Probleme de numărare -seminar 3-4
(Identităţi combinatoriale deduse prin numărare dublă)

Să se demonstreze următoarele identităţi numărând în două moduri:

1.
$$\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} = 2^n.$$

2.
$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}.$$

3.
$$k\binom{n}{k} = n\binom{n-1}{k-1}.$$

4.
$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \ldots + n\binom{n}{n} = n2^{n-1}.$$

5.
$$1 + 2 + 3 + \ldots + n = \binom{n+1}{2}.$$

6.
$$1 \cdot n + 2(n-1) + 3(n-2) + \ldots + (n-1)2 + n \cdot 1 = \binom{n+2}{3}.$$

7.
$$\binom{k}{k} + \binom{k+1}{k} + \ldots + \binom{n}{k} = \binom{n+1}{k+1}.$$

8. (Vandermonde)
$$\binom{n}{0}^2 + \binom{n}{1}^2 + \ldots + \binom{n}{n}^2 = \binom{2n}{n};$$

9. (Vandermonde)
$$\sum_{j=0}^{k} \binom{m}{j}\binom{n-m}{k-j} = \binom{n}{k}$$

10.
$$r^n = \sum_{k=1}^{n} S(n,k) r^{\underline{k}}, \qquad \text{where } r^{\underline{k}} = r(r-1) \cdot \ldots \cdot (r-k+1).$$