Math 51-52-53 topic lists

Math 51. Here is a topic list, arranged approximately by week.

(1) Vectors, dot product, length, planes in space.
(2) Span, subspaces, basis & dimension of subspaces, projection to subspaces.
(3) Orthogonal basis of planes, linear regression, multivariable functions, level sets and graphs, contour plots, partial derivatives (and tangent planes).
(4) Extrema and critical points, gradients, gradient descent, Lagrange multipliers.
(5) Linear functions, matrices, derivative matrix, linear transformations, matrix algebra.
(6) Applications of matrix algebra (Markov Chains, gambling), Chain Rule for multivariable functions, matrix inverses, multivariable Newton's method (latter on HW, not exams).
(7) Linear independence, Gram-Schmidt process, transpose, orthogonal matrices, symmetric matrices, quadratic forms, linear systems (column and null spaces).
(8) QR/LU-decompositions, eigenvalues and eigenvectors (only compute in 2-dim'l & triangular cases), Spectral Theorem, applications to quadratic forms (definiteness) & matrix powers.
(9) Higher-order partial derivatives, Hessian matrix, multivariable Second Derivative Test for local extrema via eigenvalues.
(10) (if time permits, not for HW or exams) Singular Value Decomposition via Spectral Theorem & eigenvalues.

Math 52. Here is a topic list, arranged approximately by week.

(1) Double and triple integrals over various regions.
(2) Applications of double and triple integrals.
(3) Double integrals via polar coordinates, and to compute areas and some volumes.
(4) Cylindrical and spherical coordinates, associated integration formulas.
(5) Determinants and cross products: properties, calculations, geometric meaning (and orientation). Change of Variables formula for multiple integrals.
(6) Vector fields and associated derivative operators: grad, curl, div.
(7) Parametric curves and line integrals, path independence and Fundamental Theorem.
(8) Green's theorems and applications; relation to flux, circulation, and area (planimeter).
(9) Parametric surfaces, surface area & other integrals. Tangents, normal, and orientation.
(10) Divergence Theorem, Stokes' Theorem, and applications; conservative vector fields revisited.

Math 53. Here is a topic list, arranged approximately by week.

(1) Direction fields, equilibrium solutions, first-order linear ODE, separation of variables.
(2) Existence/uniqueness theorem, analysis of equilibria, logistic growth, autonomous equations, Euler's method.
(3) Runge–Kutta, eigenvalues, eigenvectors (includes determinants, characteristic polynomials).
(4) Matrix exponential, coupled and uncoupled linear ODE systems, first-order trick, existence/uniqueness for linear systems.
(5) Solving homogeneous systems, Wronskian, complex and repeated eigenvalues, phase-space plots, asymptotic behavior.
(6) Second-degree linear ODE, damped oscillation, non-homogeneous equations, undetermined coefficients.
(7) Forced vibrations (resonance), variation of parameters, Laplace transform.
(8) Inverse Laplace transform, application to solving ODE, review of power series.
(9) Airy equation, Fourier series with examples, Laplace equation.