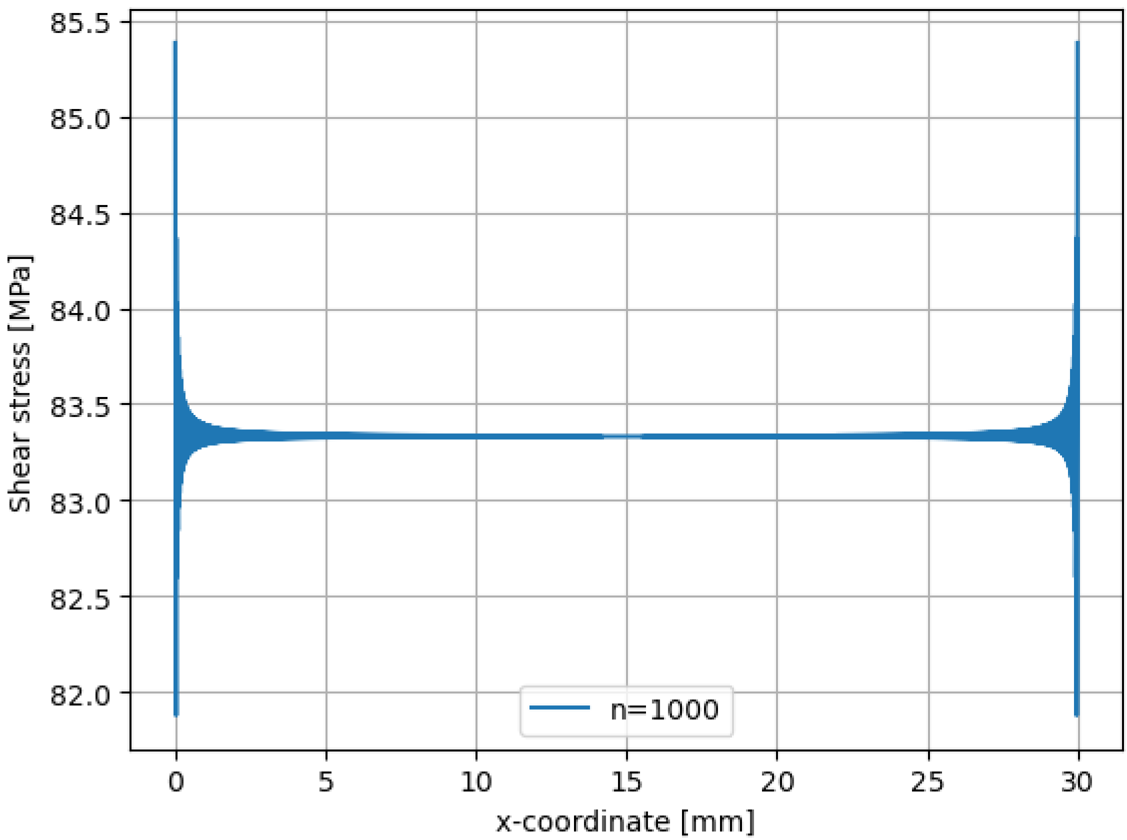


Question 4.2: What happens when n approaches infinity? Plot this using your code from question 4.1. Is there another type of joint that it would resemble?

In [ ]:

```
E1, E2, t1, t2, lu, GKl, d, F, w = 70e9, 70e9, 2e-3, 2e-3, 30e-3, 2e9, 0.
ns = [1000]
colors = ['red', 'green', 'blue', 'violet', 'orange', 'brown']
color_start = 0
for n in ns:
    tau_m = F / (w*lu)
    x0 = np.linspace(0,lu/n,100)
    x_vals, y_vals = [], []

    i = 0
    while i < n:
        x = np.linspace(i*lu/n,(i+1)*lu/n,100)
        t = (n-i)*t1/(n+1)
        for j in x:
            x_vals.append(j/1e-3)
        for k in x0:
            y_vals.append(tau_klein(k, tau_m, lu/n, GKl, E1, t, E2, t2-t,
        #plt.plot(x/1e-3, tau_klein(x0, tau_m, lu/n, GKl, E1, t, E2, t2-t
        i +=1
    plt.plot(x_vals, y_vals, label=f'n={n}')

    color_start +=1
color_start = 0

plt.legend()
plt.grid()
plt.xlabel('x-coordinate [mm]')
```

```
plt.ylabel('Shear stress [MPa]')
plt.show()
```

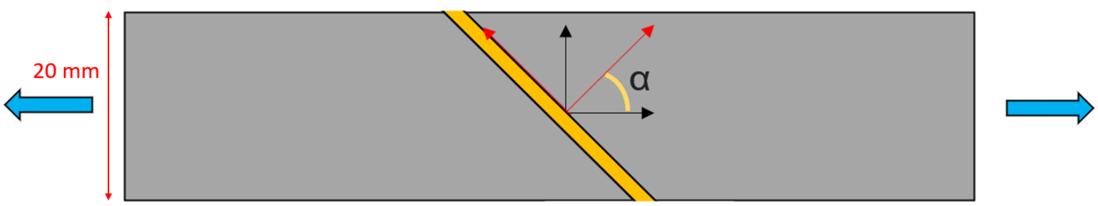


Yes, as n approaches infinity, the shear stress distribution of the stepped joint is almost constant throughout the overlap, which resembles a scarf joint.

Question 5 (6 points)

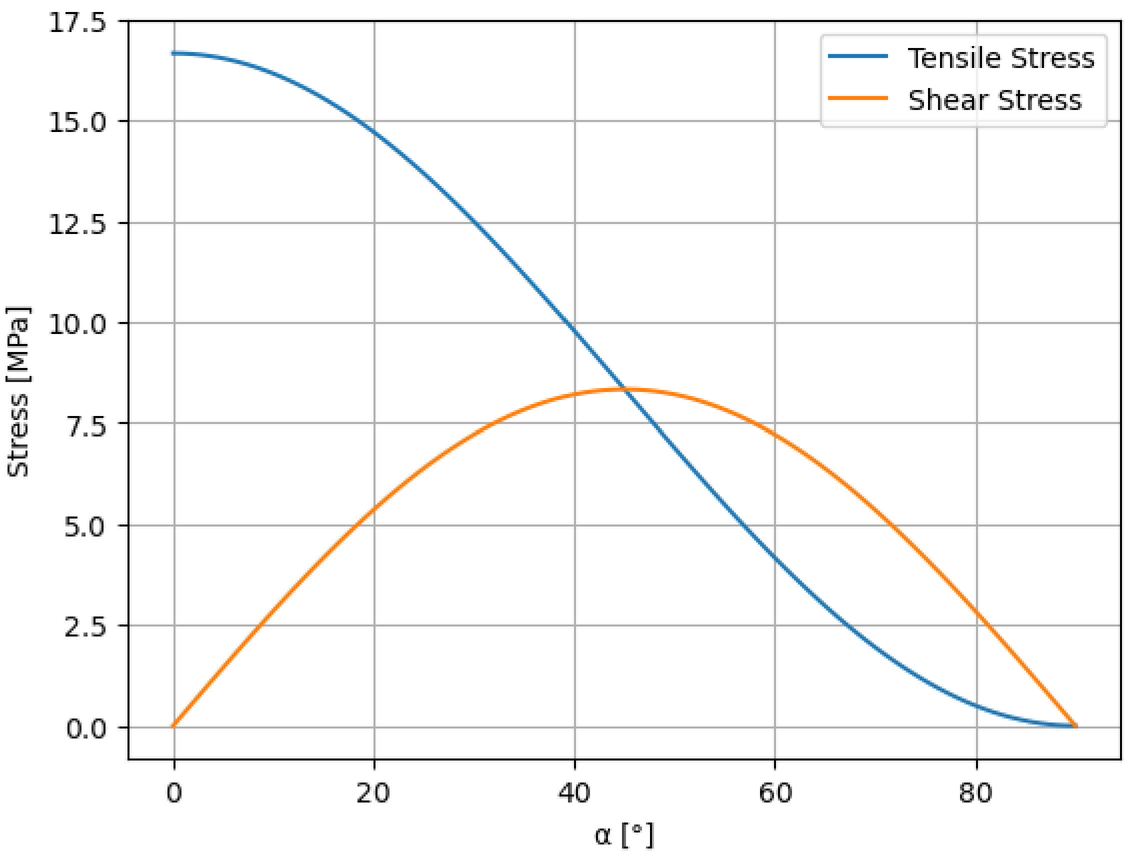
Question 5.1: Consider the scarf joint from the image below. Write a function that takes angle α as input and returns the shear- and tensile stresses in the adhesive. Then, use your function to plot the shear- and tensile stresses vs α for $0^\circ < \alpha < 90^\circ$

The thickness t of the part is 20 mm, the width b is 30 mm. The part is subjected to a tensile force of 10 kN.



In [ ]:
```
F, b, t = 10e3, 30e-3, 20e-3 #[N]

def area(a,b,t):
    return b * t / np.cos(a*np.pi/180)

def tensile_stress(F,a,b,t):
    return F*np.cos(a*np.pi/180) / area(a,b,t)
```

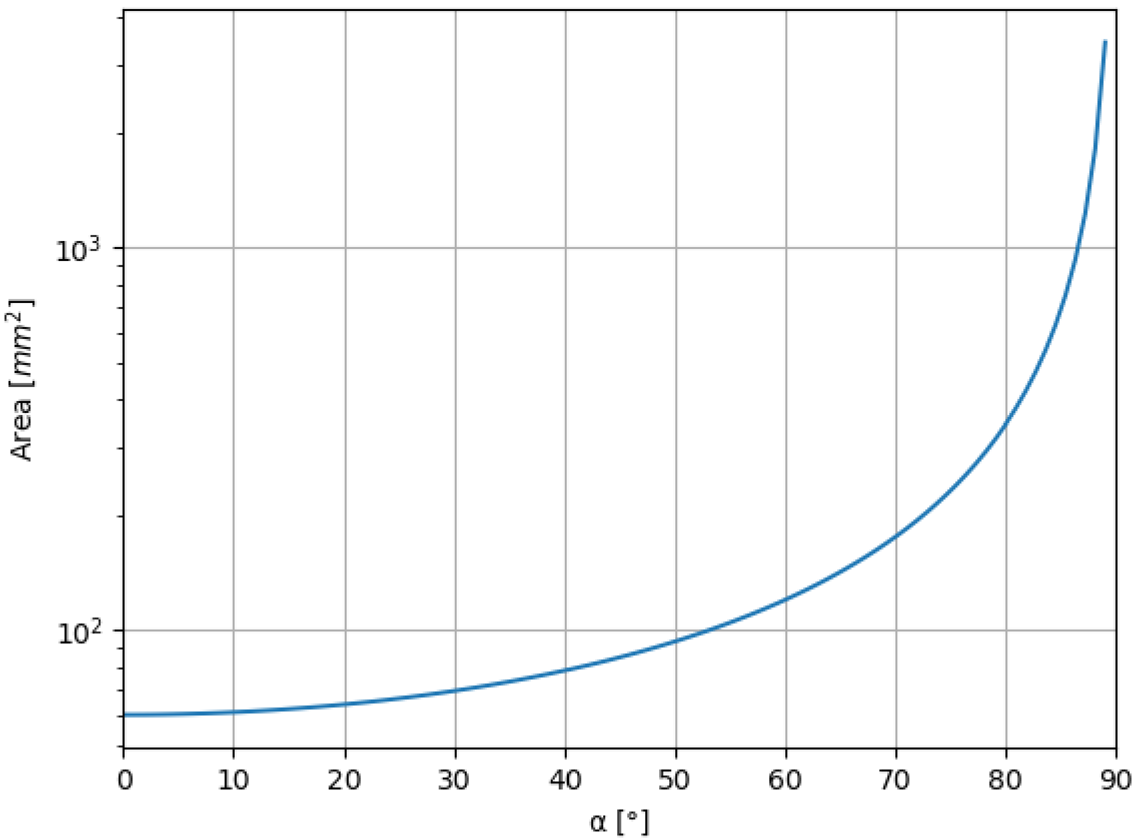
```
def shear_stress(F,a,b,t):
    return F*np.sin(a*np.pi/180) / area(a,b,t)

a = np.linspace(0,90,100)
plt.plot(a, tensile_stress(F,a,b,t)/1e6, label='Tensile Stress')
plt.plot(a, shear_stress(F,a,b,t)/1e6, label='Shear Stress')
plt.legend()
plt.grid()
plt.xlabel('α [$\degree$]')
plt.ylabel('Stress [MPa]')
plt.show()
```



Question 5.2: At which value for α does the tensile stress peak? At which value for α does the shear stress peak? Explain why this makes sense. Plot a graph, which contains bond-area vs. α, to support your explanation.

In [ ]:

```
a = np.linspace(0,89,100)
plt.plot(a, area(a,b,t))
plt.grid()

plt.xlim(0,90)
plt.ylabel('Area [$mm^2$]')
plt.xlabel('α [$\degree$]')
plt.yscale('log')
plt.show()
# Im not sure this graph is helpful haha
# Tensile peak at alpha=0, and shear peak at alpha=45, but how does the g
```



Tensile stress peaks at $\alpha = 0^{\circ}$ and shear stress peaks at $\alpha = 45^{\circ}$. This is a logical observation with the help of the charts above. At $\alpha = 0^{\circ}$, the split line between the two adherends is vertical. This means there is no shear stress and only tensile stress. The tensile stress would then be at its highest at this angle. Tensile stress is a function of cos(x)*area, and area isa function of cos(x), so this is a function of cos(x)cos(x), the maximum of which occurs at 0°

At $\alpha = 45^{\circ}$, the peak shear stress is observed and this makes sense as the shear stress is a function of sin(x)*area, which together is a function of sin(x)cos(x), the peak of which occurs at 45°.

As the scarf angle approaches 90°, bond area approaches infinity, as the split line becomes near horizontal. Stress is defined as force over area, so as area tends to infinity, the stress tends to zero.

Joining of thermoplastic composite parts

As introduced during lectures, the main advantages of TPC joining via fusion bonding are potential weight reduction, avoidance of drilling induced delaminations and stress concentrations, less labour compared to mechanical fastening. Compared to adhesive bonding, surface treatments are not needed and the process of joining can be sped up dramatically.

The process of fusion bonding entails the use of localized heating in specific areas intended for bonding, followed by a cool down under the application of pressure. After consolidation, this process creates a weld between the surfaces with no visible weld line. To achieve a successful

bond, it is necessary to heat the interface above certain temperature thresholds: the glass transition temperature (Tg) for amorphous polymers and the melting temperature (Tm) for semi-crystalline polymers. Additionally, throughout this bonding procedure, it is crucial to ensure that the maximum temperature does not exceed the polymer's degradation point

Different phases are encountered during TPC fusion bonding: intimate contact, autohesion and reptation, for which appropriate temperature, time and pressure are needed to properly consolidate the joint.

Autohesion, specifically, refers to the diffusion of polymer chains between contacted surfaces. A necessary and critical step in this is assuring intimate contact of the surfaces to bond. The degree of intimate contact is influenced by factors such as applied pressure, current temperature, surface texture, and polymer viscosity.

It is obtained through the following, simplified equation (source: Manufacturing Process Models for Thermoplastic Composites, Susan C. Mantell and George S. Springer, Journal of Composite Materials 1992 26:16, 2348-2377):

$$D_{ic} = R_c \cdot [\int_0^{t_p} \frac{p}{\mu}\, dt]^{0.2}$$

Where the parameter R_c is the initial surface roughness of the ply (geometric factor obtained through fitting), time t is expressed in seconds, p is the applied pressure (Pa) and μ (viscosity of the resin, $Pa \cdot s$) which could follow an Arrhenius type relation with temperature: $\mu = A \cdot exp(\frac{B}{T})$. The parameters A and B represent empirical constants, while T is the applied temperature, expressed in K.

Question 6 (3 points)

In this question an APC-2/AS4 (Aromatic Polymer Composite) thermoplastic composite tape, provided by Solvay, has been taken into consideration. More specifically, a standard modulus continuous carbon fiber (12K) and PEEK (polyetheretherketone) impregnated prepreg.

The following constants are given for this material: A = 643 Ns/m^2, B = 4367 K. In the following questions we will oversimplify the use of the equation by keeping the surface roughness and pressure constant during the heating process and we will only consider the heating process. Assume $R_c = 0.29$ (or specify a more suitable value derived from assumptions or from a source you found, but in any case specify any assumptions and/or resource you may be using). Use an appropriate timescale for the graphs.

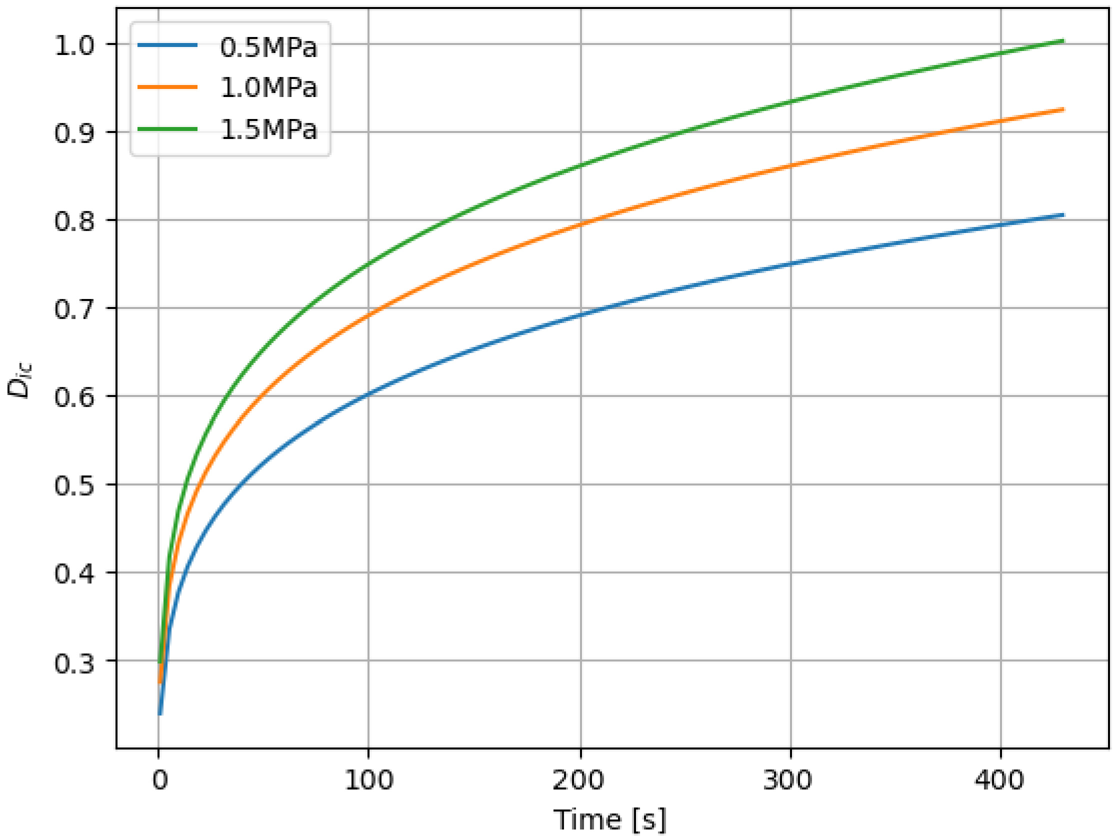
Question 6.1: Using the provided information, calculate the degree of intimate contact D_{ic} for three different applied pressures: 0.5 MPa, 1 MPa, 1.5 MPa at a constant temperature T=300°C, applied for 430 s.

Question 6.2: Using the provided information, calculate the degree of intimate contact D_{ic} for two different cure cycles: from room temperature up to 250°C, with a heating rate of 1 °C/min and a second one from RT to 350°C with a heating rate of 10 °C/min.

Note: Assume $R_c = 0.29$ (or specify a more suitable value derived from assumptions or from a source you found, but in any case specify any assumptions and/or resource you may be using). The maximum reachable value for degree of intimate contact is 1. Use an appropriate timescale for the graphs.

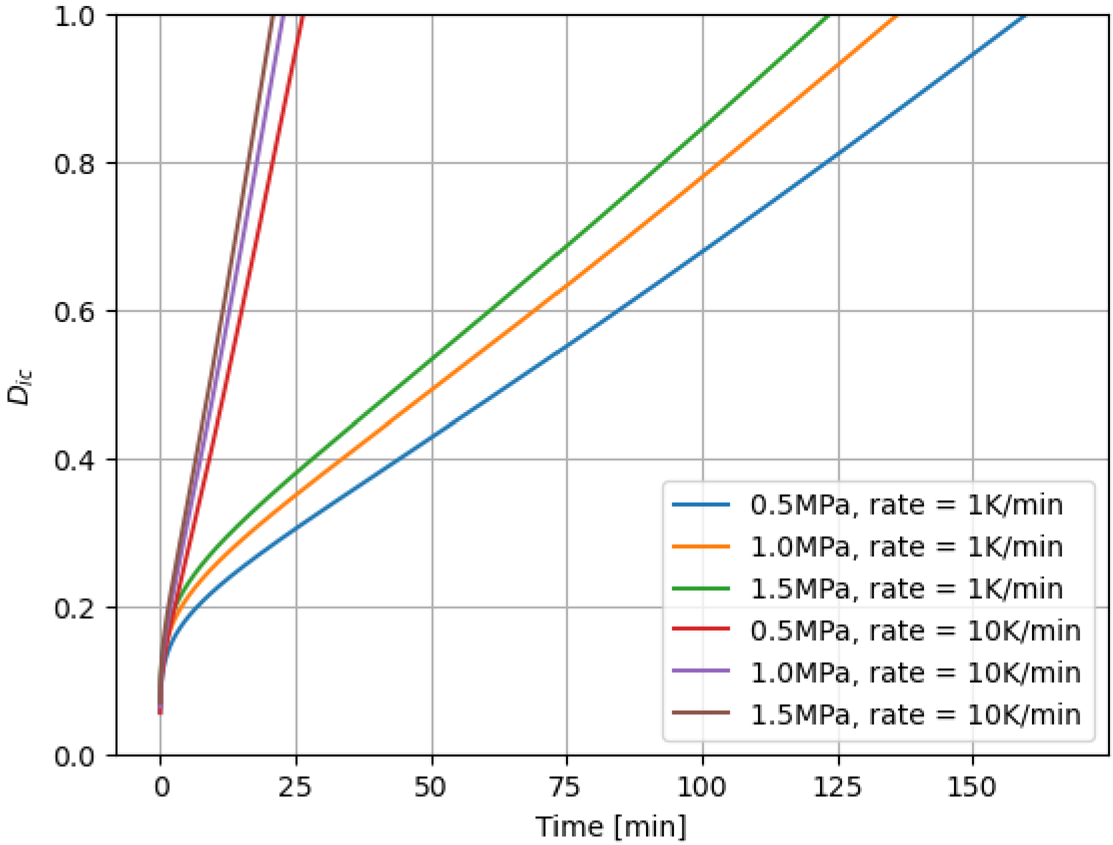
In [ ]:
```
T0 = 273.17 # [K]
A, B, Rc = 643, 4367, 0.29 # [Ns/m^2], [K], [-]
T_start, T_stop, rate = 300+T0, 300+T0, 0 # [K], [K], [K/min]
ps= [0.5e6, 1e6, 1.5e6] # [Pa], [K], [s]
t = 430 # [s]

def temperature(t,T_start,T_stop,rate):
    if rate == 0:
        return T_start

    rate_sec = rate/60
    T_delta = T_stop - T_start
    time_to_heat = T_delta / rate_sec
    if t <= time_to_heat:
        return T_start + t * rate_sec
    elif t > time_to_heat:
        return T_stop


def viscosity(T,A,B):
    return A * np.exp(B/T)

def D_ic(p,t,A,B,Rc,T_start,T_stop,rate):
    return Rc * quad(lambda x: p/viscosity(temperature(t,T_start,T_stop,r


for p in ps:
    ts = np.linspace(1,t,100)
    D = []
    for t in ts:
        D.append(D_ic(p,t,A,B,Rc,T_start,T_stop,rate))

    print(f'D_ic for pressure of {p/1e6}MPa, at {T_start-T0:.0f}C applied
    plt.plot(ts, D, label=f'{p/1e6}MPa')

plt.legend()
plt.grid()
plt.xlabel('Time [s]')
plt.ylabel('$D_{ic}$')
plt.show()
```

```
D_ic for pressure of 0.5MPa, at 300C applied for 430.0 sec is 0.8044
D_ic for pressure of 1.0MPa, at 300C applied for 430.0 sec is 0.9240
D_ic for pressure of 1.5MPa, at 300C applied for 430.0 sec is 1.0020
```



In [ ]:

```
T_start, T_stop, rate = 20+T0, 250+T0, 1 # [K], [K], [K/min]

for p in ps:
    ts = np.linspace(1,10000, 10000)
    D = []
    for t in ts:
        D.append(D_ic(p,t,A,B,Rc,T_start,T_stop,rate))
    plt.plot(ts/60, D, label=f'{p/1e6}MPa, rate = {rate}K/min')


T_start, T_stop, rate = 20+T0, 350+T0, 10 # [K], [K], [K/min]

for p in ps:
    ts = np.linspace(1,2000, 1000)
    D = []
    for t in ts:
        D.append(D_ic(p,t,A,B,Rc,T_start,T_stop,rate))
    plt.plot(ts/60, D, label=f'{p/1e6}MPa, rate = {rate}K/min')

plt.legend()
plt.grid()
plt.ylim(0,1)
plt.xlabel('Time [min]')
plt.ylabel('$D_{ic}$')
plt.show()

# these lines looked sort of weird to me, like why are they do straight?
```



Question 7 (2 points)

Mechanical properties (such as fracture toughness and strength at failure) of the resulting joint are highly influenced by the parameters applied during consolidation. Reflect on the influence of time, temperature and applied pressure on mechanical properties of co-consolidated parts.

Time, temperature, and applied pressure all affect the defects that can occur in co-consolidated parts. By optimizing these process parameters, defects can be minimized in the composite to best match modelled mechanical properties. Increasing dwell time, for example, allows better diffusion of the polymer and enables molecular entanglement in the case of thermoplastics. While process and part specific, a more cohesive structure can be obtained when joining two thermoplastic composite components.

Temperature plays another important aspect in controlling the flow of the polymer. Temperatures need to be high enough for a thermoplastic polymer to melt in order for it to flow. Controlling void content in the co-consolidated part is directly dependent on temperature.

Lastly, the applied pressure has a similar effect on voids and defects in co-consolidated parts. Appropriate pressure must be applied in order to ensure good contact between the polymer and matrix. Better fibre-matrix bonding reduces the chances of void formation at these interfaces.

Controlling these parameters to reduce void content in the composite is crucial to obtain desired fracture toughness and strength and failure. A higher void content is less material through which load can be carried or transfered.